

Nonlinear dynamics of time-variable slope circulation

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Abstract. Bottom topography strongly constrains ocean circulation in the Arctic, and both theory and numerical modeling suggest that nonlinear flow–topography interactions influence slope-following currents. Yet, how such interactions modify the circulation response to time-variable surface forcing remains poorly understood. Using idealized shallow-water simulations of flow over a corrugated slope in a re-entrant channel, we investigate how nonlinear features arise and evolve under oscillatory forcing. We observe both a prograde flow bias (aligned in the direction of topographic Rossby wave propagation) relative to linear estimates, and retrograde flow (opposing wave propagation) exhibiting flow strength saturation once the flow reaches sufficiently strong velocities. To identify the mechanisms responsible for these behaviors, we evaluate integrated momentum budgets. Which terms appear as dynamically relevant, in addition to linear surface and bottom frictional stresses, depends on the choice of integration path: when integrated along constant-depth contours, the nonlinear dynamics appear as a cross-slope relative vorticity flux, whereas integration along straight transects instead highlights momentum flux convergence and topographic form stress. These perspectives can be unified under quasi-geostrophic scaling as describing a flux of potential vorticity (PV). This PV flux is predominantly down-slope and strongest during retrograde phases, resulting in the prograde bias. When retrograde velocities approach the arrest speed of topographic Rossby waves with wavelengths comparable to the corrugation wavelength, the flux increases sharply, halting further acceleration and producing the observed asymmetry. The interplay between forcing timescale and frictional timescale shapes how these nonlinear effects manifest: when the forcing period is comparable to the dampening timescale, the response is strongly low-pass filtered, whereas for much longer forcing periods the flow has time to adjust toward distinct prograde and retrograde states. These results show how flow–topography interactions shape time-variable slope circulation, biasing the flow toward prograde states and limiting retrograde flow strength. Such effects are likely under-represented in coarse-resolution numerical simulations, and highlight the need for improved representations of unresolved topographic interactions.

1 Introduction

Bottom topography strongly steers the large-scale circulation of the Arctic Ocean and Nordic Seas (Nøst and Isachsen, 2003; Yang et al., 2016; Broomé and Nilsson, 2016; Nilsson et al., 2024), with boundary currents closely following the continental slopes and ridges (e.g. Woodgate et al., 2001; Kallmyr et al., 2025). These boundary currents are a crucial component of the

25 Arctic Ocean circulation: they transport and redistribute the warm, saline Atlantic Water through the Arctic, thereby influencing sea-ice cover, water mass transformation and, ultimately, the properties of the modified waters that are exported back to the North Atlantic. These currents further play an role in transporting freshwater out of the Arctic and into the North Atlantic (e.g., Timmermans and Marshall, 2020; Woodgate, 2013). However, despite the central role of these slope-following boundary currents, much work remains on understanding their dynamics, including their response to both steady and time-variable
30 forcing, as well as impacts of nonlinear processes.

The present study is motivated by fundamental questions about the impact of nonlinear flow–topography interactions on wind-driven ocean gyres within the Arctic. This issue has recently been raised in analyses of both high-resolution numerical models and observations by Sjur et al. (2025) and Johnston et al. (2025). Sjur et al. (2025) examined a five-year-long (2019–2024) realistic, eddy-permitting (4 km resolution) baroclinic ocean model simulation that spans the Nordic Seas and
35 Arctic Ocean and is coupled to a sea-ice model. Their analysis focused on the time variability of the circulation around closed ocean basins in the simulations, given by

$$\Gamma = \oint_C \mathbf{u} \cdot \mathbf{t} dl, \quad (1)$$

where C is a closed depth-contour, $\mathbf{u}$ is barotropic velocity, $\mathbf{t}$ is a unit vector tangential to the depth contour in the cyclonic direction, and dl is the line segment. They further compared the circulation Γ from simulations with estimates from a simplified
40 linear theory first applied by Isachsen et al. (2003). This theory applies to time scales shorter than the baroclinic adjustment time (likely to be on the order of years at high latitudes; Spall, 2016) and can be written simply as the linear shallow-water equations integrated around a closed depth contour. Given a prescribed time-variable surface stress and assuming that dissipation occurs only via a linear bottom stress, the model yields an estimate of the circulation around such a closed contour as a low-pass-filtered and lagged response to the surface forcing. Sjur et al. (2025) found that the correlation between the linear prediction
45 and circulation diagnosed from the numerical model was impressive, as shown in Fig. 1, suggesting that much of the model-simulated response to variable surface stress can be understood using linear dynamics. However, two systematic deviations from linear theory emerged from this analysis, pointing to possible nonlinear effects:

1. The numerical model circulation exhibited a response systematically shifted towards stronger cyclonic (counter-clockwise) flow around the basins relative to linear theory.
- 50 2. This shift was larger for anticyclonic than for cyclonic circulation, suggesting a directional asymmetry in the nonlinear contributions.

Johnston et al. (2025) conducted similar analyses of Argo float observations from the Lofoten Basin, an eddy-rich region in the Nordic Seas. That study found that the cyclonic circulation around a closed isobath was nearly twice as strong as the circulation obtained from the linear model forced with observed winds. In the same study, analysis of very high-resolution simulations
55 of the same region (at a horizontal resolution of 800 m) reproduced this pronounced cyclonic offset, which was substantially stronger than in the coarser simulation analyzed in Sjur et al. (2025).

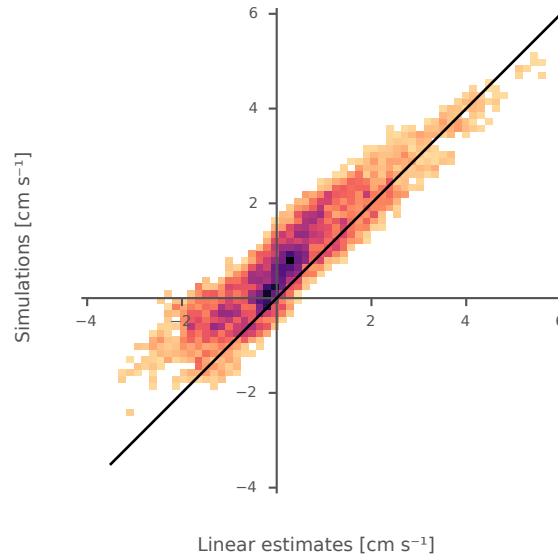


Figure 1. Two-dimensional histogram of normalized Arctic Ocean circulation around depth contours (2500, 2750, 3000, 3250, 3500 and 3750 m) from the realistic ROMS simulations of the Nordic Seas and Arctic Ocean versus the linear, idealized model (Isachsen et al., 2003) (see text). Data density increases with darker shading. Each data point represents one day from the five year long study period (2019-2024) around one contour.

In the existing literature, there are two distinct, largely disconnected perspectives on what can lead to these deviations. The first perspective is rooted in theories of geostrophic turbulence interacting with topography. These theories argue that such interactions tend to align the flow with the underlying bathymetry, a tendency sometimes referred to as the “Neptune effect” (Holloway, 1996). In particular, Bretherton and Haidvogel (1976) showed that minimization of potential enstrophy under approximate conservation of energy predicts that freely evolving, decaying turbulence will organize into cyclonic gyres within basins and anticyclonic flows over seamounts. As further argued by Salmon (1998, Ch. 6), this expected end state of freely-evolving turbulent flows can, under barotropic quasi-geostrophic (QG) conditions, be understood in terms of water columns tending to conserve their potential vorticity (PV) while also being exposed to some irreversibility, for example bottom friction. Fluid columns that are stirred back and forth over uneven topography adjust their relative vorticity to conserve PV, spinning up cyclonically as they are stretched over a basin. In the presence of irreversibility, columns lose some positive relative vorticity over deep regions and some negative vorticity over shallow regions. If these dynamics are integrated over a closed depth contour that defines an ocean basin (on the f -plane), this leads to a positive relative vorticity flux into the basin. In other words, the flux is down the mean PV gradient, tending towards PV homogenization and potential enstrophy minimization. The result of this down-topography vorticity flux is a cyclonic flow tendency around the basin. Thus, in Johnston et al. (2025) the linear model of Isachsen et al. (2003) was extended to also account for relative vorticity fluxes diagnosed from the high-resolution

numerical model of the Lofoten Basin. The vorticity flux into the studied closed depth contour was for the most part positive, and adding this flux as a forcing term to the linear model improved the agreement with the diagnosed circulation around the contour considerably.

- 75 From this perspective, the cyclonic bias observed in the eddying numerical models analyzed by both Sjur et al. (2025) and Johnston et al. (2025) could thus be tied to eddy vorticity fluxes across closed depth contours associated with a minimization of potential enstrophy. And yet, neither of these simulations described the isolated, freely evolving turbulence situation that underpins such arguments, as they were both forced by surface winds and buoyancy fluxes, as well as by flows through open lateral boundaries. Moreover, the finding by Sjur et al. (2025) that the cyclonic offset was larger during anticyclonic surface forcing is not easily explained by turbulent spin-down arguments. Finally, it should also be recognized that since topographic variations in reality extend down to meter scales, an exact constant-depth analysis is practically impossible (except in the model world). Integrations of dynamical budgets must therefore take place over some low-passed version of the actual topography. This low-pass filtering then allows interactions between the flow and topography at smaller scales to appear as forcing of the large-scale (low-passed) circulation.
- 85 The second, complementary perspective thus emerges when the dynamical balances are evaluated along smooth (low-passed) closed integration paths that cross small-scale topography rather than follow depth contours exactly. Specifically, when the depth-integrated momentum equations are evaluated along closed contours that cross variable topography, the pressure gradient term becomes a net pressure force exerted by the flow on bottom irregularities – a term known as topographic form stress. To interpret this term, it is useful to distinguish between "prograde" flow, which aligns with the intrinsic propagation direction of Rossby waves, and "retrograde" flow, which opposes it. A key point here is that topographic form stress is typically stronger for retrograde flow, where the flow can arrest Rossby waves, leading to large pressure anomalies (Brink, 1986; Holloway, 1987; Bai et al., 2021). In the Arctic Ocean, depth generally dominates over planetary vorticity in setting gradients in potential vorticity. In this setting, the prograde direction is that of topographic Rossby waves that propagate cyclonically around an ocean basin, with shallow water to their right in the Northern Hemisphere.
- 95 Topographic form stress is a retarding force, but when averaged over time-varying flows it can lead to a net cyclonic flow tendency around an ocean basin. Transient eddies are one form of time-varying flow, and Holloway (1987) argued that "Neptune" forcing of prograde mean flows is in fact due to a residual topographic form stress when averaged over eddy motions. However, time variability can also occur on larger scales, driven for example by time-variable atmospheric forcing. This point was effectively illustrated by Haidvogel and Brink (1986), who showed that barotropic flow over corrugated slopes, driven by an oscillatory along-slope surface stress with a mean value of zero, develops a time-mean residual prograde circulation along the slope. Their explanation was that while both surface and bottom stresses are directionally symmetric, the topographic form stress responds asymmetrically, exerting greater retarding stress during retrograde flow phases. The result is a weaker flow during retrograde forcing. When averaged over a forcing cycle, this manifests as a net residual flow in the prograde direction. The authors also pointed out that this result is reflected in the domain- and time-integrated momentum equation, which reduces to a balance between the form stress and the bottom frictional stress of the residual prograde flow. Brink (2010) conducted additional numerical experiments and proposed a parametrization for the residual flow as a function of cross-isobath velocity
- 105

perturbations. This acknowledges that cross-slope vorticity fluxes may be at play. Similar results of a residual prograde flow developing when flow over a corrugated slope is subject to oscillating forcing have also been derived from QG theory (Samelson and Allen, 1987) and observed in laboratory tank experiments (e.g., Zhang et al., 1996; Boyer et al., 2000).

110 As mentioned, Fig. 1 indicates that the flow response is weaker during anticyclonic than during cyclonic surface stress forcing (see also their Fig. 10) in the Arctic Ocean simulation analyzed by Sjur et al. (2025). This behavior is consistent with the arguments above that large-scale flow variability, together with an associated asymmetric topographic form stress, may be at play. Here it should be noted that if such wind-driven variability is analyzed following constant depth contours, the topographic form stress will drop out and we are left with vorticity fluxes – albeit not necessarily driven by turbulent eddy motions.

115 One of the key questions we ask is thus how the dynamics that lead to such rectification of flow appear in different diagnostic frameworks. Hydrodynamic instability and eddy motions are certainly present in the model studied by Sjur et al. (2025), but we choose here to focus on the rectification effect set up directly by time-variable atmospheric forcing. In particular, we compare integrations along depth-following versus depth-crossing closed contours. More specifically, we ask: how are vorticity fluxes and topographic form stress related when going from one perspective to the other? This relation appears to be an under-explored

120 topic in the literature.

To address this question, we will examine the flow fields and dynamical balances in idealized numerical model simulations of flow over a continental slope in a re-entrant channel, driven by oscillatory winds similar to Haidvogel and Brink (1986) and Brink (2010). The periodic channel will thus be our surrogate for the more complex geometry of a closed ocean basin. Winds in the Arctic have energy at time scales ranging from hours to years, and the scattered behavior seen in Fig. 1 reflects
125 a blend of responses to such a wide forcing spectrum. Here we will focus on two time scales only: one roughly corresponding to synoptic atmospheric time scales—which, incidentally, partially overlap with ocean eddy time scales—and another more closely associated with seasonal forcing. A reason for looking at two such different time scales is their relation to an intrinsic response scale of the integrated dynamical equations, namely the bottom friction damping time scale. One shorter forcing periods is similar to this damping time scale, whereas the other is much longer. More specifically, the linear dynamics in the
130 model of Isachsen et al. (2003) acts as a low-pass filter of the forcing, where the filter cut-off is equal to the frictional damping time scale. The expectation is thus that flows in our long-period forcing will largely pass through the low-pass filter, revealing all stages of any asymmetric response over a forcing cycle. Any asymmetric response to the shorter-period forcing, in contrast, can be expected to be somewhat smoothed out. In addition, the dynamics under long-period forcing may possibly be treated as steady, allowing for some comparison with predictions from theories of arrested topographic Rossby waves during retrograde
135 forcing.

The rest of the paper is organized as follows. In Section 2 we introduce the model configuration and forcing setup. In Section 3 we present the two momentum-budget frameworks and theoretical considerations that guide their interpretation. We also show how, under QG scaling, the two frameworks merge in that they both involve QG PV fluxes. In Section 4 we first compare linear estimates and simulations, paralleling the analysis that went into making Fig. 1 but now examined separately
140 for the two selected time scales. We then examine terms in the momentum budgets integrated separately along depth-following and depth-crossing contours, and ask whether they both can be re-interpreted as involving QG PV fluxes. Next, we consider

a case without corrugations, to further highlight the need for topographic corrugations in creating a net prograde flow along the slope. Further, we examine whether arrested-wave theory can explain the observed asymmetry. In Section 5, we discuss how our findings from this idealized study apply to more realistic settings, together with consequences for modeling, before we draw some conclusions in Section 6.

2 Numerical model

To idealize topographically steered flow under time-variable forcing, we perform a series of shallow water simulations of flow over a slope with a forcing varying sinusoidally in time. The idealized setup is similar to the approach in earlier studies (Brink, 2010; Haidvogel and Brink, 1986).

2.1 Governing equations

The shallow water equations describe the flow of a single-layer fluid with constant density, where velocity varies only in the horizontal. The horizontal velocity vector is given by $\mathbf{u} = u\mathbf{i} + v\mathbf{j}$ ($\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the x and y directions) and the total fluid depth (layer thickness) is $D = H + \eta$, where $H(x, y)$ is the fluid depth at rest and $\eta(x, y, t)$ is the free surface elevation. The governing equations are

$$\partial_t(D\mathbf{u}) + \nabla \cdot (D\mathbf{u}\mathbf{u}) + f\mathbf{k} \times (D\mathbf{u}) = -D\nabla\phi + \boldsymbol{\tau}^s - \boldsymbol{\tau}^b \quad (2)$$

and

$$\partial_t D + \nabla \cdot (D\mathbf{u}) = 0, \quad (3)$$

where f is the Coriolis parameter ($\mathbf{k}$ is the unit vector in the z direction), $\boldsymbol{\tau}^s$ and $\boldsymbol{\tau}^b$ are the surface and bottom kinematic stresses, and $\phi = g\eta$ is the dynamic pressure, where g is the gravitational acceleration. A derivative with respect to a variable x is written on the form ∂_x , so that ∂_t is a time derivative.

Depending on the diagnostic purpose, it can be convenient to rewrite the momentum equation using the identity

$$\mathbf{u} \cdot \nabla \mathbf{u} = \zeta \mathbf{k} \times \mathbf{u} + \nabla \frac{|\mathbf{u}|^2}{2}, \quad (4)$$

where

$$\zeta = \partial_x v - \partial_y u \quad (5)$$

is the relative vorticity. Using Eq. (4) and Eq. (3), we can thus rewrite Eq. (2) as

$$D\partial_t \mathbf{u} + D(\zeta + f)\mathbf{k} \times \mathbf{u} = -D\nabla \left(\phi + \frac{|\mathbf{u}|^2}{2} \right) + \boldsymbol{\tau}^s - \boldsymbol{\tau}^b. \quad (6)$$

As we will see, this formulation highlights the explicit role of vorticity fluxes.

In this study we will use a constant Coriolis parameter $f = 10^{-4} \text{ s}^{-1}$, appropriate for high-latitude regions where variations in planetary vorticity are small. The bottom frictional stress will be modeled as a linear drag,

$$170 \quad \tau^b = Ru, \quad (7)$$

where $R = 5 \times 10^{-4} \text{ m s}^{-1}$ is a constant bottom drag coefficient. This value is chosen based on typical values diagnosed in Sjur et al. (2025).

2.2 Domain geometry

We consider an idealized horizontal domain with extents of $L_x = L_y = 90 \text{ km}$, discretized using a uniform grid with resolution
 175 $\Delta x = \Delta y = 1 \text{ km}$. The domain is periodic in the x -direction. At $y = 0$ and $y = L_y$, there is a no-flux boundary condition. This configuration permits continuous circulation in the x -direction, analogous to flow around a basin.

The bottom topography consists of a large scale slope in the y -direction, with a smooth transition from a shallow shelf to a deeper basin as y increases. Superimposed are along-slope sinusoidal corrugations in the x -direction that are scaled with the y -derivative of the background profile. The full bathymetry is thus given by $H(x, y)$, and is defined as

$$180 \quad H(x, y) = H_{\text{sh}} + \frac{1}{2} (H_{\text{bs}} - H_{\text{sh}}) \left[1 + \tanh \left(\frac{\pi(y - y_c - \delta(x, y))}{W} \right) \right] \quad (8)$$

where the corrugation displacement $\delta(x, y)$ is specified as

$$\delta(x, y) = A_{\text{corr}} \sin \left(\frac{2\pi x}{\lambda} \right) \text{sech}^2 \left(\frac{\pi(y - y_c)}{W} \right). \quad (9)$$

Here, λ is the wavelength of the corrugations. Since the domain is periodic in x with length L_x the domain must contain an integer number of corrugation wavelengths, i.e. $L_x = N\lambda$ for some integer N . The slope has a width W of 30 km, corresponding
 185 to 30 grid cells at the model resolution of 1 km, so the continental slope is well resolved. Parameters relevant for the domain and bathymetry are summarized in Table 1. An illustration of the resulting bathymetry is shown in Fig. 2. While the Arctic Ocean contains basins that are substantially deeper than the 900 m basin depth used here, the idealized setup is intended to capture the essential dynamics rather than reproduce the detailed Arctic bathymetry.

In this bathymetric setup, flow in the positive x -direction is prograde, while flow in the negative x -direction is retrograde.
 190 Prograde flow corresponds here to cyclonic circulation around a basin, with shallower water to the right of the current in this Northern Hemisphere configuration.

2.3 Forcing

The system is forced by an along-channel spatially uniform surface stress that varies sinusoidally in time. It is defined as

$$\tau^s = \tau^{sx} \mathbf{i} = \tau_0 \sin \left(\frac{2\pi}{T} t \right) \mathbf{i}, \quad (10)$$

195 where T is the forcing period, and τ_0 is the maximum kinematic stress. In the base case, we set $\tau_0 = 1 \text{ cm}^4 \text{ s}^{-2}$. For a reference density of 1000 kg m^{-3} , this corresponds to a dynamic stress of 0.1 N m^{-2} , realistic for typical wind forcing.

Table 1. Standard parameters defining the model domain and bathymetry. The domain consists of a continental slope with superimposed sinusoidal corrugations. *The baseline cases use a corrugation wavelength of 45 km, while additional runs with 22.5 km and 90 km are also considered.

Symbol	Description	Value
L_x, L_y	Domain size	90km
$\Delta x, \Delta y$	Grid spacing	1km
H_{sh}	Shelf depth	0.1km
H_{bs}	Basin depth	0.9km
y_c	Slope center	45km
W	Slope width	30km
A_{corr}	Horizontal corrugation amplitude	10km
λ	Corrugation wavelength	45km*

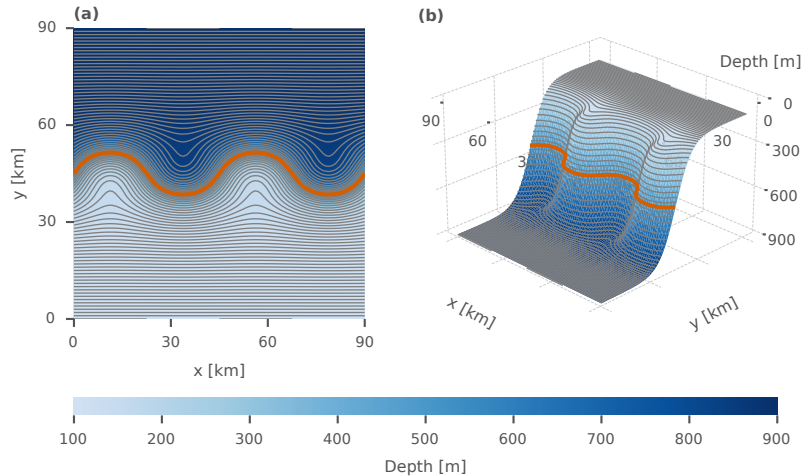


Figure 2. Model bathymetry in (a) two and (b) three dimensions. Contour lines show the constant-depth paths used in the analysis; these are not equally spaced in depth, but instead represent the mean depth associated with each y -coordinate. The mid-slope contour is highlighted.

To explore the role of forcing time scales, we contrast a short and a long forcing period. For the short-period case, we use $T = 16$ days. This value lies at the longer end of what can be considered synoptic variability, but experiments with shorter periods in our setup exhibit qualitatively similar behavior. We use 16 days as a representative fast-forcing limit, because it produces a clearer signal and facilitates interpretation of the flow response. The long-period case, with $T = 128$ days, represents slowly varying, seasonal-scale forcing. Together, these two limits bracket the essential differences between rapidly and slowly varying forcing. Animations of the flow field resulting from these two forcing periods can be found in the supplementary material.

The short-period experiment is run for 192 model days, while the long-period experiment is run for 384 model days. These durations are chosen to exceed the system's frictional adjustment time, and thus let the system lose memory of the initial state. After an initial transient, the model response becomes cycle-to-cycle repeatable, and we therefore analyze only the final full cycle of each simulation. The time step is 4 seconds, and output is saved every 3 model hours.

3 Integral balances

Which terms in the governing equation presented in Section 2.1 contribute to the along-slope flow? As indicated in the Introduction, the answer depends on how the system is diagnosed. In this section, we therefore examine the same system through two complementary frameworks: one following depth contours, the other cutting straight across the slope at constant y .

We define overbar notation to indicate path-wise averaging:

$$\overline{(\cdot)}^y \equiv \frac{1}{L_x} \int_0^{L_x} (\cdot) dx, \quad \overline{(\cdot)}^H \equiv \frac{1}{L_{C(H)}} \oint_{C(H)} (\cdot) dl. \quad (11)$$

Here, $\overline{(\cdot)}^y$ denotes an average along x at fixed y , while $\overline{(\cdot)}^H$ represents an average along a closed depth contour $C(H)$ at a fixed depth H , where $L_{C(H)}$ is the contour length and dl is the line segment.

In the theoretical derivations that follow, we adopt the rigid-lid approximation and thus neglect the contribution of the free-surface displacement η to the total layer thickness $D = H + \eta$, so that $D \approx H$. This approximation is appropriate here because the barotropic Rossby radius of deformation is much larger than the horizontal scales of the flow considered (Gill, 1982).

3.1 Momentum budget integrated along depth contours

To form an equation for the circulation around a closed depth contour, we average Eq. (6) along the contour, everywhere projecting vector components along the contour. This yields

$$H \partial_t \overline{\mathbf{u} \cdot \mathbf{t}}^H = H \overline{\zeta \mathbf{u} \cdot \mathbf{n}}^H + \overline{\boldsymbol{\tau}^s \cdot \mathbf{t}}^H - R \overline{\mathbf{u} \cdot \mathbf{t}}^H, \quad (12)$$

where $\mathbf{t}$ and $\mathbf{n}$ are the tangent and normal unit vectors to the contour, respectively. Vector $\mathbf{t}$ points along the contour in the positive x -direction, while vector $\mathbf{n}$ is defined to point towards increasing depth. In obtaining Eq. (12), we use that the projection $(\mathbf{k} \times \mathbf{u}) \cdot \mathbf{t}$ introduces the cross-contour velocity, producing the depth-integrated cross-contour vorticity flux $H \overline{\zeta \mathbf{u} \cdot \mathbf{n}}$. The associated planetary vorticity term $H f \mathbf{u} \cdot \mathbf{n}$ is proportional to the cross-contour volume flux under the f -plane approximation, and therefore vanishes under the rigid-lid assumption when integrated around the re-entrant domain. The pressure and kinetic-energy gradients do not contribute, since H can be factored out and the remaining gradient term integrates to zero around any closed contour.

The first term on the right-hand side of Eq. (12) represents the depth and contour-integrated flux of relative vorticity across the contour in the offshore direction. The second term is the average of the tangential projection of the surface stress, and the final term represents damping by linear bottom drag.

The linear version of Eq (12) is an ordinary differential equation, and can be solved to obtain an analytical model for the circulation around depth contours. Neglecting the vorticity flux term and using the integrating factor $\exp(Rt/H)$, we get

$$\overline{\mathbf{u} \cdot \mathbf{t}}_{\text{lin}}^H = \exp\left(-\frac{R}{H}t\right) \overline{\mathbf{u}_0 \cdot \mathbf{t}}^H + \int_0^t \exp\left[-\frac{R}{H}(t-t')\right] \frac{F_{\text{lin}}(t')}{H} dt', \quad (13)$$

where F_{lin} is given by the surface stress,

$$F_{\text{lin}} = \overline{\boldsymbol{\tau}^s \cdot \mathbf{t}}^H. \quad (14)$$

The first term on the right-hand side of Eq. (13) is the exponentially decaying contribution from the initial state $\overline{\mathbf{u}_0 \cdot \mathbf{t}}^H$, while the second term is a convolution between the forcing term F_{lin} and an exponential kernel. This convolution links circulation to the time-integrated forcing, and predicts the circulation to inherently carry a memory of past states over a time-scale H/R . Given our typical value for R , the factor H/R corresponds to frictional spin-down times of 2–21 days across the domain depths 100 – 900 m.

Equation (13) corresponds to the simplified linear model of time variable flow along f/H -contours of Isachsen et al. (2003), but with the additional approximation of a constant f . Eq. 13 was later used by Sjur et al. (2025) and Johnston et al. (2025) to estimate wind-driven circulation in realistic Arctic and Nordic Seas simulations and in observations. In this study, Eq. (13) will serve as a linear reference for evaluating the response. In Fig. A1, we verify this linear model against experiment with a low amplitude forcing which are expected to give a nearly linear response, and find excellent agreement. This test supports the theoretical assumptions and validates our numerical implementation.

We can further utilize Eq. (13) to estimate the maximum velocity of the linear response to the surface stress given by Eq. (10). Because the stress is purely zonal and uniform, its contour-mean projection is reduced by the geometric factor $L_x/L_{C(H)}$. Solving Eq. (13) for a periodic forcing of frequency $\omega = 2\pi/T$ gives the maximum along-contour velocity amplitude

$$u_{\text{max}}(H) = \frac{L_x}{L_{C(H)}} \frac{\tau_0}{R} \left[1 + \left(\frac{\omega H}{R} \right)^2 \right]^{-1/2}, \quad (15)$$

with derivation provided in Appendix B2.

From Eq. (15), we see that for $\omega \ll R/H$ (long forcing periods), the flow amplitude is dependent on depth only through the geometric factor, approaching $(L_x/L_{C(H)}) (\tau_0/R)$. In contrast, rapidly oscillating forcing with $\omega \gg R/H$ is expected to produce a weaker response with a stronger depth dependence.

Johnston et al. (2025) extended the linear model, and also included the nonlinear vorticity flux term from Eq. (12) as a forcing term, so that the circulation could be estimated as

$$\overline{\mathbf{u} \cdot \mathbf{t}}^H = \exp\left(-\frac{R}{H}t\right) \overline{\mathbf{u}_0 \cdot \mathbf{t}}^H + \int_0^t \exp\left[-\frac{R}{H}(t-t')\right] \frac{F(t')}{H} dt', \quad (16)$$

where

$$F = \overline{\boldsymbol{\tau}^s \cdot \mathbf{t}}^H + H \overline{\boldsymbol{\zeta} \cdot \mathbf{n}}^H. \quad (17)$$

This formulation requires that cross-contour relative vorticity fluxes $\overline{H\zeta\mathbf{u}\cdot\mathbf{n}}^H$ are diagnosed from data, or otherwise estimated. Other than this, theoretical consideration, including the role of the damping time-scale, remains the same as for the linear model in Eq. (13).

265 3.2 Momentum budget integrated along constant y transects

When integrated along straight transects, the same system is more conveniently described by a different momentum balance. Integrating Eq. (2) along the channel at fixed y , we obtain a budget for the depth-integrated x -momentum,

$$\partial_t \overline{Hu}^y = -\partial_y \overline{(Hvu)}^y - \overline{H\partial_x \phi}^y + \overline{\tau^{sx}}^y - R\overline{u}^y. \quad (18)$$

The first term on the right-hand side represents the convergence of depth-integrated flux of x -momentum. The second term
 270 measures the correlation between water depth and pressure gradients. By using the periodicity of the integration path, we can rewrite this as $-\overline{H\partial_x \phi}^y = \overline{\phi\partial_x H}^y$. The term thus also measures the correlation between the pressure and the slope of the corrugations. This term is the topographic form stress, that is the net pressure force on the corrugated bottom. Importantly, nonzero form stress requires a phase offset between pressure and the underlying bottom corrugations. Without such an offset, the integral vanishes. The third term represents the integrated wind forcing in the x -direction, and the final term accounts for
 275 linear bottom drag opposing the flow. As for the analysis along constant depth contours, the term associated with planetary vorticity (third term on left hand side of Eq. 2) becomes proportional to the cross-contour volume flux, and thus integrates to zero under the rigid-lid approximation.

While not inherently apparent from the expression $\overline{\phi\partial_x H}^y$, topographic form stress term is typically stronger for retrograde flow than for prograde flow. This asymmetry can be connected to standing Rossby waves arrested by retrograde flow, as
 280 shown for example by Bai et al. (2021), who developed a quasi-geostrophic standing-wave theory for predicting the strength of topographic form stress. We refer to Bai et al. (2021) for the full theoretical derivation, but repeat some key results here. For a mean along-slope flow U , mean depth H_0 , and corrugations given by $h \sin(kx)$, the topographic form stress is given by

$$\overline{\phi\partial_x H}^y = \frac{1}{2} \left(\frac{h}{H_0} \right)^2 \frac{RUf^2/k^2}{(U + c_k)^2 + \left(\frac{R}{kH} \right)^2}, \quad (19)$$

where c_k is the phase speed of a Rossby wave with wavenumber k . Since Rossby waves travel with shallower water to the
 285 right in the Northern Hemisphere, c_k is strictly positive. While Eq. (19) is for a steady, quasi-geostrophic system, it illustrates some key points; when retrograde flow arrests Rossby waves with a wavelength comparable to the bottom corrugation scale, corresponding to $U = -c_k$, a standing wave can form and the associated topographic form stress increases considerably in strength. Only far from resonance do we expect topographic form stress to be approximately symmetric for prograde and retrograde flow velocities. We also see Further, Eq. (19) indicates that the topographic form stress depends on the square of the
 290 topographic corrugation amplitude. We will return to these points in Sec. 4.5, where we use arrested wave theory to understand some of the simulation results.

Finally, as shown explicitly in Appendix B1, if one additionally takes the mean of Eq. (18) over time and y , the acceleration term and periodic wind stress term both vanish. Additionally, under the assumption of no momentum fluxes through the walls,

the integrated flux divergence terms also vanishes. What remains is a domain-wide balance between topographic form stress
 295 and the frictional stresses. This global balance, previously noted by Brink (2010), thus reveals that for oscillatory forcing with
 zero time mean the only way to achieve a net-zero total mass transport through the channel – or around an isolated ocean basin
 – is that it contains topographic variations that can give rise to a topographic form stress.

3.3 PV flux perspective

The two momentum budgets above emphasize different dynamics in addition to surface stress and bottom frictional stress:
 300 vorticity fluxes when the balance is evaluated along depth contours and momentum flux convergence together with topographic
 form stress when evaluated along straight transects. Such arbitrariness related to choice of integration path is unsatisfactory.
 However, as it turns out, both frameworks can be shown to reflect the same dynamics under QG assumptions, namely a down-
 gradient flux of PV across the slope.

For reference, the QG potential vorticity (QGPV) of a one-layer shallow-water system over weak topography on the f -plane,
 305 under the rigid lid assumption, is given by

$$q = \zeta + \frac{f}{H_0} h, \quad (20)$$

where the total depth has been decomposed as $H(x, y) = H_0 - h(x, y)$. Here, H_0 denotes some mean depth and h represents
 the height of variable bottom topography, assumed to be small compared to H_0 .

We now show how a flux of QGPV emerges naturally in the fixed- y momentum budget. Consider the first two terms on the
 310 right-hand side of Eq. (18), namely the convergence of depth-integrated momentum flux and the topographic form stress. We
 define their sum as $\overline{F}_q^y$, and use the identity in Eq. (4) to re-write it as

$$\overline{F}_q^y = \overline{H\zeta v}^y - \overline{H\partial_x(\phi + \frac{1}{2}|\mathbf{u}|^2)}^y. \quad (21)$$

For small Rossby numbers, geostrophic scaling implies that the pressure term dominates the kinetic energy contribution,
 $|\phi| \gg \frac{1}{2}|\mathbf{u}|^2$, allowing the latter to be neglected. Substituting the depth decomposition into Eq. (21) then yields

$$315 \quad \overline{F}_q^y = H_0 \overline{\zeta v}^y + \overline{h\zeta v}^y - \overline{h\partial_x\phi}^y. \quad (22)$$

Assuming the topographic variations are small compared to the background depth, the second term can be neglected. Finally,
 using geostrophic balance, $\partial_x\phi = fv_g$, we obtain

$$\overline{F}_q^y \approx H_0 \left(\overline{\zeta v}^y + \frac{\overline{fv_g h}^y}{H_0} \right). \quad (23)$$

Substituting $v \approx v_g$ in Eq. (23) further gives

$$320 \quad \overline{F}_q^y \approx H_0 \overline{qv_g}^y, \quad (24)$$

with q defined in Eq. (20). Thus, under QG assumptions, the combined effect of nonlinear advection and topographic form
 stress in the fixed- y momentum balance can be interpreted as a depth-integrated flux of potential vorticity across constant- y
 contours (see Held, 1983, for a treatment starting from QG).

In the depth-following framework the association with PV fluxes is trivial; since h is zero the sole contribution to the QG
 325 PV flux is the relative vorticity flux. Under QG scaling then, the momentum equation integrated along both the fixed- y and
 fixed- H paths thus take the form

$$H_0 \partial_t \overline{\mathbf{u} \cdot \mathbf{t}} = H_0 \overline{q \mathbf{u} \cdot \mathbf{n}} + \overline{\boldsymbol{\tau}^s \cdot \mathbf{t}} - R \overline{\mathbf{u} \cdot \mathbf{t}}, \quad (25)$$

where the overbar here denotes an average either along fixed- y or constant depth contours. Equation (25) corresponds to the
 Transformed Eulerian Mean (TEM) form of the depth-integrated momentum equation (see Vallis, 2017, Chapter 10, for the
 330 formulation in the multi-layer shallow-water system) with, as we emphasize here, the topographic form stress generating the
 thickness contribution to the PV flux. Further, according to Eq. (25), a flux of total PV down (up) the topographic slope, that
 is down (up) the PV gradient, either accelerates a prograde (retrograde) flow or maintains a steady prograde (retrograde) flow
 against bottom friction.

4 Simulation results

335 Having established the theoretical foundations of the two diagnostic frameworks, we now turn our attention towards analyzing
 the numerical simulations.

4.1 Comparing mid-slope circulation with linear estimates

Do the nonlinear deviations identified in realistic Arctic Ocean simulations also emerge in a stripped-down, idealized setting?
 To address this question, we apply the same diagnostic used for the realistic simulation in Fig. 1 to the idealized model
 340 experiments. Specifically, we examine scatterplots of the simulated mid-slope circulation against linear estimates given by
 Eq. (13), for both long and short forcing periods (Fig. 3). The circulation is diagnosed along the mid-slope depth contour
 (highlighted in Fig. 2), allowing direct comparison with linear theory. Perfect agreement with the linear estimates would place
 points along the 1:1 line, similar to what we see in the analysis of nearly linear simulations in Fig. A1. Note that the analytical
 estimates are integrated from the beginning of the simulation, and we discard an initial spin-up period after which the response
 345 is cycle-to-cycle repeatable and independent of the initial state.

From Fig. 3, we see that both deviations from linear theory can be reproduced, though their expression depends on forcing
 period. In particular, the long forcing period (Fig. 3a) yields a clear prograde–retrograde asymmetry, with a clear bend in the
 scatter, while the short forcing period (Fig. 3b) shows mainly a shift of the data upwards relative to the 1:1 line, i.e., a prograde
 shift. Note, however, that maximum circulation strength also differs considerably between the two experiments, a point we will
 350 return to in Sec. 4.5. In brief, only the long-period forcing generates a response whose amplitude is strong enough to arrest
 topographic Rossby waves matching the corrugation wavelength.

Further, while both forcing periods show signs of hysteresis, it is most pronounced in the long forcing case, with data
 separated into two distinct branches, one near linear theory and one displaced in the prograde direction. In contrast, the short

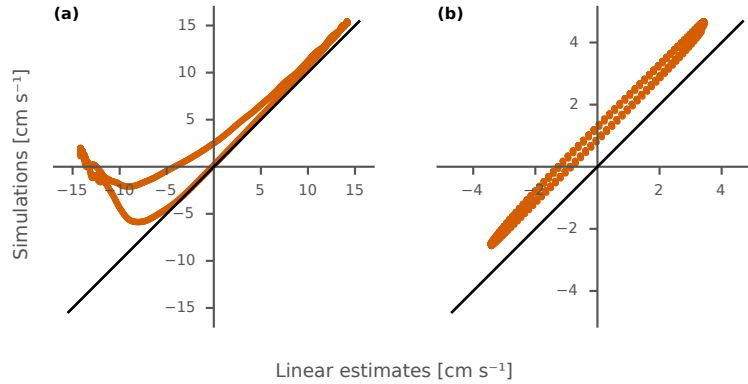


Figure 3. Scatterplots comparing circulation (normalized with contour length) along a depth contour with linear estimates, given by Eq. 13. The relevant contour is highlighted in Fig. 2. Results for a forcing period of (a) 128 days, and (b) 16 days, are shown. Black lines indicate the 1:1 reference. One forcing cycle is plotted, but the signal is repeating cycle-to-cycle after the initial spin-up period.

forcing case exhibits only a weak loop around an uniformly shifted line. These differences point to damping timescales relative
 355 to forcing timescales playing an important role in shaping the nonlinear response.

To clarify how the hysteresis patterns in Fig. 3 arise, we turn to time-series of circulation, shown in Fig. 4. For convenience
 of presentation, we shift $t = 0$ to the beginning of a retrograde forcing period. Throughout, we refer to intervals during which
 the linear estimate predicts negative circulation as the retrograde phase, and to intervals with predicted positive values as the
 prograde phase. For the long forcing period (Fig. 4a), the circulation initially follows the linear prediction as the system enters
 360 the retrograde phase. Deviation from linear theory then sets in, first gradually, then it abruptly becomes more pronounced
 (approximately 25 days into retrograde forcing), leading to a strong departure from linear estimates. At this point, the system
 enters a regime where retrograde circulation no longer strengthens, despite increasing forcing. We will refer to this behavior as
 "saturation". Animations of the flow and relative vorticity fields show that saturation coincides with the emergence of transient
 eddies (see supplementary video, or snapshot of the relative vorticity field in Fig. A2). The animated relative vorticity field
 365 exhibits a pattern that is periodic in the along-slope direction at the corrugation wavelength, with eddies forming on scales
 smaller than the topographic wavelength. This pattern is also apparent in the snapshot (Fig. A2). As the system moves into the
 prograde phase in Fig. 4a, the circulation gradually converges back toward the linear estimate. This timeseries shows that the
 lower branch of the corresponding scatterplot (Fig. 3a) stems from the prograde to retrograde transition, while the upper branch
 corresponds to the retrograde to prograde transition. This behavior is consistent with the forcing period being long compared
 370 with the frictional damping time (about 12 days at mid-slope), meaning that the system has time to approach distinct prograde
 and retrograde states as the forcing is slowly evolving.

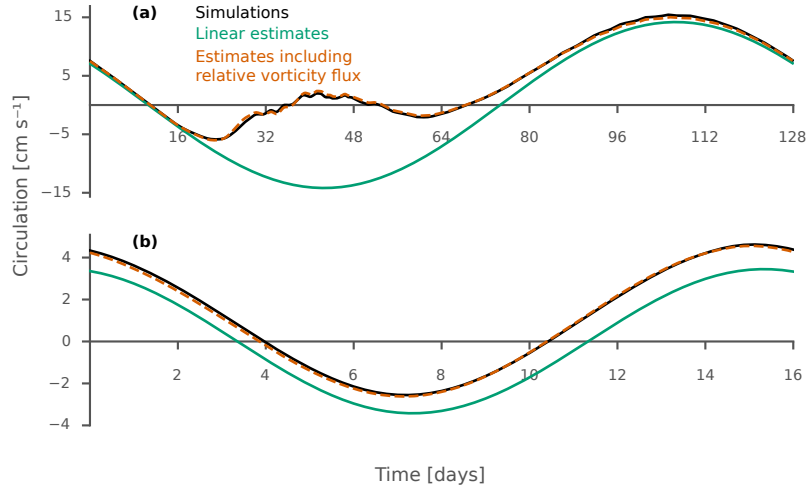


Figure 4. Time series of circulation (normalized with contour length) along a depth contour for a forcing period of (a) 128 days, and (b) 16 days. Linear estimates are shown in green, forming a sinusoidal curve around zero. Estimates including the nonlinear relative vorticity flux contribution are drawn in dashed lines, and fall on top of the circulation diagnosed from simulations. The relevant contour is highlighted in Fig. 2.

For the short forcing period, by contrast, the forcing changes on a timescale comparable to the damping time. The circulation therefore cannot fully adjust to any given forcing level before the forcing reverses. The flow behaves more in a low-pass sense, which explains the more compact hysteresis loop in Fig. 3b.

375 The nonlinear contribution relevant for depth-following circulation is a flux of relative vorticity across the depth-contour (Eq. 12). In Fig. 4, the dashed lines show estimates where diagnosed relative vorticity fluxes have been included as a forcing term (Eq. 16). For both forcing periods, including relative vorticity fluxes in the estimates almost entirely removes the discrepancy with the simulations. This agreement indicates that the observed nonlinear deviations from linear theory can indeed be described as the consequence of a single nonlinear mechanism, namely a cross-contour flux of relative vorticity. This supports
380 that Eq. (12) describes the full response of the system.

4.2 Momentum budgets in space and time

Depth-contour analysis

So far, we have focused on circulation along the mid-slope depth contour. The view is now broadened to the entire slope region, examining how the circulation and the various terms in the integrated momentum equation vary over one forcing cycle and across the slope (Fig. 5, panel a and b). As a reminder, for circulation along depth contours, the contributing terms are the
385 surface stress, bottom frictional stress, and the nonlinear relative vorticity flux term.

Panel (a) and (b) in Fig. 5, one for each forcing period, contain three components: (i) time evolution of momentum terms for the mid-slope contour, (ii) a Hovmöller diagram of circulation, with time on the horizontal axis, cross-slope position on the vertical axis, and (iii) cycle-mean profiles of momentum terms as a function of contour depth. Note that the cross-slope position is quantified by the mean y value of each depth-contour. While the top sub-panels (i) show time-series for the mid-slope depth contour only, the patterns are broadly representative of other depths as well (see Fig. A3 for full Hovmöller diagrams of the individual terms). An imbalance between the momentum terms signifies acceleration.

For both forcing periods, the depth and time-dependent structure of the circulation broadly follows the behavior predicted by linear theory (Eq. 13). In particular, the circulation lags the surface stress by an amount that increases toward deeper contours. This lag is evident, for example, in the delayed transition from prograde to retrograde circulation at greater depths. To illustrate the role of the frictional timescale H/R in setting this behavior, H/R is included as a dashed gray line in panels (ii). For the long forcing period, the lag between the circulation and the forcing scales well with this timescale at all depths (again considering, for example, how the transition from prograde to retrograde circulation scales with depth), reflecting that the flow has sufficient time to adjust to the forcing. In contrast, for the short forcing period, this scaling breaks down at depth, where the frictional timescale exceeds the forcing period. Further, the depth dependence of the response amplitude is more pronounced for the short forcing period (Fig. 5bii), consistent with Eq. (15).

Superimposed on this largely linear response, clear signs of saturation emerge for the long forcing period (Fig. 5aai). During the retrograde phase, the amplitude of retrograde circulation ceases to increase, first near the depth contour centered at $y = 40$ km, after which the saturated region gradually expands offshore. In contrast, no such saturation occurs anywhere along the slope for the short forcing period (Fig. 5bii).

The momentum diagnostics clarify the origins of the deviations from linear theory. For both long and short forcing periods, cycle-mean diagnostics (Fig. 5aiii and Fig. 5biii) show that vorticity fluxes provide a net prograde tendency across the slope, strongest at mid-slope depths. The flux is directed down the background potential vorticity gradient, from shallow toward deeper water, and is balanced by bottom friction, implying a residual prograde circulation over a forcing cycle. Compared with the long-period case, the cycle-mean flux maximum for short forcing is shifted onshore, consistent with the stronger depth dependence of the circulation response.

For the long forcing period, relative vorticity fluxes increase sharply at the onset of saturation and remain elevated throughout the retrograde phase (Fig. 5ai), capping the circulation strength. An exception from this pattern is found offshore of the slope, where relative vorticity fluxes temporarily take on retrograde values during the retrograde phase (Fig. A3). As can be seen from the supplementary animations, the abrupt rise in fluxes coincides with the emergence of transient eddies. We note that Constantinou and Young (2017) have discussed a somewhat analogous saturation phenomenon, where the emergence of transient eddies strongly reduces the mean-flow response to retrograde wind forcing. As discussed in Sec. 3.1, the system retains a memory of this elevated prograde state even after the fluxes subside, generating the hysteresis observed in the scatter plots. During strong prograde forcing, relative vorticity fluxes oscillate, producing a weak prograde tendency in that phase as well.

For the short forcing period, vorticity fluxes are weaker and alternate in sign through the cycle, but still average to a net prograde tendency. The system's memory smooths these fluctuations, leaving a residual prograde offset visible in both scatter-plots and time series. Thus, the prograde shift seen in the short forcing period is also explained by a net down-slope vorticity flux, at least when evaluated on depth-contours, even though the supplementary video shows that the flow does not develop pronounced eddies for the short forcing period.

Constant- y analysis

Having established how relative vorticity fluxes contribute to the circulation along depth contours, we now complement this with an analysis following constant- y contours (Fig. 5, panel c and d). In this framework, the nonlinear contribution is the momentum flux convergence, and topographic form stress appears explicitly. The simpler geometry also allows for area integrals to be readily calculated: panels (i) now show area-integrated momentum balances as functions of time, while crosses above panels (iii) indicate time- and area-mean values over the domain and one forcing cycle. Individual Hovmöller diagrams for each momentum term can be found in Fig. A4.

We first note that the circulation is qualitatively similar to that along depth-contours; both forcing periods show a phase lag between surface stress and circulation that increases with depth. There is also a stronger depth dependence for the short forcing period, and saturation during the retrograde phase for the long forcing period.

As expected, momentum flux convergence integrates to zero across the slope for both forcing periods (panels (ci) and (di) of Fig. 5). This result is a reminder that momentum fluxes act only to redistribute momentum along the slope, without contributing to the domain-integrated balance. The time and area-mean balance is instead between topographic form stress and bottom frictional stress (crosses above panels (ciii) and (diii) of Fig. 5), consistent with theory described in Sec. B1. This point is crucial, because it shows that although form stress does not appear in the depth-following framework, it is central for the residual transport integrated across the channel.

The time evolution of the area-integrated balances (panels (ci) and (di) of Fig. 5) highlights the role of topographic form stress in shaping the time-dependent circulation. For the long forcing period, topographic form stress is strongly prograde during the retrograde phase, with high values coinciding with saturation of the circulation, and weakly retrograde during the prograde phase. This phase-dependent asymmetry mirrors findings from steady-state analyses, where retrograde flow generates stronger topographic form stress (e.g., Bai et al., 2021), and as indicated by Eq. (19). For the short forcing period, form stress alternates in sign within each cycle and does not show the same strong phase-dependent asymmetry as for the long forcing-period. However, as evident from the time and area-mean balance, form stress still averages to a net prograde tendency, explaining the prograde bias.

Across the slope, waves or eddies redistribute momentum, giving regions of positive and negative momentum flux convergences (panels (ciii) and (diii) of Fig. 5). For the long forcing period, the time-mean momentum flux convergence is positive at mid-slope (driving prograde flow) and negative on either side (driving retrograde flow), with an additional convergence region offshore. The time-mean form stress is prograde for all y , but peaks on the top and bottom of the slope where negative momentum flux convergence occurs. These peaks likely mark regions of topographic Rossby-wave generation, which is connected

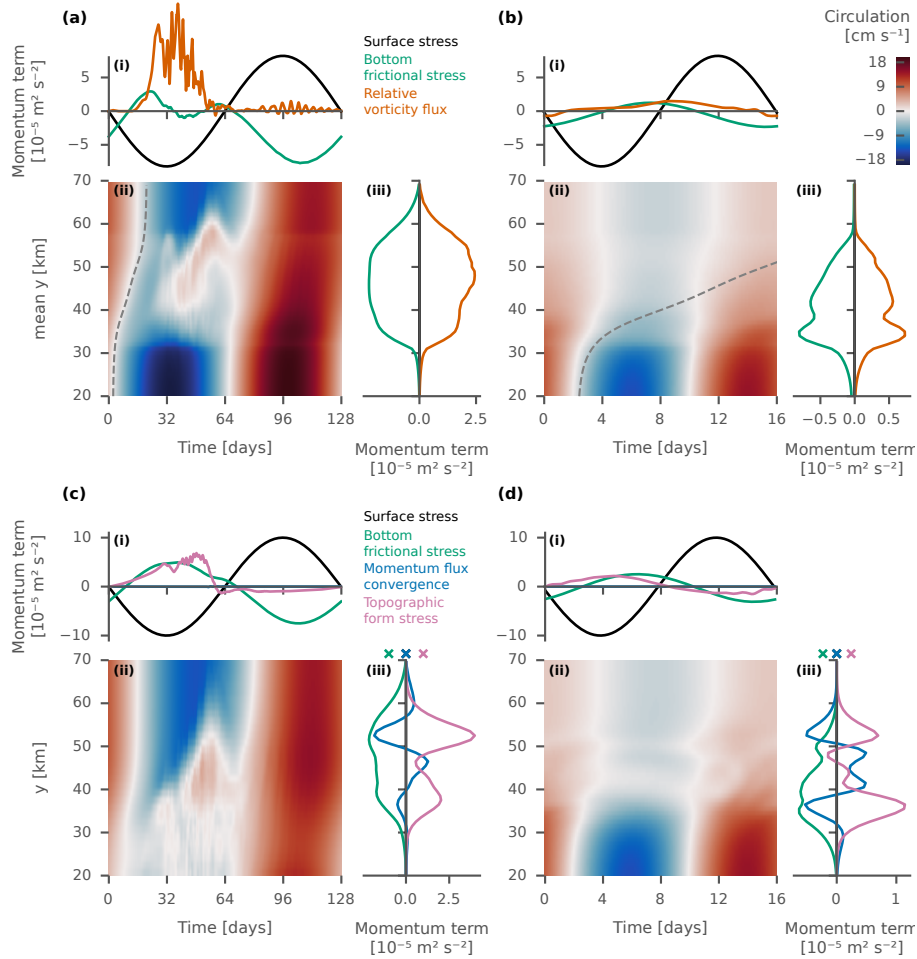


Figure 5. Momentum terms in two diagnostic frameworks: depth-following (a, b) and constant- y (c, d) contours. Results are shown for forcing periods of 128 days to the left (a, c) and 16 days to the right (b, d). Each sub-panel contains three diagnostics: (i) momentum terms as a function of time (for the mid-slope contour in a and b, and area-mean in c and d), (ii) a Hovmöller diagram of depth-integrated circulation (time on the horizontal axis, and cross-slope position on the vertical axis. Cross-slope position is quantified as the mean y -value for depth contours), and (iii) time-mean momentum terms over one forcing cycle. For panels (a, b), the momentum terms are defined in Eq. (12), and for panels (c, d), in Eq. (18). Crosses above the time-mean momentum panels in c and d indicate the area and time-mean balance. Dashed gray lines in a and b represent the damping time-scale H/R . Isobaths and depth contours are shown in Fig. 2. Note that the flat flanks of the slope have been excluded, where the response is purely linear.

455 to increased topographic form stress, (see, e.g., Held, 1983). Further, generation of Rossby waves can in turn drive negative momentum-flux convergence, as discussed in Egger (1998) and Sec. 15.1 in Vallis (2017). Over the mid-slope, positive momentum flux convergence can be connected to wave breaking and eddy potential vorticity mixing (see, e.g., Dritschel and McIntyre,

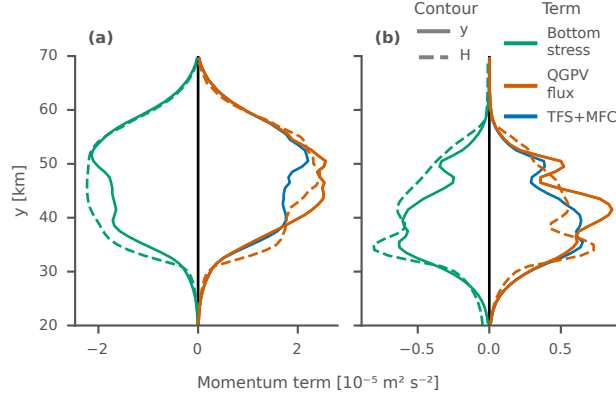


Figure 6. Time-mean quasi-geostrophic potential vorticity (QGPV) flux, bottom frictional stress and the sum of topographic form stress and the momentum flux convergence (TFS+MFC) diagnosed from the depth-following and constant- y frameworks. Results are shown for (a) a forcing period of 128 days and (b) a forcing period of 16 days. Positive QGPV flux indicates offshore transport, down the large-scale PV gradient.

2008). For the short forcing period, the overall pattern is similar, but a pronounced dip in momentum flux convergence appears over the mid-slope, coinciding with an increase in topographic form stress.

460 4.3 The unified PV flux perspective

As shown in Sec. 3.3, if QG scaling applies, both the depth-following and constant- y frameworks may be interpreted in terms of fluxes of quasi-geostrophic potential vorticity (QGPV) across the slope. Even though depth variations in our model are large, we test this interpretation by diagnosing the cross-slope QGPV flux in both frameworks and comparing its spatial structure with that of the integrated bottom frictional stress. According to theory Eq. (25), the time-averaged QGPV flux and bottom
465 frictional stress should balance. Figure 6 shows these two terms for the long (a) and short (b) forcing periods, with results from the depth-following and constant- y frameworks shown together for direct comparison. For both constant- y and constant- H contours, we calculate and plot the flux of actual QGPV, as defined in Eq. (20). As described in Sec. 3.3, in the depth-following framework the QGPV flux is entirely accomplished by the relative vorticity flux, that is, the flux plotted earlier in (a) and (b) in Fig. 5. For the constant- y framework we also plot the sum of the momentum flux convergence and the topographic form stress
470 (MFC+TFS). Under QG scaling, their sum should constitute a net flux of QGPV across the slope. There are discrepancies between the actual QGPV flux and MFC+TFS, primarily confined to the steepest part of the slope, but overall the agreement suggests that the PV-flux interpretation is a useful one.

In both forcing regimes and for both integration paths, the time-mean QGPV flux is directed offshore, down the large-scale PV gradient. It also balances the bottom frictional stress associated with the time-mean residual flow along the slope.
475 The cross-slope structure of the diagnosed QGPV flux shows strong agreement between the depth-following and constant- y frameworks, with comparable magnitudes and spatial organization across the slope. This agreement suggests that water

columns approximately maintain their PV as they are transported from a given constant- y contour to the equivalent constant- H contour, which undulates around that same y position. This, in turn, implies that the integrated thickness flux through the constant- y contour is converted, via stretching and squeezing, into a relative vorticity flux, adding to the total relative vorticity flux through the constant- H contour. In other words, some fraction of the relative vorticity flux through a constant- H contour carries the imprint of the thickness flux that passes through the associated constant- y contour. This thickness flux through the corrugations in the slope—which is a linear process—is thus essential to obtaining a net down-gradient PV flux across either integration path.

Small differences remain, which are likely related to geometric factors: a depth-following contour is generally curved and longer than a straight y -transect, leading to differences in bottom frictional stresses—and associated balancing QGPV fluxes. Nevertheless, the interpretation remains that both frameworks, integrated along constant- H and constant- y , represent a systematic down-gradient QGPV flux that maintains the residual prograde slope circulation.

4.4 The need for corrugations

So far, all simulations considered in this study have included topographic corrugations. The above results suggest that the thickness flux through these corrugations, or equivalently, the topographic form stress, is an essential ingredient in the integrated momentum budgets and in the establishment of time-mean currents. A direct theoretical implication of the absence of such along-slope depth variations, and thus of topographic form stress, is that oscillatory wind forcing with zero time mean cannot produce a domain-integrated residual flow, that is, a non-zero net mass transport through our channel (Eq. B1). To verify this theoretical constraint, we now examine a set of simulations in which the slope is smooth. These simulations share the same large-scale geometry and mean slope as the corrugated cases but lack depth variations along constant- y contours. In this configuration, analyses integrated along constant- y and along depth-following contours coincide.

We first examine the response to the standard along-slope wind forcing (Eq. 10), shown in Fig. 7a. In this case, the surface stress is spatially uniform and purely along-slope. We find that the response is well described by linear theory: the flow oscillates in time according to linear analytical estimates (Eq. 13) and produces no residual mean flow at any point along the slope. This is illustrated by the zero time-mean of all momentum terms in panel (iii).

The purely linear response may reflect the fact that the applied stress is everywhere parallel to the slope and does not induce any cross-slope motion. In the presence of topographic corrugations, however, the same along-slope forcing is observed to generate cross-slope flow. To probe whether cross-slope motion alone alters the system’s response, we therefore perform an additional experiment in which a cross-slope flow component is forced explicitly by the wind, while keeping the topography smooth.

This modified surface stress, denoted τ^{cs} , is constructed to be tangential to the bathymetric contours of the *corrugated* slope configuration used in the other experiments, while retaining the standard along-slope stress amplitude. It takes the form

$$\tau^{cs} = \tau^{sx} \mathbf{i} + \tau^{sx} \frac{\partial_x H}{\partial_y H} \mathbf{j}, \quad (26)$$

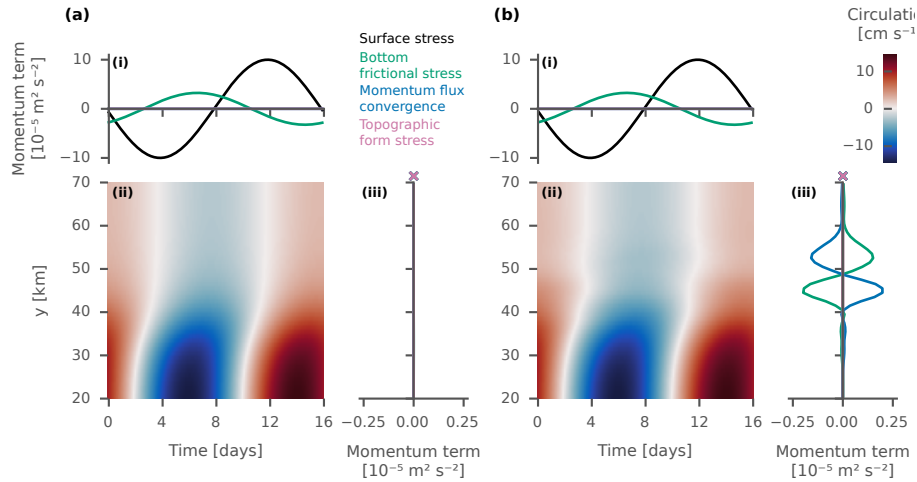


Figure 7. Momentum terms along straight depth contours for a bathymetry without corrugations. Results are shown for (a) purely along-slope wind forcing and (b) wind forcing with an imposed cross-slope component. The forcing period is 16 days in both cases. Note that relative vorticity fluxes and momentum flux convergence are identical when evaluated along straight transects without corrugations.

with H and τ^{sx} as defined in Eqs. (8) and (10). The curl of this wind stress varies on horizontal scales comparable to those of the bathymetric corrugations in the corrugated-slope experiments, representing an Ekman-driven cross-slope motion analogous to that induced by flow–topography interactions. Importantly, as in the above, the time mean is zero.

Figure 7b shows the resulting momentum balances for this experiment. Here, the circulation response takes the form of a persistent time-mean alternating jet structure over the slope (panel iii), with a prograde jet onshore and a retrograde jet offshore. This pattern is maintained by local convergence and divergence of momentum fluxes, which drive departures from the linear estimate at individual depths. The sign of these fluxes alternates across the slope, such that their contributions cancel in the domain integral. In this sense, the system permits local momentum redistribution and the formation of along-slope jets, but without corrugations there is no mechanism by which the oscillatory wind forcing can set up a time-mean net mass transport through the channel.

This flow structure can also be interpreted through the lens of potential vorticity rearrangement. Cross-slope motions driven by the time-variable wind induce local transport across the background PV gradient, leading to a redistribution of PV and the emergence of alternating along-slope jets. This pattern resembles jets associated with PV staircase formation (Dritschel and McIntyre, 2008), albeit without an explicit eddy field. Here, it is the wind forcing itself, rather than turbulent eddies, that mediates the cross-PV gradient transport.

4.5 Comparison with arrested wave theory

In earlier sections, we found that saturation strongly influences the retrograde response to long forcing periods in the presence of corrugations, whereas saturation is absent for short-period forcing. Earlier studies have shown that strong topographic form

stress occurs when topographic Rossby waves with wavelengths comparable to the bathymetric corrugations are arrested by the mean flow (Bai et al., 2021; Zhang and Lentz, 2017). Arrest corresponds to $U = -c_k$ in Eq. (19). We thus hypothesize that saturation, which is associated with strong topographic form stress (Fig. 5c), is connected to such arrest. In this section, we first focus on the response to the long forcing period and investigate the mechanism responsible for the observed saturation, before we return to why saturation is absent in the short period runs.

Wavelength dependence of saturation

If saturation is indeed connected to arrest of resonant topographic Rossby waves, the saturated flow speed is expected to approximately scale with the intrinsic phase speed of such waves with a wavelength matching the bathymetric wavelength. Because the phase speed depends on wavelength, this mechanism predicts different saturation levels for different corrugation scales. Further, the phase speed of Rossby waves also differ between the different cross-slope wave modes.

To test this idea, we compute arrest speeds of topographic Rossby waves using the linear barotropic coastal-wave model of Brink (2025). For a range of wavelengths, we calculate the phase speeds of the first three topographic Rossby wave modes and compare these with the saturation velocities diagnosed from the simulations. The coastal-wave model configuration used for the calculations is provided in the supplementary material, but in short the model is run with a free surface, closed boundaries, and the same large-scale slope as the shallow water simulations. We note that some caveats apply; the wave model assumes steady, linear dynamics and is here applied without any imposed mean along-slope velocity, whereas our simulations involve time-dependent forcing. Nevertheless, for the slowly varying cases of interest, these linear estimates provide a useful approximation to the arrest speeds.

Saturation speeds are estimated as the maximum retrograde circulation along depth contours spanning the central slope (contours with mean y -values between 40–50, corresponding to depths of roughly 360–670 m). To test whether the saturation speed follows the expected wavelength dependence, we conducted experiments with corrugation wavelengths λ of 22.5, 45, and 90 km. The theoretical arrest speeds and the diagnosed saturation speeds are shown in Fig. 8. As we see, the diagnosed saturation speeds exhibit a similar wavelength dependence as expected for the arrest of topographic Rossby waves, with increasing speed as the wavelength increases. The saturation speeds fall between mode 2 and 3, but seem to follow mode 2 better as the wavelength increases. We note that the arrest of a mode 2 wave is consistent with the findings of Zhang and Lentz (2017), who analyzed wind-driven retrograde flow over a shelf valley in terms of coastal-trapped wave characteristics. They hypothesized that the arrest of mode 2 over mode 1 was due to the lower mode being too fast to be arrested by the flow. They further argued that the disturbance introduced by bathymetry is largest mid-slope, also matching a mode 2 structure (Fig. 9a).

Cross-slope structure

To further test whether the observed saturation reflects the arrest of topographic Rossby waves, we analyze the cross-slope structure of the simulated pressure field at the onset of saturation ($t = 30$ days). The goal is to determine whether the simulated pattern resembles the mode structure expected for an arrested topographic Rossby wave. To isolate a possible mode structure, we compute residual pressure perturbations ϕ' from the model output. Obtaining a suitable ϕ' is not straightforward, since

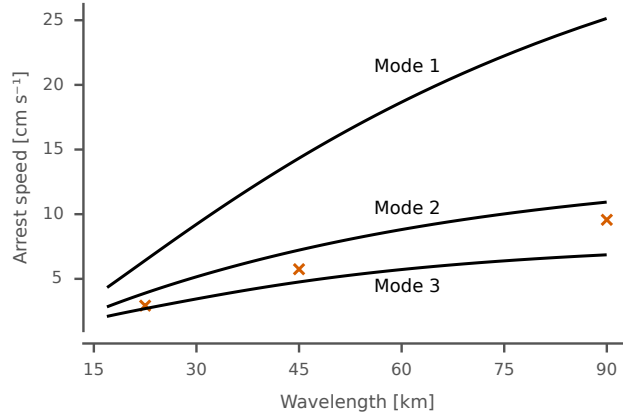


Figure 8. Theoretical arrest speed for topographic Rossby waves as a function of wavelength, and diagnosed saturation speed. Solid lines show theoretical arrest speeds for mode 1 (top), mode 2 (middle), and mode 3 (bottom), where the mode number signify the number of zero crossings in the pressure modal function. Crosses indicate the saturation speed from simulations with topographic wavelengths $\lambda = 22.5, 45$, and 90 km. Simulations were run with a forcing period of 128 days.

560 the corrugated bathymetry imprints a strong pattern on the pressure field. This imprint is not stationary, but alternates in sign between the prograde and retrograde phases, reflecting the reversal of along-slope flow. In contrast, an arrested topographic Rossby wave is expected to exist only during the retrograde phase, when the mean flow nearly matches the intrinsic prograde phase speed of the wave. Thus, the bathymetric imprint is present in both phases (with opposite sign), whereas the arrested-wave signal is confined to the retrograde phase.

565 To suppress the bathymetric imprint while retaining any trapped-wave structure, we first eliminate the large-scale along slope tilt by subtracting the cross-slope mean:

$$\tilde{\phi}(x, y, t) = \phi(x, y, t) - \bar{\phi}^y(y, t). \quad (27)$$

We then combine the retrograde state, potentially containing a trapped mode, with the prograde state (shifted by half a period T relative to the retrograde state), so that the two are in antiphase:

$$570 \quad \phi'(x, y, t) = \tilde{\phi}(x, y, t) - \tilde{\phi}(x, y, t + \frac{T}{2}). \quad (28)$$

Adding these anti-phase fields cancels the alternating topographic imprint and leaves any phase-dependent signal. We note that a small contribution from the residual flow may remain.

Figure 9 compares the residual pressure ϕ' with the theoretical cross-slope pressure structure of a 45 km mode 2 topographic Rossby wave, computed using the wave model of Brink (2025). The residual pressure ϕ' is shown at several x -positions corresponding to different phases of the same wave pattern. Note that we only plot x -positions from one of the two repeated corrugation segments. However, across the full domain, these phase profiles repeat exactly with the corrugation wavelength, indicating that the pressure field is periodic at the corrugation wavelength in the along-slope direction. The simulation ϕ'

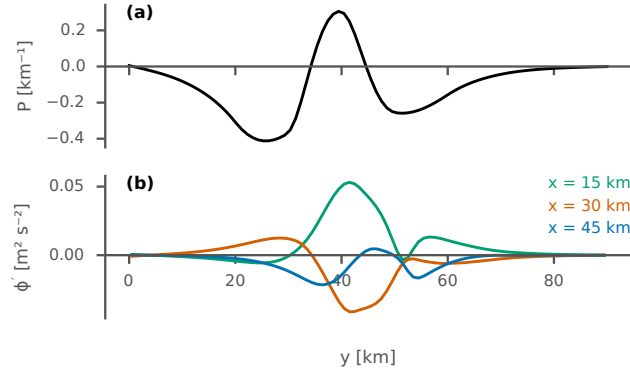


Figure 9. Panel (a) shows the theoretical cross-slope mode 2 structure for a wavelength of 45 km. Panel (b) shows the residual dynamic pressure from simulations for a wavelength of 45 km at different x -positions, used as a proxy for the mode structure. The procedure for estimating this diagnostic is described in the main text. Simulations were run with a forcing period of 128 days.

reveals a pattern similar to that of a mode 2 wave, with alternating maxima and minima across the slope that resemble the theoretical structure. The match is not exact, with curves slightly displaced in the vertical (likely reflecting contamination by the residual flow), but the extrema align well with theory. Albeit not perfect, this structural match supports the interpretation that the observed saturation arises from the arrest of topographic Rossby waves of mode 2.

Varying the corrugation amplitude

Considering the QG expression for topographic form stress given in Eq. (19), we see that QG-theory predicts that topographic form stress depends on both mean velocity U and the corrugation amplitude h . One can further ask what the role of corrugation height in maintaining saturation is, as forcing strength varies over a forcing cycle. Near the arrest condition of $U = -c_k$, the form stress is very sensitive to the mean velocity U . Here, small changes in U can balance relatively large changes in surface stress, which we assume to underlie the saturation behavior we find. Whether such saturation persists as the surface stress is increased further depends on whether the maximum attainable form stress at arrest is large enough to balance the maximum surface stress, and this in turn is expected to depend on the corrugation amplitude. Thus, in principle, the Rossby wave arrest mechanism itself should not be sensitive to corrugation amplitude; but whether topographic form stress can sustain a persistent saturation as the surface stress further increases is expected to rely on the corrugation amplitude being sufficiently large. A possible complicating factor is that different wave modes have different arrest speeds, as we see in Fig. 8. For the same forcing strength but smaller corrugation amplitude, arrest at mode 2 might not result in topographic form stress strong enough to balance the surface stress. At velocities closer to arrest conditions of a mode 1 wave, however, the bottom frictional stress is stronger, and thus weaker topographic form stress is required to achieve balance. Thus, as the corrugation height changes for the same forcing strength, saturation might not happen at mode 2, but instead at mode 1.

To investigate the role of corrugation amplitude, we conducted a series of simulations where we vary λ in Eq. (15), retaining the standard long-period surface stress. Figure 10 shows the maximum retrograde and prograde circulations along depth con-

tours spanning the central slope (mean y -values between 40–50, as in Fig. 8) as a function of vertical corrugation amplitude. For reference, the base case has a vertical corrugation amplitude of 311 m. Dashed gray lines in the figure indicate the phase speeds of mode 1 and mode 2 topographic Rossby waves.

Focusing first on the maximum retrograde flow speed, which represents the saturation velocity, we observe a stepwise dependence on corrugation amplitude. As the corrugation height decreases relative to the largest amplitude (moving from right to left in Fig. 10), the saturation velocity initially increases gradually. This behavior aligns with the expectation from Eq. (19), where a reduction in corrugation height h can be compensated by an increase in velocity U . When the corrugation amplitude is further reduced such that the maximum retrograde speed crosses the arrest condition for a mode 2 wave, the saturation velocity exhibits a pronounced increase before stabilizing at a value below the arrest speed of mode 1. We note that the flow strength is still considerably weaker for retrograde than for prograde flow for these corrugation amplitudes. At very low corrugation amplitudes, the saturation velocity abruptly transitions to a value characteristic of no saturation (i.e., comparable to the maximum prograde flow speed, shown by the orange line in Fig. 10). Maximum prograde flow speed, on the other hand, is only weakly sensitive to corrugation amplitude. There is a slight decrease in maximum prograde circulation strength with increasing corrugation height, likely connected to slightly longer integration paths as the corrugations become more prominent.

These results demonstrate that corrugation amplitude plays an important role in modulating the saturation behavior, though the relationship is not straightforward. The observed stepwise changes in saturation velocity suggest that corrugation amplitude influences which wave mode dominates the arrest condition. Specifically, our findings suggests that if topographic form stress is not sufficiently strong at arrest of mode 2 to balance the surface stress, the flow velocity will increase until arrest of mode 1 occurs. Notably, the arrest condition does not occur precisely at the phase speed of the Rossby waves, but rather slightly below it. This can at least partly be understood from Eq. (19); since topographic form stress increases rapidly as the velocity approach the arrest condition, form stress might be strong enough to balance the surface stress for velocities slightly below the arrest condition.

Varying the forcing strength

We now turn to investigate why saturation is absent in the simulations with rapidly oscillating forcing. Building on the long-period results, where saturation can be linked to the arrest of topographic Rossby waves, we hypothesize that the shorter-period forcing simply does not generate flow strong enough to arrest such waves. To test this, we perform an additional set of simulations in which the surface forcing amplitude is systematically changed, so that the short-period runs reach the theoretical arrest speeds.

The results are summarized in Fig. 11. Panel (a) shows the maximum prograde circulation along three different depth contours at the mid-slope and its flanks. Theoretical linear predictions from Eq. (15) are also included, shown as shaded areas; the bottom branch corresponds to shorter forcing periods, while the top branch corresponds to longer forcing periods. The simulated velocities scale linearly with forcing strength, consistent with linear predictions, but are consistently larger. This result further illustrates that nonlinearities result in stronger prograde flow.

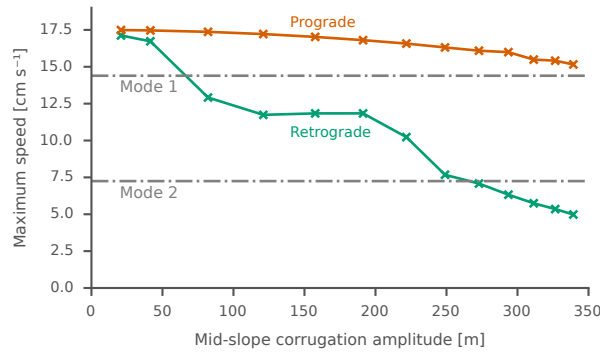


Figure 10. Maximum prograde and retrograde along-slope speed over the central slope (mean y -values between 40-50) as a function of mid-slope corrugation amplitude. Arrest speeds of a mode 1 and mode 2 topographic Rossby wave with wavelength matching the horizontal corrugation wavelength are marked by gray dashed-dotted lines.

Figure 11b shows the maximum retrograde circulation as a function of forcing strength for both short and long forcing periods. The arrest speed of a mode 2 Rossby wave for a wavelength matching the bathymetry, which can be taken as an estimate for the saturation velocity, is also included. At weak forcing, retrograde speeds increase nearly linearly, but at higher amplitudes the response levels off, with the limiting speed varying by depth. At shallow contours, the speed declines as the forcing further strengthens, whereas at deeper contours it continues to increase before eventually plateauing. This offshore progression indicates that the strongest retrograde flow shifts towards deeper water as the forcing intensifies. Both forcing periods display this behavior. However, saturation occurs at relatively low speeds for the short-period case, as can be seen from the solid lines plateauing well below the arrest speed of a mode 2 Rossby wave. The relatively low maximum retrograde speed for short forcing likely reflects that these runs do not reach a steady arrested state, and are thus not fully understood by steady-state arrested-wave theory. Nonetheless, the analysis demonstrates that retrograde flow in the short-period simulations also saturates when the forcing is sufficiently strong, in the sense that a maximum velocity exists (given, as indicated by Fig. 10, sufficiently pronounced corrugations).

It is, however, worth noting that for the short forcing period, saturation does not manifest as the pronounced bend in the scatterplot seen for the long-period case in Fig. 3a. Instead, the rapid alternation of the forcing, combined with the low-pass character of the response, leads to a response qualitatively similar to that in Fig. 3a, but with a more pronounced prograde shift (not shown).

5 Discussion

Previous realistic simulations of the Nordic Seas and Arctic Ocean have hinted that interactions between flow and topography may play an important role in shaping the circulation around basins in these high-latitude regions, as evidenced by two nonlinear features: a general prograde (cyclonic) offset of the flow strength relative to linear estimates, and an enhanced offset under

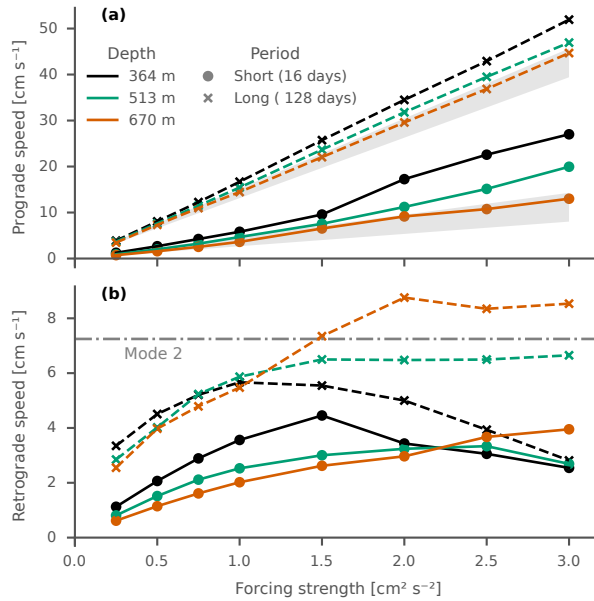


Figure 11. Maximum prograde and retrograde velocity along depth-contours as function of forcing strength. Panel (a) shows maximum prograde velocity along selected depth contours along the mid-slope and its flanks, and panel (b) shows maximum retrograde velocity along the same contours. Solid lines denote the 16 day forcing period and dashed lines the 128 day period. Analytical predictions for the maximum linear flow strength across the depth range, given by Eq. (15), are shown as gray shaded areas in panel (a); the top branch corresponds to 128 days forcing period, while the bottom branch corresponds to 16 days forcing period. The arrest speed of a mode 2 topographic Rossby wave with wavelength matching that of the bathymetry is included as a dashed-dotted line in panel (b).

655 persistent anticyclonic wind forcing compared to cyclonic wind forcing. Having now examined the underlying dynamics in an idealized setting, we can return to these features and consider what our results suggest about their expression in a more realistic setting.

660 A first consideration concerns the range of forcing timescales. In our simplified experiments, each forcing period produced a characteristic response; a short forcing period maintained a nearly steady prograde bias over the forcing period, whereas a long forcing period allowed partial relaxation toward the linear estimate and flow saturation during strong retrograde forcing. The transition to saturation was found to depend on flow strength, but with stronger forcing required to reach saturation under short-period forcing. In the realistic system, where surface stress varies continuously in time and along the slope, these behaviors would coexist. For example, periods of slow, quasi-steady prograde forcing could permit relaxation towards a linear response, while periods of strong retrograde flow could induce transient saturation. Together with along-slope variations in wind stress strength, such mixed conditions would produce a broader spectrum of circulation responses, helping to explain the spread seen in the realistic simulations (Fig. 1).

665 A second consideration relates to the topography. The slopes in our idealized experiments have a single corrugation scale, whereas natural bathymetry spans a wide range of wavelengths and amplitudes. The arrest speed of topographic Rossby waves depends on their wavelength, so different topographic wavelengths are associated with different arrest speeds, and therefore result in saturation at different flow magnitudes. Fine-scale roughness is associated with short-wavelength waves that arrest at relatively weak retrograde flow. Bathymetric features represented by longer wavelengths interact with longer waves that require
670 stronger flow before arrest occurs. Further, saturation might not be sustained as forcing strength increases beyond maximum attainable topographic form stress given the corrugation amplitude. However, since the real bathymetry of the Arctic consists of a spectrum of wavelengths, we expect arrest of topographic Rossby waves to be likely, resulting in the nonlinear behavior studied here to be present.

Beyond differences in temporal and spatial scales, a realistic setting includes several processes absent from our simplified
675 model, most notably stratification and vertical shear. Stratification is expected to alter the vertical structure of the response, tending to concentrate topographic interactions near the bottom, as found in stratified rectification studies (Brink, 2011). This suggests that the mechanism identified here may still operate, but with a depth-dependent signature.

Further, the role of irreversible processes is worth mentioning. As introduced in Sec. 1, the rectifying effect we observe in simulations can be understood in terms of water columns tending to conserve their PV while also being subject to irreversibility,
680 so that PV-perturbations accumulate. In simulations presented here, irreversibility is included through the linear bottom drag. The steady QG expression of Bai et al. (2021), reproduced in Eq. (19) here, suggests that for prograde flows topographic form stress vanishes for $R = 0$. For retrograde flows near resonance, wave amplitudes would get very large as $R \rightarrow 0$. Such waves will become unstable and turbulent, as shown by e.g. Constantinou and Young (2017). A thorough investigation of this scenario is beyond the scope of this study. But it is also worth mentioning that linear bottom drag, beyond being a parameterization, is
685 only one of many possible irreversible processes. Additional mechanisms, such as wave radiation or wave breaking, may also contribute in realistic settings.

Finally, we have focused on a system where there is a local generation of eddies by flow–topography interactions. An open question is how additional sources of eddy activity, such as those arising from baroclinic instability, would influence the circulation. We hypothesize that a cyclonic bias will be established, where momentum is removed from the system by
690 topographic form stress when eddies induce temporary retrograde flow over topographic irregularities. Related behavior has been documented in idealized studies, where an externally generated eddy field interacts with variable bathymetry, producing mean flows aligned with topography (Eurin, 1999; Miyama et al., 2018). This question is particularly interesting in the context of a future “blue Arctic,” where reduced sea-ice cover is projected to enhance eddy activity substantially (Li et al., 2024), potentially amplifying the role of eddy-topography interactions in shaping slope circulation.

695 6 Conclusions

This study examined the nonlinear dynamics of flow over corrugated slopes under time-variable wind forcing, and investigated how these interactions shape the relationship between circulation and forcing. The analysis was conducted in an idealized

shallow-water model using two complementary frameworks – momentum budgets integrated along depth contours and along straight transects – which reveal how the same dynamics manifest themselves from different perspectives.

700 Nonlinear flow–topography interactions introduce a systematic prograde bias in the circulation response, in the sense that the response is shifted towards prograde values relative to linear estimates. As the circulation strengthens, the retrograde flow reaches a critical velocity beyond which its strength saturates, while prograde flow can continue to increase, leading to a pronounced asymmetry between prograde and retrograde circulation.

The forcing period influences the persistence of this offset: rapid oscillations maintain an almost steady prograde bias, 705 whereas slower forcing allows partial relaxation toward the linear response during prograde phases, and a strong nonlinear expression during retrograde phases. This behavior reflects the inertial low-pass filtering of the systems. Saturation, however, depends primarily on flow strength; once retrograde flow arrests topographic Rossby waves with wavelengths matching the along-slope topography, further acceleration is suppressed, resulting in saturation largely independent of timescale. This mechanism relies on the presence of along slope corrugations.

710 In the depth-following framework, down-slope vorticity fluxes quantify the nonlinear contribution and provide a clean diagnostic of the mechanisms driving the circulation offset and asymmetry. In the momentum budget integrated along straight transects (which includes variations in depth), the same dynamics appear as a combination of momentum-flux convergence and topographic form stress, which represents a sink of momentum when integrated over the domain. In accordance with previous studies, form-stress closes the domain-integrated balance and clarifies why the net bias in circulation is prograde (Haidvogel 715 and Brink, 1986; Brink, 2010).

Importantly, under QG assumptions, these two perspectives can be unified as describing a flux of potential vorticity. In both frameworks, the residual time-mean prograde flow arises from a systematic flux of PV down the large-scale PV gradient: in the presence of corrugations, flow–topography interactions generate topographic form stress which, to the extent that QG scaling applies, can also be interpreted as the thickness contribution to the PV flux. When corrugations are absent, this linear 720 contribution to the PV flux vanishes and no net (domain-integrated) PV flux or residual flow is produced. QG scaling is not exactly applicable to our simulations, particularly since they have steep slopes and finite topographic corrugations. Still, the QG-based estimate captured the simulated flow–topography interactions, suggesting that it indeed provides a useful conceptual framework for relating the two diagnostic approaches. In particular, it brings topographic form stress into the context of the down-gradient PV fluxes that are central to the minimum potential enstrophy arguments of Bretherton and Haidvogel (1976). 725 As far as we know, this has not been pointed out before.

Overall, our results confirm that mesoscale topographic corrugations can induce a systematic prograde bias in time-variable slope currents and can limit anticyclonic flow strength through Rossby wave arrest. The contribution of this study lies in examining how these nonlinear effects evolve with changing forcing, how they depend on the forcing period and strength, and how they can be interpreted consistently across diagnostic frameworks as a down-gradient PV flux (under QG scaling). Future 730 work should examine how stratification, vertical shear, and externally generated eddies modify or reinforce this mechanism in more realistic settings.

Turning to implications for large-scale models, the mesoscale flow–topography interactions identified here require that the bathymetry be well-resolved. Thus, they may not be explicitly represented in coarse-resolution ocean models. In the context of Arctic circulation, this limitation becomes particularly relevant for future projections. Recent CMIP6 ensemble analyses indicate that surface stress over the Arctic Ocean is expected to intensify (Mulwijk et al., 2024), implying a stronger dynamical imprint of surface forcing on slope currents. As surface stress increases, the nonlinear mechanisms highlighted here are likely to play a larger role, meaning that coarse-resolution models may increasingly misrepresent the resulting boundary currents. Finally it should be mentioned that, while motivated by Arctic Ocean dynamics, the present analysis is relevant for any closed f/H basin in the world oceans.

Code and data availability. Simulations are made using the Oceananigans.jl software package (Wagner et al., 2025). Code and data needed to reproduce the simulations, analysis and figures are available in a Zenodo repository (Sjur, 2026).

Appendix A: Additional figures

In Fig. A1, we verify the linear theory developed in Sec. 3.1 against a nearly linear numerical simulation. We perform a model experiment with the wind-stress amplitude reduced by a factor of 10 and a forcing period of 128 days, so that the flow remains weak and laminar. We compare the simulated circulation to the linear predictions given by Eq. (13), evaluated along the mid-slope depth contour (highlighted in Fig. 2). The resulting scatterplot shows excellent agreement between the simulated circulation and the linear estimates, with data closely following the 1:1 line.

While animations of the time-evolution of relative vorticity and velocity fields in the two base-case simulations are included in the supplementary video material, a snapshot of the relative vorticity field for a highly nonlinear state is included for reference in Fig. A2.

Figures A3 and A4 show the complete spatiotemporal evolution of the momentum-budget terms. Figure A3 shows analysis on depth-contours, with terms defined in Eq. (12). Figure A4 show analysis on constant- y sections, with terms defined in Eq. (18).

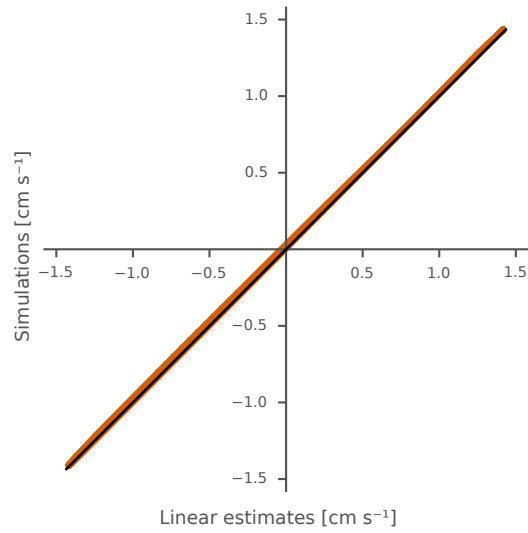


Figure A1. Scatterplot comparing circulation (normalized with contour length) along a depth contour in a nearly linear numerical simulation with linear estimates, given by Eq. (13). The relevant contour is highlighted in Fig. 2.

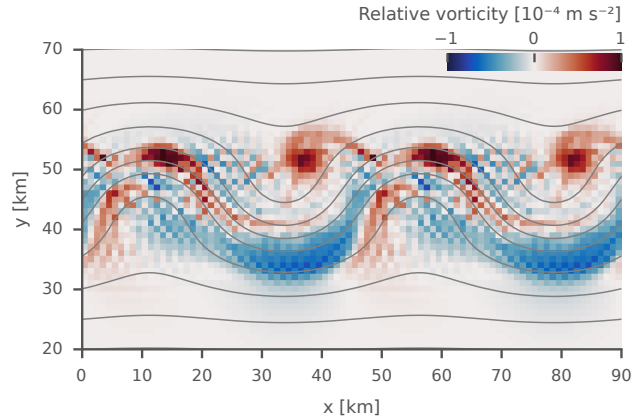


Figure A2. Snapshot of the relative vorticity field in simulations with a long forcing period (128 days). The snapshot is from a highly nonlinear state (t=32 days in Fig. 4). Gray lines indicate isobaths. Full animations of the relative vorticity field can be found in the supplementary video material.

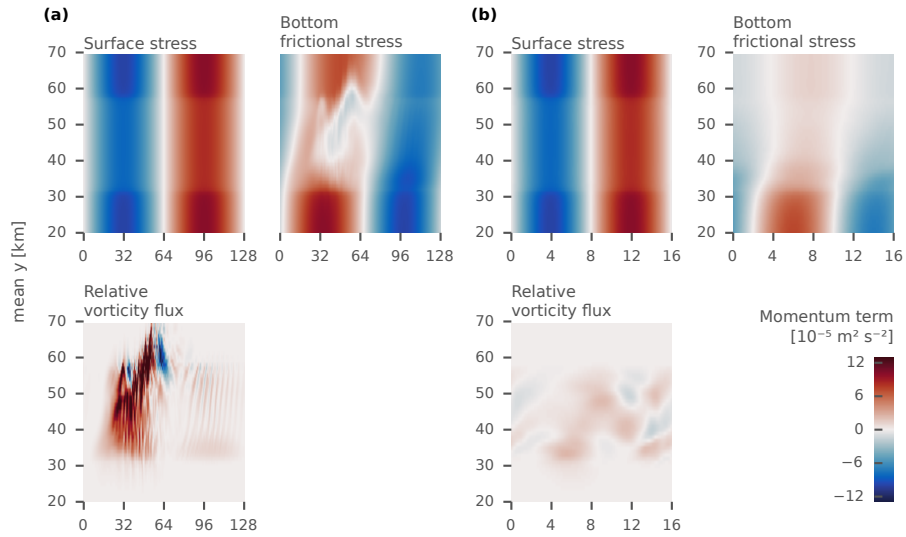


Figure A3. Hovmöller diagrams of momentum terms evaluated along depth-contours. Results for a forcing period of (a) 128 days, and (b) 16 days are shown. Each sub-figure displays surface stress, bottom frictional stress (proportional to the circulation itself), and the nonlinear relative vorticity flux (positive offshore). The terms are defined in Eq. (12). Time is shown on the x -axis and depth the y -axis. Note that depth-contours are equally spaced in y , not in depth; the corresponding contours are shown in Fig. 2.

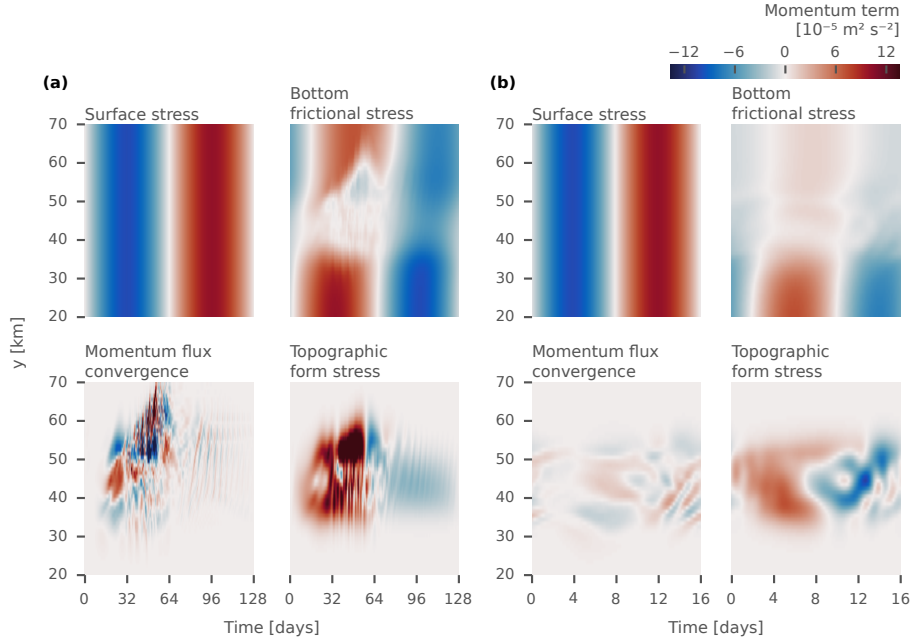


Figure A4. Hovmöller diagrams of momentum terms evaluated along constant- y sections. Results for a forcing period of (a) 128 days, and (b) 16 days are shown. Each sub-figure displays surface stress, bottom frictional stress (proportional to the along-slope circulation), the convergence of nonlinear momentum flux, and the topographic form stress. The terms are defined in Eq. (18). Time is shown on the x -axis and cross-slope position y on the y -axis.

Appendix B: Additional derivations

755 B1 Domain-integrated transport

We will extend our theoretical analysis to examine the total transport through the domain, which is the equivalent of the total transport around an ocean gyre. To do so, we integrate Eq. (18) in the y -direction, yielding:

$$\partial_t \langle Hu \rangle = -\langle H \partial_x \phi \rangle + \langle \tau^{sx} \rangle - R \langle u \rangle, \quad (\text{B1})$$

where

$$760 \quad \langle (\cdot) \rangle \equiv \frac{1}{A} \iint (\cdot) dx dy \quad (\text{B2})$$

denotes the area average over the domain. Here, A is the total area. This equation expresses the time evolution of the domain-averaged transport as dependent on three terms: the domain-averaged topographic form stress, the domain-averaged surface stress, and the domain-averaged bottom drag, all in the x -direction. We have here assumed a no-flux condition at the y -

boundaries. Notably, the nonlinear momentum flux convergence in Eq. (18) then integrates to zero, reflecting that nonlinear
765 advection only redistributes momentum within the domain.

Let us now consider the long-term evolution of the total transport, following Brink (2010). We integrate Eq. (B1) from $t = 0$ to $t = T$ and dividing by T , thereby calculating the time average. Taking the limit $T \rightarrow \infty$, and assuming that the transport remains finite, the left-hand side vanishes. Furthermore, the applied wind stress is purely periodic and has zero mean, implying that its long-time average is zero. The mean momentum balance therefore reduces to

$$770 \quad -\overline{\langle H \partial_x \phi \rangle} = \overline{R \langle u \rangle} \quad (\text{B3})$$

where the overbar denotes a long-time average. Thus, in the statistically steady state, a residual transport can be sustained by a balance between topographic form stress and bottom drag. In the absence of form stress, no residual transport can be maintained.

Given its central role in the domain-integrated budget, the apparent absence of topographic form stress in the depth-following
775 framework demands further comment. To address this, we return to the pressure term in Eq. (2) and evaluate it along depth-following contours $C(H)$, but this time integrating over the full domain. To do so, we need to express an area element in depth-following coordinates. We utilize that neighboring contours separated by a depth increment dH are spaced a normal distance $dn = dH/|\nabla H|$ apart. The area element in the depth-following coordinates is thus given by

$$dA = dl \, dn = dl \frac{dH}{|\nabla H|}, \quad (\text{B4})$$

780 where dl is a line segment along a depth contour. Integrating first along the contours and then over the full depth range yields

$$\iint_A H \nabla \phi \cdot \mathbf{t} \, dA = \int_{H_y=0}^{H_y=L_y} H \left[\oint_{C(H)} |\nabla H|^{-1} \nabla \phi \cdot \mathbf{t} \, dl \right] dH. \quad (\text{B5})$$

In the last expression, the pressure contribution is weighted by the local inverse slope $|\nabla H|^{-1}$, and the inner contour integral does not generally vanish. Thus, while topographic form stress cancels when integrated along a single depth contour, it still enters the domain-integrated balance. In practice, however, evaluating this expression is numerically challenging, as the inte-
785 gration along depth contours does not align with the discrete model grid, and great care must be taken to precisely cover the full domain.

B2 Amplitude of linear response to periodic forcing

We here derive the maximum amplitude of the solution to the linear depth following momentum equation (Eq. (12), retaining only linear terms) forced by the sinusoidal surface stress (Eq. (10)). The solution $U(H, t) = \overline{\mathbf{u} \cdot \mathbf{t}}^H$ satisfies

$$790 \quad \partial_t U + (R/H)U = F(t)/H, \quad (\text{B6})$$

where $F(t) = \overline{\tau^s \cdot \mathbf{t}}^H$. For a purely zonal and spatially uniform stress $\tau^{sx}(t)\mathbf{i}$, the projection onto the contour reduces by the geometric factor $L_x/L_{C(H)}$, so that

$$F(t) = \frac{L_x}{L_{C(H)}} \tau^{sx}(t). \quad (\text{B7})$$

Writing the surface stress in polar form, we have

$$795 \quad \tau^{sx}(t) = \tau_0 \cos(\omega t) = \Re\{\tau_0 e^{i\omega t}\}, \quad (\text{B8})$$

where $\Re\{\cdot\}$ is the real part. We seek a periodic solution of the form

$$U(H, t) = \Re\{\hat{u}(H) e^{i\omega t}\}. \quad (\text{B9})$$

Substituting Eqs. (B8) and (B9) into Eq. (B6) gives the relation

$$(i\omega + R/H) \hat{u}(H) = \frac{L_x}{L_{C(H)}} \frac{\tau_0}{H}, \quad (\text{B10})$$

800 and therefore

$$\hat{u}(H) = \frac{L_x}{L_{C(H)}} \frac{\tau_0/H}{(R/H) + i\omega}. \quad (\text{B11})$$

Taking the magnitude yields

$$u_{\max}(H) = |\hat{u}(H)| = \frac{L_x}{L_{C(H)}} \frac{\tau_0}{H \sqrt{(R/H)^2 + \omega^2}}. \quad (\text{B12})$$

Factoring out R/H , we obtain the form used in Eq. (15) in the main text,

$$805 \quad u_{\max}(H) = \frac{L_x}{L_{C(H)}} \frac{\tau_0}{R} \left[1 + \left(\frac{\omega H}{R} \right)^2 \right]^{-1/2}. \quad (\text{B13})$$

Author contributions. ALPS and PEI designed the experimental framework. ALPS implemented the experiments, carried out the simulations and analyses, and produced the figures. All authors contributed to the theoretical development and interpretation of results. ALPS prepared the original manuscript, and all authors revised and approved the final version.

Competing interests. The authors declare that they have no competing interests.

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