

THEMIS: Spin Axis Magnetic Field Reconstruction

by

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The study presents a method for reconstructing the missing component of the measured magnetic field on one of the space probes within the THEMIS mission. Its absence is a consequence of the failure of the instrument, the sensor of which is oriented along the spin axis of the probe. The reconstruction of the magnetic field component is possible thanks to the relatively favourable geometric configuration of the measuring device, which provides two additional components measured in a plane slightly deviated from the spin plane, and also thanks to the knowledge of the probe's spin.

The manuscript is organized as follows: The introductory part describes the conditions and motivation of the measurements performed, the problem that is the subject of the solution and the circumstances of its emergence, and the proposal of the solution itself. The second part presents the mathematical approach to the solution. In the third part, the solution is implemented on measured data, the fourth part presents the results and their comparisons, the fifth part is devoted to the discussion, and the sixth part formulates the conclusions. The study is supplemented by an appendix that addresses additional factors relevant to the issue and justifies the chosen approach.

The presented results indicate that, with the availability of 2 additional components, the presented approach can obtain a relatively accurate image of the 3-dimensional magnetic field even if one of the sensors fails. The significance of the study lies mainly in two respects:

1. It proposes an emergency solution for maintaining the continuity of measured data in a situation where, due to the impossibility of intervention in the form of repair/replacement/ adjustment of the instrument, it is inevitable to face a measurement failure. This solution therefore largely saves the success of the mission and brings significant scientific and economic value.
2. It may lead to a more general proposal for a more advantageous geometric arrangement of the instruments, so that the eventual failure of any component does not cause significant loss of data or their accuracy.

This revised version is substantially clearer than the original manuscript, and I appreciate the authors' efforts to clarify ambiguities and support their study's findings with additional analyses. Addressing such a complex problem requires a presentation that allows the professional community to appreciate the work's ingenuity and potentially utilize it in the future.

I understand that the authors rely on earlier studies, but I consider it beneficial for the explanation to be sufficiently clear without the need to consult additional sources. Since it is easy to get confused while following the authors' steps, I suggest making some minor corrections to ensure the work's rigor and readability.

In summary: the manuscript requires technical corrections.

1. **Introduction**, p. 2, l. 28: Given that some of the listed steps are also carried out in isolation depending on the purpose, I would opt for the wording: "*The complete transformation into $\mathbf{B}$ involves several procedures:...*"
2. p. 2, l. 49: I consider it appropriate to clarify the information regarding equation (2): "*Assuming that the spin plane offsets have been accurately estimated, the magnitude of the magnetic field*

component along the spin axis, B_{SA} , projected onto the axes of sensors $S1$ and $S2$, can be expressed as
... (2),
where α (α_{S1} or α_{S2}) is the angle between the axis of the respective sensor and the spin axis.”

3. I find it beneficial to expand the introductory part to the **Section 2** and slightly reorganize and enrich the text of the subsections. I offer my idea below.
4. I consider the explanation regarding the transformation using the $\mathbf{\Gamma}$ matrix to be the most significant shortcoming. You present the $\mathbf{\Gamma}$ matrix as the inverse of the transformation matrix, implicitly assuming, I guess, that the transposed matrix is equal to the inverse matrix. Yet, this holds true only for orthogonal matrices, and the transformation in question is not orthogonal. Consequently, applying the inverse of the transformation described by equation (1) would merely yield a vector of coefficients (B_{S1} , B_{S2} , B_{S3}) that lacks physical vector significance and the standard relation for the dot product doesn't work. For this reason, the explanation regarding the $\mathbf{\Gamma}$ matrix transformation is somewhat confusing, which also affects **Subsection 2.2**. Please, see my further clarifications below.
5. Regarding the next two rotations that form the transformation matrix $\mathbf{\Sigma}$, it is appropriate to briefly comment on this procedure, please also see below.
6. It is worth noting that the transformation using the $\mathbf{\Phi}$ matrix is presented solely for the sake of completeness and does not affect the determination of the angles α (the effect of this transformation does not manifest in the final relationship (8)).
7. In relation (11), within the argument of the cosine function, ω_s multiplies only τ and not φ , i.e. $\cos(\omega_s \tau - \varphi)$.
8. p. 15, l. 286: ... δB ...
9. **Appendix:** The labels for the axes and scales in the figures are too small and difficult to read.
10. p. 21, l. 426: Is $2\Delta B_A(t)$ or $2\Delta B_D(t)$ used in equation (A7), then? Please, clarify.

2. Mathematical Background

The mathematical approach to the problem consists of two parts. The first is the geometric transformation of the non-orthogonal system associated with the measuring device into an orthogonal system that is related to the reference frame of the spinning space probe. It is realized in several steps, while here we are only interested in the directional nature of the measured output in order to obtain the angles between the sensors and the spin axis. The second part is devoted to the conversion of the two available primary sensor outputs into plausible records of the magnetic field in the respective directions. Taking into account the fact that the magnetic field oriented along the spin axis persistently contributes to these results, it is possible to separate this component and determine it with sufficient accuracy.

The orientation of the three non-orthogonal sensor axes, $S1$, $S2$ and $S3$, is illustrated in red in Fig. 1a. The raw magnetometer output along these axes is transformed into an orthogonal spin axis-aligned coordinate system (x, y and z) with physical meaningful units using the coupling matrix $\mathbf{C}$ (see Eq. 1). The coupling matrix can be expressed as

$$\mathbf{C} = \mathbf{\Phi} \cdot \mathbf{\Sigma} \cdot \mathbf{\Gamma} \cdot \mathbf{G}$$

where each submatrix accounts for a specific step in the complete transformation. We first introduce the coordinate systems and derive an expression for α , before describing the determination of B_{pSA}

from the spin plane measurements.

2.1 Coordinate System and Transformation Matrices

To clarify the geometry of the problem, we follow the formalism introduced by Plaschke et al. (2019), which builds on Kepko et al. (1996). We adopt their notation for consistency, while noting that the final implementation of our coupling matrix $\mathbf{C}$ differs slightly in form while yielding identical results. The conversion matrix $\mathbf{\Gamma}$ transforms from the non-orthogonal sensor axes coordinate system (red axes in Fig. 1a) into an orthogonal sensor package coordinate system (P_x , P_y , P_z), represented by the blue axes in Fig. 1:

$$\begin{aligned}\mathbf{S1} &= \sin \theta_{S1} \hat{\mathbf{P}}\mathbf{x} + \cos \theta_{S1} \hat{\mathbf{P}}\mathbf{z} \\ \mathbf{S2} &= \cos \phi_{S12} \sin \theta_{S2} \hat{\mathbf{P}}\mathbf{x} + \sin \phi_{S12} \sin \theta_{S2} \hat{\mathbf{P}}\mathbf{y} + \cos \theta_{S2} \hat{\mathbf{P}}\mathbf{z} \\ \mathbf{S3} &= \hat{\mathbf{P}}\mathbf{z}\end{aligned}$$

and thus

$$\mathbf{\Gamma} = \begin{pmatrix} \sin \theta_{S1} & \sin \theta_{S2} \cos \phi_{S12} & 0 \\ 0 & \sin \theta_{S2} \sin \phi_{S12} & 0 \\ \cos \theta_{S1} & \cos \theta_{S2} & 1 \end{pmatrix}.$$

The angles ...

The subsequent matrix $\mathbf{\Sigma}$ determines the orientation of the spin axis - marked in black in Fig. 1b - relative to the sensor-package system and consists of a (counterclockwise) rotation by an angle σ_{P_y} about the P_x -axis and a (clockwise) rotation by an angle $-\sigma_{P_x}$ about the P_y -axis:

...

2.2 Determination of the Sensor-to-Spin-Axis Angles

The angles α_{S1} , α_{S2} , and α_{S3} (see Fig. 1c) can be obtained from the projection of the spin axis onto the sensor axes using the geometric definition of the scalar product of two vectors, $\cos \alpha = \mathbf{u} \cdot \mathbf{v} / (|\mathbf{u}| |\mathbf{v}|)$. In the orthogonal spin axis-aligned coordinate system, the spin axis unit vector is given by $\mathbf{z} = [0, 0, 1]^T$. Using the transformation matrices, the calculation of the angles can be expressed as:

$$\begin{pmatrix} \alpha_{S1} \\ \alpha_{S2} \\ \alpha_{S3} \end{pmatrix} = \cos^{-1} ((\mathbf{\Phi} \cdot \mathbf{\Sigma} \cdot \mathbf{\Gamma})^T \cdot \mathbf{z}) = \dots$$

2.3 Gain Matrix and Determination of the Projected Spin Axis Contribution

The gain matrix $\mathbf{G}$ applies the sensor-specific conversion factors into meaningful physical units:

$$\mathbf{G} = \begin{pmatrix} gG_p & 0 & 0 \\ 0 & G_p/g & 0 \\ 0 & 0 & G_a \end{pmatrix},$$

where g is the ratio of the spin plane gains G_{S1}/G_{S2} , and $G_p (= \sqrt{G_{S1}G_{S2}})$ and G_a are the absolute gains for the spin plane and the spin axis sensors, respectively (Plaschke et al., 2019). To determine the projected spin axis contribution B_{pSA} from the measurements of the spin plane sensor axes data B_{S1} and B_{S2} , ...

Scientific significance: Does the manuscript represent a substantial contribution to scientific progress within the scope of Geoscientific Instrumentation, Methods and Data Systems (substantial new concepts, ideas, methods, or data)?	Yes (1)
Scientific quality: Are the scientific approach and applied methods valid? Are the results discussed in an appropriate and balanced way (consideration of related work, including appropriate references)?	Yes (2)
Presentation quality: Are the scientific results and conclusions presented in a clear, concise, and well-structured way (number and quality of figures/tables, appropriate use of English language)?	Yes (1)