



## On some factors controlling fission-track etch rates in apatite

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### Abstract

1 Fission tracks in apatite must be etched to be counted and measured with an optical microscope.  
2 Track etching is described with the aid of two velocities: the track etch rate  $v_T$  and the apatite etch  
3 rate  $v_R$ . On their values depend the efficiencies of track counts, the selection of confined tracks for  
4 measurement, and the shapes of etched tracks;  $v_R$  is an indicator of the apatite composition and  
5 annealing kinetics. In spite of this, direct measurements of  $v_T$  and  $v_R$  have long been wanting. This  
6 work provides numerical data and investigates factors controlling  $v_T$  and  $v_R$ . The results indicate  
7 that  $v_T$  is constant over most of the ion- and fission-fragment ranges in apatite (constant core).  
8 We observed that, when using a step-etch protocol, the etchant advances at a high, but finite, rate  
9 along tracks etched in a preceding step. Thus, step-etching broadens the effective-etch-time dis-  
10 tribution, and the summed etch time overestimates the aggregate effect of the procedure. Neither  
11 annealing of induced tracks nor ageing of fossil tracks has a measurable effect on  $v_T$ . Since the  $v_T$ -  
12 value along a particular track section does not depend on its radius or on the work expended to  
13 create it, it can be considered characteristic of an apatite instead of a track. This could account for  
14 the different  $v_T$  of the Duluth and Durango apatites. The identical distributions of the host-con-  
15 fined-track intersections,  $c$ -axis angles, and track-etch rates across all experiments suggests that  
16 confined-track selection is little dependent on the track densities or lengths, but dominated by a  
17 condition that is unaffected by annealing. In contrast to  $v_T$ ,  $v_R$  exhibits clear variation related to  
18 radiation damage from natural background radiation in the case of fossil tracks and from neutron  
19 irradiation in case of induced. Greater  $v_R$  shorten the access times to confined tracks, increasing  
20 their effective etch times. This implies a need for caution when using  $D_{par}$  to assess the annealing  
21 kinetics.

### Keywords

Apatite, fission track, etching, track etch rate, apatite etch rate

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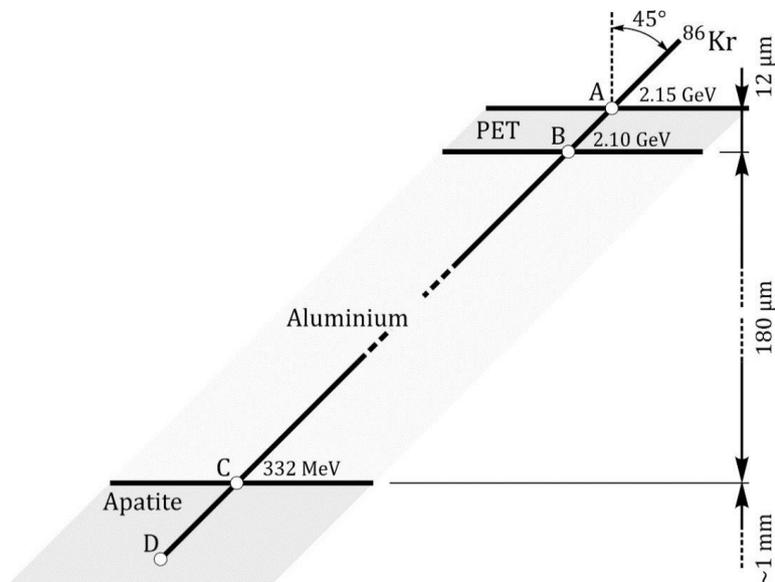
## 1. Introduction

22 The fission-track method is a useful tool for dating rocks and retracing their low-temperature  
23 thermal histories, based on counting and measuring the damage trails from uranium fission in  
24 minerals. Apatite is the most studied mineral, and the focus of this investigation. Fission tracks in  
25 apatite shorten and fade through a range of temperatures relevant to geological processes such  
26 as basin subsidence and uplift, erosional and tectonic unroofing, thrust tectonics, and hydrocar-  
27 bon maturation. Apatite grains separated from a rock are mounted in resin, polished and etched  
28 for examination with a microscope. Etching creates straight channels along the damage trails left  
29 by the fission fragments. In contrast to unetched tracks, which have nanometre diametres, etched  
30 tracks have micrometre diameters, suitable for counting and measuring. From the beginning,  
31 track etching has been described in terms of two velocities: the etch rate,  $v_T$ , of the damaged lat-  
32 tice along the track and the bulk etch rate,  $v_B$ , of the undamaged apatite in all other directions  
33 (Fleischer and Price, 1964; Hurford, 2019). Their values determine the etching and counting effi-  
34 ciencies of different crystal faces (Gleadow, 1981; Tagami and O'Sullivan, 2005), and the round-  
35 edness of the confined track tips (Ketcham and Tamer, 2021). These factors determine which  
36 grains are suitable for counting and dating and which tracks are suitable for measurement and  
37 Tt-modelling.

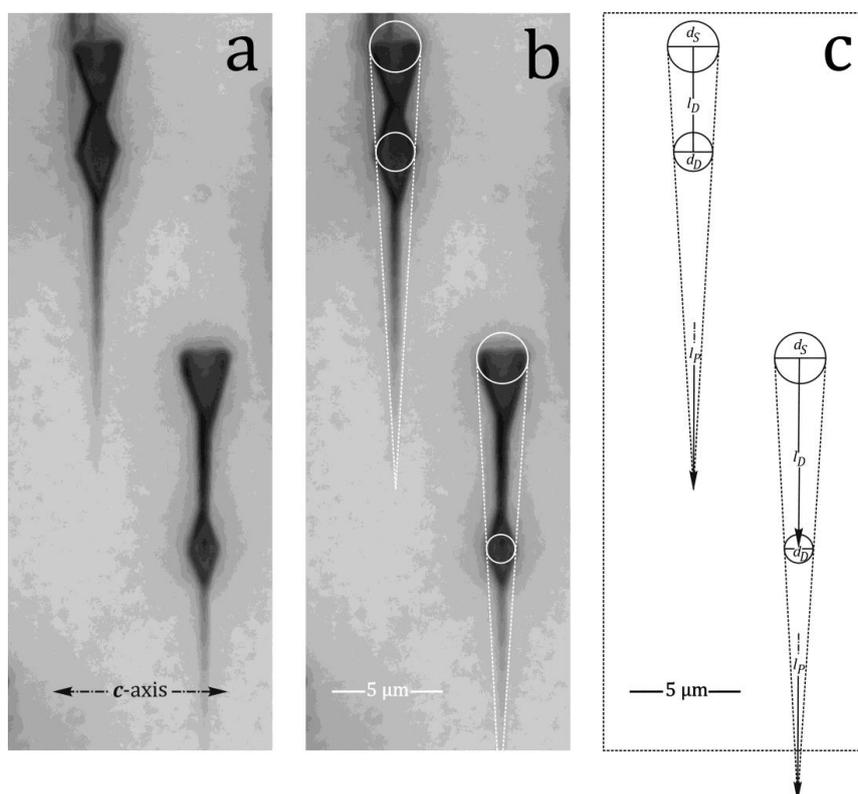
38 Little is known for certain about the factors that affect the apatite and track etch rates. It is known  
39 that  $v_B$  is anisotropic (Gleadow, 1981), and also depends on the apatite composition (Burtner et  
40 al., 1994; Carlson et al., 1999). We know less about  $v_T$ , which is assumed constant (Laslett et al.,  
41 1984) or related to the track radius or damage density (Ketcham and Tamer, 2021; Li et al., 2023).  
42 Earlier, following established concepts of crystal dissolution (Gross, 1918; Heimann, 1975; 1982),  
43 we redefined the apatite etch rate as the rate of displacement of a lattice plane ( $v_R$ ; Jonckheere et  
44 al., 2019; 2022) instead of that of each point on a surface ( $v_B$ ). We investigated how  $v_R$  varies with  
45 composition (Trilsch et al. 2023; Fu et al., 2024) and with orientation, noting steep minima par-  
46 allel and perpendicular to the  $c$ -axis (Aslanian et al., 2021). Here we investigate factors affecting  
47  $v_R$  and  $v_T$ . Section 2 focusses on the variation of  $v_T$  along tracks, section 3 on the effect of anneal-  
48 ing, and section 4 on that of ageing on  $v_R$  and  $v_T$ . The discussion is based on the cumulative fre-  
49 quencies of the measured quantities. Additional scatter plots and histograms are presented in the  
50 supplements.

## 2. Variation of $v_T$

51 We mounted a prism section of Durango apatite in resin, ground, and polished it with 6- $\mu\text{m}$ ,  
52 3- $\mu\text{m}$ , and 1- $\mu\text{m}$  diamond suspensions and a 0.04- $\mu\text{m}$  silica suspension to expose an even inter-  
53 nal surface. We covered the mount with 180- $\mu\text{m}$  Al degrader and 12- $\mu\text{m}$  PET film, and irradiated  
54 it perpendicular to the  $c$ -axis and at 45° dip angle with 25.0 MeV.amu<sup>-1</sup> <sup>86</sup>Kr-ions at the HIRFL  
55 in Lanzhou, operated by the Institute of Modern Physics of the Chinese Academy of Sciences (Figure  
56 1). We etched the mount in 5.5 M HNO<sub>3</sub> at 21 °C for 20 s + 10 s + 10 s to reveal the tracks (Figure  
57 2). The sample had a sealed natural crack extending from the surface to tens of microns below  
58 it. Although no etchant could penetrate along it, the crack nevertheless constituted a break in  
59 the apatite lattice. The intersections of the Kr-tracks with the crack are therefore convex, like  
60 their intersections with the surface, causing etch figures bounded by the fastest etching faces to  
61 develop (Aslanian et al., 2021; Jonckheere et al., 2022). This produces uniform etch pits at the  
62 apatite surface, one side of which, with length  $D_{\text{par}}$ , is parallel to the  $c$ -axis (Burtner et al., 1994).  
63 The same process takes place on either side of the crack, giving rise to diamond-shaped etch  
64 figures, which decrease in size at increasing distances along the track from its surface intersec-  
65 tion (Figure 2a).



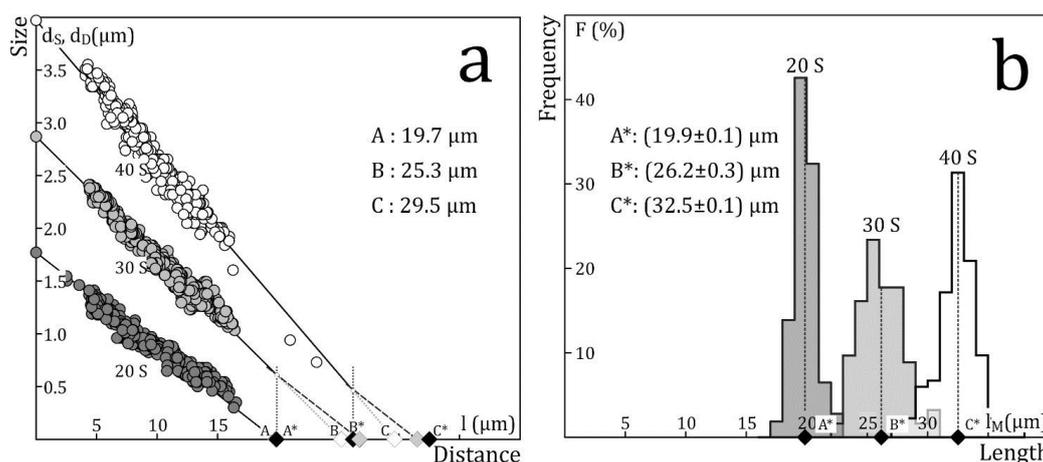
66 **Figure 1.** Kr irradiation: a prism section of an unannealed single Durango apatite is covered with a 180  
 67  $\mu\text{m}$  Al degrader and a 12  $\mu\text{m}$  PET film, and irradiated perpendicular to the apatite *c*-axis and at 45° dip  
 68 angle with 25.0 MeV/amu  $^{86}\text{Kr}$ . The samples were etched in 5.5 M  $\text{HNO}_3$  at 21 °C for 20 s + 10 s + 10 s to  
 69 reveal the tracks.



70 **Figure 2.** Kr tracks etched in 5.5 M  $\text{HNO}_3$  at 21 °C for 20 s (a, b), the measurements are indicated in (c).



71 From their respective sizes ( $d_s$ ,  $d_b$ ) and the immersion times ( $t_i$ ) we calculated the effective etch  
 72 times  $t_E = (d_D / d_s) t_i$ , and access times,  $t_A = t_i - t_E$ , of the diamond shapes, i.e., the time required  
 73 for the etchant to advance along the track from the surface to the crack. Figure 2b illustrates the  
 74 measurements of  $d_s$ ,  $d_D$ , and  $l = l_D / \cos(1/4\pi)$  and the total projected length  $l_M = l_P / \cos(1/4\pi)$ . Table  
 75 1 reports the measurements for the 20-, 30-, and 40-s cumulative immersion times. Figure 3a  
 76 plots  $d_D$  against  $l$ ; Figure 3b shows histograms of the calculated lengths  $l_M$ . The solid lines in Fig-  
 77 ure 3a are geometric mean regression lines anchored at the mean values of  $d_s$ . These means have  
 78 negligible error compared to the individual  $d_D$ -values, because  $d_s$  exhibits almost no variation  
 79 and was measured for each track and immersion time. Table 1 lists the slopes ( $s$ ) of the regres-  
 80 sion lines, along with their  $d$ -axis and  $l$ -axis intercepts, and the calculated mean track etch rates:  
 81  $v_T = -d_s / (s t_i)$ .



82 **Figure 3.** Results of the ion-track length and size measurements. **(a)** Diamond sizes ( $d_D$ ) plotted against  
 83 distance along the track ( $l = l_D / \cos(45^\circ)$ ) for 20 s (dark), 30 s (light), and 40 s (white) immersion.  
 84 The data at  $l = 0$  ( $d_s$ ) are the average track openings ( $D_{PAR}$ ). The solid black lines are regression lines anchored  
 85 at  $d_s$ . The intersections of the regression lines with the  $l$ -axis at A (20 s), B (30 s), and C (40 s) are extrap-  
 86 olated total etched lengths at each immersion time (white). The black symbols at  $A^*$  (20 s),  $B^*$  (30 s),  
 87 and  $C^*$  (40 s) are the corresponding measured values. The dashed lines are extrapolations of the 30 s,  
 88 and 40 s data at  $A^*$  and  $B^*$ , assuming the track rate calculated from the 20 s data applies to the unetched  
 89 sections beyond  $A^*$  and  $B^*$ ; **(b)** Histograms of the measured track lengths ( $l_M = l_P / \cos(45^\circ)$ ) after etching  
 90 for 20 s, (20+ 10) s, and (30+ 10) s in 5.5 M  $\text{HNO}_3$  at 21 °C.  $A^*$ ,  $B^*$ , and  $C^*$  are the mean lengths plotted in  
 91 (a) (black symbols).

$t_i$ (s)	$n_T$	$s$	$v_T$	$d_l$	$l_l$	$l_M$	$l_E$	$t_M$
20	467	-0.090	59.6	1.77	19.7	19.9	19.8	20.1
30	514	-0.113	276	2.87	25.3	26.2	26.7	32.1
40	363	-0.135	324	3.97	29.5	32.5	31.1	40.8

92 **Table 1.** Measurements of Kr-tracks in a Durango apatite prism face etched for 20,  
 93 30, and 40 s in 5.5 M  $\text{HNO}_3$  at 21 °C.  $t_i$  (s): immersion time;  $n_T$ : number of tracks;  $s$ :  
 94 slopes of geometrical mean regression lines fitted to  $d_D$  vs.  $l$  ( $d_D$  ( $\mu\text{m}$ ) and  $l$  ( $\mu\text{m}$ ):  
 95 size of the diamond-shaped etch figure and its distance from the surface; Figure 2);  
 96  $v_T$  ( $\mu\text{m}/\text{min}$ ): rate of etchant penetration, calculated from the slopes  $s$ ;  $d_l$  ( $\mu\text{m}$ ),  $l_l$   
 97 ( $\mu\text{m}$ ):  $d$ - and  $l$ -axis intercepts of the regression lines;  $l_M$  ( $\mu\text{m}$ ),  $l_E$  ( $\mu\text{m}$ ): measured  
 98 (Figure 3b) and calculated (Figure 3a) mean etched length, assuming a constant  
 99 track etch rate  $v_T$ ;  $t_M$  (s): immersion times corresponding to the measured mean  
 100 lengths and calculated etch rates.



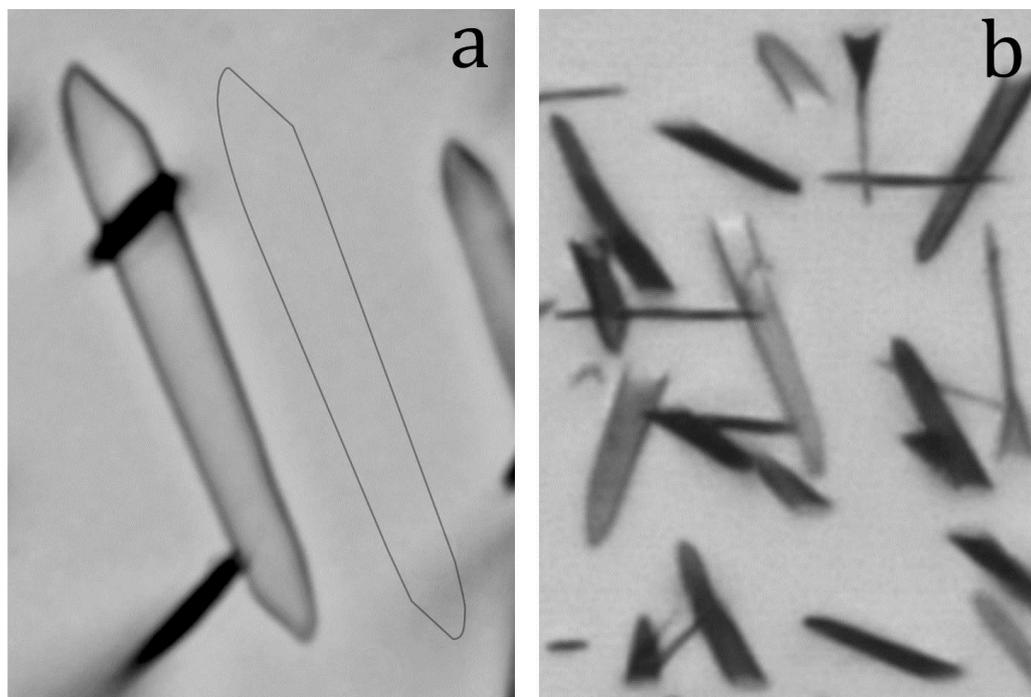
101 The  $v_T$ -value for the initial 20-s immersion ( $v_T \approx 60 \mu\text{m}\cdot\text{min}^{-1}$ ) is the actual track etch rate,  
102 whereas the values for 30 s and 40 s measure the fast but finite rate of etchant advance along a  
103 pre-etched track. The  $v_T$ -estimate is consistent with results for induced fission tracks at high an-  
104 gles to the  $c$ -axis measured in a prism section of Durango apatite (Jonckheere et al., 2024). The  
105 latter results indicate that the sum of the immersion times of a step-etch experiment overesti-  
106 mates the true cumulative etch time. For the Kr-tracks, this amounts to  $\geq 4$  s for the 10-s step  
107 following the initial 20-s immersion; i.e., the time to travel the length ( $\approx 20 \mu\text{m}$ ) of pre-etched  
108 track section at  $v_T \approx 275 \mu\text{m}\cdot\text{min}^{-1}$  ( $20 \mu\text{m} / 4.6 \mu\text{m}\cdot\text{s}^{-1} \approx 4.3$  s). The effect is less for shorter fission  
109 tracks.

110 The difference ( $l_M - l_i$ ) between the measured mean lengths and the  $l$ -axis intercepts of the regres-  
111 sion line is almost negligible after the initial 20-s etch, but increases with each subsequent step  
112 (Table 1; Figure 2). This is because the increased penetration rates during later steps relate to the  
113 track sections etched during the previous step, but not to the unetched sections beyond, which  
114 etch at the track etch rate  $v_T$ . We revised the expected lengths ( $l_E$ ) based on the measured lengths  
115 ( $l_M$ ) and etch rates ( $v_T$ ) in Table 1. The agreement between the measured and expected lengths ( $l_M$   
116 -  $l_E$ ) and between the estimated and actual immersion times ( $t_M - t_i$ ) is good considering the un-  
117 certainties. In particular the enhanced rates of penetration weigh on the calculations because they  
118 are derived from the small differences between the slopes of the regression lines fitted to the 20-  
119 30-, and 40-s data. The calculation assumes that  $v_T$  is constant over the entire etched track length  
120 up to 40 s (constant core; Tamer and Ketcham, 2020). The data in Table 1 are interpreted as sup-  
121 porting this.

122 Constant  $v_T$  agrees with the fact that fission tracks in apatite have straight sides, lacking any cur-  
123 vature due to a variable  $v_T$  (Figures 4a and 4b; 'motorboat effect'; Fleischer et al., 1969; 1975;  
124 Paretzke et al., 1973; Somogyi and Szalay, 1973; Durrani and Bull, 1987; Nikezić, 2000; Nikezić  
125 and Yu, 2003; 2004). This means that  $v_T$  is unrelated to  $(dE/dx)$  (Tamer and Ketcham, 2020;  
126 Ketcham and Tamer, 2021), or to the radius of the latent track (Li et al., 2012; 2014). The most  
127 compelling indication is that etched confined tracks show no constriction, interruption or other  
128 modification in their midsections, even though the effective fission fragment charges, and thus  
129  $(dE/dx)$  and the track radius, are assumed to be near zero at the locus of the fissioned nucleus  
130 (Li et al., 2023). We have earlier reported that the average fission track etch rate varies with  $D_{\text{par}}$   
131 (Fu et al., 2024) and with the track orientation, with minima parallel and perpendicular to the  $c$ -  
132 axis, analogous to the apatite etch rate  $v_R$  (Jonckheere et al., 2024). This indicates that  $v_T$  is more  
133 related to the chemical characteristics of the track detector than to the properties of the fission  
134 fragments.

### 3. Effect of annealing

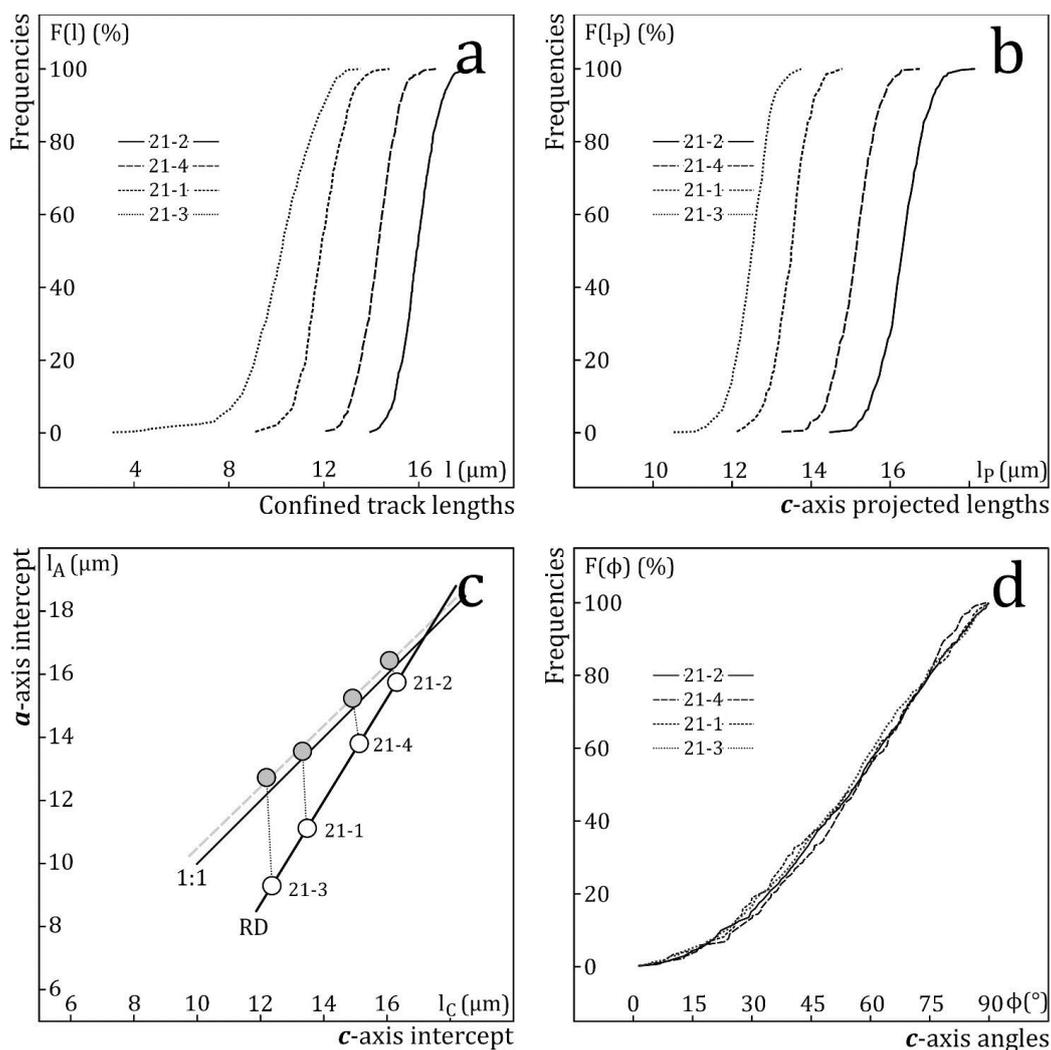
135 To investigate the effects of partial annealing on  $v_T$  and  $v_R$ , we carried out track measurements on  
136 four prism sections of Durango apatite that had been pre-annealed, neutron-irradiated and - all  
137 bar one - re-annealed to create induced-track populations with mean lengths of  $\sim 16 \mu\text{m}$  (sample  
138 21-2),  $\sim 14 \mu\text{m}$  (21-4),  $\sim 12 \mu\text{m}$  (21-1), and  $\sim 10 \mu\text{m}$  (21-3). The irradiation and annealing condi-  
139 tions are detailed in Ketcham et al. (2015). The track densities measured in transmitted and re-  
140 flected light are reported in Aslanian et al. (2022). The samples were mounted in resin, polished  
141 with 6- $\mu\text{m}$ , 3- $\mu\text{m}$ , and 1- $\mu\text{m}$  diamond and 0.04- $\mu\text{m}$  silica suspensions, and etched for 20 s in 5.5 M  
142  $\text{HNO}_3$  at 21  $^\circ\text{C}$  (Carlson et al., 1999). The confined tracks were imaged and measured as in Jonck-  
143 heere (2023). The measurements included the track lengths, orientations, widths, cone-angles,  
144 the positions of the host track intersections, and  $D_{\text{par}}$ . From the results we calculated the effective  
145 etch times and the apatite and track etch rates, using the equations of Aslanian et al. (2021) and  
146 Fu et al. (2024).



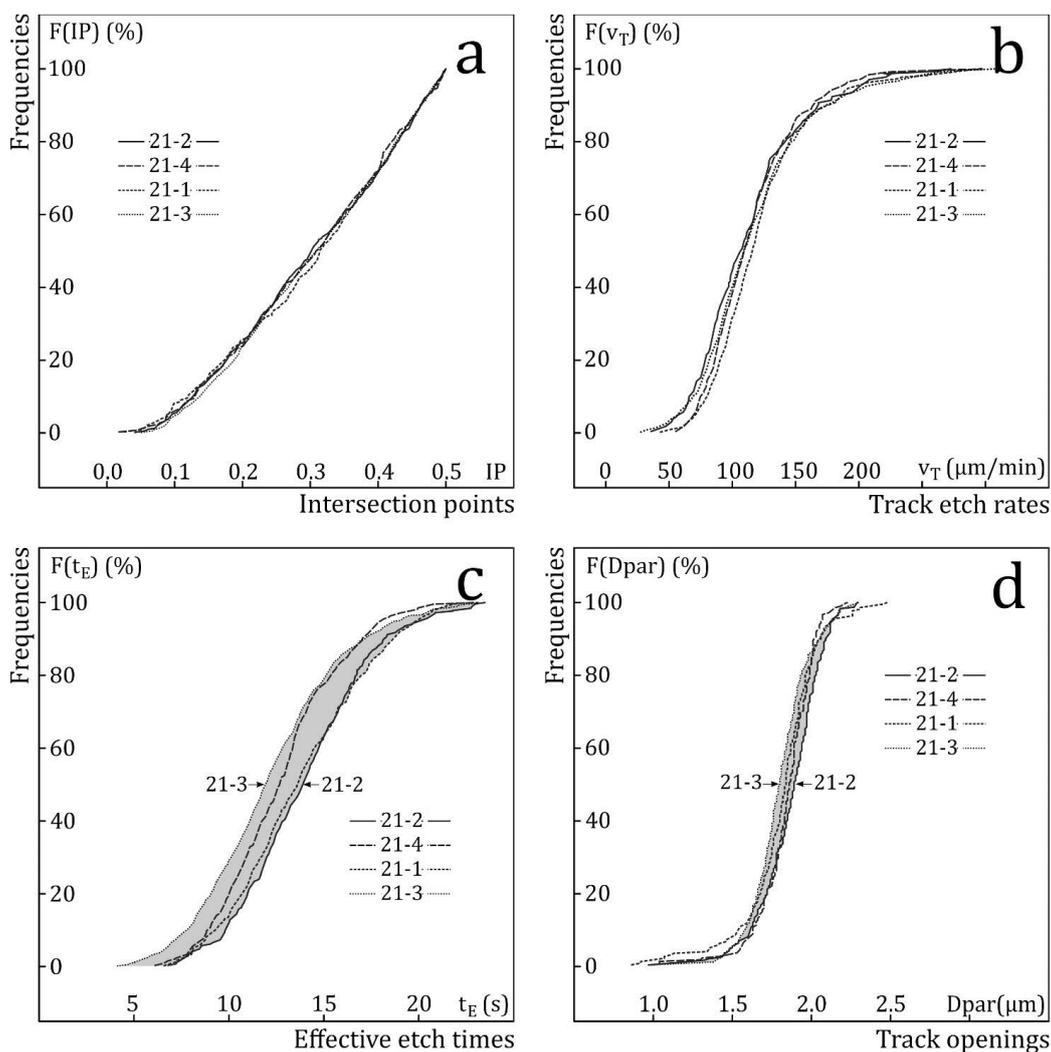
147 **Figure 4.** Etched fission tracks in an apatite prism face showing the characteristic straight-sided tapering  
148 outline, due to a constant track etch rate  $v_T$ : **(a)** horizontal confined track; **(b)** compressed image of surface  
149 tracks (etched 20 s in 5.5 M  $\text{HNO}_3$  at 21°C).

150 **Figures 5 and 6** show the cumulative distributions of the measured track properties for different  
151 annealing stages; **Table 2** lists the main statistics. It is surprising to observe that the angular  
152 (azimuth) distribution remains unchanged on annealing (Figure 5d). This is similar to the re-  
153 sults of Ketcham (2003), except that our data cover mean lengths from 15.9  $\mu\text{m}$  to 10.1  $\mu\text{m}$ ,  
154 before *c*-axis projection (Supplement Figure 1d). The intersection-point distribution (from host  
155 track to track tip) is also unaffected by annealing (Figure 6a). In each sample, the number of  
156 intersections decreases from the centre towards the track tips (Supplement Figure S4) but there  
157 is no evidence that, in relative terms, short tracks are differently affected than long tracks. There  
158 appears to be no obvious rationale for the invariance of the angular frequencies and of the in-  
159 tersection lengths across the annealing conditions of our samples. These observations are coun-  
160 ter to expectation based on length bias and anisotropic annealing (Galbraith et al., 1990; Donel-  
161 ick et al., 1999; Galbraith, 2002; Ketcham, 2003). It has been established that track selection is  
162 not just a matter of geometrical biases, but that etching and personal biases must also be taken  
163 into consideration. According to Aslanian et al. (2021) and Jonckheere (2023), the selection of  
164 a confined track depends on whether it exceeds a threshold width, which is not affected by an-  
165 nealing.

166 The cumulative  $v_T$ -distributions are the same after 10-h annealing at 240 °C (21-4), 288 °C (21-  
167 1), and 310 °C (21-3; Figure 5b) as that of unannealed induced tracks (21-2), despite experiencing  
168 mean length reductions of  $\sim 2$ ,  $\sim 4$ , and  $\sim 6$   $\mu\text{m}$ . This contradicts the finding that annealing induced  
169 tracks at temperatures of 235 °C to 280 °C causes up to a doubling of  $v_T$  (Ketcham and Tamer,  
170 2021). Taken together with the fact that  $v_T$  exhibits no variation along the tracks (section 2) and  
171 is unaffected by lab-annealing, the observations that  $v_T$  correlates with  $D_{\text{par}}$  (Fu et al., 2024) and  
172 varies with the angle to the apatite *c*-axis (Jonckheere et al., 2024a) underscore the fact that  $v_T$  is  
173 more a function of the detector composition than of the damage densities along the fission frag-  
174 ment trajectories.



175 **Figure 5. (a)** Cumulative frequencies of the lengths of horizontal induced confined tracks in prism faces  
 176 of four Durango apatites with increasing degrees of annealing, distributed as part of an inter-lab exper-  
 177 iment (Ketcham et al., 2015; 21-2: unannealed; 21-4: annealed 10 h at 240 °C; 21-1: annealed 10 h at  
 178 288 °C; 21-3: annealed 10 h at 310 °C). **(b)** Cumulative frequencies of the *c*-axis projected lengths. **(c)**  
 179 The *a*-axis vs. *c*-axis intercepts of ellipses fitted to the length vs. *c*-axis angle data (white; RD:  $l_A = 1.632$   
 180  $l_C - 10.879$ ; empirical relationship of Donelick et al., 1999), and of regression lines fitted to the *c*-axis  
 181 projected length vs. true *c*-axis angle (grey; 1:1: isotropic line:  $l_A = l_C$ ); **(d)** cumulative frequencies of the  
 182 *c*-axis angles.



183 **Figure 6. (a)** Cumulative frequencies of the intersections between host-track and confined track, measured from the intersection to the closest track tip and normalised to the confined track length. **(b)** Cumulative frequencies of the calculated track etch rates  $v_T$ . **(c)** Cumulative frequencies of the calculated effective etch times  $t_E$  **(d)** cumulative frequencies of the size of the track surface intersections parallel to the  $c$ -axis (Dpar).  
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 185  
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LSTD	N <sub>C</sub>	l <sub>M</sub> (σ)	S <sub>M</sub>	l <sub>50</sub>	l <sub>P</sub> (σ)	S <sub>P</sub>	l <sub>P50</sub>	l <sub>C</sub> (σ)	l <sub>A</sub> (σ)	l <sub>PC</sub> (σ)	l <sub>PA</sub> (σ)
21-2	572	15.9(1)	0.75	15.9	16.3(1)	0.56	16.3	16.3(1)	15.7(1)	16.1(1)	16.4(1)
21-4	370	14.2(1)	0.77	14.3	15.1(1)	0.52	15.1	15.1(1)	13.8(1)	14.9(1)	15.2(1)
21-1	332	11.9(1)	0.90	11.8	13.5(1)	0.46	13.5	13.5(1)	11.1(1)	13.3(1)	13.6(1)
21-3	801	10.1(1)	1.49	10.3	12.5(1)	0.47	12.5	12.4(1)	9.3(1)	12.2(1)	12.7(1)

LSTD	N <sub>E</sub>	t <sub>EM</sub> (σ)	S <sub>TE</sub>	t <sub>E50</sub>	N <sub>VT</sub>	v <sub>TM</sub> (σ)	S <sub>VT</sub>	v <sub>T50</sub>	N <sub>D</sub>	Dpar(σ)	S <sub>D</sub>	D <sub>50</sub>
21-2	568	13.3(1)	3.4	12.4	607	127(2)	44	114	219	1.81(2)	0.25	1.89
21-4	368	12.2(2)	3.0	11.4	426	123(2)	41	111	231	1.87(2)	0.20	1.86
21-1	329	13.2(2)	3.2	12.4	328	123(3)	47	107	239	1.80(1)	0.19	1.84
21-3	799	11.5(1)	3.5	10.4	697	123(2)	48	105	212	1.84(1)	0.18	1.80

188 **Table 2.** Measurements of confined tracks in unannealed and part-annealed prism faces of Durango  
 189 apatite etched for 20s in 5.5 M HNO<sub>3</sub> at 21 °C. N<sub>C</sub>: number of confined track lengths; l<sub>M</sub>: mean length  
 190 (μm); S<sub>M</sub>: standard deviation (μm); l<sub>50</sub>: median; l<sub>P</sub>: mean *c*-axis-projected length (μm); S<sub>P</sub>: standard deviation  
 191 of the projected lengths (μm); l<sub>P50</sub>: median projected length; (l<sub>C</sub>, l<sub>A</sub>): *c*-axis and *a*-axis intercepts of  
 192 ellipses fitted to length vs. orientation data (μm); (l<sub>PC</sub>, l<sub>PA</sub>): *c*-axis and *a*-axis intercepts of regression lines  
 193 to projected length vs. orientation data (μm). N<sub>D</sub>: Number of Dpar-measurements; Dpar: mean Dpar  
 194 (μm); S<sub>D</sub>: standard deviation (μm); D<sub>50</sub>: median (μm); N<sub>E</sub>: number of effective-etch-time measurements;  
 195 t<sub>EM</sub>: mean effective etch time (s); S<sub>TE</sub>: standard deviation (s); t<sub>E50</sub>: median effective etch time (s); N<sub>VT</sub>:  
 196 number of track etch rate measurements; v<sub>TM</sub>: mean track etch rate (μm.min<sup>-1</sup>); S<sub>VT</sub>: standard deviation  
 197 (μm.min<sup>-1</sup>); v<sub>T50</sub>: median track etch rate (μm.min<sup>-1</sup>). Numbers in brackets are standard errors of the cor-  
 198 responding statistic.

199 The median effective etch time of the unannealed tracks (21-2) is 2 s longer than that of the most  
 200 annealed (21-3; Figure 6c). Both intermediate samples lie between the extremes, albeit in reverse  
 201 order. Given that their track and apatite etch rates are almost the same, one could expect that t<sub>E</sub>  
 202 correlates with the reciprocal track densities. That of 21-3 is ~2/3 of 21-2 (Aslanian et al., 2022);  
 203 the average separation between a host track and a potential confined track in 21-3 is thus greater  
 204 than in 21-2. However, this must not be the case for the measured tracks, selected on the basis of  
 205 their widths. For an average v<sub>T</sub> of ~2 μm.s<sup>-1</sup> (Table 2: ~120 μm.min<sup>-1</sup>), a ~2-s t<sub>E</sub>-difference corre-  
 206 sponds to a ~4-μm difference of the etchant trajectories from the host track intersections to the  
 207 confined track tips. This is in broad agreement with the ~6 μm mean length difference between  
 208 21-2 and 21-3.

209 All samples have narrow Dpar-distributions with medians (D<sub>50</sub>) decreasing from 1.89 μm for the  
 210 unannealed apatite (21-2) to 1.80 μm for the most annealed (21-3). The difference is small but  
 211 reflects a systematic shift of the Dpar-distribution to lower values with increasing annealing  
 212 (Figure 5d). Because Dpar is proportional to the apatite etch rate, it suggests a thermal relaxa-  
 213 tion of irradiation-induced strain from recoils following (n,γ)-reactions or uranium fission and  
 214 track formation (Rivera et al., 2011; Dufour and Toulemonde, 2016; Schauries, 2018; Rymzhanov  
 215 et al., 2019). Assuming Arrhenius kinetics, the D<sub>50</sub>-ratio (1.80/1.89 = 0.953) for the most an-  
 216 nealed and unannealed samples represents an increase of the activation energy for apatite etch-  
 217 ing of 118 J.mol<sup>-1</sup>, or 1.22 meV per atom. If an individual point defect stores ~1 eV, the lowering  
 218 of the apatite etch-rate results from annealing one in a thousand (F<sub>D</sub> = 1.22 10<sup>-3</sup>) defects created  
 219 by irradiation.

220 For Arrhenius kinetics, the rate factor (k<sub>D</sub>) and activation energy (E<sub>D</sub>) for defect annealing are  
 221 given by:

$$k_D = \frac{\ln(1-F_D)}{t_A} = 3.3910^{-8} s^{-1} \quad (1)$$



222 wherein  $t_A = 36000$  s is the annealing time (10 h). For a pre-exponential factor  $k_0 = 10^{13} \text{ s}^{-1}$ , this  
 223 gives:

$$E_D = RT_A \ln \left( \frac{k_0}{k_D} \right) = 228 \text{ kJ} \cdot \text{mol}^{-1} \quad (2)$$

224 wherein  $R = 8.314 \text{ J} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}$  is the gas constant and  $T_A = 310 \text{ }^\circ\text{C}$  the annealing temperature of  
 225 sample 21-3. For comparison, we calculated similar rate factors ( $k_F$ ) and activation energies ( $E_F$ )  
 226 for fission-track annealing using the fractional length reduction,  $F_F = 1 - l/l_0$ , as the measure of  
 227 progress. This shows that  $E_F$  increases from  $182 \text{ kJ} \cdot \text{mol}^{-1}$  at  $240 \text{ }^\circ\text{C}$  to  $200 \text{ kJ} \cdot \text{mol}^{-1}$  at  $310 \text{ }^\circ\text{C}$  (Table  
 228 3). Without attaching undue meaning to the specific numbers, it bears out that fission-track an-  
 229 nealing and the partial elimination of point defects that causes a decrease of the apatite etch rate  
 230 are processes that occur in parallel under the annealing conditions to which the samples were  
 231 subjected. Point defects begin to anneal at  $T_t$ -conditions at which fission-track annealing is well  
 232 advanced.

LSTD	$T_A$ ( $^\circ\text{C}$ )	$t_A$ (h)	$T_A$ (K)	$t_A$ (s)	$F_F$	$k_F$ ( $10^{-6} \cdot \text{s}^{-1}$ )	$E_F$ ( $\text{kJ} \cdot \text{mol}^{-1}$ )
21-4	240	10	513	36000	0.11	3.14	182
21-1	288	10	561	36000	0.25	8.05	194
21-3	310	10	583	36000	0.36	12.6	200

233 **Table 3.** Activation energies for fission-track annealing.  $T_A$ : annealing temperature in  $^\circ\text{C}$  and  
 234 in K;  $t_A$ : annealing time in hours and in seconds;  $F_F$ : fractional annealing  $F_F = 1 - l/l_0$ ;  $k_F$ : rate  
 235 factor;  $E_F$ : activation energy.

236 For a constant-core model and high average track etch rate ( $v_T \approx 120 \text{ } \mu\text{m} \cdot \text{min}^{-1}$ ; Table 2), the exact  
 237 value of  $v_T$  and the precise point from where a confined track begins to etch are of little consequence.  
 238 For the same reason, the effect of multiple host track intersections, even occurring at the same  
 239 time and far apart, is minimal. This is not so for a linear  $v_T$ -profile with a central maximum ( $\sim 1.7$   
 240  $\mu\text{m} \cdot \text{s}^{-1}$ ; unannealed induced tracks) decreasing to  $v_R$  towards the track tips (Ketcham and Tamer,  
 241 2021). In that case, short tracks intersected towards one end have a higher likelihood than longer  
 242 tracks to be etched at the opposite end to a point that satisfies an operator's selection criteria.  
 243 This would result in a positive correlation between the track length and the position of the host  
 244 track intersection (Figure 8 of Jonckheere, 2023). The correlation would decrease depending on  
 245 the extent of length reduction and the increased track etch rate after partial annealing (Ketcham  
 246 and Tamer, 2021).

247 Supplement Figures S07 - S09 show the dependence of the  $c$ -axis-projected lengths ( $l_P$ ) of individ-  
 248 ual tracks in each sample on  $IP$ ,  $v_T$ , and  $t_E$ . Two regression lines to  $l_P$  vs.  $IP$  have small positive and  
 249 two have small negative slopes (Table 4), suggesting the absence of significant correlation. The  
 250 regression lines to  $l_P$  vs.  $v_T$  have small positive slopes. Because of the large  $v_T$ -range (Supplement  
 251 Figure S5), this nonetheless adds  $\sim 0.1 - 0.3 \text{ } \mu\text{m}$  to the lengths of fast-etching compared to slow-  
 252 etching tracks. We reported positive correlations between  $l_P$  and  $t_E$  of fossil (Aslanian et al., 2021)  
 253 and induced tracks (Jonckheere et al., 2024a) in Durango apatite, based on repeat measurements  
 254 of individual step-etched tracks. In contrast, Supplement Figure S9 compares  $l_P$  and  $t_E$  measure-  
 255 ments of different tracks in a population etched once. The fact that the plots indicate far less cor-  
 256 relation (none in 21-4) is in part due to the limited  $t_E$ -range (Supplement Figure S6) and statistical  
 257 factors. The correlation adds  $\sim 0.1 - 0.3 \text{ } \mu\text{m}$  to the lengths of well-etched ( $t_E \approx 20$  s) compared to  
 258 underetched ( $t_E \approx 5$  s) tracks. We nevertheless conclude that, for practical purposes, neither the  
 259 intersection point, the track etch rate or the effective etch time have a significant effect on length  
 260 measurements of confined tracks etched once (20 s; 5.5 M  $\text{HNO}_3$ ;  $21 \text{ }^\circ\text{C}$ ). This does however not  
 261 mean that these factors also have negligible effect on the selection of confined tracks for meas-  
 262 urement.



LSTD	$I(v_T)$	$S(v_T)$	$\Delta(v_T)$	$\Delta\%$	$I(t_E)$	$S(t_E)$	$\Delta(t_E)$	$\Delta\%$
21-2	16.25	0.001	0.13	0.8	16.21	0.007	0.11	0.7
21-4	14.93	0.001	0.23	1.5	15.11	-0.001	-0.01	-0.1
21-1	13.38	0.001	0.22	1.6	13.24	0.017	0.28	2.1
21-3	12.36	0.001	0.30	2.4	12.41	0.006	0.11	0.9
DULU	$I(v_T)$	$S(v_T)$	$\Delta(v_T)$	$\Delta\%$	$I(t_E)$	$S(t_E)$	$\Delta(t_E)$	$\Delta\%$
FC1-I	16.22	0.0004	0.10	0.6	15.95	0.030	0.58	3.6
FC1-S	13.48	0.0031	0.67	4.9	13.90	0.007	0.14	1.1
AS3-I	16.22	0.0006	0.15	0.9	15.83	0.034	0.66	4.2
AS3-S	13.94	0.0008	0.17	1.2	13.69	0.027	0.57	4.5

263 **Table 4.** Relationship between  $c$ -axis-projected lengths ( $l_p$ ), track etch rates ( $v_T$ ), and effective etch times ( $t_E$ ).  $I(\dots)$  and  $S(\dots)$ : intercepts and slopes of regression lines fitted to  $l_p$  versus  $v_T$  and  $l_p$  versus  $t_E$ ;  $\Delta(\dots)$ : maximum difference over the range of  $v_T$ - and  $t_E$ - values;  $\Delta\%$ : percent difference.  
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#### 4. Effect of ageing

267 We analysed the FC1 and AS3 apatites from the Duluth intrusive complex in Minnesota to investigate the effects of geological ageing. These well-studied samples are reference materials and proposed age standards (Iwano et al., 2019; Härtel et al., 2022; 2023). Their interest lies in the fact that they are old and have long low-temperature thermal histories, permitting the accumulation of extensive radiation damage, the most important factor with respect to ageing. Based on data of Härtel et al. (2023), the total alpha dose for FC1 is  $\sim 7.10^{16} \text{ a.g}^{-1}$  (0.8-1.0 dpa), although it is not known which fraction of the accumulated damage survived till the present. We annealed an aliquot of FC1 and AS3 for 24 h at 450 °C, and irradiated it with thermal neutrons in channel Y4 of the BR1 reactor in Mol, Belgium. We mounted an irradiated and an untreated fraction of each in resin, and ground, polished and etched them using the techniques described in section 3. As there also, we measured the track lengths, orientations, widths, cone-angles, and  $D_{par}$ , but not the host track intersections, and calculated the effective etch times and the apatite and track etch rates.  
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280 Consistent with their similar  $D_{par}$ -values, the induced-track length distributions of FC1 and AS3 are almost identical. The fossil tracks in both samples underwent >20% shortening; both fossil-track length distributions are also similar, albeit that those in FC1 are somewhat shorter. We interpret this difference as significant;  $c$ -axis projection highlights the greater fraction of short tracks (Table 5; Figure 7a,b). The semi-axes of ellipses fitted to the induced-track data are close to the trend for unannealed apatites (Donelick et al., 1999); those fitted to the fossil-track data reveal them to be somewhat less anisotropic than induced tracks of the same mean length (Figure 7c). This could be due to different length bias affecting high- and low-angle tracks, different angular distributions, which control the relative weights of the constituent populations, or under-etching (Jonckheere, 2023; Jonckheere et al., 2024). However, the angular distributions of the fossil, as well as induced, tracks are almost identical (Figure 7d) and the effective etch times of the fossil tracks are 1-2 s longer than those of the induced (Table 3; Figure 8b). This argues against experimental factors. This leaves length-bias or unknown differences between fossil and induced tracks.  
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DULU	N <sub>c</sub>	l <sub>M</sub> (σ)	S <sub>M</sub>	l <sub>50</sub>	l <sub>p</sub> (σ)	S <sub>p</sub>	l <sub>p50</sub>	l <sub>c</sub> (σ)	l <sub>A</sub> (σ)	l <sub>pC</sub> (σ)	l <sub>pA</sub> (σ)
FC1-I	536	15.9(1)	0.83	15.9	16.3(1)	0.60	16.3	16.1(1)	15.8(1)	16.1(1)	16.4(1)
FC1-S	527	12.4(1)	1.49	12.5	13.8(1)	1.00	13.9	13.2(2)	12.0(1)	13.3(1)	14.2(1)
AS3-I	346	15.8(1)	0.86	15.8	16.2(1)	0.63	16.3	16.1(1)	15.6(1)	16.1(1)	16.3(1)
AS3-S	309	12.6(1)	1.36	12.7	14.0(1)	0.88	14.0	13.6(2)	12.1(1)	13.7(1)	14.2(1)

DULU	N <sub>D</sub>	Dpar(σ)	S <sub>D</sub>	D <sub>50</sub>	N <sub>E</sub>	t <sub>EM</sub> (σ)	S <sub>E</sub>	t <sub>E50</sub>	N <sub>VT</sub>	V <sub>TM</sub> (σ)	S <sub>VT</sub>	V <sub>T50</sub>
FC1-I	406	2.06(2)	0.34	2.11	460	10.6(2)	3.8	10.6	460	105(2)	42	99
FC1-S	384	2.41(2)	0.30	2.47	513	13.1(2)	4.1	13.1	503	99(2)	39	97
AS3-I	381	1.92(1)	0.26	1.97	300	11.2(2)	3.7	10.9	300	107(2)	41	100
AS3-S	390	2.23(1)	0.28	2.27	268	12.2(3)	4.0	12.0	267	99(3)	42	93

294 **Table 5.** Measurements of fossil and induced confined tracks in FC1 and AS3 apatite, etched for 20s in  
 295 5.5 M HNO<sub>3</sub> at 21 °C. See Table 1 for symbols.

296 As in the case of the annealed Durango samples, the track-etch-rate distributions of all the Du-  
 297 luth samples are almost identical (Figure 8a). The ~5% difference between the means for the  
 298 fossil and the induced tracks (Table 3) is too small to call it significant at this stage. The overall  
 299 agreement is striking given that the track etch rates  $v_T$  are calculated from the apatite etch rates  
 300  $v_R$  (Fu et al., 2024):

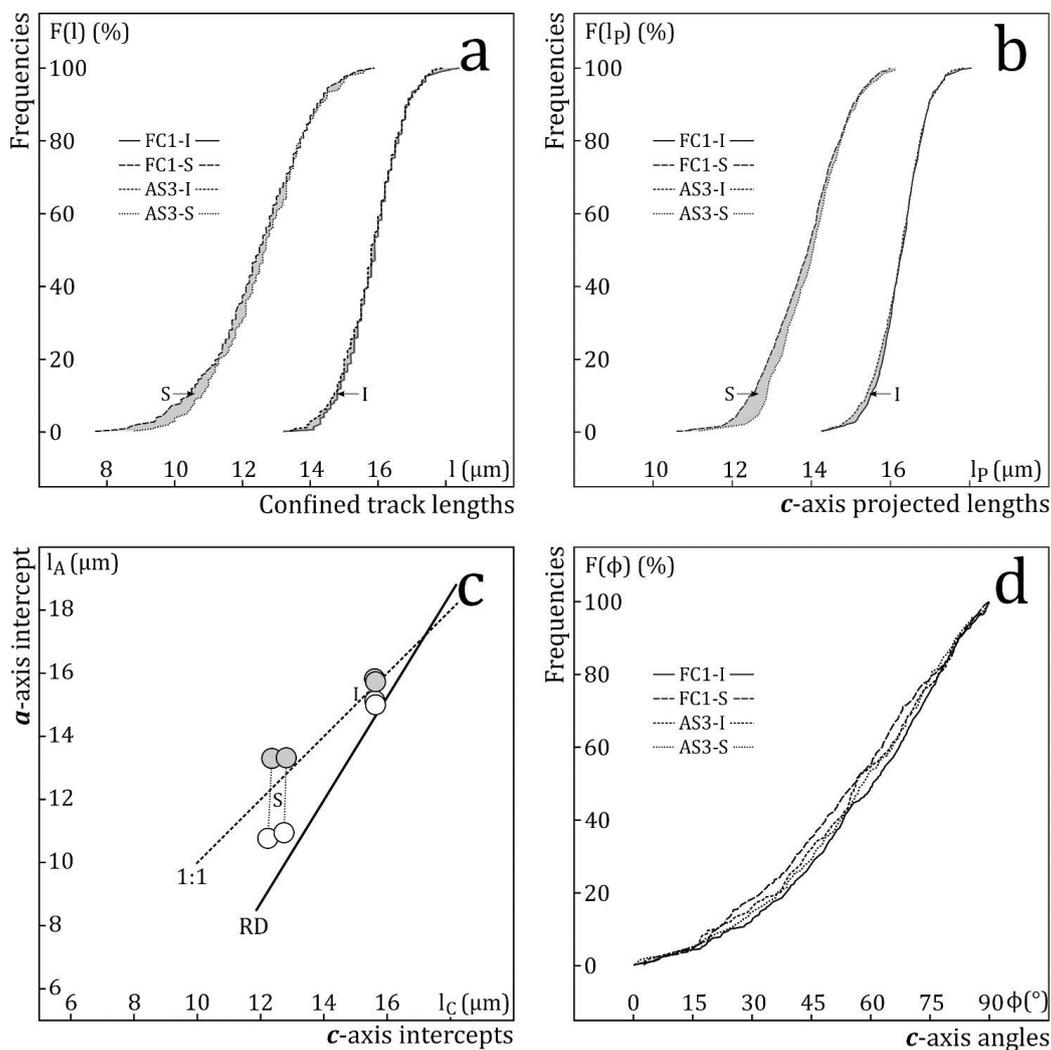
$$v_T = \frac{S_{12}}{(D_1 - D_2)} v_{R,S} \quad (3),$$

301 wherein  $S_{12}$  is the distance between two inscribed circles, with diametres  $D_1$  and  $D_2$ , touching  
 302 facing sides of the confined track, and  $v_{R,S}$  is the apatite etch rate perpendicular to the track axis.  
 303 Dpar-measurements indicate that the apatite etch rate differs 7 - 8% between FC1 and AS3 and  
 304 16-17% between the fossil and induced tracks (Table 3). We applied the empirical correlation  
 305 between Dpar and  $v_R$  (Fu et al., 2024; Trilsch et al., 2024) for calculating the apatite etch rates  
 306 of each sample:

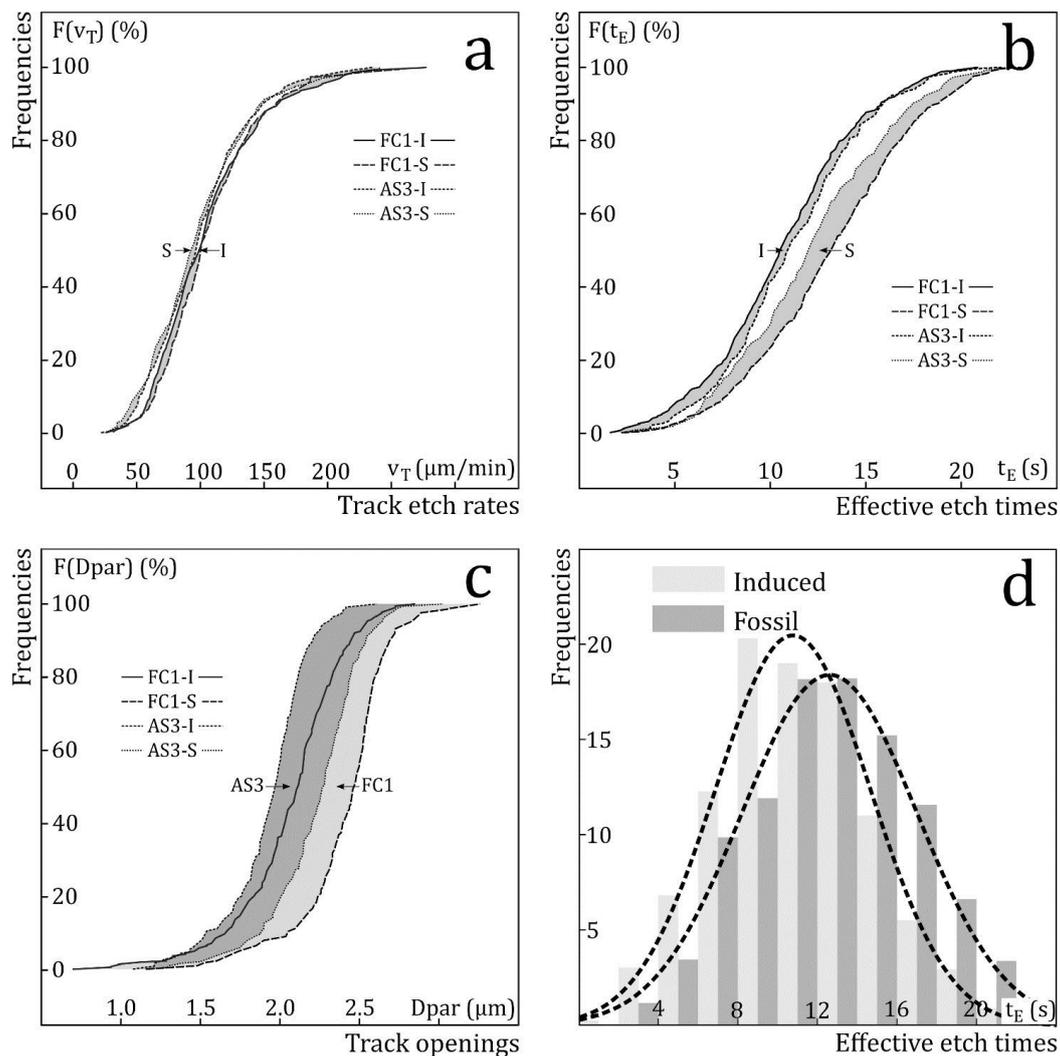
$$v_{R,S} = \frac{Dpar_S}{Dpar_D} v_{R,D} \quad (4).$$

307 Dpar<sub>S</sub> and Dpar<sub>D</sub> refer to the sample and to Durango apatite, and  $v_{R,D}$  is the etch rate of Durango  
 308 (Aslanian et al., 2021). The agreement between the  $v_T$ -distributions of FC1 and AS3 is perhaps  
 309 unsurprising. Both are from the Duluth complex anorthosite series, albeit from locations >100  
 310 km apart, and, according to their ages and track length distributions, both had similar thermal  
 311 histories. The less obvious fact is that, in contrast to the substantial length reduction, ageing, or  
 312 low-temperature geological annealing, appears to have a minimal effect on the etch rates of con-  
 313 fined tracks.

314 As in the case of the annealed Durango samples, the average effective etch-times ( $t_E$ ) of FC1 and  
 315 AS3 are just over half the immersion time ( $t_i = 20$  s; Table 3). The means, medians and standard  
 316 deviations for both induced-track samples are similar, those for both fossil-track samples some-  
 317 what less so, reflecting different geological histories. There is, on the other hand, a distinct sep-  
 318 aration between the  $t_E$ -distributions of the fossil and induced tracks (Figure 8b,d), which is not  
 319 due to  $v_T$ . The longer effective etch times of the fossil tracks are instead due to the greater  $v_R$  of  
 320 the fossil-track grains, which enable faster etchant access to the confined tracks. There are con-  
 321 tributing factors: the much higher fossil than induced track densities (Jonckheere et al., 2024a)  
 322 lowers the average separation between the host tracks and confined tracks; on the other hand,  
 323 the shorter fossil- than induced-track lengths have an opposite effect. It is difficult to separate  
 324 these factors because the stock of confined tracks is filtered through the scientist's selection  
 325 criteria.



326 **Figure 7. (a)** Cumulative frequencies of the lengths of fossil (S) and induced (I) confined tracks in prism  
 327 faces of FC-1 and AS-3 apatites from the Duluth complex, Minnesota, USA (Iwano et al., 2019; Härtel et  
 328 al., 2023). **(b)** Cumulative frequencies of the  $c$ -axis projected lengths of FC-1 and AS-3. **(c)**  $a$ -axis vs.  $c$ -  
 329 axis intercepts of ellipses fitted to the lengths and regression lines fitted to the  $c$ -axis projected length  
 330 vs.  $c$ -axis angle (white: induced; grey: fossil; RD and 1:1 as in Figure 5); **(d)** cumulative frequencies of  
 331 the  $c$ -axis angles.

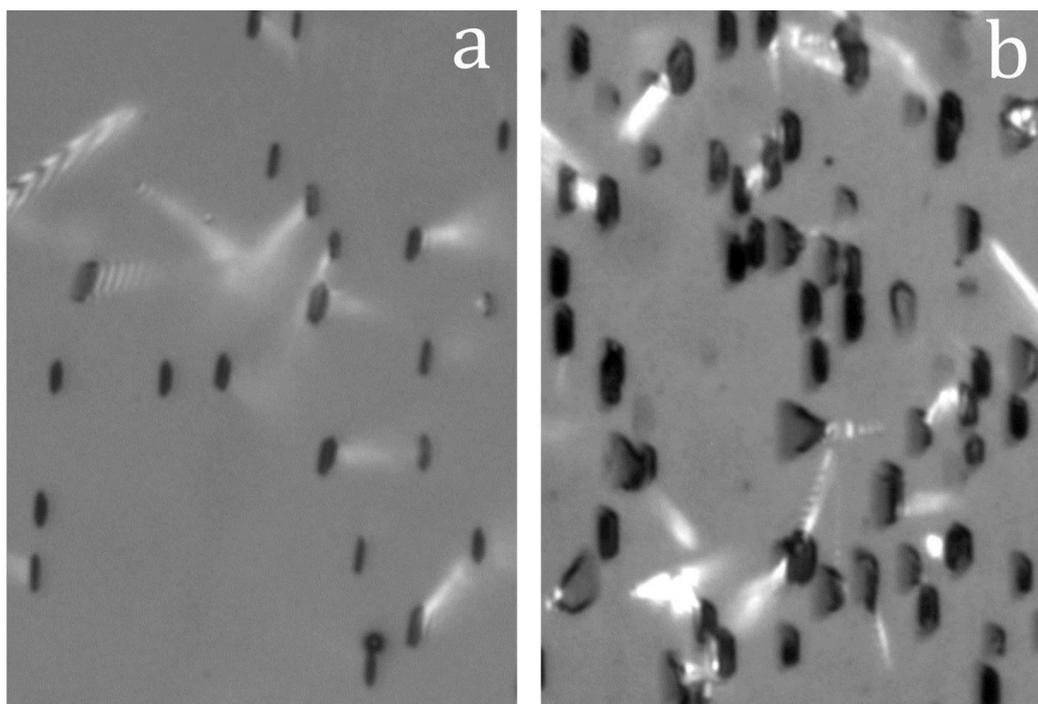


332 **Figure 8.** (a) Cumulative frequencies of the track etch rates of fossil (S) and induced (I) confined tracks  
 333 in the FC-1 and AS-3 apatites (b) Cumulative frequencies of the effective etch times of fossil and induced  
 334 tracks (c) cumulative frequencies of the sizes of the track surface intersections ( $D_{par}$ ) of fossil and  
 335 induced tracks. (d) Histograms of the effective etch times of the combined fossil and induced tracks of FC-  
 336 1 and AS-3.

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338 The clearest distinction between FC1 and AS3, and between fossil and induced tracks, is in their  
339 Dpar's (Table 3; Figure 8c). The Dpar's of fossil-tracks are 16%(AS3)-17%(FC1) higher than those  
340 of induced tracks. We propose that this is due to distributed radiation damage from  $\alpha$ -disintegration  
341 of  $^{238}\text{U}$ ,  $^{235}\text{U}$ ,  $^{232}\text{Th}$ , and  $^{146}\text{Sm}$ , and collateral recoils from  $^{238}\text{U}$ -fission, accumulated over geo-  
342 logical time. The resulting lattice damage increases the overall apatite etch rate, leading to larger  
343 Dpar. **Figure 9** shows the Dpar's of fossil and induced tracks in FC1. The ageing of the fossil-track  
344 samples appears to have a much greater effect on Dper than on Dpar. This supports the radiation  
345 damage concept: Dpar depends on the etch rate of fast etching apatite faces while Dper is controlled  
346 by the slow-etching prism faces flanking the track on either side (Jonckheere et al., 2022).  
347 Slow-etching faces are more susceptible to radiation damage than fast etching faces (Jonckheere  
348 et al., 2019), which accounts for the doubling of Dper, while Dpar increases <20% on average  
349 (Figure 9b).



350 **Figure 9.** Reflected-light images of the openings of induced **(a)** and fossil **(b)** tracks in prism faces of the  
351 FC1 apatite (etched 20 s in 5.5 M  $\text{HNO}_3$  at 21 °C).

#### 4. Conclusion

352 Since the earliest fission track studies, all etch models have been based on two velocities:  $v_T$ , the  
353 rate of advance along the damaged core, and  $v_R$ , the rate of retreat of the undamaged material.  
354 These velocities are more than conceptual: their values define the efficiencies of the track counts  
355 (Tagami and O'Sullivan, 2005) and set the thresholds for confined-track selection (Ketcham and  
356 Tamer, 2021). They form furthermore the basis for explaining the shapes of etched tracks (Jonck-  
357 heere et al., 2022). On its own,  $v_R$  is an important indicator of chemical composition and anneal-  
358 ing kinetics (Dpar; Burtner et al., 1994). Despite this, measurements of  $v_T$  and  $v_R$  have long been  
359 lacking. This work aims to fill this gap to some extent and to investigate some factors controlling  
360  $v_R$  and  $v_T$ .



361 Direct measurements and microscopic observation indicate that  $v_T$  is constant over most of the  
362 length of ion and fission tracks in apatite (constant core model; Tamer and Ketcham, 2020). This  
363 suggests that there is a threshold damage above which  $v_T$  does not increase, e.g., an amorphous  
364 core raises  $v_T$  to a value that does not depend on the work expended to create it (amorphous is  
365 amorphous). This is however not a conclusion within the compass of our investigation. On the  
366 other hand, reconciling straight-sided etched surface and confined tracks with something other  
367 than a constant  $v_T$  requires a different and more complex etch model than any proposed to date.  
368 Our results revealed that the etchant advances at a high but finite rate along pre-etched tracks.  
369 This implies that the summed immersion times of step-etch experiments are overestimates, and  
370 that step-etching broadens the effective-etch-time distribution (Figure 6a of Jonckheere et al.,  
371 2024a). It also implies that successive etch steps cause  $v_T$  to be underestimated to an increasing  
372 extent (Figure 5c of Jonckheere et al., 2024a). The low  $v_T$  of Aslanian et al. (2021;  $\sim 75 \mu\text{m}\cdot\text{min}^{-1}$ )  
373 for fossil tracks in Durango apatite must therefore be read as an underestimate, consistent with  
374 the present results.

375 The distributions of host-track intersections (IP),  $c$ -axis angles ( $\phi$ ), and track-etch rates ( $v_T$ ) are  
376 almost identical across all measurements. There is a notable offset between the average  $v_T$  of  
377 the Durango and Duluth samples, but this need not mean that it is also of fundamental signifi-  
378 cance. The calculation of  $v_T$  requires  $v_R$ , which was measured for Durango (Aslanian et al., 2021)  
379 and other apatites (Trilsch et al., 2023; Fu et al., 2024), but not for FC1 and AS3, whose etch  
380 rates were estimated from their  $D_{\text{par}}$ 's, relative to that of Durango. The  $v_T$ -difference between  
381 Durango and Duluth could be within the scope of this approximation, which could also account  
382 for the small difference between the fossil and induced tracks in FC1 and AS3. The near-identical  
383 IP-,  $\phi$ -, and  $v_T$ -distributions are difficult to explain, but suggest that the confined-track selection  
384 is dominated by a criterion that is more or less independent of the track densities or lengths.  
385 The confined-track width is such a criterion that is moreover little affected by annealing (Jonck-  
386 heere, 2023).

387 In contrast to the track etch rate  $v_T$ , the apatite etch rate  $v_R$  shows marked variation. This is most  
388 obvious from the  $D_{\text{par}}$ 's of the fossil and induced tracks in FC1 and AS3, but also from those of  
389 the part-annealed induced tracks in Durango. In the case of the fossil tracks, the  $D_{\text{per}}/D_{\text{par}}$ -ratio  
390 increases as well as, and faster than  $D_{\text{par}}$ . It is therefore probable that the increases are related to  
391 radiation damage, from the natural background radiation in the fossil-track samples and neutron  
392 irradiation in the induced-track samples.  $D_{\text{per}}/D_{\text{par}}$  offers a means of assessing damage densi-  
393 ties in apatite if we can separate their effect from the chemical control of  $D_{\text{par}}$ . On the other hand,  
394 this  $D_{\text{par}}$ -variation reveals the need for caution when using  $D_{\text{par}}$  for estimating apatite annealing  
395 kinetics. The increased  $v_R$  of the Duluth samples with fossil tracks increases the rate of advance  
396 of the etchant along the slowest section of its course down the host track ( $v_T$ ), across to the con-  
397 fined track ( $v_R$ ; widening of the host track) and along its length to each tip ( $v_T$ ). Confined tracks in  
398 apatites with higher  $v_R$  therefore have shorter access times and, in consequence, longer effective  
399 etch times.

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402

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