

Nonlinear dynamics of time-variable slope circulation

Anna Lina Petruseviciute Sjur^{1,2}, Pål Erik Isachsen^{1,2}, Johan Nilsson³, and Susan Elizabeth Allen⁴

¹University of Oslo, Oslo, Norway

²Norwegian Meteorological Institute, Oslo, Norway

³Stockholm University, Stockholm, Sweden

⁴University of British Columbia, Vancouver, Canada

Correspondence: Anna Lina Petruseviciute Sjur (anna.l.sjur@met.no)

Abstract. Bottom topography strongly constrains ocean circulation in the Arctic, and both theory and numerical modeling suggest that nonlinear flow–topography interactions influence slope-following currents. Yet, how such interactions modify the circulation response to time-variable surface forcing remains poorly understood. Using idealized shallow-water simulations of flow over a corrugated slope in a re-entrant channel, we investigate how nonlinear features arise and evolve under oscillatory forcing. We observe both a ~~persistent~~ prograde flow bias (aligned with-in the direction of topographic Rossby wave propagation) relative to linear estimates, and ~~an asymmetry in the circulation response, with~~ retrograde flow (opposing wave propagation) exhibiting ~~a saturation of flow strength~~ flow strength saturation once the flow reaches sufficiently strong velocities. To identify the mechanisms responsible for these behaviors, we evaluate integrated momentum budgets. Which terms appear as dynamically relevant, in addition to linear surface and bottom frictional stresses, depends on the choice of integration path: when integrated along constant-depth contours, the nonlinear dynamics appear as a cross-slope relative vorticity flux, whereas integration along straight transects instead highlights momentum flux convergence and topographic form stress. These perspectives can be unified under quasi-geostrophic scaling as describing a flux of potential vorticity (PV). This PV flux is predominantly down-slope and strongest during retrograde ~~flow and predominantly down-slope, explaining phases, resulting in~~ the prograde bias. When retrograde velocities approach the arrest speed of topographic Rossby waves with wavelengths comparable to the corrugation scalewavelength, the flux increases sharply, halting further acceleration and producing the observed asymmetry. The interplay between forcing timescale and frictional timescale shapes how these nonlinear effects manifest: when the forcing period is comparable to the dampening timescale, the response is strongly low-pass filtered, whereas for much longer forcing periods the flow has time to adjust toward distinct prograde and retrograde states. These results show how flow–topography interactions shape time-variable slope circulation, biasing the flow toward prograde states and limiting retrograde flow strength. Such effects are likely ~~underrepresented-under-represented~~ in coarse-resolution numerical simulations, and highlight the need for improved representations of unresolved topographic interactions.

1 Introduction

~~The Bottom topography strongly steers the~~ large-scale ~~time-mean~~ circulation of the Arctic Ocean ~~is strongly topographically steered-and Nordic Seas~~ (Nøst and Isachsen, 2003; Yang et al., 2016; Broomé and Nilsson, 2016; Nilsson et al., 2024), with

25 boundary currents closely following the continental slopes and ridges (e.g. Woodgate et al., 2001; Kallmyr et al., 2025). These boundary currents are a crucial component of the Arctic Ocean circulation: they ~~carry-transport~~ and redistribute the warm, saline Atlantic Water ~~that enters the Arctic Ocean through the Arctic~~, thereby influencing sea-ice cover, ~~water-mass transformation, and the export of modified waters~~ water mass transformation and, ultimately, the properties of the modified waters that are exported back to the North Atlantic ~~(e.g. Timmermans and Marshall, 2020; Woodgate, 2013). However, despite their central~~
 30 ~~role in Arctic Ocean circulation, the dynamics governing~~. These currents further play an role in transporting freshwater out of the Arctic and into the North Atlantic (e.g., Timmermans and Marshall, 2020; Woodgate, 2013). However, despite the central role of these slope-following boundary currents, ~~including the contribution of nonlinear flow-topography interactions, remain incompletely understood~~ much work remains on understanding their dynamics, including their response to both steady and time-variable forcing, as well as impacts of nonlinear processes.

35 ~~This~~ The present study is motivated by fundamental questions ~~about the impact of nonlinear flow-topography interactions on flow-topography interactions, which were highlighted in recent analyses of a realistic Arctic Ocean simulation presented by Sjur et al.~~ wind-driven ocean gyres within the Arctic. This issue has recently been raised in analyses of both high-resolution numerical models and observations by Sjur et al. (2025) and Johnston et al. (2025). Sjur et al. (2025) examined a five-year-long (2019–2024) realistic, eddy-permitting (2025). They examined an eddy-permitting, 4 km resolution baroclinic ocean circulation
 40 ~~model configuration (ROMS)-) baroclinic ocean model simulation~~ that spans the Nordic Seas and Arctic Ocean and is coupled to a sea-ice model. ~~It captures the major slope-following currents and provides daily output for 2014–2019. In these simulations, time-variable basin~~ Their analysis focused on the time variability of the circulation around closed ~~depth contours was strongly correlated~~ ocean basins in the simulations, given by

$$\Gamma = \oint_C \mathbf{u} \cdot \mathbf{t} dl, \quad (1)$$

45 where C is a closed depth-contour, $\mathbf{u}$ is barotropic velocity, $\mathbf{t}$ is a unit vector tangential to the depth contour in the cyclonic direction, and dl is the line segment. They further compared the circulation Γ from simulations with estimates from a simplified linear theory (Fig. 1). In this simplified theory, first developed in Isachsen et al. (2003), the time evolution of circulation around closed f/H -contours (corresponding to depth contours under the f -plane approximation) is governed solely by first applied by Isachsen et al. (2003). This theory applies to time scales shorter than the baroclinic adjustment time (likely to be on the order of years at high
 50 and can be written simply as the linear shallow-water equations integrated around a closed depth contour. Given a prescribed time-variable surface stress and assuming that dissipation occurs only via a linear bottom drag, yielding stress, the model yields an estimate of ~~circulation as a low-pass filtered~~ the circulation around such a closed contour as a low-pass-filtered and lagged response to the surface forcing. ~~The close agreement between linear theory and realistic numerical simulations thus suggested~~ Sjur et al. (2025) found that the correlation between the linear prediction and circulation diagnosed from the numerical model
 55 was impressive, as shown in Fig. 1, suggesting that much of the ~~model-simulated Arctic Ocean model-simulated~~ response to variable surface stress can be understood using linear dynamics.

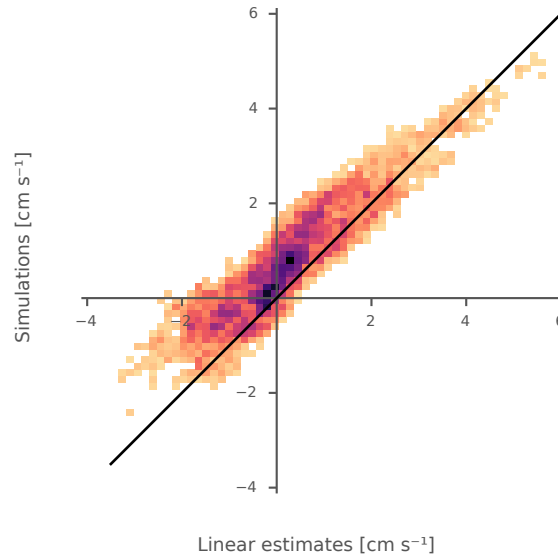


Figure 1. Two-dimensional histogram of normalized Arctic Ocean circulation around depth contours ~~in several subbasins (2500, 2750, 3000, 3250, 3500 and 3750 m)~~ from ~~a the realistic ROMS simulations of the Nordic Seas and Arctic Ocean versus the~~ linear, idealized model ~~versus realistic, numerical simulations (Isachsen et al., 2003) (see text)~~. Data density increases with darker shading. ~~Each data point represents one day from the five year long study period (2019-2024) around one contour.~~

However, two systematic deviations from linear theory emerged from this analysis, ~~as seen in Fig. 1. Specifically, pointing to possible nonlinear effects:~~

1. ~~the~~ The numerical model circulation exhibited a ~~systematically more cyclonic response than predicted by linear theory, and response systematically shifted towards stronger cyclonic (counter-clockwise) flow around the basins relative to linear theory.~~
2. ~~the deviations were larger for anti-cyclonic~~ This shift was larger for anticyclonic than for cyclonic ~~forcing, indicating circulation, suggesting~~ a directional asymmetry in the ~~response, nonlinear contributions.~~

~~While these deviations were relatively modest in the pan-Arctic simulations studied in Sjur et al. (2025), Johnston et al. (2025)~~ conducted similar analyses of Argo float observations from the Lofoten Basin, an eddy-rich region in the Nordic Seas ~~revealed cyclonic flow along the isobath that. That study found that the cyclonic circulation around a closed isobath~~ was nearly twice as strong as the flow circulation obtained from the linear model ~~, when forced with observed winds (Johnston et al., 2025)~~. In the same study, analysis of a very high-resolution ~~simulation of the region linked the offset to an eddy-driven flux of relative vorticity into the basin, which, by Gauss's theorem, implies a net cyclonic tendency. These simulations were substantially~~ higher in resolution (simulations of the same region (at a horizontal resolution of 800m) than those m) reproduced this

pronounced cyclonic offset, which was substantially stronger than in the coarser simulation analyzed in Sjur et al. (2025), and the cyclonic offset for the same region was also more pronounced. Together, these findings suggest that nonlinear interactions drive a cyclonic tendency, and that the effect is particularly pronounced in eddy-active regions such as the Lofoten Basin. The stronger offset in the higher-resolution simulation further indicates that resolving mesoscale eddies is essential for capturing this effect.

This persistent bias towards cyclonic flow in basins is consistent with theoretical models predicting that eddies. In the existing literature, there are two distinct, largely disconnected perspectives on what can lead to these deviations. The first perspective is rooted in theories of geostrophic turbulence interacting with topography drive the flow toward a state aligned, in a prograde sense. These theories argue that such interactions tend to align the flow with the underlying bathymetry—a tendency often, a tendency sometimes referred to as the “Neptune effect” (Holloway, 1996). In particular, the minimum potential enstrophy state described by Bretherton and Haidvogel (1976) and the maximum entropy framework of Salmon et al. (1976) both predict the end state of Bretherton and Haidvogel (1976) showed that minimization of potential enstrophy under approximate conservation of energy predicts that freely evolving, decaying turbulence as anticyclones over bumps and cyclonic gyres in basins. It is worth noting, however, that these theories do not account for external forcing or the mechanisms by which the system reaches this state. In will organize into cyclonic gyres within basins and anticyclonic flows over seamounts. As further argued by Salmon (1998, Ch. 6), this expected end state of freely-evolving turbulent flows can, under barotropic quasi-geostrophic (QG) conditions, be understood in terms of water columns tending to conserve their potential vorticity (PV) while also being exposed to some irreversibility, for example bottom friction. Fluid columns that are stirred back and forth over uneven topography adjust their relative vorticity to conserve PV, spinning up cyclonically as they are stretched over a basin. In the presence of irreversibility, columns lose some positive relative vorticity over deep regions and some negative vorticity over shallow regions. If these dynamics are integrated over a closed depth contour that defines an ocean basin (on the f -plane), this leads to a positive relative vorticity flux into the basin. In other words, the flux is down the mean PV gradient, tending towards PV homogenization and potential enstrophy minimization. The result of this down-topography vorticity flux is a cyclonic flow tendency around the basin. Thus, in Johnston et al. (2025) the Lofoten Basin simulations in Johnston et al. (2025), the mechanism was explicitly diagnosed as the eddy-driven relative vorticity linear model of Isachsen et al. (2003) was extended to also account for relative vorticity fluxes diagnosed from the high-resolution numerical model of the Lofoten Basin. The vorticity flux into the basin, as introduced above, studied closed depth contour was for the most part positive, and adding this flux as a forcing term to the linear model improved the agreement with the diagnosed circulation around the contour considerably.

The Lofoten study discussed above considers a regime with mainly cyclonic wind forcing, and therefore could not shed light on the second feature, namely a directional asymmetry. A systematically weaker response of slope currents to retrograde forcing is, however, consistent with literature linking such asymmetries to topographic form stress (e.g. Brink, 1986), which is the net pressure force exerted by the flow against topographic irregularities. Topographic form stress is typically stronger for retrograde flow (see i.e., Bai et al., 2021, for an idealized study of how topographic form stress depends on the forcing direction). While form stress itself is linear in the equations of motion, asymmetries in its magnitude can arise from nonlinear flow-topography interaction. In particular, when retrograde flow becomes strong enough to arrest Rossby waves matching the dominant wavelength of

~~the underlying topography (recall that retrograde flow opposes Rossby wave propagation), the resulting large-amplitude pressure gradients across topographic features considerably enhance the~~ From this perspective, the cyclonic bias observed in the eddying numerical models analyzed by both Sjur et al. (2025) and Johnston et al. (2025) could thus be tied to eddy vorticity fluxes across closed depth contours associated with a minimization of potential enstrophy. And yet, neither of these simulations described the isolated, freely evolving turbulence situation that underpins such arguments, as they were both forced by surface winds and buoyancy fluxes, as well as by flows through open lateral boundaries. Moreover, the finding by Sjur et al. (2025) that the cyclonic offset was larger during anticyclonic surface forcing is not easily explained by turbulent spin-down arguments. Finally, it should also be recognized that since topographic variations in reality extend down to meter scales, an exact constant-depth analysis is practically impossible (except in the model world). Integrations of dynamical budgets must therefore take place over some low-passed version of the actual topography. This low-pass filtering then allows interactions between the flow and topography at smaller scales to appear as forcing of the large-scale (low-passed) circulation.

The second, complementary perspective thus emerges when the dynamical balances are evaluated along smooth (low-passed) closed integration paths that cross small-scale topography rather than follow depth contours exactly. Specifically, when the depth-integrated momentum equations are evaluated along closed contours that cross variable topography, the pressure gradient term becomes a net pressure force exerted by the flow on bottom irregularities – a term known as topographic form stress. ~~As an example,~~ To interpret this term, it is useful to distinguish between "prograde" flow, which aligns with the intrinsic propagation direction of Rossby waves, and "retrograde" flow, which opposes it. A key point here is that topographic form stress ~~has been shown to nearly balance the surface wind stress along the Antarctic Circumpolar Current~~ is typically stronger for retrograde flow, where the eastward flow arrests planetary Rossby waves (e.g., Munk and Palmén, 1951; Stevens and Ivehenko, 1997; Ward and Hogg, 2004). ~~flow can arrest Rossby waves, leading to large pressure anomalies (Brink, 1986; Holloway, 1987; Bai et al., 2021). In the Arctic, by contrast, the steep continental slopes and strongly-steered boundary currents suggest that the relevant mechanism is the arrest of topographic Rossby waves. Nevertheless, the overall contribution of topographic form stress to the Arctic momentum balance remains poorly understood~~ Ocean, depth generally dominates over planetary vorticity in setting gradients in potential vorticity. In this setting, the prograde direction is that of topographic Rossby waves that propagate cyclonically around an ocean basin, with shallow water to their right in the Northern Hemisphere.

The ocean surface stress in the Arctic is determined by surface winds and sea-ice cover, and it exhibits strong short-term synoptic fluctuations superimposed on a seasonal cycle (see e.g., Fig. 5 in Sjur et al., 2025). This highly variable surface stress highlights the need to examine the effect of nonlinearities under time-dependent forcing. Nevertheless, studies using steady wind forcing can offer some insight into the mechanisms causing the asymmetry between prograde and retrograde flows. Notably, Holloway (1987) presented a series of steady-state simulations of flow over topography, spanning a range of forcing strengths in both the prograde and retrograde direction. These simulations reproduced both a cyclonic offset and a directional asymmetry in flow strength, closely resembling the nonlinear features found by Sjur et al. (2025). Further, he argued that Topographic form stress is a retarding force, but when averaged over time-varying flows it can lead to a net cyclonic flow tendency around an ocean basin. Transient eddies are one form of time-varying flow, and Holloway (1987) argued that "Neptune"

140 forcing of prograde mean flows is in fact due to a residual topographic form stress ~~provides the physical mechanism underlying both features.~~

~~A few studies have explored the role of topographic form stress under~~ when averaged over eddy motions. However, time variability can also occur on larger scales, driven for example by time-variable ~~forcing.~~ Haidvogel and Brink (1986) atmospheric forcing. This point was effectively illustrated by Haidvogel and Brink (1986), who showed that barotropic flow over corrugated slopes, driven by an oscillatory along-slope surface stress with a mean value of zero, develops a time-mean residual prograde circulation. ~~This result can be interpreted via the domain and time-integrated momentum balance: while the surface stress is along the slope. Their explanation was that while both surface and bottom stresses are~~ directionally symmetric, the topographic form stress responds asymmetrically, exerting greater retarding stress during retrograde ~~phases.~~ This asymmetry is balanced by a stronger bottom frictional stress, and thus a stronger flow, flow phases. The result is a weaker flow during retrograde forcing. When averaged over a forcing cycle, this manifests as a net residual flow in the prograde ~~phase, giving rise to a residual prograde circulation.~~ direction. The authors also pointed out that this result is reflected in the domain- and time-integrated momentum equation, which reduces to a balance between the form stress and the bottom frictional stress of the residual prograde flow. Brink (2010) conducted additional numerical experiments and proposed a parametrization for the residual flow as a function of cross-isobath velocity perturbations. This acknowledges that cross-slope vorticity fluxes may be at play. Similar results of a residual prograde flow ~~have~~ developing when flow over a corrugated slope is subject to oscillating forcing have also been derived from quasi-geostrophic QG theory (Samelson and Allen, 1987) and observed in laboratory tank experiments (e.g., Zhang et al., 1996; Boyer et al., 2000). ~~Brink (2010) conducted related numerical experiments, and proposed a parametrization for the residual flow as a function of across-isobath velocity perturbations.~~

~~Building on the ideas introduced above,~~ we seek to understand the deviations from linear theory suggested by the realistic simulation analyzed by Sjur et al. (2025), in particular the persistent prograde offset and the asymmetric response to prograde and retrograde forcing. To examine these deviations, we analyze how nonlinear contributions evolve under oscillating forcing in a set of idealized shallow-water simulations. This time-dependent perspective separates the two signatures of nonlinearity that would otherwise merge in time-mean analyses, where a time-dependent asymmetry would average to a mean offset. We also examine how the response depends on the forcing period, reflecting the broad range of surface stress variability in the Arctic Ocean. Moreover, because explanations for departures from linear theory have mainly emphasized either form stress asymmetry or eddy-mediated vorticity fluxes, we make an effort to reconcile these perspectives by applying both diagnostics to the same system. Specifically, we ask: How do nonlinearities and flow-topography interactions modify the relationship between surface forcing and circulation? To what extent does the forcing period control whether the system exhibits a persistent offset or an asymmetric response? How do two complementary diagnostic perspectives ~~As mentioned, Fig. 1 indicates that the~~ flow response is weaker during anticyclonic than during cyclonic surface stress forcing (see also their Fig. 10) in the Arctic Ocean simulation analyzed by Sjur et al. (2025). This behavior is consistent with the arguments above that large-scale flow variability, together with an associated asymmetric topographic form stress, may be at play. Here it should be noted that if such wind-driven variability is analyzed following constant depth contours, the topographic form stress will drop out and we are left with vorticity fluxes ~~one based on straight transects and the other on~~ albeit not necessarily driven by turbulent eddy motions.

175 One of the key questions we ask is thus how the dynamics that lead to such rectification of flow appear in different diagnostic frameworks. Hydrodynamic instability and eddy motions are certainly present in the model studied by Sjur et al. (2025), but we choose here to focus on the rectification effect set up directly by time-variable atmospheric forcing. In particular, we compare integrations along depth-following contours, in which topographic form stress and vorticity fluxes play distinct roles—capture and jointly explain these nonlinear effects?—versus depth-crossing closed contours. More specifically, we ask: how are vorticity fluxes and topographic form stress related when going from one perspective to the other? This relation appears to be an under-explored topic in the literature.

To address this question, we will examine the flow fields and dynamical balances in idealized numerical model simulations of flow over a continental slope in a re-entrant channel, driven by oscillatory winds similar to Haidvogel and Brink (1986) and Brink (2010). The periodic channel will thus be our surrogate for the more complex geometry of a closed ocean basin. Winds in the Arctic have energy at time scales ranging from hours to years, and the scattered behavior seen in Fig. 1 reflects a blend of responses to such a wide forcing spectrum. Here we will focus on two time scales only: one roughly corresponding to synoptic atmospheric time scales—which, incidentally, partially overlap with ocean eddy time scales—and another more closely associated with seasonal forcing. A reason for looking at two such different time scales is their relation to an intrinsic response scale of the integrated dynamical equations, namely the bottom friction damping time scale. One shorter forcing periods is similar to this damping time scale, whereas the other is much longer. More specifically, the linear dynamics in the model of Isachsen et al. (2003) acts as a low-pass filter of the forcing, where the filter cut-off is equal to the frictional damping time scale. The expectation is thus that flows in our long-period forcing will largely pass through the low-pass filter, revealing all stages of any asymmetric response over a forcing cycle. Any asymmetric response to the shorter-period forcing, in contrast, can be expected to be somewhat smoothed out. In addition, the dynamics under long-period forcing may possibly be treated as steady, allowing for some comparison with predictions from theories of arrested topographic Rossby waves during retrograde forcing.

The rest of the paper is organized as follows; in Section 2 we introduce the model configuration and forcing setup. In Section 3 we present the two momentum-budget frameworks and theoretical considerations that guide their interpretation, as well as an unifying PV perspective. We also show how, under QG scaling, the two frameworks merge in that they both involve QG PV fluxes. In Section 4, we first compare linear estimates and simulations, paralleling the analysis behind that went into making Fig. 1, before presenting analyses in both momentum-budget frameworks. Further, we examine whether arrested-wave theory can explain the observed asymmetry. We also but now examined separately for the two selected time scales. We then examine terms in the momentum budgets integrated separately along depth-following and depth-crossing contours, and ask whether they both can be re-interpreted as involving QG PV fluxes. Next, we consider a case without corrugations, to further isolate the contribution of flow-topography interactions, highlight the need for topographic corrugations in creating a net prograde flow along the slope. Further, we examine whether arrested-wave theory can explain the observed asymmetry. In Section 5, we discuss how our findings from this idealized study applies to more realistic settings, together with consequences for modeling, before we conclude draw some conclusions in Section 6, by returning to questions 1-3 above.

2 Numerical model

210 To idealize topographically steered flow under time-variable forcing, we preform a series of shallow water simulations of flow over a slope with a forcing varying sinusoidally in time. The idealized setup is similar to the approach in earlier studies (Brink, 2010; Haidvogel and Brink, 1986). ~~Simulations are implemented using the Oceananigans.jl software package (Wagner et al., 2025). The full model configuration files can be found in add reference to zenodo, once code is published~~

2.1 Governing equations

215 The shallow water equations describe the flow of a single-layer fluid with constant density, where velocity varies only in the horizontal ~~plane~~. The horizontal velocity vector is given by $\mathbf{u} = u\mathbf{i} + v\mathbf{j}$ ~~, and the~~ ($\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the x and y directions) and the total fluid depth (layer thickness) is $D = H + \eta$, where $H(x, y)$ is the fluid depth at rest and $\eta(x, y, t)$ is the free surface elevation. ~~Unit vectors in the x , y and z directions are written as $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$.~~ The governing equations are

$$\partial_t (D\mathbf{u}) + \nabla \cdot (D\mathbf{u}\mathbf{u}) + f\mathbf{k} \times (D\mathbf{u}) = -D\nabla\phi + \tau_{\underline{s}}^s - \tau_{\underline{b}}^b \quad (2)$$

220 and

$$\partial_t D + \nabla \cdot (D\mathbf{u}) = 0, \quad (3)$$

where f is the Coriolis parameter ~~, and τ_s^s and τ_b^b~~ ($\mathbf{k}$ is the unit vector in the z direction), τ_s^s and τ_b^b are the surface and bottom kinematic stresses, respectively. Additionally, and $\phi = g\eta$ is the dynamic pressure, where g is the gravitational acceleration. A derivative with respect to a variable x is written on the form ∂_x , so that ∂_t is a time derivative.

225 Depending on the diagnostic purpose, it can be convenient to rewrite the momentum equation ~~in its vector-invariant form. Using the identity using the identity~~

$$\mathbf{u} \cdot \nabla \mathbf{u} = \zeta \mathbf{k} \times \mathbf{u} + \frac{1}{2} \nabla \cdot \frac{|\mathbf{u}|^2}{2}, \quad (4)$$

where

$$\zeta = \partial_x v - \partial_y u \quad (5)$$

230 is the relative vorticity. Using Eq. (4) and Eq. (3), we can thus rewrite Eq. ~~(??) as (2) as~~

$$D\partial_t \mathbf{u} + D(\zeta + f)\mathbf{k} \times \mathbf{u} = -D\nabla \left(\phi + \frac{1}{2} \cdot \frac{|\mathbf{u}|^2}{2} \right) + \tau_{\underline{s}}^s - \tau_{\underline{b}}^b. \quad (6)$$

As we will ~~come back to see~~, this formulation highlights the explicit role of vorticity fluxes.

~~We further assume~~ In this study we will use a constant Coriolis parameter ~~$f = 10^{-4} \text{s}^{-1}$~~ $f = 10^{-4} \text{s}^{-1}$, appropriate for high-latitude regions where variations in planetary vorticity are small. The bottom ~~stress is frictional stress will be~~

235 linear drag,

$$\tau_{\underline{b}}^b = R\mathbf{u}, \quad (7)$$

where $R = 5 \times 10^{-4} \text{ m s}^{-1}$ is a constant bottom drag coefficient. This value is chosen based on typical values diagnosed in Sjur et al. (2025).

2.2 Domain geometry

240 We consider an idealized horizontal domain with extents of $L_x = L_y = 90 \text{ km}$, discretized using a uniform grid with resolution $\Delta x = \Delta y = 1 \text{ km}$. The domain is periodic in the x -direction, and bounded by solid, no-flux walls at $y = 0$ and $y = L_y$. This configuration permits continuous circulation in the x -direction, analogous to a slope current flow around a basin.

The bottom topography consists of a slope-independent of x large scale slope in the y -direction, with a smooth transition from a shallow shelf to a deeper basin as y increases. Superimposed are along-slope sinusoidal corrugations, in the x -direction that are scaled with the local steepness. The bathymetry is y -derivative of the background profile. The full bathymetry is thus given by $H(x, y)$, and is defined as

$$H(x, y) = H_{\text{sh}} + \frac{1}{2} (H_{\text{bs}} - H_{\text{sh}}) \left[1 + \tanh \left(\frac{\pi(y - y_c - \delta(x, y))}{W} \right) \right] \quad (8)$$

where the corrugation displacement $\delta(x, y)$ is specified as

$$250 \quad \delta(x, y) = A_{\text{corr}} \sin \left(\frac{2\pi x}{\lambda} \right) \text{sech}^2 \left(\frac{\pi(y - y_c)}{W} \right). \quad (9)$$

Here, λ is the wavelength of the corrugations. Since the domain is periodic in x with length L_x , the domain must contain an integer number of corrugation wavelengths, i.e. $L_x = N\lambda$ for some integer N . The slope has a width W of 30 km, corresponding to 30 grid cells at the model resolution of 1 km, so the continental slope is well resolved. Parameters relevant for the domain and bathymetry are summarized in Table 1. An illustration of the resulting bathymetry is shown in Fig. 2.

255 While the Arctic Ocean contains basins that are substantially deeper than the 900 m basin depth used here, the idealized setup is intended to capture the essential dynamics rather than reproduce the detailed Arctic bathymetry.

In this bathymetric setup, flow in the positive x -direction is prograde, while flow in the negative x -direction is retrograde. Prograde flow corresponds here to cyclonic circulation around a basin, with shallower water to the right of the current in this Northern Hemisphere configuration.

2.3 Forcing

The system is forced by an along-channel spatially uniform surface stress that varies sinusoidally in time. It is defined as

$$\tau_{\underline{s}}^s = \tau_{\underline{s}\underline{x}}^{\underline{s}\underline{x}} \hat{i} = \tau_0 \sin \left(\frac{2\pi}{T} t \right) \hat{i}, \quad (10)$$

where T is the forcing period, and τ_0 is the maximum kinematic forcing stress. In the base case, we set $\tau_0 = 10^{-4} \text{ m}^2 \text{ s}^{-2}$, following values used in comparable idealized studies (e.g. Brink, 2010) $\tau_0 = 1 \text{ cm}^4 \text{ s}^{-2}$. For a reference density of 1000 kg m^{-3} , this corresponds to a dynamic stress of 0.1 N m^{-2} , realistic for typical wind forcing.

Table 1. ~~Parameters~~Standard parameters defining the model domain and bathymetry. The domain consists of a continental slope with superimposed sinusoidal corrugations. *The baseline cases use a corrugation wavelength of 45 km, while additional runs with 22.5 km and 90 km are also considered.

Symbol	Description	Value
L_x, L_y	Domain size	90km
$\Delta x, \Delta y$	Grid spacing	1km
H_{sh}	Shelf depth	0.1km
H_{bs}	Basin depth	0.9km
y_c	Slope center	45km
W	Slope width	30km
A_{corr}	Corrugation <u>Horizontal corrugation</u> amplitude	10km
λ	Corrugation wavelength	45km*

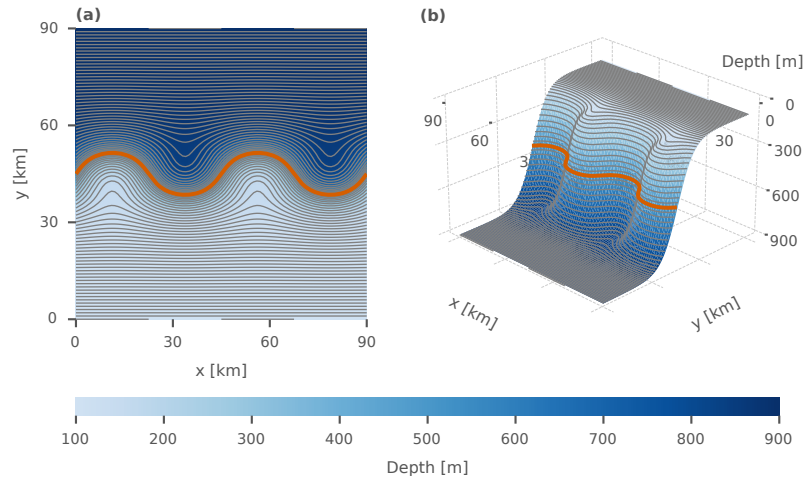


Figure 2. Model bathymetry in (a) two and (b) three dimensions. Contour lines show the constant-depth paths used in the analysis; these are not equally spaced in depth, but instead represent the mean depth associated with each y -coordinate. The mid-slope contour is highlighted.

To explore the role of forcing time scales, we contrast a short and a long forcing period. For the short-period case, we use $T = 16$ days. This value lies at the longer end of what can be considered synoptic variability, but experiments with shorter periods in our setup exhibit qualitatively similar behavior. We ~~therefore~~ use 16 days as a representative fast-forcing limit, because it produces a clearer signal and facilitates interpretation of the flow response. The long-period case, with $T = 128$ days, represents slowly varying, seasonal-scale forcing. Together, these two limits bracket the essential differences between rapidly and slowly varying forcing. [Animations of the flow field resulting from these two forcing periods can be found in the supplementary material.](#)

The short-period experiment is run for 192 model days, while the long-period experiment is run for 384 model days. These durations are chosen to exceed the system's frictional adjustment time, and thus let the system lose memory of the initial state.

275 After an initial transient, the model response becomes cycle-to-cycle repeatable, and we therefore analyze only the final ~~128~~ days-full cycle of each simulation. The time step is 4 seconds, and output is saved every 3 model hours.

3 Integral balances

Which terms in the governing equation presented in Section 2.1 contribute to the along-slope flow? As ~~it turns out~~ indicated in the Introduction, the answer depends on how the system is diagnosed. In this section, we therefore examine the same system

280 through two complementary frameworks: one following depth contours, the other cutting straight across the slope at constant y .

We define overbar notation to indicate ~~pathwise~~ path-wise averaging:

$$\overline{(\cdot)}^y \equiv \frac{1}{L_x} \int_0^{L_x} (\cdot) dx, \quad \overline{(\cdot)}^H \equiv \frac{1}{L_{C(H)}} \oint_{C(H)} (\cdot) dl. \quad (11)$$

Here, $\overline{(\cdot)}^y$ denotes an average along x at fixed y , while $\overline{(\cdot)}^H$ represents an average along a closed depth contour $C(H)$ at a

285 fixed depth H , where $L_{C(H)}$ is the contour length and dl is ~~a~~ the line segment.

In the theoretical derivations that follow, we adopt the rigid-lid approximation and thus neglect the contribution of the free-surface displacement η to the total layer thickness $D = H + \eta$, so that $D \approx H$. This approximation is appropriate here because the barotropic Rossby radius of deformation is much larger than the horizontal scales of the flow considered (Gill, 1982).

3.1 Momentum budget integrated along depth contours

290 ~~Along closed depth contours, the momentum balance takes on a particularly simple form. Integrating the momentum equation given by~~ To form an equation for the circulation around a closed depth contour, we average Eq. (??) along a closed depth contour $C(H)$ yields the contour-integrated momentum balance (6) along the contour, everywhere projecting vector components along the contour. This yields

$$H \partial_t \overline{\mathbf{u} \cdot \mathbf{t}}^H = H \zeta \overline{\mathbf{u} \cdot \mathbf{n}}^H + \overline{\boldsymbol{\tau}^s \cdot \mathbf{t}}^H - R \overline{\mathbf{u} \cdot \mathbf{t}}^H, \quad (12)$$

295 where $\mathbf{t}$ and $\mathbf{n}$ are the tangent and normal unit vectors to the contour, respectively. ~~The normal unit~~ Vector $\mathbf{t}$ points along the contour in the positive x -direction, while vector $\mathbf{n}$ is defined to point ~~in the direction of~~ towards increasing depth.

In obtaining Eq. (12), we use that the projection $(\mathbf{k} \times \mathbf{u}) \cdot \mathbf{t}$ introduces the cross-contour velocity, producing the depth-integrated cross-contour vorticity flux $H \zeta \overline{\mathbf{u} \cdot \mathbf{n}}$. The associated planetary vorticity term $H f \overline{\mathbf{u} \cdot \mathbf{n}}$ is proportional to the cross-contour volume flux under the f -plane approximation, and therefore vanishes under the rigid-lid assumption when integrated

300 around the re-entrant domain. The pressure and kinetic-energy gradients do not contribute, since H can be factored out and the remaining gradient term integrates to zero ~~along~~ around any closed contour.

The first term on the right-hand side of Eq. (12) represents the ~~depth-integrated~~ depth and contour-integrated flux of relative vorticity across the contour in the offshore direction. The second term ~~reflects the~~ is the average of the tangential projection of the surface stress, and the final term represents damping by linear bottom drag.

305 The linear version of Eq (12) is an ordinary differential equation, and can be solved to obtain an analytical model for the circulation around depth contours. Neglecting the ~~nonlinear~~ vorticity flux term and using the integrating factor $\exp(Rt/H)$, we get

$$\overline{\mathbf{u} \cdot \mathbf{t}}_{\text{lin}}^H = \exp\left(-\frac{R}{H}t\right) \overline{\mathbf{u}_0 \cdot \mathbf{t}}^H + \int_0^t \exp\left[-\frac{R}{H}(t-t')\right] \frac{F}{H} \frac{F_{\text{lin}}(t')}{H} dt', \quad (13)$$

where

310 $\underline{F = \overline{\tau_s \cdot \mathbf{t}}^H}$

~~is the forcing by surface stress.~~ F_{lin} is given by the surface stress,

$$\underline{F_{\text{lin}} = \overline{\tau^s \cdot \mathbf{t}}^H}. \quad (14)$$

The first term on the right-hand side of Eq. ~~(??)~~ (13) is the exponentially decaying contribution from the initial state $\overline{\mathbf{u}_0 \cdot \mathbf{t}}^H$, while the second term is a convolution between the forcing term $\underline{F}$ F_{lin} and an exponential kernel. ~~In other words, Eq. (??)~~
 315 ~~shows that the linear ocean response is a lagged and smoothed version of the forcing.~~ This convolution links circulation to the time-integrated forcing, and predicts the circulation to inherently carry a memory of past states over a time-scale H/R . Given our typical value for R , the factor H/R corresponds to frictional spin-down times of 2–21 days across the domain depths 100 – 900 m, ~~indicating how quickly the system adjusts to forcing.~~

Equation ~~(??)~~ (13) corresponds to the simplified linear model of time variable flow along f/H -contours of Isachsen et al.
 320 (2003), but with the additional approximation of a constant f . ~~The same diagnostic~~ Eq. 13 was later used by Sjur et al. (2025) and Johnston et al. (2025) to estimate wind-driven circulation in realistic Arctic and Nordic Seas simulations and in observations. ~~The latter study also diagnosed the nonlinear vorticity flux term in high-resolution simulations and included it as a forcing term, so that~~

$$\underline{F = \overline{\tau_s \cdot \mathbf{t}}^H + H \zeta \overline{\mathbf{u} \cdot \mathbf{n}}^H}.$$

325 In this study, ~~Eqs. (??) and (14)~~ Eq. (13) will serve as a linear reference for evaluating the response. In Fig. A1, we verify this linear model against experiment with a low amplitude forcing which are expected to give a nearly linear response, and find excellent agreement. This test supports the theoretical assumptions and validates our numerical implementation.

We can further utilize ~~the linear version of~~ Eq. (12)-(13) to estimate the maximum velocity of the linear response to the surface stress given by Eq. (10). Because the stress is purely zonal and uniform, its contour-mean projection is reduced by
 330 the geometric factor $L_x/L_{C(H)}$. Solving Eq. ~~(12)-(13)~~ (13) for a periodic forcing of frequency $\omega = 2\pi/T$ gives the maximum

along-contour velocity amplitude

$$u_{\max}(H) = \frac{L_x}{L_{C(H)}} \frac{\tau_0}{R} \left[1 + \left(\frac{\omega H}{R} \right)^2 \right]^{-1/2}, \quad (15)$$

with derivation provided in Appendix B2.

From Eq. (15), we see that for $\omega \ll R/H$ (long forcing periods), the flow amplitude is dependent on depth only through the geometric factor, approaching $(L_x/L_{C(H)}) (\tau_0/R)$. In contrast, rapidly oscillating forcing with $\omega \gg R/H$ is expected to produce a weaker response with a stronger depth dependence.

Johnston et al. (2025) extended the linear model, and also included the nonlinear vorticity flux term from Eq. (12) as a forcing term, so that the circulation could be estimated as

$$\overline{\mathbf{u} \cdot \mathbf{t}}^H = \exp \left(-\frac{R}{H} t \right) \overline{\mathbf{u}_0 \cdot \mathbf{t}}^H + \int_0^t \exp \left[-\frac{R}{H} (t - t') \right] \frac{F(t')}{H} dt', \quad (16)$$

where

$$F = \overline{\boldsymbol{\tau}^s \cdot \mathbf{t}}^H + H \overline{\zeta \mathbf{u} \cdot \mathbf{n}}^H. \quad (17)$$

This formulation requires that cross-contour relative vorticity fluxes $H \overline{\zeta \mathbf{u} \cdot \mathbf{n}}^H$ are diagnosed from data, or otherwise estimated. Other than this, theoretical consideration, including the role of the damping time-scale, remains the same as for the linear model in Eq. (13).

3.2 Momentum budget integrated along constant y transects

When integrated along straight transects, the same system is more conveniently described by a different momentum balance when integrated along straight transects. Integrating Eq. (2) along the channel at fixed y , we obtain a budget for the depth-integrated x -momentum,

$$\partial_t \overline{H u}^y = -\overline{\partial_y (H v u)}^y - \overline{H \partial_x \phi}^y + \overline{\tau^{sx}}^y - R \overline{u}^y. \quad (18)$$

The first term on the right-hand side represents the convergence of depth-integrated momentum flux of x -momentum. The second term, involving measures the correlation between water depth and pressure gradients. By using the periodicity of the integration path, we can rewrite this as $-\overline{H \partial_x \phi}^y = \overline{\phi \partial_x H}^y$. The term thus also measures the correlation between the dynamic pressure and bottom slope, corresponds to pressure and the slope of the corrugations. This term is the topographic form stress, that is the pressure work done against net pressure force on the corrugated bottom. Importantly, nonzero form stress requires a phase offset between pressure and the underlying bottom corrugations. Without such an offset, the integral vanishes. The third term represents direct the integrated wind forcing in the x direction, and the final term accounts for linear bottom drag opposing the flow. As for the analysis along constant depth contours, the term associated with planetary vorticity (third term on

left hand side of Eq. 2) becomes proportional to the cross-contour volume flux, and thus integrates to zero under the rigid-lid approximation.

360 While not inherently apparent from the expression $\overline{\phi \partial_x H^y}$, topographic form stress term is typically stronger for retrograde flow than for prograde flow. This asymmetry can be connected to standing Rossby waves arrested by retrograde flow, as shown for example by Bai et al. (2021), who developed a quasi-geostrophic standing-wave theory for predicting the strength of topographic form stress. We refer to Bai et al. (2021) for the full theoretical derivation, but repeat some key results here. For a mean along-slope flow U , mean depth H_0 , and corrugations given by $h \sin(kx)$, the topographic form stress is given by

$$365 \quad \overline{\phi \partial_x H^y} = \frac{1}{2} \left(\frac{h}{H_0} \right)^2 \frac{RU f^2 / k^2}{(U + c_k)^2 + \left(\frac{R}{kH} \right)^2}, \quad (19)$$

where c_k is the phase speed of a Rossby wave with wavenumber k . Since Rossby waves travel with shallower water to the right in the Northern Hemisphere, c_k is strictly positive. While Eq. (19) is for a steady, quasi-geostrophic system, it illustrates some key points: when retrograde flow arrests Rossby waves with a wavelength comparable to the bottom corrugation scale, corresponding to $U = -c_k$, a standing wave can form and the associated topographic form stress increases considerably in strength. Only far from resonance do we expect topographic form stress to be approximately symmetric for prograde and retrograde flow velocities. We also see Further, Eq. (19) indicates that the topographic form stress depends on the square of the topographic corrugation amplitude. We will return to these points in Sec. 4.5, where we use arrested wave theory to understand some of the simulation results.

375 Finally, as shown explicitly in Appendix B1, if one additionally takes the mean of Eq. (18) over time and y , the acceleration term and periodic wind stress term both vanish. Additionally, under the assumption of no momentum fluxes through the walls, the integrated flux divergence terms also vanishes. What remains is a domain-wide balance between topographic form stress and the frictional stresses. This global balance, previously noted by Brink (2010), thus reveals that for oscillatory forcing with zero time mean the only way to achieve a net-zero total mass transport through the channel – or around an isolated ocean basin – is that it contains topographic variations that can give rise to a topographic form stress.

380 3.3 ~~Potential vorticity~~ **PV flux** perspective

~~At first glance, the~~ The two momentum budgets ~~emphasize different mechanisms above~~ emphasize different dynamics in addition to surface ~~and bottom stresses~~ stress and bottom frictional stress: vorticity fluxes when the balance is evaluated along depth contours ~~, and form stress together with~~ and momentum flux convergence ~~together with topographic form stress~~ when evaluated along straight transects. ~~As~~ Such arbitrariness related to choice of integration path is unsatisfactory. However, as it turns out, ~~however, these terms can be interpreted both frameworks can be shown~~ to reflect the same mechanism. Specifically, under quasi-geostrophic assumptions, both frameworks can be interpreted to describe a flux of potential vorticity (PV) dynamics under QG assumptions, namely a down-gradient flux of PV across the slope.

For reference, the ~~quasi-geostrophic-QG~~ potential vorticity (QGPV) of a one-layer shallow-water system over weak topography ~~on the f -plane, under the rigid lid assumption,~~ is given by

$$390 \quad q = \zeta + \frac{fh}{H_0} \frac{f}{H_0} h, \quad (20)$$

where the total depth has been decomposed as ~~$H(x, y) = H_0(y) - h(x, y)$~~ $H(x, y) = H_0 - h(x, y)$. Here, H_0 denotes ~~the mean depth along constant y contours~~ some mean depth and h represents ~~deviations associated with topographic corrugations~~ the height of variable bottom topography, assumed to be small compared to H_0 .

We now show how ~~this a flux of~~ QGPV emerges naturally in the fixed- y momentum budget. Consider the first two terms on
395 the right-hand side of Eq. (18), namely the convergence of depth-integrated momentum flux and the topographic form stress. We define their sum as $\overline{Q^y F_q^y}$, and use the identity in Eq. (4) to re-write it as

$$\overline{F_q^y} = \overline{H \zeta v^y} - \overline{H \partial_x \left(\phi + \frac{1}{2} |\mathbf{u}|^2 \right)^y}. \quad (21)$$

For small Rossby numbers, geostrophic scaling implies that the pressure term dominates the ~~kinetic-energy~~ kinetic energy contribution, $|\phi| \gg \frac{1}{2} |\mathbf{u}|^2$, allowing the latter to be neglected. Substituting the depth decomposition into Eq. (21) then yields

$$400 \quad \overline{F_q^y} = H_0 \overline{\zeta v^y} - \overline{h \zeta v^y} - \overline{h \partial_x \phi^y}. \quad (22)$$

~~Since Assuming~~ the topographic variations are ~~assumed to be~~ small compared to the background depth, the ~~final~~ second term can be neglected. Finally, using geostrophic balance, $\partial_x \phi = f v_g$, we obtain

$$\overline{F_q^y} \approx H_0 \left(\overline{\zeta v^y} + \frac{f v_g \overline{h^y}}{H_0} \right). \quad (23)$$

~~The bracketed term~~ Substituting $v \approx v_g$ in Eq. (23) ~~has the form of a QGPV flux across constant y contours. In particular,~~
405 ~~under quasi-geostrophic scaling, the advecting velocity may be taken as geostrophic to leading order, such that the combination of terms can be written as~~ further gives

$$\overline{F_q^y} \approx H_0 \overline{q v_g^y}, \quad (24)$$

with q defined in Eq. (20). Thus, under ~~quasi-geostrophic-QG~~ assumptions, the combined effect of nonlinear advection and topographic form stress in the fixed- y momentum balance can be interpreted as a depth-integrated flux of potential vorticity
410 across constant- y contours ~~-(see Held, 1983, for a treatment starting from QG).~~

In the depth-following ~~formulation, the situation simplifies further. Along contours of constant depth, the topographic contribution fh/H_0 to the QGPV is constant by construction, and the corresponding flux is therefore proportional to the cross-contour volume transport. The topographic~~ framework the association with PV fluxes is trivial; since h is zero the sole
415 ~~contribution to the flux is thus zero under the rigid-lid assumption, and the QGPV flux reduces to a flux of relative vorticity alone. This is precisely the nonlinear term appearing in the contour-integrated momentum balance (Eq. 12). Interpreting the dynamics in terms of PV fluxes thus provides a unifying framework, showing that the~~ QG PV flux is the relative vorticity

flux. Under QG scaling then, the momentum equation integrated along both the fixed- y and depth-contour diagnostics can be understood as reflecting the same underlying PV flux. We can hence write the momentum balance in a general form as fixed- H paths thus take the form

$$420 \quad H_0 \partial_t \overline{\mathbf{u} \cdot \mathbf{t}} = H_0 \overline{q \mathbf{u} \cdot \mathbf{n}} + \overline{\boldsymbol{\tau}^s \cdot \mathbf{t}} - R \overline{\mathbf{u} \cdot \mathbf{t}}, \quad (25)$$

where the overbar [here](#) denotes an average either along fixed- y or constant depth contours. Further, although topographic effects do not appear explicitly in the depth-following formulation, their influence on the flow is included implicitly in the relative vorticity flux through the conservation of q . Equation (25) corresponds to the Transformed Eulerian Mean (TEM) form of the depth-integrated momentum equation (see Vallis, 2017, Chapter 10, for the formulation in the multi-layer shallow-water system) with, as we emphasize here, the topographic form stress generating the thickness contribution to the PV flux. Further, according to Eq. (25), a flux of total PV down (up) the topographic slope, that is down (up) the PV gradient, either accelerates a prograde (retrograde) flow or maintains a steady prograde (retrograde) flow against bottom friction.

We emphasize that this PV-based interpretation relies on quasi-geostrophic assumptions, including small Rossby number and weak topographic slopes. While not exact, it provides a useful conceptual framework for relating the two diagnostic approaches.

3.4 Domain-integrated transport

We will extend our theoretical analysis to examine the total transport through the domain. To do so, we integrate Eq. (18) in the y -direction, yielding:

$$\partial_t \langle H u \rangle = - \langle H \partial_x \phi \rangle + \langle \tau_{sx} \rangle - R \langle u \rangle,$$

where

$$\langle (\cdot) \rangle \equiv \frac{1}{A} \iint (\cdot) dx dy$$

denotes the area average over the domain. Here, A is the total area. This equation expresses the time evolution of the domain-averaged transport as dependent on three terms: the domain-averaged topographic form stress, the domain-averaged surface stress, and the domain-averaged bottom drag, all in the x -direction. We have here utilized the no-flux condition at the y -boundaries. Notably, the nonlinear momentum flux convergence in Eq. (18) then integrates to zero, reflecting that nonlinear advection only redistributes momentum within the domain.

Let's now consider the evolution of the total transport over a forcing period T , following Brink (2010). If the surface stress integrates to zero over T , and there is no slow drift so that $\partial_t \langle H u \rangle$ also integrates to zero, then the residual balance is solely between bottom drag, proportional to a residual flow, and topographic form stress. In other words, form stress is the mechanism that can sustain a residual transport.

Given its central role in the domain-integrated budget, the apparent absence of topographic form stress in the depth-following framework demands further comment. To address this, we return to the pressure term in Eq. (??) and evaluate it along

depth-following contours $C(H)$, but this time integrating over the full domain. To do so, we need to express an area element in depth-following coordinates. We utilize that neighboring contours separated by a depth increment dH are spaced a normal distance $dn = dH/|\nabla H|$ apart. The area element in the depth-following coordinates is thus given by

$$dA = dl dn = dl \frac{dH}{|\nabla H|},$$

where dl is a line segment along a depth contour. Integrating first along the contours and then over the full depth range yields

$$\iint_A H \nabla \phi \cdot \mathbf{t} dA = \int_{H_y=0}^{H_y=L_y} H \left[\oint_{C(H)} |\nabla H|^{-1} \nabla \phi \cdot \mathbf{t} dl \right] dH.$$

In the last expression, the pressure contribution is weighted by the local inverse slope $|\nabla H|^{-1}$, and the inner contour integral does not generally vanish. Thus, while topographic form stress cancels when integrated along a single depth contour, it still enters the domain-integrated balance. In practice, however, evaluating this expression is numerically challenging, as the integration along depth contours does not align with the discrete model grid, and great care must be taken to precisely cover the full domain.

4 Simulation results

Having established the theoretical foundations of the two diagnostic frameworks, we now turn our attention towards analyzing the numerical simulations.

4.1 Comparing mid-slope circulation with linear estimates

Do the nonlinear deviations identified in realistic Arctic Ocean simulations also emerge in a stripped-down, idealized setting? To address this question, we apply the same diagnostic used for the realistic simulation in Fig. 1 to the idealized model experiments. Specifically, we examine scatterplots of the simulated mid-slope circulation against linear estimates given by Eqs. (??) and (14) Eq. (13), for both long and short forcing periods (Fig. 3). The circulation is diagnosed along the mid-slope depth contour (highlighted in Fig. 2), allowing direct comparison with linear theory. Perfect agreement with the linear estimates would place points along the 1:1 line, similar to what we see in the analysis of nearly linear simulations in Fig. A1. Note that the analytical estimates are integrated from the beginning of the simulation, and we discard an initial spin-up period after which the response is cycle-to-cycle repeatable and independent of the initial state.

From Fig. 3, we see that both deviations from linear theory can be reproduced, though their expression depends on forcing period. In particular, the long forcing period (Fig. 3a) yields a clear prograde-retrograde asymmetry, with a clear bend in the scatter, while the short forcing period (Fig. 3b) shows only a sustained prograde shift. Thus, we see indication of the nonlinear contribution being dependent on forcing period, mainly a shift of the data upwards relative to the 1:1 line, i.e., a prograde shift. Note, however, that maximum circulation strength also differs considerably between the two experiments, a point we

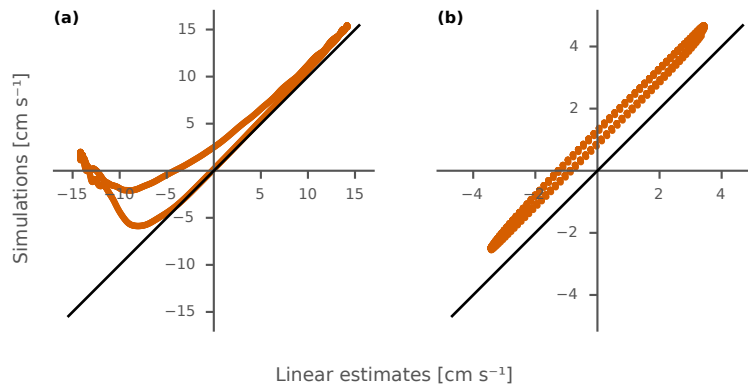


Figure 3. Scatterplots comparing circulation (normalized with contour length) along a depth contour with linear estimates, [given by Eq. 13](#). The relevant contour is highlighted in Fig. 2. Results for a forcing period of (a) 128 days, and (b) 16 days, are shown. Black lines indicate the 1:1 reference. [One forcing cycle is plotted, but the signal is repeating cycle-to-cycle after the initial spin-up period.](#)

will return to [later](#). [Notably, in Sec. 4.6. In brief, only the long-period forcing generates a response whose amplitude is strong enough to arrest topographic Rossby waves matching the corrugation wavelength.](#)

[Further, while both forcing periods show signs of hysteresis. The hysteresis, it is most pronounced in the long forcing case, with data separated into two distinct branches, one near linear theory and one displaced in the prograde direction. In contrast, the short forcing case exhibits only a weak loop around an uniformly shifted line. These differences point to damping timescales relative to forcing timescales playing an important role in shaping the nonlinear response.](#)

To [investigate when in the forcing cycle nonlinear deviations emerge](#) clarify how the hysteresis patterns in Fig. 3 arise, we turn to time-series of circulation, shown in Fig. 4. [We set For convenience of presentation, we shift \$t = 0\$ to be at the beginning of a retrograde forcing period.](#) Throughout, we refer to intervals during which the linear estimate predicts negative circulation as the retrograde phase, and to intervals with predicted positive values as the prograde phase. For the long forcing period (Fig. 4a), the circulation initially follows the linear prediction as the system enters the [prograde-retrograde](#) phase. Deviation from linear theory then sets in, first gradually, then it abruptly becomes more pronounced (approximately 25 days into retrograde forcing), leading to a strong departure [from linear estimates](#). At this point, the system enters a regime where retrograde circulation no longer strengthens, despite increasing forcing. We will refer to this [behaviour as saturation. behavior as "saturation". Animations of the flow and relative vorticity fields show that saturation coincides with the emergence of transient eddies \(see supplementary video, or snapshot of the relative vorticity field in Fig. A2\). The animated relative vorticity field exhibits a pattern that is periodic in the along-slope direction at the corrugation wavelength, with eddies forming on scales smaller than the topographic wavelength. This pattern is also apparent in the snapshot \(Fig. A2\). As the system moves into the](#)

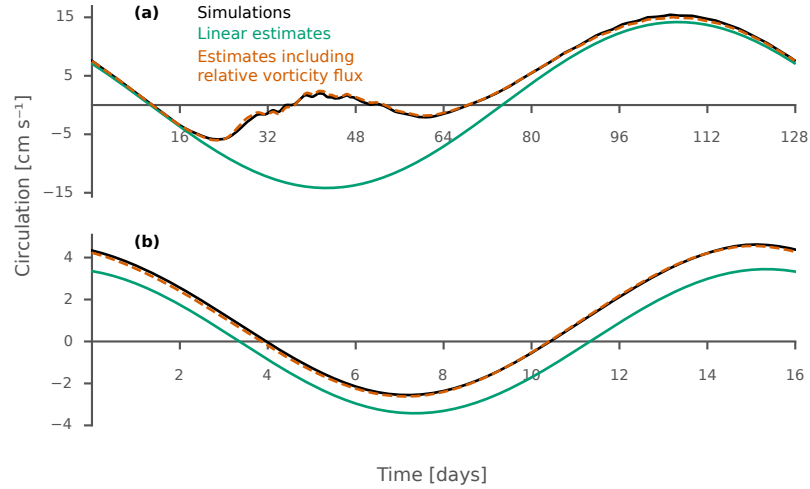


Figure 4. Time series of circulation (normalized with contour length) along a depth contour for a forcing period of (a) 128 days, and (b) 16 days. Linear estimates are shown in green, forming a sinusoidal curve around zero. Estimates including the nonlinear relative vorticity flux contribution are drawn in dashed lines, and fall on top of the circulation diagnosed from simulations. The relevant contour is highlighted in Fig. 2.

prograde phase in Fig. 4a, the circulation gradually converges back toward the linear estimate. This timeseries shows that the
495 lower branch of the corresponding scatterplot (Fig. 3a) stems from the prograde to retrograde transition, while the upper branch corresponds to the retrograde to prograde transition. This behavior is consistent with the forcing period being long compared with the frictional damping time (about 12 days at mid-slope), meaning that the system has time to approach distinct prograde and retrograde states as the forcing is slowly evolving.

For the short forcing period (Fig. 4b), the circulation remains closer to the linear estimate throughout the forcing cycle,
500 consistent with the lack of bending seen in the scatterplots. As noted, the main feature here is a sustained prograde shift. However, there are slightly larger offsets during transitions from the retrograde to the prograde phase than in the opposite direction. In other words, the system is without signs of saturation, but with an increased offset at the end of the retrograde phase, giving rise to the hysteresis seen in the scatterplot. short forcing period, by contrast, the forcing changes on a timescale comparable to the damping time. The circulation therefore cannot fully adjust to any given forcing level before the forcing
505 reverses. The flow behaves more in a low-pass sense, which explains the more compact hysteresis loop in Fig. 3b.

The nonlinear contribution relevant for depth-following circulation is a flux of relative vorticity across the depth-contour (Eq. 12). In Fig. 4, the dashed lines show estimates where diagnosed relative vorticity fluxes have been included as a forcing term (using Eq. 17 as forcing term in Eq. ??16). For both forcing periods, including relative vorticity fluxes in the estimates almost entirely removes the discrepancy with the simulations. This agreement indicates that the observed nonlinear deviations

510 from linear theory can indeed be ~~understood~~described as the consequence of a single nonlinear mechanism, namely a cross-contour ~~fluxes~~flux of relative vorticity. This supports that Eq. (12) describes the full response of the system.

4.2 ~~Spatial~~Momentum budgets in space and ~~temporal-dependence~~time

Depth-contour analysis

So far, we have focused on circulation along the mid-slope depth contour. The view is now broadened to the entire ~~domain~~slope
515 region, examining how the circulation and the various terms in the integrated momentum equation vary over one forcing cycle and across the slope (Fig. ~~??5~~ panel a and b). As a reminder, for circulation along depth contours, the contributing terms are the surface stress, bottom frictional stress, and the nonlinear relative vorticity flux term.

Panel (a) and (b) in Fig. ~~??5~~ , one for each forcing period, contain three components: (i) time evolution of momentum terms for the mid-slope contour, (ii) a Hovmöller diagram of circulation, with time on the horizontal axis, ~~depth~~cross-slope position
520 on the vertical axis, and (iii) cycle-mean profiles of momentum terms as a function of contour depth. Note that the cross-slope position is quantified by the mean y value of each depth-contour. While the top sub-panels (i) show time-series for the mid-slope depth contour only, the patterns are broadly representative of other depths as well (see Fig. A3 for full Hovmöller diagrams of the individual terms). An imbalance between the momentum ~~term signify~~terms signifies acceleration.

For both forcing periods, the depth and time-dependent structure of the circulation broadly follows the behavior predicted
525 by linear theory (Eq. ~~??13~~). In particular, the circulation lags the surface stress by an amount that increases toward deeper contours, ~~as expected from the linear response.~~ This lag is evident, for example, in the delayed transition from prograde to retrograde circulation at greater depths. ~~The depth~~To illustrate the role of the frictional timescale H/R in setting this behavior, H/R is included as a dashed gray line in panels (ii). For the long forcing period, the lag between the circulation and the forcing scales well with this timescale at all depths (again considering, for example, how the transition from prograde to retrograde
530 circulation scales with depth), reflecting that the flow has sufficient time to adjust to the forcing. In contrast, for the short forcing period, this scaling breaks down at depth, where the frictional timescale exceeds the forcing period. Further, the depth dependence of the response amplitude is more pronounced for the short forcing period (Fig. ~~??5~~bii), consistent with Eq. (15).

Superimposed on this largely linear response, clear signs of saturation emerge for the long forcing period (Fig. ~~??a~~5aii). During the retrograde phase, the amplitude of retrograde circulation ceases to increase, first near ~~364-m depth~~the depth
535 contour centered at $y = 40$ km, after which the saturated region gradually expands offshore. In contrast, no such saturation occurs anywhere along the slope for the short forcing period (Fig. ~~??b~~5bii).

The momentum diagnostics clarify the origins of the deviations from linear theory. For both long and short forcing periods, cycle-mean diagnostics (Fig. ~~??5~~aiii and Fig. ~~??5~~biii) show that vorticity fluxes provide a net prograde tendency across the slope, strongest at mid-slope depths. The flux is directed down the background potential vorticity gradient, from shallow toward
540 deeper water, and is balanced by bottom friction, implying a residual prograde circulation over a forcing cycle. Compared with the long-period case, the cycle-mean flux maximum for short forcing is shifted onshore, consistent with the stronger depth dependence of the circulation response.

For the long forcing period, relative vorticity fluxes increase sharply at the onset of saturation and remain elevated throughout the retrograde phase (Fig. ??5ai), capping the circulation strength. An exception from this pattern is found offshore of the slope, where relative vorticity fluxes temporarily take on retrograde values during the retrograde phase (Fig. A3). Animations of the flow show that As can be seen from the supplementary animations, the abrupt rise in fluxes coincides with the emergence of transient eddies, consistent with vorticity being exported across depth contours (see supplementary video). We note that Constantinou and Young (2017) have discussed a somewhat analogous saturation phenomenon, where the emergence of transient eddies strongly reduces the mean-flow response to retrograde wind forcing. Because of inertia As discussed in Sec. 3.1, the system retains a memory of this elevated prograde state even after the fluxes subside, generating the hysteresis observed in the scatterplots. This behavior follows directly from Eq. (??), which links circulation to the time-integrated forcing, and thus predicts the circulation to inherently carry a memory of past states over a time-scale H/R . scatter plots. During strong prograde forcing, relative vorticity fluxes oscillate, producing a weak prograde tendency in that phase as well.

For the short forcing period, vorticity fluxes are weaker and alternate in sign through the cycle, but still average to a net prograde tendency, with the highest prograde values during the retrograde phase. Exactly when this maximum occurs during the retrograde phase depends on depth (Fig. A3). The system's memory smooths these fluctuations, leaving a residual prograde offset visible in both scatterplots and time series. Thus, the prograde shift seen in the short forcing period is also explained by a net down-slope vorticity flux, at least when evaluated on depth contours, even though the supplementary video shows that the flow does not develop pronounced eddies for the short forcing period.

560 Constant- y analysis

Having established how relative vorticity fluxes contribute to the circulation along depth contours, we now complement this with an analysis following constant- y contours (Fig. ??5, panel c and d). In this framework, the nonlinear contribution is the full-momentum flux convergence, and topographic form stress appears explicitly. The simpler geometry also allows for area integrals :the top panels to be readily calculated: panels (i) now show area-integrated momentum balances as functions of time, while crosses above the bottom-right panels panels (iii) indicate time- and area-mean values over the domain and one forcing cycle. Individual Hovmöller diagrams for each momentum term can be found in Fig. A4.

Momentum terms along constant- y contours. Results for a forcing period of (a) 128 days, and (b) 16 days are shown. Each sub-figure contains the following diagnostics: (i) area-mean momentum terms as a function of time; (ii) a Hovmöller diagram of depth-integrated circulation (time on the horizontal axis, y on the vertical axis); and (iii) time-mean momentum terms over one forcing cycle, with crosses above the panels indicating the area and time-mean balance. The momentum terms are defined in Eq. (18).

We first note that the circulation is qualitatively similar to that along depth contours; both forcing periods show a phase lag between surface stress and circulation that increases with depth. There is also a stronger depth dependence for the short forcing period, and saturation during the retrograde phase for the long forcing period.

A key result here is that As expected, momentum flux convergence integrates to zero across the slope for both forcing periods (Fig. ??ai and panels (ci) and (di) of Fig. ??bi5). This result is a reminder that it acts momentum fluxes act only to redistribute

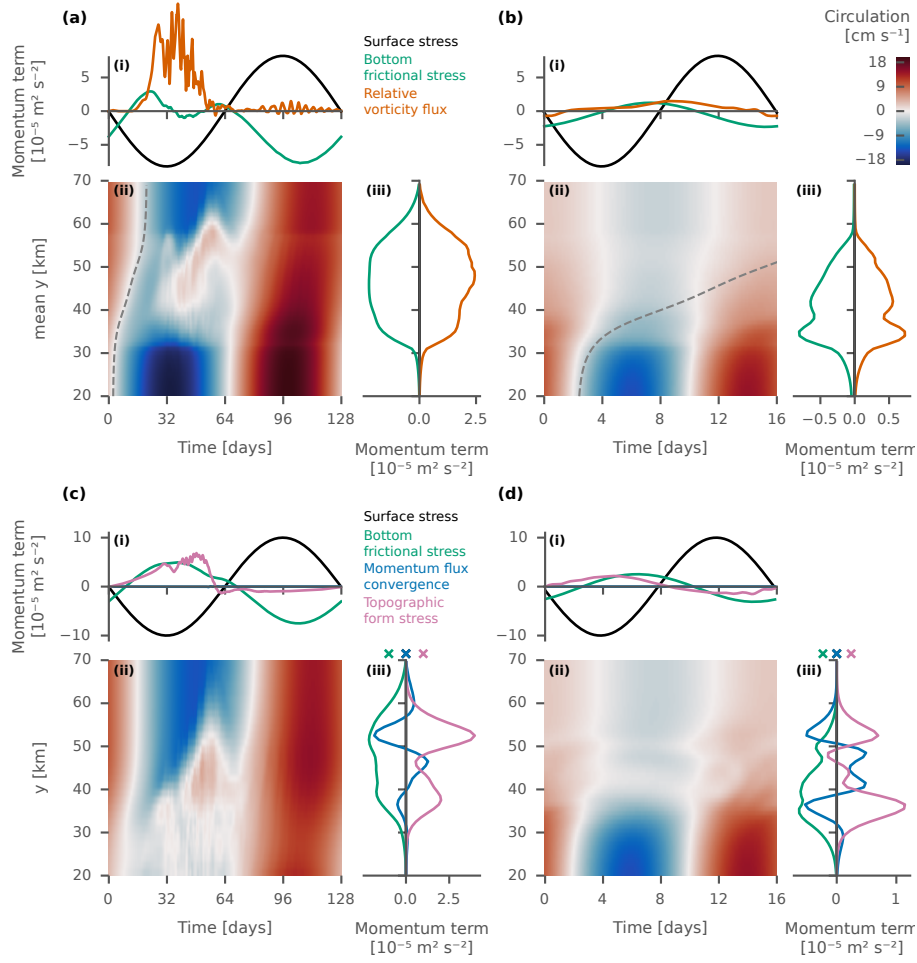


Figure 5. Momentum terms along-depth in two diagnostic frameworks: depth-following (a, b) and constant- y (c, d) contours. Results are shown for a forcing period-periods of (a) 128 days to the left (a, c) and 16 days to the right (b, d) 16 days are shown. Each sub-figure-sub-panel contains three diagnostics: (i) momentum terms for the mid-slope depth contour as a function of time ;(for the mid-slope contour in a and b, and area-mean in c and d), (ii) a Hovmöller diagram of depth-integrated circulation (time on the horizontal axis, depth and cross-slope position on the vertical axis. Cross-slope position is quantified as the mean y -value for depth contours);, and (iii) time-mean momentum terms over one forcing cycle as a function of depth. The For panels (a, b), the momentum terms are defined in Eq. (12). Note that the depth contours are equally spaced in y , and for panels (not depth, d), and correspond to the y -values in Fig Eq. ??(18). Depth-Crosses above the time-mean momentum panels in c and d indicate the area and time-mean balance. Dashed gray lines in a and b represent the damping time-scale H/R . Isobaths and depth contours are shown in Fig. 2. Note that the flat flanks of the slope have been excluded, where the response is purely linear.

momentum along the slope, without contributing to the domain-integrated balance. The time and area-mean balance is instead between topographic form stress and bottom frictional stress (crosses above Fig. ??a(ii) and panels (ciii) and (diii) of Fig. ??b(ii)5),

consistent with theory described in Sec. B1. This point is crucial, because it shows that although form stress does not appear
580 in the depth-following framework, it is central for the residual transport integrated across the channel.

The time evolution of the area-integrated balances (~~top panels panels (ci) and (di)~~ of Fig. ??5) highlights the role of topographic form stress in shaping the time-dependent circulation. For the long forcing period, topographic form stress is strongly prograde during the retrograde phase, with high values coinciding with saturation of the circulation, and weakly retrograde during the prograde phase. This phase-dependent asymmetry mirrors findings from steady-state analyses, where retrograde
585 flow generates stronger topographic form stress (e.g., Bai et al., 2021). ~~and as indicated by Eq. (19).~~ For the short forcing period, form stress alternates in sign within each cycle ~~but and does not show the same strong phase-dependent asymmetry as for the long forcing period.~~ However, as evident from the time and area-mean balance, form stress still averages to a ~~slight prograde bias, consistent with a residual prograde tendency~~ net prograde tendency, explaining the prograde bias.

Across the slope, ~~momentum flux redistributes momentum~~ (Fig. ??aiii ~~and waves or eddies redistribute momentum, giving~~
590 ~~regions of positive and negative momentum flux convergences~~ (panels (ciii) and (diii) of Fig. ??biii5). For the long forcing period, the time-mean momentum flux convergence is ~~prograde positive~~ at mid-slope (driving prograde flow) and ~~retrograde negative~~ on either side (driving retrograde flow), with an additional convergence region offshore. ~~The time-variable momentum flux convergence shows a similar pattern mainly during the retrograde phase (Fig. A4), indicating that the~~ The time-mean structure captures a persistent, phase-dependent aspect of the flow. The time-mean form stress is prograde for all y , but
595 peaks on the ~~flanks where retrograde~~ top and bottom of the slope where negative momentum flux convergence occurs. These peaks likely mark regions of topographic Rossby-wave generation, which ~~is connected to increased topographic form stress,~~ (see, e.g., Held, 1983). Further, ~~generation of Rossby waves can~~ in turn drive ~~retrograde negative~~ momentum-flux convergence, ~~whereas over as discussed in Egger (1998) and Sec. 15.1 in Vallis (2017).~~ Over the mid-slope, ~~positive momentum flux convergence can be connected to~~ wave breaking and eddy potential vorticity mixing ~~promote prograde convergence~~ (see, e.g.,
600 Dritschel and McIntyre, 2008). For the short forcing period, the overall pattern is similar, but a pronounced dip in momentum flux convergence appears over the mid-slope, coinciding with an increase in topographic form stress.

4.3 Unified analysis in terms of ~~The unified~~ PV fluxes flux perspective

As shown in Sec. 3.3, ~~if QG scaling applies,~~ both the depth-following and constant- y ~~framework can frameworks may~~ be interpreted in ~~light terms~~ of fluxes of quasi-geostrophic potential vorticity (QGPV) across the slope. ~~We now~~ Even though depth
605 ~~variations in our model are large, we~~ test this interpretation by diagnosing the cross-slope QGPV flux in both frameworks and comparing ~~their spatial structure and magnitude.~~ Figure 6 shows the time-mean ~~its spatial structure with that of the integrated bottom frictional stress.~~ According to theory Eq. (25), the time-averaged QGPV flux and ~~the bottom stress, which according to theory balances the flux,~~ bottom frictional stress should balance. Figure 6 shows these two terms for the long (a) and short (b) forcing periods. ~~Results,~~ with results from the depth-following and constant- y frameworks ~~are~~ shown together for
610 direct comparison. ~~For both constant- y and constant- H contours, we calculate and plot the flux of actual QGPV, as defined in Eq. (20).~~ As described in Sec. 3.3, in the depth-following framework ~~the~~ the QGPV flux is entirely accomplished by the relative vorticity flux. ~~In,~~ that is, the flux plotted earlier in (a) and (b) in Fig. 5. For the constant- y framework ~~the QGPV flux can be~~

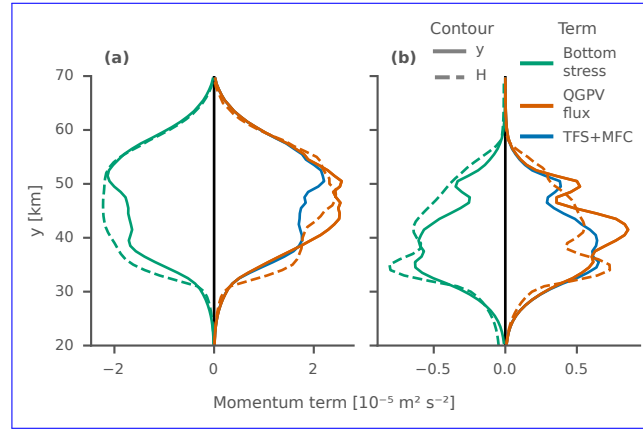


Figure 6. Time-mean quasi-geostrophic potential vorticity (QGPV) flux and bottom frictional stress and the sum of topographic form stress and the momentum flux convergence (TFS+MFC) diagnosed from the depth-following and constant- y frameworks. Results are shown for (a) a forcing period of 128 days and (b) a forcing period of 16 days. Positive QGPV flux indicates offshore transport, down the large-scale PV gradient.

approximated by combining we also plot the sum of the momentum flux convergence and the topographic form stress. These two contributions tend to be anti-correlated (see Fig. ??). Their sum, however, constitutes (MFC+TFS). Under QG scaling, their sum should constitute a net flux of QGPV across the slope, which is the quantity shown in Fig. 6. We verified this interpretation by also computing the cross-contour flux of QGPV (as defined in Eq. 20) directly. This direct calculation yields qualitatively similar spatial patterns and magnitudes, with discrepancies confined primarily. There are discrepancies between the actual QGPV flux and MFC+TFS, primarily confined to the steepest part of the slope, but overall the agreement suggests that the PV-flux interpretation is a useful one.

In both forcing regimes, and for both integration paths, the time-mean QGPV flux is directed offshore, down the large-scale PV gradient, and approximately balances the vorticity removal by bottom. It also balances the bottom frictional stress associated with the time-mean residual flow along the slope. The cross-slope structure of the diagnosed QGPV flux shows a strong structural strong agreement between the depth-following and constant- y frameworks, with comparable magnitudes and spatial organization across the slope. This agreement suggests that water columns approximately maintain their PV as they are transported from a given constant- y contour to the equivalent constant- H contour, which undulates around that same y position. This, in turn, implies that the integrated thickness flux through the constant- y contour is converted, via stretching and squeezing, into a relative vorticity flux, adding to the total relative vorticity flux through the constant- H contour. In other words, some fraction of the relative vorticity flux through a constant- H contour carries the imprint of the thickness flux that passes through the associated constant- y contour. This thickness flux through the corrugations in the slope—which is a linear process—is thus essential to obtaining a net down-gradient PV flux across either integration path.

Small differences remain, which are likely related to geometric factors: a depth-following contour is generally curved and longer than a straight y -transect, leading to differences in bottom stresses frictional stresses—and associated balancing

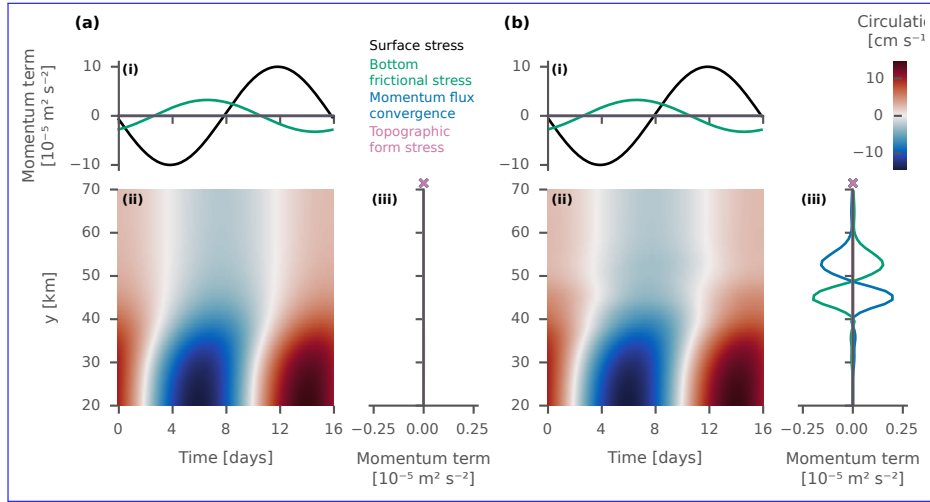


Figure 7. Momentum terms along straight depth contours for a bathymetry without corrugations. Results are shown for (a) purely along-slope wind forcing and (b) wind forcing with an imposed cross-slope component. The forcing period is 16 days in both cases. Note that relative vorticity fluxes and momentum flux convergence are identical when evaluated along straight transects without corrugations.

QGPV fluxes. Nevertheless, the ~~two diagnostics consistently identify a systematic offshore~~ interpretation remains that both frameworks, integrated along constant- H and constant- y , represent a systematic down-gradient QGPV flux that maintains the residual prograde slope circulation.

4.4 The need for corrugations

So far, all simulations considered in this study have included topographic corrugations. The above results suggest that the thickness flux through these corrugations, or equivalently, the topographic form stress, is an essential ingredient in the integrated momentum budgets and in the establishment of time-mean currents. A direct theoretical implication of the absence of such along-slope depth variations, and thus of topographic form stress, is that oscillatory wind forcing with zero time mean cannot produce a domain-integrated residual flow, that is, a non-zero net mass transport through our channel (Eq. B1). To verify this theoretical constraint, we now examine a set of simulations in which the slope is smooth. These simulations share the same large-scale geometry and mean slope as the corrugated cases but lack depth variations along constant- y contours. In this configuration, analyses integrated along constant- y and along depth-following contours coincide.

We first examine the response to the standard along-slope wind forcing (Eq. 10), shown in Fig. 7a. In this case, the surface stress is spatially uniform and purely along-slope. We find that the response is well described by linear theory: the flow oscillates in time according to linear analytical estimates (Eq. 13) and produces no residual mean flow at any point along the slope. This is illustrated by the zero time-mean of all momentum terms in panel (iii).

The purely linear response may reflect the fact that the applied stress is everywhere parallel to the slope and does not induce any cross-slope motion. In the presence of topographic corrugations, however, the same along-slope forcing is observed to

generate cross-slope flow. To probe whether cross-slope motion alone alters the system's response, we therefore perform an additional experiment in which a cross-slope flow component is forced explicitly by the wind, while keeping the topography smooth.

This modified surface stress, denoted τ^{cs} , is constructed to be tangential to the bathymetric contours of the *corrugated* slope configuration used in the other experiments, while retaining the standard along-slope stress amplitude. It takes the form

$$\tau^{cs} = \tau^{sx} \mathbf{i} + \tau^{sx} \frac{\partial_x H}{\partial_y H} \mathbf{j}, \quad (26)$$

with H and τ^{sx} as defined in Eqs. (8) and (10). The curl of this wind stress varies on horizontal scales comparable to those of the bathymetric corrugations in the corrugated-slope experiments, representing an Ekman-driven cross-slope motion analogous to that induced by flow–topography interactions. Importantly, as in the above, the time mean is zero.

Figure 7b shows the resulting momentum balances for this experiment. Here, the circulation response takes the form of a persistent time-mean alternating jet structure over the slope (panel iii), with a prograde jet onshore and a retrograde jet offshore. This pattern is maintained by local convergence and divergence of momentum fluxes, which drive departures from the linear estimate at individual depths. The sign of these fluxes alternates across the slope, such that their contributions cancel in the domain integral. In this sense, the system permits local momentum redistribution and the formation of along-slope jets, but without corrugations there is no mechanism by which the oscillatory wind forcing can set up a time-mean net mass transport through the channel.

This flow structure can also be interpreted through the lens of potential vorticity rearrangement. Cross-slope motions driven by the time-variable wind induce local transport across the background PV gradient, leading to a redistribution of PV and the emergence of alternating along-slope jets. This pattern resembles jets associated with PV staircase formation (Dritschel and McIntyre, 2008), albeit without an explicit eddy field. Here, it is the wind forcing itself, rather than turbulent eddies, that mediates the cross-PV gradient transport.

4.5 Comparison with arrested wave theory

In earlier sections, we found that saturation strongly influences the retrograde response to long forcing periods, ~~whereas it in the presence of corrugations, whereas saturation~~ is absent for short-period forcing. Earlier studies have shown that strong topographic form stress occurs when topographic Rossby waves with wavelengths comparable to the bathymetric corrugations are arrested by the mean flow (Bai et al., 2021; Zhang and Lentz, 2017). Arrest corresponds to $U = -c_k$ in Eq. (19). We thus hypothesize that saturation, which is associated with strong topographic form stress (Fig. 5c), is connected to such arrest. In this section, we first focus on the response to the long forcing period and investigate the mechanism responsible for the observed saturation, ~~before we return to why saturation is absent in the short period runs.~~

~~Leaning on earlier steady-state studies (Bai et al., 2021; Zhang and Lentz, 2017), we hypothesize that saturation occurs when~~

Wavelength dependence of saturation

If saturation is indeed connected to arrest of resonant topographic Rossby waves ~~with wavelengths comparable to the bathymetric corrugations are arrested by the mean flow. In this case,~~ the saturated flow speed is expected to approximately scale with the intrinsic phase speed of ~~these waves such waves with a wavelength matching the bathymetric wavelength.~~ Because the phase speed depends on wavelength, this mechanism predicts different saturation levels for different corrugation scales. Further, the phase speed of Rossby waves also differ between the different cross-slope wave modes.

To test this idea, we compute arrest speeds of topographic Rossby waves using the linear barotropic coastal-wave model of Brink (2025). For a range of wavelengths, we calculate the phase speeds of the first three topographic Rossby wave modes and compare these with the saturation velocities diagnosed from the simulations. The coastal-wave model configuration used for the calculations is provided in the supplementary material, but in short the model is run with a free surface, closed boundaries, and the same large-scale slope as the shallow water simulations. We note that some caveats apply; the wave model assumes steady, linear dynamics and is here applied without any imposed mean along-slope velocity, whereas our simulations involve time-dependent forcing. Nevertheless, for the slowly varying cases of interest, these linear estimates provide a useful approximation to the arrest speeds.

Saturation speeds are estimated as the maximum retrograde circulation along depth contours spanning the central slope (~~contour indices 40–50~~ contours with mean y -values between 40–50, corresponding to depths of roughly 360–670 m), ~~providing an upper-bound estimate.~~ To test whether the saturation speed follows the expected wavelength dependence, we conducted experiments with corrugation wavelengths λ of 22.5, 45, and 90 km. The theoretical arrest speeds and the diagnosed saturation speeds are shown in Fig. 8. As we see, the diagnosed saturation speeds exhibit a similar wavelength dependence as expected for the arrest of topographic Rossby waves, with increasing speed as the wavelength increases. The arrest-saturation speeds fall between mode 2 and 3, but seem to follow mode 2 better as the wavelength increases. We note that the arrest of a mode 2 wave is consistent with the findings of Zhang and Lentz (2017), who analyzed wind-driven retrograde flow over a shelf valley in terms of coastal-trapped wave characteristics. They hypothesized that the arrest of mode 2 over mode 1 was due to the lower mode being too fast to be arrested by the flow. They further argued that the disturbance introduced by bathymetry is largest mid-slope, also matching a mode 2 structure (Fig. 9a).

Cross-slope structure

To further test whether the observed saturation reflects the arrest of topographic Rossby waves, we analyze the cross-slope structure of the simulated pressure field at the onset of saturation ($t = 30$ days). The goal is to determine whether the simulated pattern resembles the mode structure expected for an arrested topographic Rossby wave. To isolate a possible mode structure, we compute residual pressure perturbations ϕ' from the model output. Obtaining a suitable ϕ' is not straightforward, since the corrugated bathymetry imprints a strong ~~-cycle-dependent~~ pattern on the pressure field. This imprint is not stationary, but alternates in sign between the prograde and retrograde phases, reflecting the reversal of along-slope flow. ~~To suppress this~~ In contrast, an arrested topographic Rossby wave is expected to exist only during the retrograde phase, when the mean flow nearly

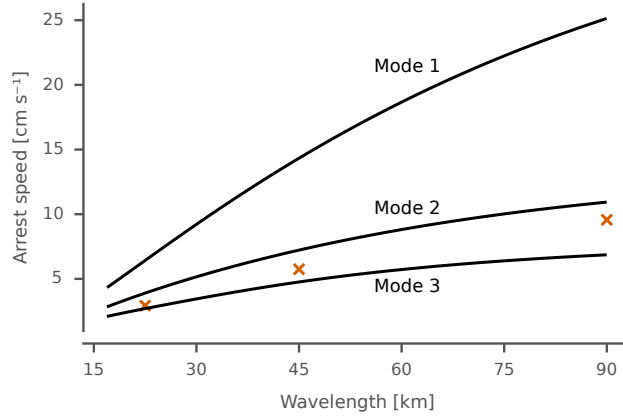


Figure 8. Theoretical arrest speed for topographic Rossby waves as a function of wavelength, and diagnosed saturation speed. Solid lines show theoretical arrest speeds for mode 1 (top), mode 2 (middle), and mode 3 (bottom), where the mode number signify the number of zero crossings in the pressure modal function. Crosses indicate the saturation speed from simulations with topographic wavelengths $\lambda = 22.5, 45$, and 90 km. Simulations were run with a forcing period of 128 days.

715 matches the intrinsic prograde phase speed of the wave. Thus, the bathymetric imprint is present in both phases (with opposite sign), whereas the arrested-wave signal is confined to the retrograde phase.

To suppress the bathymetric imprint while retaining any coherent-trapped-wave structure, we first eliminate the large-scale along slope tilt by subtracting the cross-slope mean:

$$\tilde{\phi}(x, y, t) = \phi(x, y, t) - \bar{\phi}^y(y, t). \quad (27)$$

720 We then combine the retrograde state, potentially containing a trapped mode, with the prograde state (shifted by half a period T relative to the retrograde state), so that the two are in ~~exact antiphase.~~ antiphase:

$$\phi'(x, y, t) = \tilde{\phi}(x, y, t) - \tilde{\phi}(x, y, t + \frac{T}{2}). \quad (28)$$

Adding these anti-phase fields cancels the alternating topographic imprint and leaves any phase-dependent signal. We note that a small contribution from the residual flow may remain. ~~Additionally, the cross-slope mean is removed from each snapshot of ϕ to eliminate the large-scale along-slope tilt.~~

730 Figure 9 compares the residual pressure ϕ' with the theoretical cross-slope pressure structure of a 45 km mode 2 topographic Rossby wave, computed using the wave model of Brink (2025). The residual pressure ϕ' is shown at several x -positions corresponding to different phases of the same wave pattern. Note that we only plot x -positions from one of the two repeated corrugation segments. However, across the full domain, these phase profiles repeat exactly with the corrugation wavelength, indicating that the pressure field is periodic at the corrugation wavelength in the along-slope direction. The simulation ϕ' reveals a pattern similar to that of a mode 2 wave, with alternating maxima and minima across the slope that resemble the theoretical structure. ~~Further, we see different phases of the same wave pattern for different x -positions.~~ The match is not exact, with

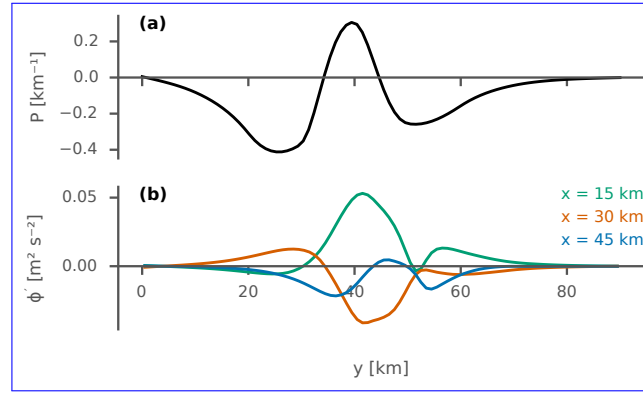


Figure 9. Panel (a) shows the theoretical cross-slope mode 2 structure for a wavelength of 45 km. Panel (b) shows the residual dynamic pressure from simulations for a wavelength of 45 km at different x -positions, used as a proxy for the mode structure. The procedure for estimating this diagnostic is described in the main text. Simulations were run with a forcing period of 128 days.

curves slightly displaced in the vertical (likely reflecting contamination by the residual flow), but the extrema align well with theory. Albeit not perfect, this structural match supports the interpretation that the observed saturation arises from the arrest of topographic Rossby waves ~~of mode 2.~~

~~We note that the arrest of a~~

Varying the corrugation amplitude

Considering the QG expression for topographic form stress given in Eq. (19), we see that QG-theory predicts that topographic form stress depends on both mean velocity U and the corrugation amplitude h . One can further ask what the role of corrugation height in maintaining saturation is, as forcing strength varies over a forcing cycle. Near the arrest condition of $U = -c_k$, the form stress is very sensitive to the mean velocity U . Here, small changes in U can balance relatively large changes in surface stress, which we assume to underlie the saturation behavior we find. Whether such saturation persists as the surface stress is increased further depends on whether the maximum attainable form stress at arrest is large enough to balance the maximum surface stress, and this in turn is expected to depend on the corrugation amplitude. Thus, in principle, the Rossby wave arrest mechanism itself should not be sensitive to corrugation amplitude; but whether topographic form stress can sustain a persistent saturation as the surface stress further increases is expected to rely on the corrugation amplitude being sufficiently large. A possible complicating factor is that different wave modes have different arrest speeds, as we see in Fig. 8. For the same forcing strength but smaller corrugation amplitude, arrest at mode 2 ~~wave is consistent with the findings of Zhang and Lentz (2017), who analyzed wind-driven retrograde flow over a shelf valley in terms of coastal-trapped wave characteristics. In their study,~~ the bathymetry consisted of a single valley, allowing the cross-slope structure of the resulting lee waves to be compared more directly with theoretical mode structure; ~~might not result in topographic form stress strong enough to balance the surface stress.~~ At velocities closer to arrest conditions of a mode 1 wave, however, the bottom frictional stress is stronger, and thus weaker

topographic form stress is required to achieve balance. Thus, as the corrugation height changes for the same forcing strength, saturation might not happen at mode 2, but instead at mode 1.

755 To investigate the role of corrugation amplitude, we conducted a series of simulations where we vary λ in Eq. (15), retaining the standard long-period surface stress. Figure 10 shows the maximum retrograde and prograde circulations along depth contours spanning the central slope (mean y -values between 40–50, as in Fig. 8) as a function of vertical corrugation amplitude. For reference, the base case has a vertical corrugation amplitude of 311 m. Dashed gray lines in the figure indicate the phase speeds of mode 1 and mode 2 topographic Rossby waves.

760 Focusing first on the maximum retrograde flow speed, which represents the saturation velocity, we observe a stepwise dependence on corrugation amplitude. As the corrugation height decreases relative to the largest amplitude (moving from right to left in Fig. 10), the saturation velocity initially increases gradually. This behavior aligns with the expectation from Eq. (19), where a reduction in corrugation height h can be compensated by an increase in velocity U . When the corrugation amplitude is further reduced such that the maximum retrograde speed crosses the arrest condition for a mode 2 wave, the
765 saturation velocity exhibits a pronounced increase before stabilizing at a value below the arrest speed of mode 1. We note that the flow strength is still considerably weaker for retrograde than for prograde flow for these corrugation amplitudes. At very low corrugation amplitudes, the saturation velocity abruptly transitions to a value characteristic of no saturation (i.e., comparable to the maximum prograde flow speed, shown by the orange line in Fig. 10). Maximum prograde flow speed, on the other hand, is only weakly sensitive to corrugation amplitude. There is a slight decrease in maximum prograde circulation
770 strength with increasing corrugation height, likely connected to slightly longer integration paths as the corrugations become more prominent.

These results demonstrate that corrugation amplitude plays an important role in modulating the saturation behavior, though the relationship is not straightforward. The observed stepwise changes in saturation velocity suggest that corrugation amplitude influences which wave mode dominates the arrest condition. Specifically, our findings suggests that if topographic form stress
775 is not sufficiently strong at arrest of mode 2 to balance the surface stress, the flow velocity will increase until arrest of mode 1 occurs. Notably, the arrest condition does not occur precisely at the phase speed of the Rossby waves, but rather slightly below it. This can at least partly be understood from Eq. (19); since topographic form stress increases rapidly as the velocity approach the arrest condition, form stress might be strong enough to balance the surface stress for velocities slightly below the arrest condition.

780 4.6 Varying the forcing strength

Varying the forcing strength

We now turn to investigate why saturation is absent in the simulations with rapidly oscillating forcing. Building on the long-period results, where saturation can be linked to the arrest of topographic Rossby waves, we hypothesize that the shorter-period forcing simply does not generate flow strong enough to arrest such waves. To test this, we perform an additional set of

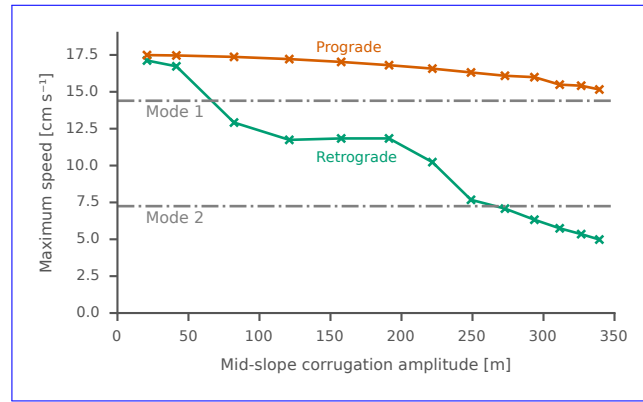


Figure 10. Panel (a) shows the maximum prograde and retrograde along-slope speed over the central slope (a mean y -values between 40–50) as a function of mid-slope corrugation amplitude. Panel (b) shows the residual dynamic pressure from simulations for a wavelength of 45 km at different x -positions, used as a proxy for the mode structure. The procedure for estimating this diagnostic is described in 1 and mode 2 topographic Rossby wave with wavelength matching the main horizontal corrugation wavelength are marked by gray dashed-dotted lines. Simulations were run with a forcing period of 128 days.

785 simulations in which the surface forcing amplitude is systematically increased, so that the short-period runs reach the theoretical arrest speeds.

The results are summarized in Fig. 11. Panel (a) shows the maximum prograde circulation along three different depth contours at the mid-slope and its flanks. These velocities scale linearly with forcing strength, consistent with the theoretical linear prediction. Theoretical linear predictions from Eq. (15) are also included, shown as shaded areas; the bottom branch corresponds to shorter forcing periods, with the depth dependence especially pronounced for the short-period case, as expected from linear theory, while the top branch corresponds to longer forcing periods. The simulated velocities scale linearly with forcing strength, consistent with linear predictions, but are consistently larger. This result further illustrates that nonlinearities result in stronger prograde flow.

790 and for longer are also included, shown as shaded areas; the bottom branch corresponds to shorter forcing periods, with the depth dependence especially pronounced for the short-period case, as expected from linear theory, while the top branch corresponds to longer forcing periods. The simulated velocities scale linearly with forcing strength, consistent with linear predictions, but are consistently larger. This result further illustrates that nonlinearities result in stronger prograde flow.

Figure 11b shows the maximum retrograde circulation as a function of forcing strength, where the signature of saturation becomes evident for both short and long forcing periods. The arrest speed of a mode 2 Rossby wave for a wavelength matching the bathymetry, which can be taken as an estimate for the saturation velocity, is also included. At weak forcing, retrograde speeds increase nearly linearly, but at higher amplitudes the response levels off, with the limiting speed varying by depth. At shallow contours, the speed begins to decline as the forcing further strengthens, whereas at deeper contours it continues to increase before eventually plateauing. This offshore progression indicates that the strongest retrograde flow shifts towards deeper water as the forcing intensifies. Both forcing periods display this behavior, but. However, saturation occurs at lower relatively low speeds for the short-period case. The difference in, as can be seen from the solid lines plateauing well below the arrest speed of a mode 2 Rossby wave. The relatively low maximum retrograde speed for short forcing likely reflects that the short-period runs do not reach a steady arrested state, whereas the long-period runs approach steady conditions.

795 becomes evident for both short and long forcing periods. The arrest speed of a mode 2 Rossby wave for a wavelength matching the bathymetry, which can be taken as an estimate for the saturation velocity, is also included. At weak forcing, retrograde speeds increase nearly linearly, but at higher amplitudes the response levels off, with the limiting speed varying by depth. At shallow contours, the speed begins to decline as the forcing further strengthens, whereas at deeper contours it continues to increase before eventually plateauing. This offshore progression indicates that the strongest retrograde flow shifts towards deeper water as the forcing intensifies. Both forcing periods display this behavior, but. However, saturation occurs at lower relatively low speeds for the short-period case. The difference in, as can be seen from the solid lines plateauing well below the arrest speed of a mode 2 Rossby wave. The relatively low maximum retrograde speed for short forcing likely reflects that the short-period runs do not reach a steady arrested state, whereas the long-period runs approach steady conditions.

800 deeper water as the forcing intensifies. Both forcing periods display this behavior, but. However, saturation occurs at lower relatively low speeds for the short-period case. The difference in, as can be seen from the solid lines plateauing well below the arrest speed of a mode 2 Rossby wave. The relatively low maximum retrograde speed for short forcing likely reflects that the short-period runs do not reach a steady arrested state, whereas the long-period runs approach steady conditions.

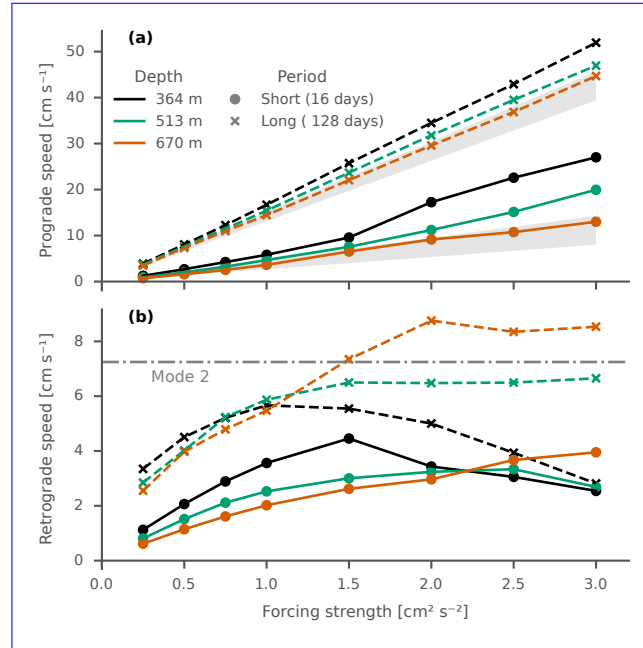


Figure 11. Maximum prograde and retrograde ~~circulation-velocity along depth-contours~~ as function of forcing strength. Panel (a) shows maximum prograde velocity along selected depth contours along the mid-slope and its flanks, and panel (b) shows maximum retrograde velocity along the same contours. Solid lines denote the 16 day forcing period and dashed lines the 128 day period. Analytical predictions for the maximum linear flow strength across the depth range, given by Eq. (15), are shown as gray shaded areas in panel (a); the top branch corresponds to 128 days forcing period, while the bottom branch corresponds to 16 days forcing period. The arrest speed of a mode 2 topographic Rossby wave with wavelength matching that of the bathymetry is included as a dashed-dotted line in panel (b).

and are thus not fully understood by steady-state arrested-wave theory. Nonetheless, the analysis demonstrates that retrograde
805 flow in the short-period simulations also saturates when the forcing is sufficiently strong, ~~accompanied by an offshore shift in the flow maximum,~~ in the sense that a maximum velocity exists (given, as indicated by Fig. 10, sufficiently pronounced corrugations).

4.6 ~~The need for corrugations~~

So far, all simulations considered in this study have included topographic corrugations. A direct theoretical implication of the
810 ~~absence of such along-slope depth variations, and thus of topographic form stress, is that oscillatory wind forcing with zero time mean cannot produce a domain-mean residual flow, as indicated by the domain-integrated momentum balance (Eq. B1). To illustrate this theoretical constraint explicitly, we now examine a set of simulations in which the slope is smooth. These simulations share the same large-scale geometry and mean slope as the corrugated cases, but lack depth variations along constant- y contours. In this configuration, analyses along constant- y and along depth-following contours coincide.~~

815 Momentum terms along straight depth contours for a bathymetry without corrugations. Results are shown for (a) purely along-slope wind forcing and (b) wind forcing with an imposed cross-slope component. The forcing period is 16 days in both cases. Note that relative vorticity fluxes and momentum flux convergence are identical when evaluated along straight transects without corrugations.

We first examine the response to the standard along-slope wind forcing (Eq. 10), shown It is, however, worth noting that for
 820 the short forcing period, saturation does not manifest as the pronounced bend in the scatterplot seen for the long-period case in
 Fig. 7a. In this case, the surface stress is spatially uniform and purely along-slope. We find that the response is well described by linear theory: the flow oscillates in time according to linear analytical estimates (Eq. ??) and produces no residual mean flow at any point along the slope. This is illustrated by the zero-time mean of all momentum terms in panel (iii). 3a. Instead, the
rapid alternation of the forcing, combined with the low-pass character of the response, leads to a response qualitatively similar
 825 to that in Fig. 3a, but with a more pronounced prograde shift (not shown).

The purely linear response may reflect the fact that the applied stress is everywhere parallel to the slope and does not induce any cross-slope motion. In the presence of topographic corrugations, however, the same along-slope forcing is observed to generates cross-slope flow. To probe whether cross-slope motion alone alter the system's response, we therefore perform an additional experiment in which a cross-slope flow component is forced explicitly by the wind, while keeping the topography
 830 smooth.

This modified surface stress, denoted τ_s^{cs} , is constructed to be tangential to the bathymetric contours of the *corrugated* slope configuration used in the other experiments, while retaining the standard along-slope stress amplitude. It takes the form

$$\tau_s^{cs} = \tau_{sx} \hat{i} + \tau_{sx} \frac{\partial_x H}{\partial_y H} \hat{j},$$

with H and τ_{sx} as defined in Eqs. (8) and (10). The curl of this wind stress varies on horizontal scales comparable to those of
 835 the bathymetric corrugations in the corrugated-slope experiments, representing an Ekman-driven cross-slope motion analogous to that induced by flow-topography interactions. Importantly, the time mean is zero.

Figure 7b shows the resulting momentum balances for this experiment. Here, the circulation response takes the form of a persistent time-mean alternating jet structure over the slope (panel iii), with a prograde jet onshore and a retrograde jet offshore. This pattern is maintained by local convergence and divergence of momentum fluxes, which drive departures from
 840 the linear estimate at individual depths. The sign of these fluxes alternates across the slope, such that their contributions cancel in the domain integral. In this sense, the system permits local momentum redistribution and the formation of along-slope jets, but without corrugations there is no mechanism by which the oscillatory wind forcing can be converted into a net momentum transfer.

This flow structure can also be interpreted through the lens of potential vorticity rearrangement. Cross-slope motions driven by the time-variable wind induce local transport across the background PV gradient, leading to a redistribution of
 845 PV and the emergence of alternating along-slope jets. This pattern resembles jets associated with PV staircase formation (Dritschel and McIntyre, 2008), albeit without an explicit eddy field. Here, it is the wind forcing itself, rather than turbulent eddies, that mediates the cross-PV gradient transport.

5 Discussion

850 Previous realistic ~~Arctic Ocean simulations~~ simulations of the Nordic Seas and Arctic Ocean have hinted that interactions between flow and topography may play an important role in shaping ~~boundary currents around basins~~ the circulation around basins in these high-latitude regions, as evidenced by two nonlinear features: a general prograde (cyclonic) offset of the flow strength relative to linear estimates, and an enhanced offset under persistent anticyclonic wind forcing compared to cyclonic wind forcing. Having now examined the underlying dynamics in an idealized setting, we can return to these features and
855 consider what our results suggest about their expression in a more realistic setting.

A first consideration concerns the range of forcing timescales. In our simplified experiments, each forcing period produced a characteristic response; a short forcing period maintained a nearly steady prograde bias over the forcing period, whereas a long forcing period allowed partial relaxation toward the linear estimate and flow saturation during strong retrograde forcing. The transition to saturation ~~, however,~~ was found to depend ~~mainly~~ on flow strength, but with stronger forcing required to reach
860 ~~arrest saturation~~ under short-period forcing. In the realistic system, where surface stress varies continuously ~~across frequencies~~ in time and along the slope, these behaviors would coexist. ~~Periods~~ For example, periods of slow, quasi-steady prograde forcing could permit relaxation ~~, while energetic retrograde events towards a linear response, while periods of strong retrograde flow~~ could induce transient saturation. Together with along-slope variations in wind stress strength, such mixed conditions would produce a broader spectrum of circulation responses, helping to explain the spread seen in the realistic simulations (Fig. 1).

865 A second consideration relates to the topography. The slopes in our idealized experiments have a single ~~dominant~~-corrugation scale, whereas natural bathymetry spans a wide range of wavelengths and amplitudes. The arrest speed of topographic Rossby waves depends on their wavelength, so different topographic ~~scales~~ wavelengths are associated with different arrest speeds, and therefore result in saturation at different flow magnitudes. Fine-scale roughness is associated with short-wavelength waves that arrest at relatively weak retrograde flow, ~~whereas broader features~~. Bathymetric features represented
870 by longer wavelengths interact with longer waves that require stronger flow before arrest occurs. ~~The resulting patchwork of local arrest conditions can be expected to smooth the sharp asymmetry found in our experiments, while retaining the same underlying nonlinear mechanism~~ Further, saturation might not be sustained as forcing strength increases beyond maximum attainable topographic form stress given the corrugation amplitude. However, since the real bathymetry of the Arctic consists of a spectrum of wavelengths, we expect arrest of topographic Rossby waves to be likely, resulting in the nonlinear behavior
875 studied here to be present.

Beyond differences in temporal and spatial scales, a realistic setting includes several processes absent from our simplified model, most notably stratification and vertical shear. Stratification is expected to alter the vertical structure of the response, tending to concentrate topographic interactions near the bottom, as found in stratified rectification studies (Brink, 2011). This suggests that the mechanism identified here may still operate, but with a depth-dependent signature.

880 ~~Turning to implications for large-scale models, the mesoscale flow-topography interactions identified here require that the bathymetry is well-resolved. Thus, they may not be explicitly represented in coarse-resolution ocean models. In the context of Arctic circulation, this limitation becomes particularly relevant for future projections. Recent CMIP6 ensemble analyses~~

indicate that surface stress over the Arctic Ocean is expected to intensify (Muilwijk et al., 2024), implying a stronger dynamical imprint of surface forcing on slope currents. As surface stress increases, the nonlinear mechanisms highlighted here are likely to play a larger role, meaning that coarse-resolution models may increasingly misrepresent the resulting boundary currents. Further, the role of irreversible processes is worth mentioning. As introduced in Sec. 1, the rectifying effect we observe in simulations can be understood in terms of water columns tending to conserve their PV while also being subject to irreversibility, so that PV-perturbations accumulate. In simulations presented here, irreversibility is included through the linear bottom drag. The steady QG expression of Bai et al. (2021), reproduced in Eq. (19) here, suggests that for prograde flows topographic form stress vanishes for $R = 0$. For retrograde flows near resonance, wave amplitudes would get very large as $R \rightarrow 0$. Such waves will become unstable and turbulent, as shown by e.g. Constantinou and Young (2017). A thorough investigation of this scenario is beyond the scope of this study. But it is also worth mentioning that linear bottom drag, beyond being a parameterization, is only one of many possible irreversible processes. Additional mechanisms, such as wave radiation or wave breaking, may also contribute in realistic settings.

Finally, we have focused on a system where there is a local generation of eddies by flow–topography interactions. An open question is how additional sources of eddy activity, such as those arising from baroclinic instability, ~~as well as remotely generated eddies entering the system,~~ would influence the circulation. We hypothesize that a cyclonic bias will be established, where momentum is removed from the system by topographic form stress when eddies induce temporary retrograde flow over topographic irregularities. Related behavior has been documented in idealized studies, where an externally generated eddy field interacts with variable bathymetry, producing mean flows aligned with topography (Eurin, 1999) (Eurin, 1999; Miyama et al., 2018). This question is particularly interesting in the context of a future “blue Arctic,” where reduced sea-ice cover is projected to enhance eddy activity substantially (Li et al., 2024), potentially amplifying the role of eddy-topography interactions in shaping slope circulation.

6 Conclusions

This study examined the nonlinear dynamics of flow over corrugated slopes under time-variable wind forcing, and investigated how these interactions shape the relationship between circulation and surface forcing. The analysis was conducted in an idealized shallow-water model using two complementary frameworks – momentum budgets integrated along depth contours and along straight transects – which reveal how the same dynamics manifest themselves from different perspectives. ~~We now return to the three questions posed in the introduction.~~

First, nonlinear flow–topography interactions introduce a systematic prograde bias in the circulation response, in the sense that the response is shifted towards prograde values relative to linear estimates. As the circulation strengthens, the retrograde flow reaches a critical velocity beyond which its strength saturates, while prograde flow can continue to increase, leading to a pronounced asymmetry between prograde and retrograde circulation. ~~In the depth-following framework, both features originate from a net downslope flux of relative vorticity, which reaches its maximum during retrograde flow.~~

915 ~~Second, the~~ The forcing period influences the persistence of this offset: rapid oscillations maintain an almost steady prograde bias, whereas slower forcing allows partial relaxation toward the linear response during prograde ~~forcing phases~~ phases, ~~and a strong nonlinear expression during retrograde phases.~~ This behavior reflects the inertial low-pass filtering of the ~~system combined with the net positive downslope vorticity flux, with its peak during retrograde flow.~~ systems. Saturation, however, depends primarily on flow strength; once retrograde flow arrests topographic Rossby waves with wavelengths matching the along-slope topography, further acceleration is suppressed, resulting in saturation largely independent of timescale. This mechanism relies on the presence of along slope corrugations.

~~Third, the two diagnostic frameworks capture complementary aspects of the same process.~~ In the depth-following framework, ~~downslope~~ down-slope vorticity fluxes quantify the nonlinear contribution and provide a clean diagnostic of the mechanisms driving the circulation offset and asymmetry. ~~Importantly, we also show that topographic form stress does not necessarily vanish in this framework when integrated over an area (Eq. B5), even though it is zero along individual contours.~~ In the momentum budget integrated along straight transects (which includes variations in depth), the same dynamics appear as a combination of momentum-flux convergence and topographic form stress, which represents a sink of momentum when integrated over the domain. In accordance with previous studies, form-stress closes the domain-integrated balance and clarifies why the net bias in circulation is prograde (Haidvogel and Brink, 1986; Brink, 2010).

930 Importantly, under ~~quasi-geostrophic~~ QG assumptions, these two perspectives can be unified as describing a flux of potential vorticity. In both frameworks, the residual time-mean prograde flow arises from a systematic flux of ~~potential vorticity~~ PV down the large-scale ~~potential vorticity gradient~~ PV gradient: in the presence of corrugations, flow–topography interactions generate topographic form stress ~~that, through a chain of processes involving topographic Rossby wave dynamics, yield an integrated potential vorticity flux directed toward deeper water which, to the extent that QG scaling applies, can also be interpreted as~~ the thickness contribution to the PV flux. When corrugations are absent, this ~~mechanism~~ linear contribution to the PV flux vanishes and no net ~~potential vorticity (domain-integrated) PV flux or residual flow is produced, consistent with the absence of form stress.~~ QG scaling is not exactly applicable to our simulations, particularly since they have steep slopes and finite topographic corrugations. Still, the QG-based estimate captured the simulated flow–topography interactions, suggesting that it indeed provides a useful conceptual framework for relating the two diagnostic approaches. In particular, it brings topographic

940 form stress into the context of the down-gradient PV fluxes that are central to the minimum potential enstrophy arguments of Bretherton and Haidvogel (1976). As far as we know, this has not been pointed out before.

Overall, our results confirm that mesoscale topographic corrugations can induce a systematic prograde bias in time-variable slope currents and can ~~sharply~~ limit anticyclonic flow strength through Rossby wave arrest. The contribution of this study lies in examining how these nonlinear effects evolve with changing forcing, how they depend on the forcing period and strength, and

945 how they can be interpreted consistently across diagnostic frameworks as a down-gradient PV flux (under QG scaling). Future work should examine how stratification, vertical shear, and externally generated eddies modify or reinforce this mechanism in more realistic settings.

Turning to implications for large-scale models, the mesoscale flow–topography interactions identified here require that the bathymetry be well-resolved. Thus, they may not be explicitly represented in coarse-resolution ocean models. In the context

950 of Arctic circulation, this limitation becomes particularly relevant for future projections. Recent CMIP6 ensemble analyses indicate that surface stress over the Arctic Ocean is expected to intensify (Mulwijk et al., 2024), implying a stronger dynamical imprint of surface forcing on slope currents. As surface stress increases, the nonlinear mechanisms highlighted here are likely to play a larger role, meaning that coarse-resolution models may increasingly misrepresent the resulting boundary currents. Finally it should be mentioned that, while motivated by Arctic Ocean dynamics, the present analysis is relevant for any closed
955 f/H basin in the world oceans.

Code and data availability. Simulations are made using the Oceananigans.jl software package (Wagner et al., 2025). Code and data needed to reproduce the simulations, analysis and figures are available in a Zenodo repository (Sjur, 2026).

Appendix A: Additional figures

In Fig. A1, we verify the linear theory developed in Sec. 3.1 against a nearly linear numerical simulation. We perform a
960 model experiment with the wind-stress amplitude reduced by a factor of 10 and a forcing period of 128 days, so that the flow remains weak and laminar. We compare the simulated circulation to the linear predictions given by Eq. (13), evaluated along the mid-slope depth contour (highlighted in Fig. 2). The resulting scatterplot shows excellent agreement between the simulated circulation and the linear estimates, with data closely following the 1:1 line.

While animations of the time-evolution of relative vorticity and velocity fields in the two base-case simulations are included
965 in the supplementary video material, a snapshot of the relative vorticity field for a highly nonlinear state is included for reference in Fig. A2.

Figures A3 and A4 show the complete spatiotemporal evolution of the momentum-budget terms. Figure A3 ~~show~~ shows analysis on depth-contours, with terms defined in Eq. (12). Figure A4 show analysis on constant- y sections, with terms defined in Eq. (18).

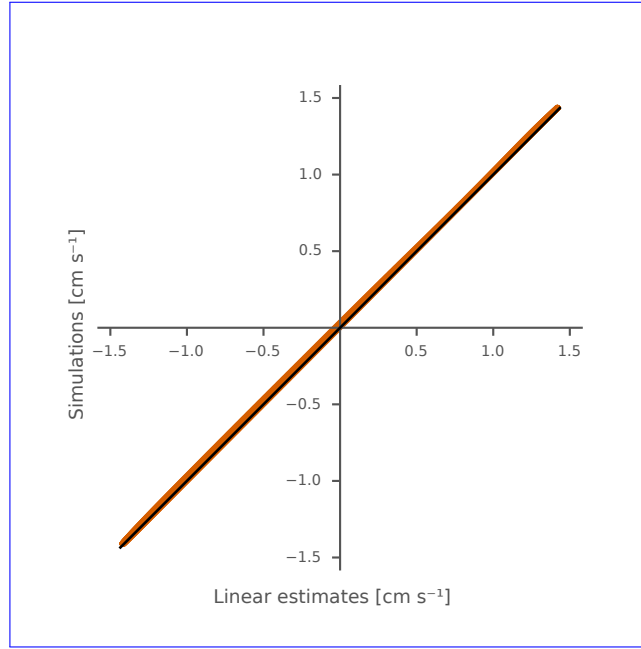


Figure A1. Scatterplot comparing circulation (normalized with contour length) along a depth contour in a nearly linear numerical simulation with linear estimates, given by Eq. (13). The relevant contour is highlighted in Fig. 2.

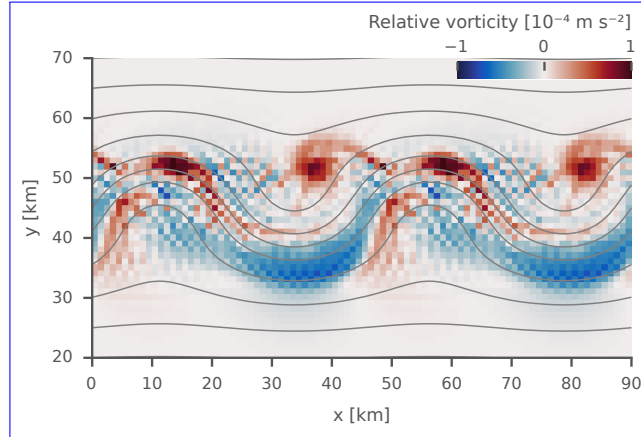


Figure A2. Snapshot of the relative vorticity field in simulations with a long forcing period (128 days). The snapshot is from a highly nonlinear state ($t=32$ days in Fig. 4). Gray lines indicate isobaths. Full animations of the relative vorticity field can be found in the supplementary video material.

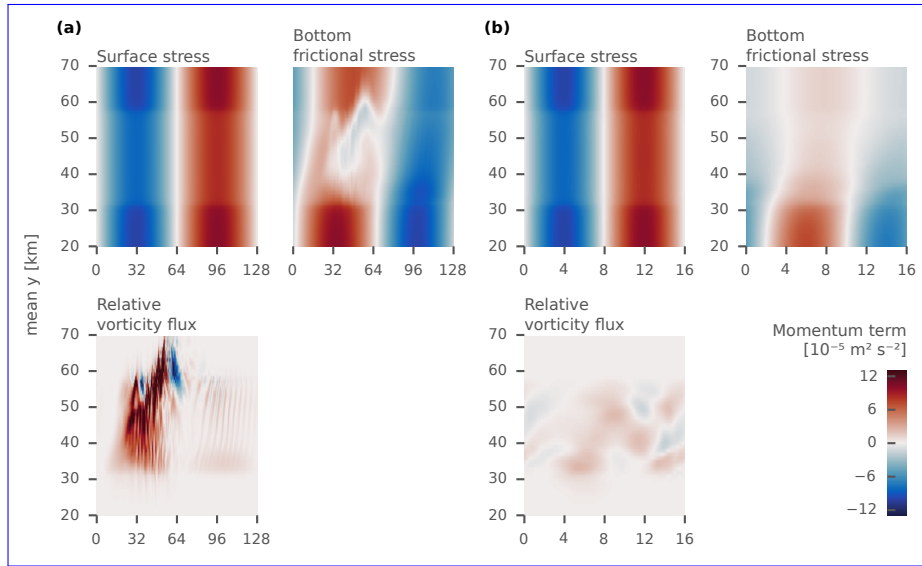


Figure A3. Hovmöller diagrams of momentum terms evaluated along depth-contours. Results for a forcing period of (a) 128 days, and (b) 16 days are shown. Each sub-figure displays surface stress, bottom frictional stress (proportional to the circulation itself), and the nonlinear relative vorticity flux (positive offshore). The terms are defined in Eq. (12). Time is shown on the x -axis and depth the y -axis. Note that depth-contours are equally spaced in y , not in depth; the corresponding contours are shown in Fig. 2.

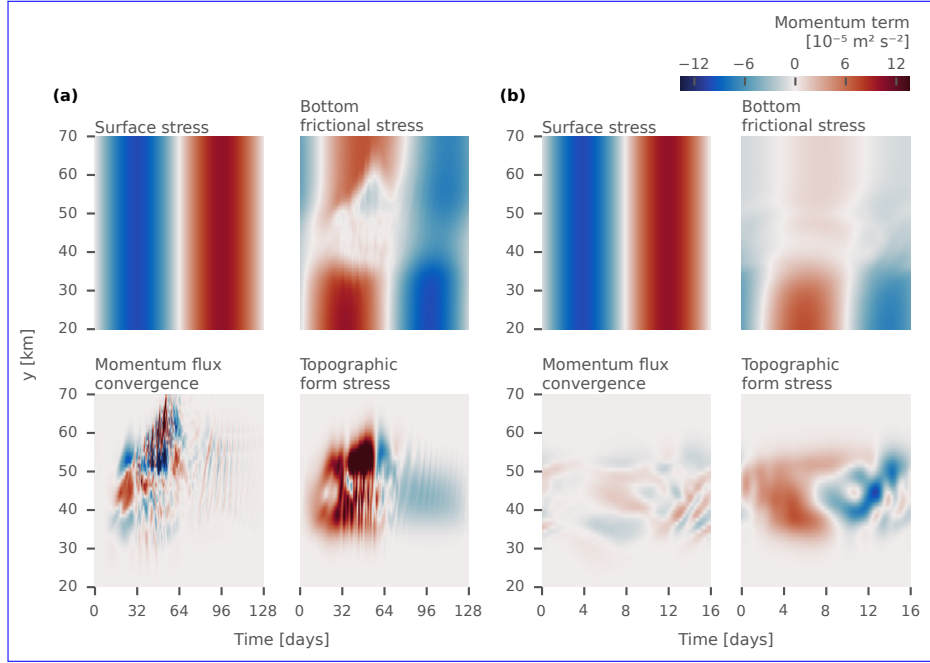


Figure A4. Hovmöller diagrams of momentum terms evaluated along constant- y sections. Results for a forcing period of (a) 128 days, and (b) 16 days are shown. Each sub-figure displays surface stress, bottom frictional stress (proportional to the along-slope circulation), the convergence of nonlinear momentum flux, and the topographic form stress. The terms are defined in Eq. (18). Time is shown on the x -axis and cross-slope position y on the y -axis.

970 Appendix B: Additional derivations

B1 Domain-integrated transport

We will extend our theoretical analysis to examine the total transport through the domain, which is the equivalent of the total transport around an ocean gyre. To do so, we integrate Eq. (18) in the y -direction, yielding:

$$\partial_t \langle Hu \rangle = -\langle H \partial_x \phi \rangle + \langle \tau^{sx} \rangle - R \langle u \rangle, \quad (\text{B1})$$

975 where

$$\langle (\cdot) \rangle \equiv \frac{1}{A} \iint (\cdot) dx dy \quad (\text{B2})$$

denotes the area average over the domain. Here, A is the total area. This equation expresses the time evolution of the domain-averaged transport as dependent on three terms: the domain-averaged topographic form stress, the domain-averaged surface stress, and the domain-averaged bottom drag, all in the x -direction. We have here assumed a no-flux condition at the

980 y -boundaries. Notably, the nonlinear momentum flux convergence in Eq. (18) then integrates to zero, reflecting that nonlinear advection only redistributes momentum within the domain.

Let us now consider the long-term evolution of the total transport, following Brink (2010). We integrate Eq. (B1) from $t = 0$ to $t = T$ and dividing by T , thereby calculating the time average. Taking the limit $T \rightarrow \infty$, and assuming that the transport remains finite, the left-hand side vanishes. Furthermore, the applied wind stress is purely periodic and has zero mean, implying
 985 that its long-time average is zero. The mean momentum balance therefore reduces to

$$-\overline{\langle H \partial_x \phi \rangle} = \overline{R \langle u \rangle} \quad (\text{B3})$$

where the overbar denotes a long-time average. Thus, in the statistically steady state, a residual transport can be sustained by a balance between topographic form stress and bottom drag. In the absence of form stress, no residual transport can be maintained.

990 Given its central role in the domain-integrated budget, the apparent absence of topographic form stress in the depth-following framework demands further comment. To address this, we return to the pressure term in Eq. (2) and evaluate it along depth-following contours $C(H)$, but this time integrating over the full domain. To do so, we need to express an area element in depth-following coordinates. We utilize that neighboring contours separated by a depth increment dH are spaced a normal distance $dn = dH/|\nabla H|$ apart. The area element in the depth-following coordinates is thus given by

$$995 \quad dA = dl \, dn = dl \frac{dH}{|\nabla H|}, \quad (\text{B4})$$

where dl is a line segment along a depth contour. Integrating first along the contours and then over the full depth range yields

$$\iint_A H \nabla \phi \cdot \mathbf{t} \, dA = \int_{H_y=0}^{H_y=L_y} H \left[\oint_{C(H)} |\nabla H|^{-1} \nabla \phi \cdot \mathbf{t} \, dl \right] dH. \quad (\text{B5})$$

In the last expression, the pressure contribution is weighted by the local inverse slope $|\nabla H|^{-1}$, and the inner contour integral does not generally vanish. Thus, while topographic form stress cancels when integrated along a single depth contour, it
 1000 still enters the domain-integrated balance. In practice, however, evaluating this expression is numerically challenging, as the integration along depth contours does not align with the discrete model grid, and great care must be taken to precisely cover the full domain.

B2 Amplitude of linear response to periodic forcing

We here derive the maximum amplitude of the solution to the linear depth following momentum equation (Eq. (12), retaining
 1005 only linear terms) forced by the sinusoidal surface stress (Eq. (10)). The solution $U(H, t) = \overline{\mathbf{u} \cdot \mathbf{t}}^H$ satisfies

$$\partial_t U + (R/H)U = F(t)/H, \quad (\text{B6})$$

where $\overline{F(t)} = \overline{\tau_s \cdot t^H} F(t) = \overline{\tau^s \cdot t^H}$. For a purely zonal and spatially uniform stress $\tau_{sx}(t) \hat{i} \tau^{sx}(t) \hat{i}$, the projection onto the contour reduces by the geometric factor $L_x/L_{C(H)}$, so that

$$F(t) = \frac{L_x}{L_{C(H)}} \tau_{sx}^{sx}(t). \quad (\text{B7})$$

1010 Writing the surface stress in polar form, we have

$$\tau_{sx}^{sx}(t) = \tau_0 \sin \cos(\omega t) = \Re\{-i\tau_0 e^{i\omega t}\}, \quad (\text{B8})$$

where $\Re\{\cdot\}$ is the real part. We seek a periodic solution of the form

$$U(H, t) = \Re\{\hat{u}(H) e^{i\omega t}\}. \quad (\text{B9})$$

Substituting Eqs. (B8) and (B9) into Eq. (B6) gives the relation

$$1015 \quad (i\omega + R/H) \hat{u}(H) = \frac{L_x}{L_{C(H)}} \frac{-i\tau_0}{H} \frac{\tau_0}{H}, \quad (\text{B10})$$

and therefore

$$\hat{u}(H) = \frac{L_x}{L_{C(H)}} \frac{-i\tau_0/H}{(R/H) + i\omega} \frac{\tau_0/H}{(R/H) + i\omega}. \quad (\text{B11})$$

Taking the magnitude yields

$$u_{\max}(H) = |\hat{u}(H)| = \frac{L_x}{L_{C(H)}} \frac{\tau_0}{H \sqrt{(R/H)^2 + \omega^2}}. \quad (\text{B12})$$

1020 Factoring out R/H , we obtain the form used in Eq. (15) in the main text,

$$u_{\max}(H) = \frac{L_x}{L_{C(H)}} \frac{\tau_0}{R} \left[1 + \left(\frac{\omega H}{R} \right)^2 \right]^{-1/2}. \quad (\text{B13})$$

Author contributions. ALPS and PEI designed the experimental framework. ALPS implemented the experiments, carried out the simulations and analyses, and produced the figures. All authors contributed to the theoretical development and interpretation of results. ALPS prepared the original manuscript, and all authors revised and approved the final version.

1025 *Competing interests.* The authors declare that they have no competing interests.

Acknowledgements. This work was supported by the Research Council of Norway through the TopArctic project (grant 314826). We thank Kenneth Brink for making the coastal trapped-wave code publicly available. ~~AI-tools~~ AI-tools have been used exclusively for language editing and coding support; all scientific analysis, data processing, interpretation, and conclusions were carried out by the authors.

References

- 1030 Bai, Y., Wang, Y., and Stewart, A. L.: Does Topographic Form Stress Impede Prograde Ocean Currents?, *Journal of Physical Oceanography*, 51, 2617–2638, <https://doi.org/10.1175/JPO-D-20-0189.1>, number: 8, 2021.
- Boyer, D. L., Zhang, X., and Pérenne, N.: Laboratory observations of rotating, stratified flow in the vicinity of a submarine canyon, *Dynamics of Atmospheres and Oceans*, 31, 47–72, [https://doi.org/10.1016/S0377-0265\(99\)00028-7](https://doi.org/10.1016/S0377-0265(99)00028-7), 2000.
- Bretherton, F. P. and Haidvogel, D. B.: Two-dimensional turbulence above topography, *Journal of Fluid Mechanics*, 78, 129–154, <https://doi.org/10.1017/S002211207600236X>, 1976.
- 1035 Brink, K. H.: Topographic Drag Due to Barotropic Flow over the Continental Shelf and Slope, *Journal of Physical Oceanography*, 16, 2150–2158, [https://doi.org/10.1175/1520-0485\(1986\)016<2150:TDDTBF>2.0.CO;2](https://doi.org/10.1175/1520-0485(1986)016<2150:TDDTBF>2.0.CO;2), 1986.
- Brink, K. H.: Topographic rectification in a forced, dissipative, barotropic ocean, *Journal of Marine Research*, 68, 337–368, <https://doi.org/10.1357/002224010794657209>, number: 3-4, 2010.
- 1040 Brink, K. H.: Topographic rectification in a stratified ocean, *Journal of Marine Research*, 69, 483–499, <https://doi.org/10.1357/002224011799849354>, 2011.
- Brink, K. H.: Stable barotropic coastal trapped wave modes: edge, shelf and Kelvin waves (version 3), <https://hdl.handle.net/1912/26311.3>, 2025.
- Broomé, S. and Nilsson, J.: Stationary Sea Surface Height Anomalies in Cyclonic Boundary Currents: Conservation of Potential Vorticity and Deviations from Strict Topographic Steering, *Journal of Physical Oceanography*, 46, 2437–2456, <https://doi.org/10.1175/JPO-D-15-0219.1>, 2016.
- 1045 Constantinou, N. C. and Young, W. R.: Beta-plane turbulence above monoscale topography, *Journal of Fluid Mechanics*, 827, 415–447, <https://doi.org/10.1017/jfm.2017.482>, 2017.
- Dritschel, D. G. and McIntyre, M. E.: Multiple Jets as PV Staircases: The Phillips Effect and the Resilience of Eddy-Transport Barriers, *Journal of the Atmospheric Sciences*, <https://doi.org/10.1175/2007JAS2227.1>, 2008.
- 1050 Egger, J.: Mountain torque and Rossby wave radiation, *Journal of the atmospheric sciences*, 55, 2937–2945, 1998.
- Eurin, D.: Circulation and cross-shelf exchanges over an irregular coastal topography, Ph.D. thesis, University of British Columbia, <https://doi.org/10.14288/1.0053157>, 1999.
- Gill, A. E.: Atmosphere–Ocean Dynamics, no. 30 in International Geophysics Series, Academic Press, San Diego, ISBN 0-12-283522-0, 1982.
- 1055 Haidvogel, D. B. and Brink, K. H.: Mean Currents Driven by Topographic Drag over the Continental Shelf and Slope, *Journal of Physical Oceanography*, 16, 2159–2171, [https://doi.org/10.1175/1520-0485\(1986\)016<2159:MCDBTD>2.0.CO;2](https://doi.org/10.1175/1520-0485(1986)016<2159:MCDBTD>2.0.CO;2), 1986.
- Held, I. M.: Stationary and quasi-stationary eddies in the extra-tropical atmosphere, in: Large-scale dynamical processes in the atmosphere, edited by Pearce, R. P. and Hoskins, B. J., pp. 127–168, Academic Press, 1983.
- 1060 Holloway, G.: Systematic forcing of large-scale geophysical flows by eddy-topography interaction, *Journal of Fluid Mechanics*, 184, 463–476, <https://doi.org/10.1017/S0022112087002970>, 1987.
- Holloway, G.: Neptune effect: statistical mechanical forcing of ocean circulation, in: Stochastic Modelling in Physical Oceanography, edited by Adler, R. J., Müller, P., and Rozovskii, B. L., pp. 207–219, Birkhäuser Boston, Boston, MA, ISBN 978-1-4612-7533-6 978-1-4612-2430-3, https://doi.org/10.1007/978-1-4612-2430-3_8, 1996.

- 1065 Isachsen, P. E., LaCasce, J. H., Mauritzen, C., and Häkkinen, S.: Wind-Driven Variability of the Large-Scale Recirculating Flow in the Nordic Seas and Arctic Ocean, *Journal of Physical Oceanography*, 33, 2534–2550, [https://doi.org/10.1175/1520-0485\(2003\)033<2534:WVOTLR>2.0.CO;2](https://doi.org/10.1175/1520-0485(2003)033<2534:WVOTLR>2.0.CO;2), 2003.
- Johnston, T. M. S., Sjur, A. L. P., Isachsen, P. E., and LaCasce, J. H.: Eddy- and Wind-Driven Circulation in the Enclosed Basins of the Norwegian Sea Evaluated Using a Model and Absolute Geostrophic Flow From Argo, *Journal of Geophysical Research: Oceans*, 130, e2024JC021990, <https://doi.org/10.1029/2024JC021990>, 2025.
- 1070 Kallmyr, J.-A. H., Nilsson, J., Chafik, L., and Isachsen, P. E.: The Time-Mean Arctic Ocean Circulation as Seen Through Satellite Altimetry and Hydrography, *Journal of Geophysical Research: Oceans*, 130, e2024JC022203, <https://doi.org/10.1029/2024JC022203>, 2025.
- Li, X., Wang, Q., Danilov, S., Koldunov, N., Liu, C., Müller, V., Sidorenko, D., and Jung, T.: Eddy activity in the Arctic Ocean projected to surge in a warming world, *Nature Climate Change*, 14, 156–162, <https://doi.org/10.1038/s41558-023-01908-w>, publisher: Nature Publishing Group, 2024.
- 1075 Masich, J., Chereskin, T. K., and Mazloff, M. R.: Topographic form stress in the Southern Ocean State Estimate, *Journal of Geophysical Research: Oceans*, 120, 7919–7933, <https://doi.org/10.1002/2015JC011143>, 2015.
- Miyama, T., Mitsudera, H., Nishigaki, H., and Furue, R.: Dynamics of a Quasi-Stationary Jet along the Subarctic Front in the North Pacific Ocean (the Western Isoguchi Jet): An Ideal Two-Layer Model, *Journal of Physical Oceanography*, 48, 807–830, <https://doi.org/10.1175/JPO-D-17-0086.1>, 2018.
- 1080 Muilwijk, M., Hattermann, T., Martin, T., and Granskog, M. A.: Future sea ice weakening amplifies wind-driven trends in surface stress and Arctic Ocean spin-up, *Nature Communications*, 15, 6889, <https://doi.org/10.1038/s41467-024-50874-0>, 2024.
- Munk, W. H. and Palmén, E.: Note on the Dynamics of the Antarctic Circumpolar Current, *Tellus A: Dynamic Meteorology and Oceanography*, 3, <https://doi.org/10.3402/tellusa.v3i1.8609>, 1951.
- 1085 Nilsson, J., Kallmyr, J.-A. H., and Isachsen, P. E.: Topographic Steering of the Upper Arctic Ocean Circulation by Deep Flows, *Tellus A: Dynamic Meteorology and Oceanography*, 76, 206–226, <https://doi.org/10.16993/tellusa.4072>, 2024.
- Nøst, O. A. and Isachsen, P. E.: The large-scale time-mean ocean circulation in the Nordic Seas and Arctic Ocean estimated from simplified dynamics, *Journal of Marine Research*, 61, 175–210, <https://doi.org/10.1357/002224003322005069>, 2003.
- Salmon, R.: *Lectures on Geophysical Fluid Mechanics*, Oxford University Press, ISBN 0-19-510808-6, 1998.
- 1090 Salmon, R., Holloway, G., and Hendershott, M. C.: The equilibrium statistical mechanics of simple quasi-geostrophic models, *Journal of Fluid Mechanics*, 75, 691–703, <https://doi.org/10.1017/S0022112076000463>, 1976.
- Samelson, R. M. and Allen, J. S.: Quasi-Geostrophic Topographically Generated Mean Flow over the Continental Margin, *Journal of Physical Oceanography*, 17, 2043–2064, [https://doi.org/10.1175/1520-0485\(1987\)017<2043:QGTGMF>2.0.CO;2](https://doi.org/10.1175/1520-0485(1987)017<2043:QGTGMF>2.0.CO;2), 1987.
- Sjur, A. L.: [alpsjur/temporal-topo-flow: v1.1.0](https://doi.org/10.5281/zenodo.20973293), <https://doi.org/10.5281/zenodo.20973293>, 2026.
- 1095 Sjur, A. L. P., Isachsen, P. E., Nilsson, J., LaCasce, J. H., and Ryseth, M. D.: The Wind-Driven Time-Variable Circulation in the Arctic Mediterranean, *Journal of Geophysical Research: Oceans*, 130, e2024JC021713, <https://doi.org/10.1029/2024JC021713>, 2025.
- Spall, M. A.: Wind-driven flow over topography, *Journal of Marine Research*, 74, 229–248, 2016.
- Stevens, D. P. and Ivchenko, V. O.: The zonal momentum balance in an eddy-resolving general-circulation model of the southern ocean, *Quarterly Journal of the Royal Meteorological Society*, 123, 929–951, <https://doi.org/10.1002/qj.49712354008>, 1997.
- 1100 Stewart, A. L. and Hogg, A. M.: Reshaping the Antarctic Circumpolar Current via Antarctic Bottom Water Export, *Journal of Physical Oceanography*, 47, 2577–2601, <https://doi.org/10.1175/JPO-D-17-0007.1>, 2017.

- Timmermans, M. and Marshall, J.: Understanding Arctic Ocean Circulation: A Review of Ocean Dynamics in a Changing Climate, *Journal of Geophysical Research: Oceans*, 125, e2018JC014378, <https://doi.org/10.1029/2018JC014378>, 2020.
- Vallis, G. K.: *Atmospheric and Oceanic Fluid Dynamics*, Cambridge University Press, Cambridge, 2 edn., ISBN 9781108400157, 2017.
- 1105 Wagner, G. L., Silvestri, S., Constantinou, N. C., Ramadhan, A., Campin, J.-M., Hill, C., Chor, T., Strong-Wright, J., Lee, X. K., Poulin, F., Souza, A., Burns, K. J., Marshall, J., and Ferrari, R.: High-level, high-resolution ocean modeling at all scales with Oceananigans, <https://arxiv.org/abs/2502.14148>, 2025.
- Ward, M. L. and Hogg, A. M.: Establishment of momentum balance by form stress in a wind-driven channel, *Ocean Modelling*, 40, 133–146, <https://doi.org/10.1016/j.ocemod.2011.08.004>, 2011.
- 1110 Woodgate, R.: Arctic Ocean Circulation: Going Around at the Top of the World, *Nature Education Knowledge*, 4, 8, <https://www.nature.com/scitable/knowledge/library/arctic-ocean-circulation-going-around-at-the-102811553/>, 2013.
- Woodgate, R. A., Aagaard, K., Muench, R. D., Gunn, J., Björk, G., Rudels, B., Roach, A. T., and Schauer, U.: The Arctic Ocean Boundary Current along the Eurasian slope and the adjacent Lomonosov Ridge: Water mass properties, transports and transformations from moored instruments, *Deep Sea Research Part I: Oceanographic Research Papers*, 48, 1757–1792, [https://doi.org/10.1016/S0967-0637\(00\)00091-1](https://doi.org/10.1016/S0967-0637(00)00091-1),
 1115 2001.
- Yang, J., Proshutinsky, A., and Lin, X.: Dynamics of an idealized Beaufort Gyre: 1. The effect of a small beta and lack of western boundaries, *Journal of Geophysical Research: Oceans*, 121, 1249–1261, <https://doi.org/10.1002/2015JC011296>, 2016.
- Zhang, W. and Lentz, S. J.: Wind-Driven Circulation in a Shelf Valley. Part I: Mechanism of the Asymmetrical Response to Along-Shelf Winds in Opposite Directions, *Journal of Physical Oceanography*, <https://doi.org/10.1175/JPO-D-17-0083.1>, 2017.
- 1120 Zhang, X., Boyer, D. L., Pérenne, N., and Renouard, D. P.: Mean flow generation along a sloping region in a rotating homogeneous fluid, *Journal of Geophysical Research: Oceans*, 101, 28 597–28 614, <https://doi.org/10.1029/96JC02834>, 1996.