

Supplement of "Emergent multiscale interdependence of land and ocean temperatures"

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S1 Empirical Mode Decomposition for SST and Land Temperatures

S2 Common-trend surrogate test

We tested whether the observed multiscale interdependence could arise solely from the simultaneous evolution of the nonlinear land-air and sea-surface temperature trends. Let

$$\mathcal{X}(t) = \{c_1^X(t), \dots, c_{K-1}^X(t), r^X(t)\} \quad (\text{S1})$$

and

$$\mathcal{Y}(t) = \{c_1^Y(t), \dots, c_{K-1}^Y(t), r^Y(t)\} \quad (\text{S2})$$

denote the empirical-mode-decomposition representations of the two records. Here, c_k denotes the k th oscillatory intrinsic mode function (IMF), while $r(t)$ is the nonlinear residual trend.

- 10 At multiscale level k , the first k components were treated as coordinates of the corresponding state vector, following exactly the construction employed in the observed dynamical analysis. Specifically, the state vectors were defined as

$$\mathbf{x}_k(t) = [c_1^X(t), \dots, c_k^X(t)], \quad (\text{S3})$$

$$\mathbf{y}_k(t) = [c_1^Y(t), \dots, c_k^Y(t)]. \quad (\text{S4})$$

The components were therefore not summed into scalar cumulative reconstructions.

- 15 For each surrogate realization, a stationary block-bootstrap index sequence was generated independently for the two climate variables. Within each variable, the same index sequence was applied to all oscillatory IMFs. This construction preserves their

internal cross-scale alignment and short-range temporal dependence. By contrast, the use of independent index sequences for land-air and sea-surface temperature disrupts their cross-variable covariability. The observed residual trends $r^X(t)$ and $r^Y(t)$ were retained without modification. Consequently, every surrogate contains the original pair of nonlinear warming trends but
 20 lacks the observed organization between the oscillatory components of the two records.

We generated $N_{\text{surr}} = 500$ surrogate pairs using a mean block length of 120 months. The complete dynamical analysis was repeated for every realization, yielding

$$\alpha^{(b)}(s, t), \quad b = 1, \dots, N_{\text{surr}}, \quad (\text{S5})$$

where s denotes the EMD-associated timescale. The mean of the null distribution was calculated as

$$25 \quad \mu_\alpha(s, t) = \frac{1}{N_{\text{surr}}} \sum_{b=1}^{N_{\text{surr}}} \alpha^{(b)}(s, t), \quad (\text{S6})$$

and its standard deviation as

$$\sigma_\alpha(s, t) = \left[\frac{1}{N_{\text{surr}} - 1} \sum_{b=1}^{N_{\text{surr}}} \left(\alpha^{(b)}(s, t) - \mu_\alpha(s, t) \right)^2 \right]^{1/2}. \quad (\text{S7})$$

The standardized departure of the observed value from the null distribution was then defined as

$$z_\alpha(s, t) = \frac{\alpha_{\text{obs}}(s, t) - \mu_\alpha(s, t)}{\sigma_\alpha(s, t)}. \quad (\text{S8})$$

30 Because the hypothesis of interest concerns an enhancement of interdependence, significance was evaluated using the one-sided empirical probability

$$p_+(s, t) = \frac{1 + \sum_{b=1}^{N_{\text{surr}}} \mathbb{I} \left[\alpha^{(b)}(s, t) \geq \alpha_{\text{obs}}(s, t) \right]}{N_{\text{surr}} + 1}, \quad (\text{S9})$$

where $\mathbb{I}[\cdot]$ is the indicator function. Colored cells in Fig. S3 satisfy both $p_+ < 0.05$ and $z_\alpha > 0$. Gray cells indicate that the pointwise null hypothesis could not be rejected. These probabilities are pointwise and do not constitute a field-wide or false-
 35 discovery-rate-corrected significance statement.

S2.1 Results of the common-trend test

The common-trend null produces substantially weaker interdependence than that observed in the original records. Before approximately the mid-twentieth century, positive departures are comparatively sparse and intermittent, although isolated significant episodes occur at several timescales. From approximately the 1950s onward, significant positive departures become
 40 more frequent and temporally persistent. Their clearest concentration occurs at decadal and multidecadal timescales, with several intervals reaching $z_\alpha \gtrsim 3$. Significant departures at shorter timescales remain more fragmented.

This temporal and scale-dependent structure is inconsistent with an explanation based exclusively on the smooth coevolution of the two nonlinear trends. Because both observed trends are retained in every surrogate, their common warming tendency

contributes to both the observed and null calculations. The excess interdependence therefore involves the organization of the
45 oscillatory variability that is destroyed when the two sets of IMFs are independently block-resampled. The test does not establish a direction of influence or identify a specific physical mechanism. Instead, it provides statistical evidence for dynamical interdependence beyond that induced by the common trends.

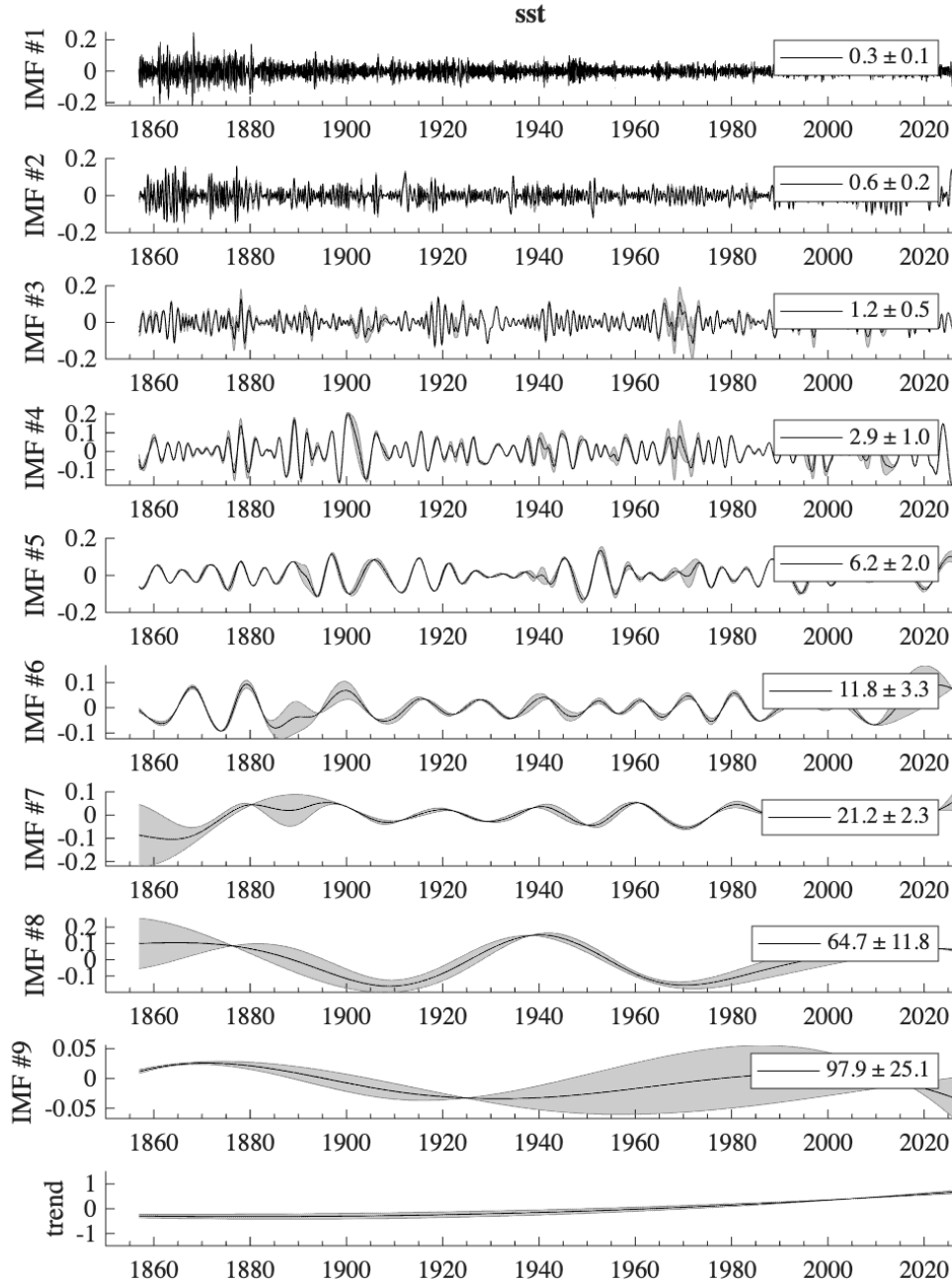


Figure S1. Intrinsic Mode Functions (IMFs) and the trend of the SST. The legend in each panel reports the average period and its associated error in years. The gray shaded area marks the standard deviation of IMFs derived from the ensemble of decompositions.

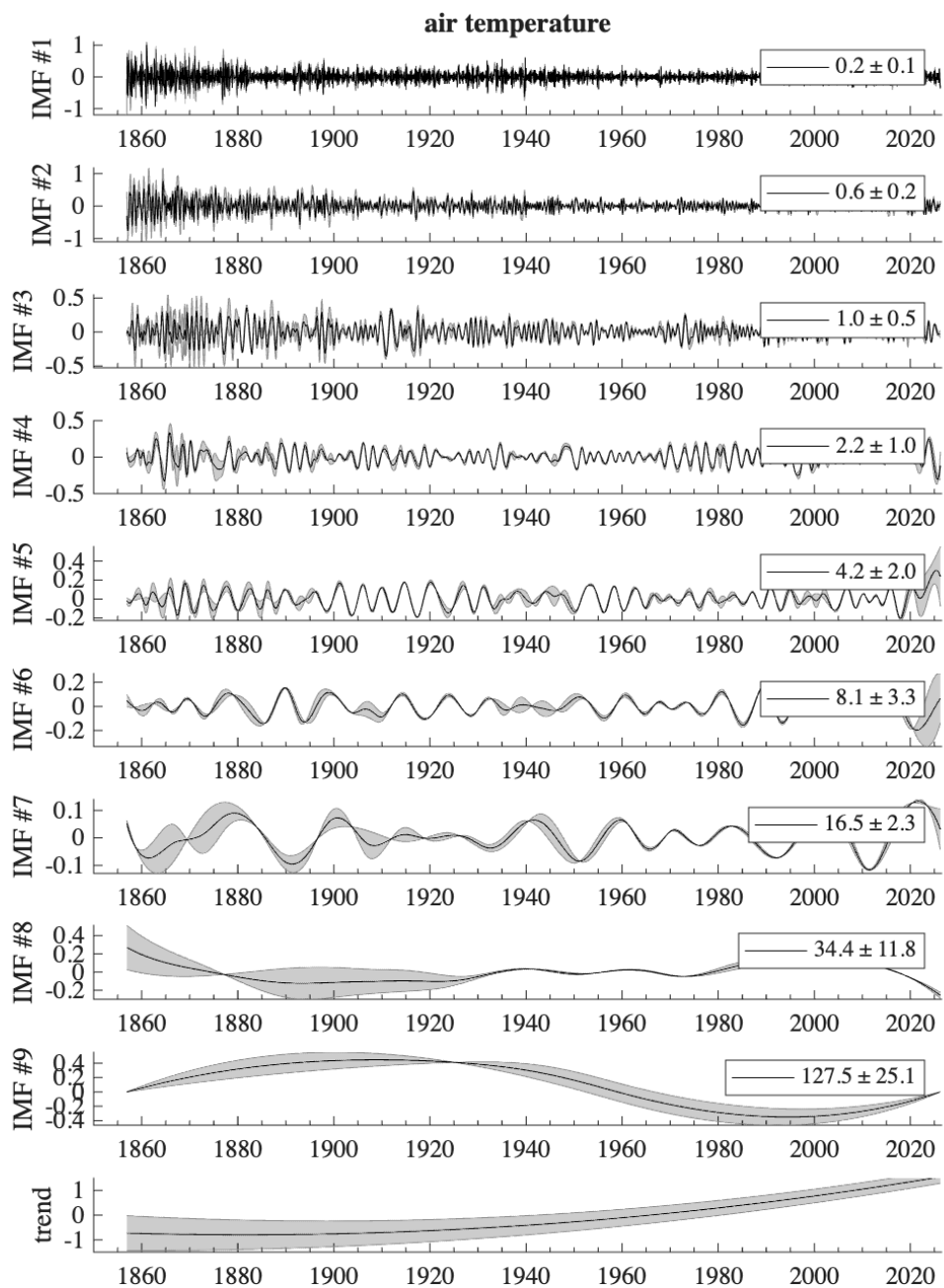


Figure S2 Same as Fig. S1 but for land temperatures.

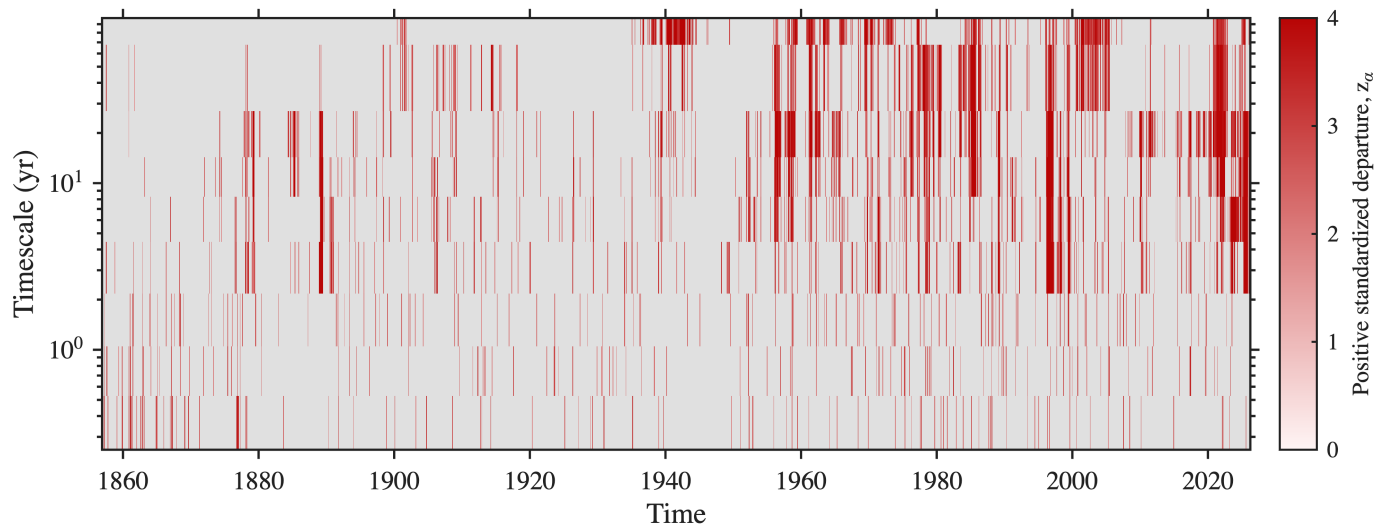


Figure S3. Dynamical interdependence beyond the common-trend null. Positive standardized departures of the observed interdependence α from the common-trend surrogate distribution are shown where the one-sided empirical test gives $p < 0.05$; nonsignificant cells are gray. The surrogates preserve the nonlinear trends of both records while independently block-resampling their oscillatory components. Significant departures become more frequent and persistent after the mid-twentieth century, predominantly at decadal to multidecadal timescales. Significance is pointwise and is not corrected for multiple comparisons.