1 **Supplementary Information (SI)**

2 **Supplementary Information 1**

3 The supplementary S1 contains the power spectral densities calculated for the length of day,

4 atmospheric angular momentum, and residual length of day observations corrected for

5 atmospheric angular momentum. The confidence level corresponds to the percentile of 10,000

6 red noise models, whose total power and auto-correlation coefficient match those of the

7 considered products.

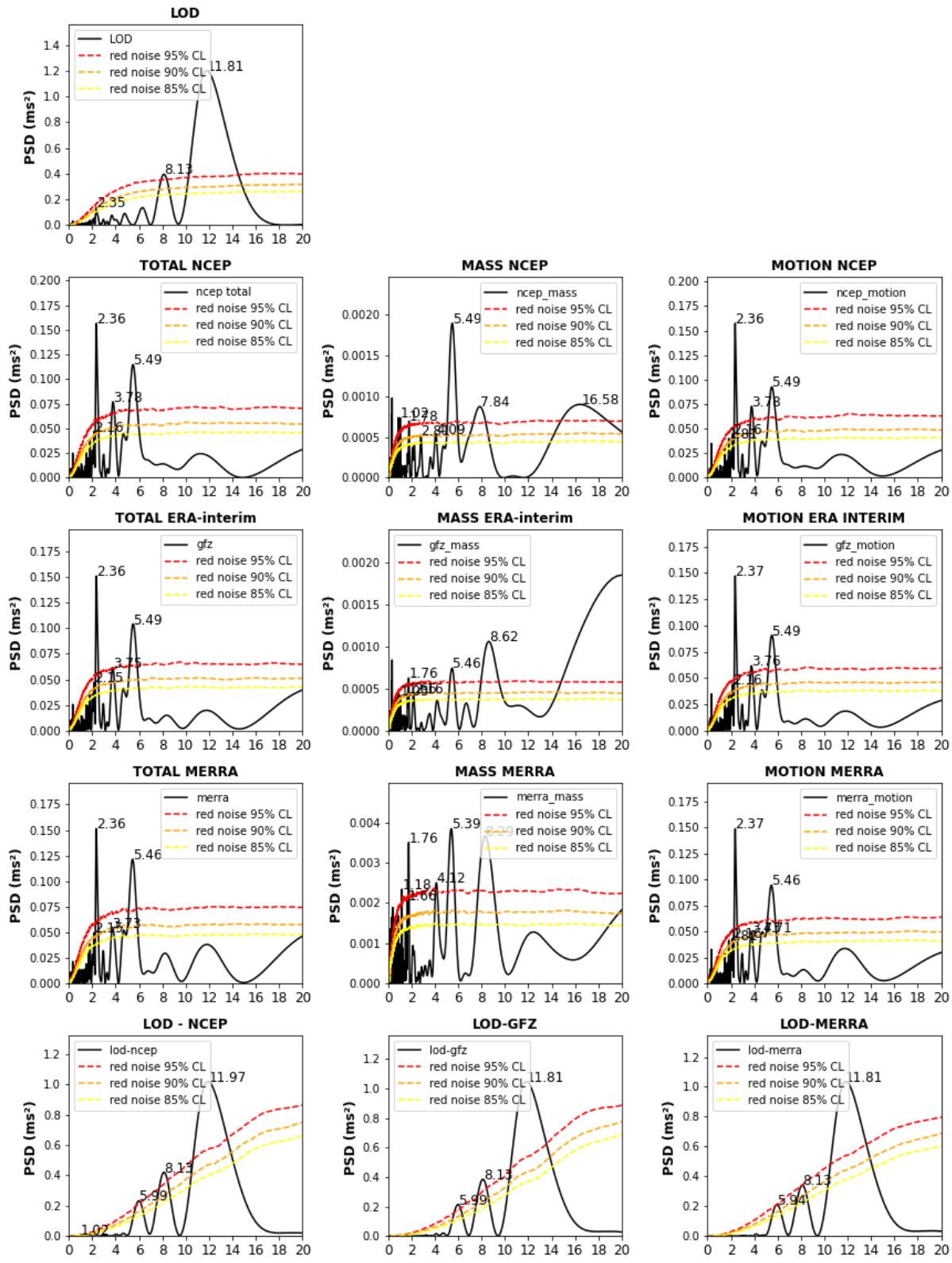



Figure S1: Power Spectral Densities (PSD) of the length of day (LOD), atmospheric angular momentum, and residual LOD corrected for atmospheric angular momentum using different atmospheric reanalyses (NCEP; ERA Interim/GFZ and MERRA). Dashed colored lines represent the confidence levels with red noise assumption.

13   **Supplementary Information 2**

14   The S2 supplementary contains the 6-yr cycles extracted from the mass and motion components

15   of the effective atmospheric angular momentum (AAM) functions calculated for NCEP, ERA-

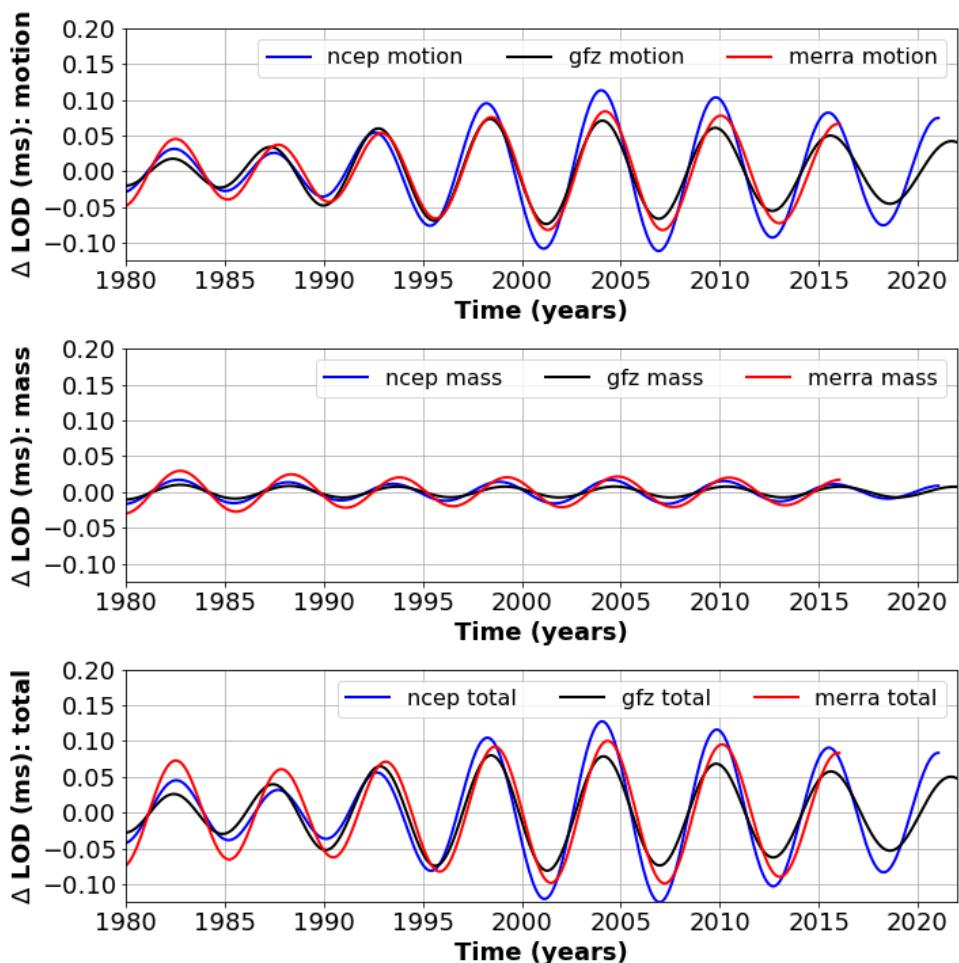
16   interim, and MERRA. The AAM is divided into two components: a mass term, associated with

17   atmospheric mass distribution (pressure variations), and a motion term, linked to the wind field

18   (zonal and meridional winds). The mass term dominates the mean angular momentum transfer

19   from the atmosphere to the solid Earth, exceeding the motion term by two orders of magnitude.

20   However, short-term, seasonal, and interannual fluctuations in AAM are primarily driven by

21   the motion term, as wind speeds vary more significantly than atmospheric mass.

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23   *Figure S2. 6-yr cycles in the total, mass, and motion components of the Atmospheric Angular*
24   *Momentum (AAM) between 1980 and 2016/2022 based on NCEP (blue), ERA-Interim/GFZ*
25   *(black), and MERRA (red) reanalyses. The 6-yr cycles were extracted using a fifth-order*
26   *Butterworth filter with a 5.1 to 7.2-year period band.*

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30 **Supplementary Information 3**

31 The ensemble of core flow models from Gillet et al., (2022) was analyzed to detect the

32 occurrence, significance, and characteristics of a potential 6-yr cycle in the core angular

33 momentum (CAM). All ensemble members exhibited power maxima between 5.2 and 6.6

34 years, with amplitudes ranging from 0.04 to 0.32 m/s (Fig. S3). The significance of the power

35 maxima exceeded 50% CL in 33 out of 50 realizations and 80% CL in 20 out of 50 realizations

36 (Fig. S3). A significant (>85% CL) six-year cycle was also detected in the CAM ensemble

37 mean with a period peaking at 5.8 years and an amplitude of 0.13 ms. For comparison, the 6-

38 yr cycle in LOD residuals peaks at 5.94 to 5.99 years with an amplitude of around 0.11 ms

39 (Fig. S3). The LOD observations uncorrected for the effect of the atmosphere exhibit a less

40 significant (68% CL) power maximum at 6.3 years with an amplitude of 0.086 ms (Fig. S3).

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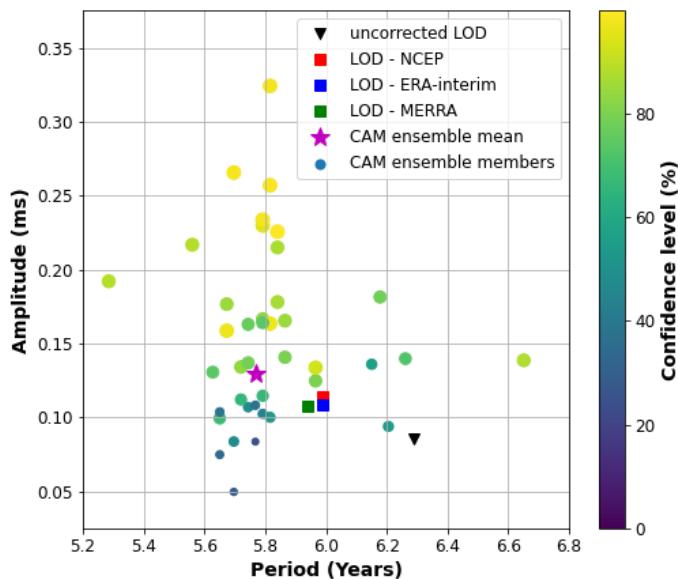
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44 *Figure S3. Characteristics of the power maxima closest to six years in the CAM model*
45 *ensemble and LOD observations, uncorrected and corrected for the atmospheric angular*
46 *momentum using the NCEP, ERA-Interim, and MERRA reanalyses.*

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