Including CO_2 on the x-axis

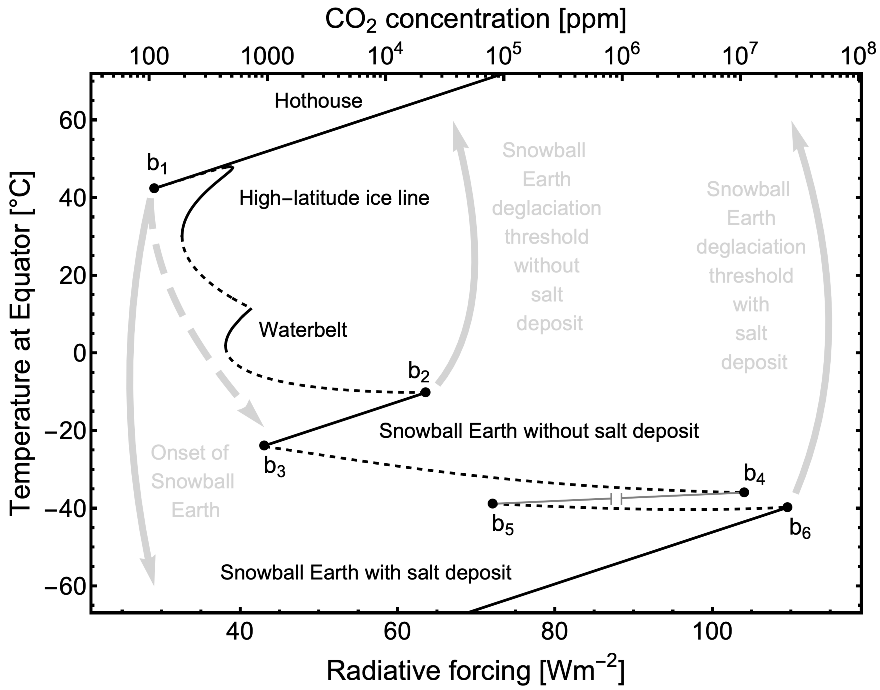

Figure 1: Bifurcation diagram for the EBM with a salt-albedo feedback (Eq. (1) with the albedo function in Eq. (2) from the manuscript). Radiative forcing on the x-axis is defined as $\Delta A = A_0 - A$, where A_0 is the reference value given in Tab. 1. The top x-axis shows approximate atmospheric CO_2 concentrations (derived in below). Black solid lines indicate stable equilibrium states and dashed lines indicate unstable states. Stable climate states are labeled. Gray solid arrows show transitions in the Snowball Earth hysteresis. The dashed arrow indicates the unfeasible transition from a warm climate to a Snowball Earth without salt deposits. Model parameters are given in Tab. 1. The gray line between point b_4 and b_5 represents a truncation where Snowball Earth states with multiple salt deposits are omitted, the situation here is analogous to the complicated ice distributions reported in Samuelsberg and Jakobsen (2025). These solutions are unstable.

Estimating CO_2 concentrations from radiative forcing

We can relate the radiative forcing ΔA to an approximate increase in atmospheric CO_2. The increase in radiative forcing ΔA associated with changing the CO_2 concentrations from a reference level pC_0 (in ppm) to pC_1, can be estimated by

$$\Delta A = k \ln\left(\frac{pC_1}{pC_0}\right), \tag{1}$$

where $k = 6.0$ W m^{-2} (Liu and Peltier 2010). Choosing a suitable reference level with Neoproterozic model parameters is problematic. Here we have chosen to use the modern climate as a reference point. Following North (1975), we fix the ice-line latitude at 72°N, corresponding to the modern climate, and assign the associated A value to the reference level pC_0. We set $pC_0 = 330$ ppm and determine pC_1 using Eq. (1), which is displayed on the top x-axis in Fig. 1. The reliability of these concentrations is questionable, considering the changes in solar insolation from the Neoproterozoic to today. Adjusting the A value associated with pC_0 to account for these changes was not found to improve the CO_2 estimates. We observe that the bifurcation point b_2 in Fig. 1 is consistent with the range of deglaciation thresholds reported in the simulations of Wu and Liu (2024). However, the concentrations at the Snowball Earth initiation b_1 are likely too low. Initial concentrations for Snowball Earth are uncertain, but estimates have found it to be around 10^3 ppm (Mills et al. 2019).

Considering these issues, we believe the second x-axis in Fig. 1 with CO_2 concentrations can be misleading to readers. Reading Fig. 1 with the second axis, one may get the impression that specific CO_2 concentrations correspond to points in the bifurcation diagram, representing a given model output for a given CO_2 input. Including a realistic CO_2 treatment would require extending the model formulation, which lies beyond the scope of the present work. We therefore choose to omit considerations of CO_2 concentrations in the final manuscript.

References

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