Author response to RC1

Referee comments in blue
Author comments in black

General comments

> With this manuscript, the authors present a new estimate of the thickness of a glacier in High Mountain Asia (the Bhote Kosi glacier) using a high-order, deep-learning-based model (the Instructed Glacier Model, or IGM) that improves upon existing, less reliable low-order models. The primary broader goal of this effort is to establish an optimized case study that can then be applied to thickness estimates of other mountain glacier systems. The authors develop an inversion method to derive glacier thickness from ice surface velocity alone, then validate this thickness estimate with previously published thickness observations derived from airborne radar. Finally, the authors investigate the role that the spatial distribution of thickness observations plays in the calibration. The authors ultimately find that the velocity-only inversion with IGM is generally successful in reproducing the ice thickness field for Bhote Kosi glacier with errors comparable to the global thickness inversion estimates of Millan et al. (2022). Notably, the higher-order physics of IGM improve upon the Millan study for one branch of the Bhote Kosi glacier, potentially by including non-negligible stress components that are omitted in the Shallow Ice Approximation.
>
> This is a well-motivated, well-written manuscript that makes an interesting contribution to the growing literature on deep-learning techniques applied to mountain glacier settings. The discussion of relevant parameters, results, implications for other glaciers, and recommendations for future observation locations is generally thorough and thoughtful. I found the IGM description and methodology to be relatively accessible and well explained. My suggestions for improvement are minor.
>
> I recommend the manuscript be accepted for publication pending the authors' attention to the comments below at their discretion.

Thank you for your thoughtful response and your constructive comments which will help to improve the manuscript.

Specific comments

> 1. Although the description of the development of ice thickness estimation methods (lines 62–102) in the Introduction is informative, it reads as somewhat encyclopedic and could be made more concise.

Taking this comment into consideration, along with the comment from reviewer #2 that the Introduction is educational for readers with less knowledge of mountain glacier research, we will make these paragraphs more concise in such a way that is intended to convey the same information.

> 2. Although the choice of α_h among $\alpha_h = 2, 3, 6, 10$ does not substantively affect the MAE value, the authors note a strong volume dependence on α_h. The authors should therefore consider presenting results in the Appendix of one or more of the key experiments specifically with $\alpha_h = 2, 3, 10$, not just $\alpha_h = 6$, for the sake of completeness. The authors also note that valid arguments could be made for choosing any one of $\alpha_h = 2, 3, 6, 10$, so it would be beneficial to include a brief discussion of the arguments favoring each one of these four particular values.

Qualitative differences between the results for $\alpha_h = 2, 3, 6, 10$ are minimal, and quantitative differences integrated across the glacier are already shown in Figure 2. We have chosen not to include the resulting thickness maps for each in the manuscript, as they look similar and we feel it would add little value. However, we show them in Figure AR1-1 in response to this comment.

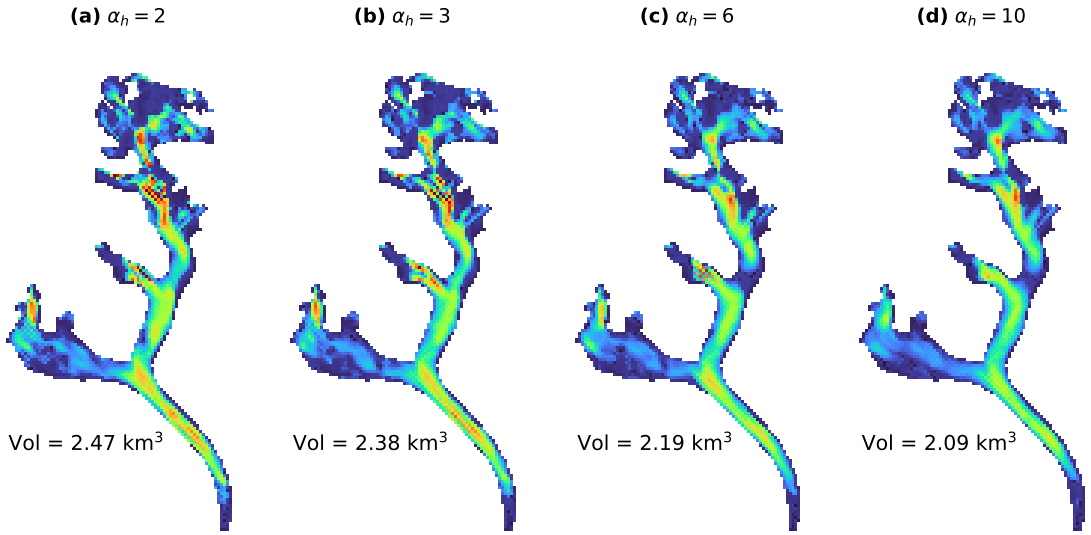

Figure AR1-1: Thickness maps of Bhote Kosi glacier for four different choices of the thickness regularization parameter α_h: (a) $\alpha_h = 2$, (b) $\alpha_h = 3$, (c) $\alpha_h = 6$, (d) $\alpha_h = 10$. The colour scale for all panels is identical to that of Fig. 6(a,b) in the manuscript.

The "valid arguments" paragraph will be reworded for clarity. It is not that there are entirely separate arguments for choosing each of the α_h, but that the "best" value can be chosen taking multiple factors into account (regularization cost, velocity cost, MAE, MBE, and total volume). As the L-curve is a heuristic method, any of the values $\alpha_h = 2, 3, 6, 10$ are in some sense valid, and the choice of any one of them over the others is subjective.

> 3. The biases in Figure 3 (namely: thickest-parts, flattest-parts, and low-elevation) would be made clearer if presented quantitatively. That is, it would be useful to give the readers a quantitative sense of the observed thicknesses, surface slopes, and elevations in question within the same figure. This information could potentially be visualized in a map overlay, or in a separate row of subplots.

The thickest-parts and flattest-parts constraining sets were chosen by sorting the 23 profiles by their maximum thickness and mean slope respectively, and the top 5 or 6 profiles were chosen as the constraining set. The low-elevation constraining set was chosen by selecting profiles with at least one point in the lowest 10% of the glacier's elevation range. For each constraining set, profiles intersecting with profiles already in the constraining set were also included. Figure 3 will be updated to show the ranges of observed thicknesses, surface slopes, and elevations considered in each constraining set.

While updating Figure 3, it was noted that the maximum observed thickness on Bhote Kosi was incorrectly stated later in the text as 287 m. The text will be updated to reflect the correct value, which is 292 m.

> 4. The title of the manuscript is somewhat generic and slightly undersells the contribution of this work. I suggest the authors consider changing the title to be more informative, perhaps emphasizing the study's result of deriving glacier thickness from velocity using a deep-learning-based modeling framework.

Thank you for this suggestion. The title of the manuscript will be amended to reflect the role of the neural network emulator in our methodology.

Technical corrections

> [Line 70] The comma after "ice thickness" should be removed.

> [Line 104] The comma after "glacier model" should be removed.

> [Line 126] Only the acronym "IGM" is needed here, as it was already defined in the Introduction.

> [Line 132] Only the acronym "CNN" is needed here, as it was already defined in the previous paragraph.

> [Line 229] The comma after "dataset" should be removed.

> [Line 237] The word "performing" should be "perform".

> [Figure 3 caption] To maintain consistency with other figure captions, "partitions" after (a–d) and "mean" after (e–h) should be capitalized.

Thank you for these technical corrections; the text will be updated as requested.