

# Author comment in reply to Referee 1

NPG, egusphere-2026-753

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I thank Referee 1 for an encouraging and constructive reading, and in particular for identifying that the main weakness is one of ordering rather than content. I agree with the structural suggestions and group my responses accordingly. All of the changes described below will be made available in the revised version.

*Overall reorganization.* I restructure so that intuition precedes results. The Mittag-Leffler material formerly in Section 4.6 moves into Section 2.2.2 where it is first needed, and the physical interpretation formerly in lines 261 to 267 moves forward into Section 2.3. Section 4 now opens with a single table listing every numerical configuration, grid size,  $N_t$ , the value of  $\beta$  or  $H$ , the value of  $\alpha$ , and the number of realizations, before any result is presented, so that configurations and outcomes are no longer interleaved. The fractional Brownian results formerly split across Sections 4.1, 4.4 and 4.5 are consolidated into one section.

*Section 2.2.2.* I add an opening that explains why a generalization of the exponential is required, namely that anomalous relaxation departs from exponential decay and the Mittag-Leffler function interpolates between stretched exponential and power law behavior, which links to the operators introduced next. I expand the description of the two parameters and promote the relaxation figure, formerly Figure 6, so that it appears here as the first figure rather than at the end of the paper.

*Line 137 and the role of  $\alpha$ .* I introduce memory at this point, since it distinguishes Fickian from anomalous transport, and I add intuition for  $\alpha$  as the parameter governing the spatial jump distribution before it is used quantitatively.

*Lines 149 and 152 to 155.* I sharpen the distinction between space and time. I agree that the statement about not requiring a separate spatial operator was unclear and overstated, and I have rewritten it so that the roles of  $\mu$  and  $\alpha$  are kept distinct and neither is presented as substituting for the other. This point, together with the later question of whether  $H$  and  $\alpha$  are related, touches a matter also raised by Referee 2, and the revised version addresses it consistently. The two parameters introduce memory in different channels,  $\alpha$  in the spatial correlation and  $\mu$  in the temporal relaxation, and they are independent.

*Section 3.1.* I add the interpretation requested, that small eigenvalues correspond to smooth large scale modes and large eigenvalues to small scale modes, and I note the Fiedler value as the connectivity scale. This also clarifies lines 171 to 177 and 226 to 230, which I cross link so that the fractional weighting is understood as the operation that re-ranks these scales.

*References to environments, lines 198 and 375.* I reference equations directly. The scaling result is equation (17), and the result formerly cited as a theorem is referred to by its equation number in the text.

*Steepening, lines 204 to 209.* I state plainly that steepening concentrates the reweighted variance into the leading modes, and, in light of Referee 2, I specify that this concentration is in the scale weighted metric rather than a claim about Euclidean representation of the field.

*Figures 1 and 2.* I superimpose the reference slopes on both the standard and the fractional curves, and I correct the color mismatch between the two panels of the relaxation figure.

*Treatment of time, lines 294, 311 to 313, 342 to 344 and 363.* The referee is correct that the time dimension was underspecified. For the spatial validations the fields are white in time with independent snapshots, so the spatial spectra are effectively single field quantities and ensemble averaging is used only to estimate uncertainty. I state this explicitly, move the parameter definitions to the configuration table, and separate method from results in Section 4.2.

*Lines 478 to 479 and 583 to 585.* The  $L^{2\alpha}$  scaling reflects self similar, scale free correlation structure, and I make this connection explicit. On the Caputo derivative, I add the comparison requested in the regime where both operators are defined,  $\mu \in (0, 1)$ , where the asymptotic scaling coincides and the operators differ only in the handling of the initial transient.

*Sections 4.4, 4.5 and 4.6.* Sections 4.4 and 4.5 are merged into the unified fractional Brownian section, and the Mittag-Leffler material of Section 4.6 moves to Section 2.2.2 as noted above, so that the results section contains only results.

I thank the referee for these suggestions, which improve the readability of the manuscript. All of the changes above will be made available in the revised version.