Authors' Response to Reviews of

Convolution Based Techniques for Computing Self Attraction and Loading in MOM6

A. Chen, H. Wang, B. Arbic, and R. Krasny

RE: egusphere-2026-654

Title: Convolution Based Techniques for Computing Self Attraction and Loading in MOM6

Dr. Qiang Wang

Topic Editor

EGU Geoscientific Model Development

Dear Dr. Qiang Wang

Thank you for the second round of reviews of our article referenced above. The article has been revised to address the comments of the Referees. The revised manuscript has been submitted. Here is a list of changes that we have made in response to the referee comments:

- We have also implemented and tested a Cesaro regularized tabular Green's function. The error results and plots through the paper have been updated accordingly.
- Various citations have been added and removed
- Various wording changes have been made

I have uploaded an updated draft of the paper, as well as a LaTex diff copy highlighting the changes between the updated draft of the paper and the previous version. In the following pages, detailed responses to the referee comments are provided.

Sincerely,

Anthony Chen, on behalf of co-authors:

He Wang

Robert Krasny

Brian Arbic

1. Reviewer 1

RC: I'm sorry to say that during the first round of reviews I did not pick up on a major flaw in this work. It is only after reading the authors' response to reviewer comments and their revised paper did I realize this. Not only does the paper need substantial changes, but their coding that has been put into MOM6 also needs substantial changes.

Their implementation of the convolution method is the problem. Evidently to evaluate the convolution integral (Eqn.9), they think they need to use the summation in Eq (10) to evaluate the Green's function *at every integration point*. That is clearly expensive, so they replace the correct Green's function (Eq.10) with their approximation (Eq.13), which has incorrect Love numbers (as I noted in my first review).

The problem is that Eq(10) does NOT need to be evaluated at every integration point. In fact, the evaluation of the Green's function (as a function of angular distance psi) is done ONE TIME, and it could certainly be done outside MOM6. You then have a Green's function table that gives G(psi) for all values of psi between 0° and 180° – just as Farrell did in his original paper (see his Table A3). It is that tabular Green's function that should be used to evaluate the convolution integral (Eq.9).

There is thus no reason to use an incorrect Green's function. You keep the correct loading Love numbers h',k', and you take the sum in Eq(10) to very high degree – just as Farrell described it. (And Martens discusses this summation extensively, too.)

All of the geodetic papers you now cite (e.g., Agnew, Martens, etc.) do this. They compute their Green's function one time, taking as much CPU time as needed since it's only done one time, and that (numerically tabulated) Green's function is then stored and used for all applications. In this case, you use the same tabulated function for every integration point and at every MOM6 time step.

Once this is done, it will be interesting to see the revised Figures 2-6. As it is now, I'm really not sure what the "correct" SAL field should look like, whether it is the SHT(N=200) or the Convolution with the flawed Green's function or something else.

Again, I'm sorry I did not spot this during the first round of reviewing.

AR: We thank Referee 1 for their second round of comments in this review process.

Following the suggestion of Referee 1, we have implemented a tabular Green's function in MOM6 as well, and tested it with the CSFMM to obtain error results. For this testing, we tried four different tabular Green's functions. Starting with the Legendre series

$$\nabla G_{\mathrm{SAL}}(\vec{x}\cdot\vec{y}) = \nabla \cos\gamma(\vec{x},\vec{y}) \sum_{n=0}^{\infty} \frac{3\rho_w(1+k'_n-h'_n)}{4\pi\rho_e} P'_n(\vec{x}\cdot\vec{y}), \tag{1}$$

which is Eq. 17 in the paper, we define

$$s(\mathbf{x}\cdot\mathbf{y}) = \sum_{n=0}^{\infty}(1+k'_n-h'_n)P'_n(\mathbf{x}\cdot\mathbf{y}), \tag{2}$$

as the sum which needs to be tabulated. Working directly with this sum is not feasible, as it is very highly oscillatory, and because of this, we tested four different regularization procedures. The first tabular Green's

function was computed by directly applying Cesàro summation to this sum. The remaining three were based on Kummer's transformation, where the sum was transformed as

$$s(\mathbf{x}\cdot\mathbf{y}) = \sum_{n=1}^{\infty}\left((1-b_0) + \frac{a_1-b_1}{n} + \left(k'_n - \frac{a_1}{n} - h'_n + b_0 + \frac{b_1}{n}\right)\right) P'_n(\mathbf{x}\cdot\mathbf{y}). \tag{3}$$

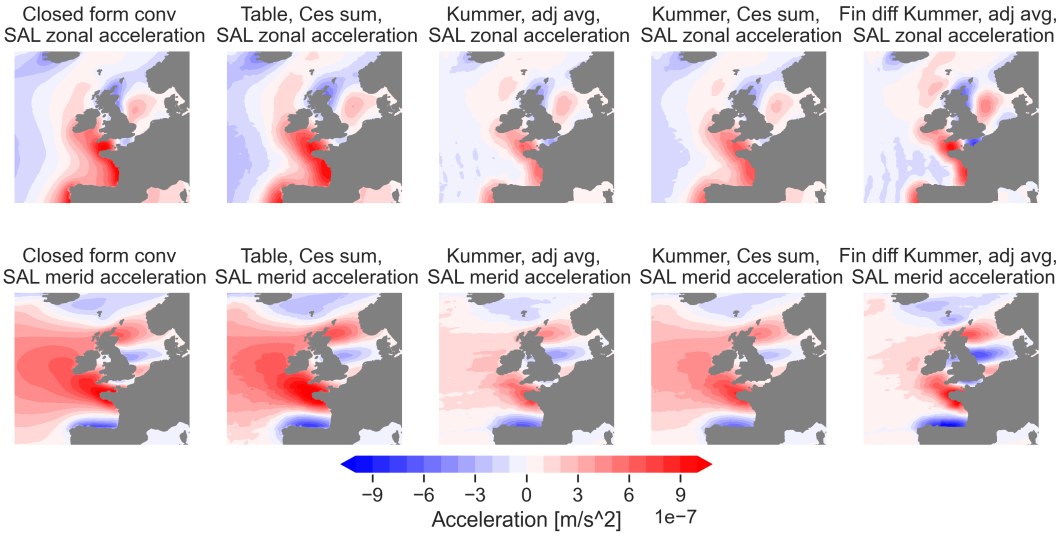
This is similar to the procedure performed in the paper to compute the approximate closed form Green's function. The remainder term is then defined as

$$r(\mathbf{x}\cdot\mathbf{y}) = \sum_{n=1}^{\infty}\left(k'_n - \frac{a_1}{n} - h'_n + b_0 + \frac{b_1}{n}\right) P'_n(\mathbf{x}\cdot\mathbf{y}), \tag{4}$$

as the sum of interest. To regularize this, three different techniques are used. Guo et. al., 2004, presents a regularization procedure based on iteratively averaging adjacent terms in the sum:

$$(c_0, c_1, c_2, c_3, c_4, c_5, \ldots) \longrightarrow \left(\frac{c_0}{2}, \frac{c_0+c_1}{2}, \frac{c_1+c_2}{2}, \frac{c_2+c_3}{2}, \frac{c_3+c_4}{2}, \frac{c_4+c_5}{2}, \ldots\right). \tag{5}$$

This damps high wavenumbers, and was used to compute the second Tabular Green's function. The third tabular Green's function was based on Cesàro regularization applied to $r(\mathbf{x}\cdot\mathbf{y})$, and the fourth tabular Green's function was based on applying finite differencing to the adjacent term averaged Legendre series pre-differentiation. When applied to the SSH snapshot in Fig. 1, these all lead to different SAL acceleration profiles.



In this, we can see that the Green's functions derived using the adjacent term averaging procedure are quite different in spatial structure. The Cesàro sum applied to the sum directly has the most agreement with the closed form Green's function, while doing Kummer's transformation and Cesàro regularizing the remainder term is similar but has some differences in magnitude.

These all lead to different error results as well, when run in the tidal experiment configuration as described in the paper. On the 1/3rd degree grid, the deep non-polar M2 tidal RMSE was 4.10 cm, using the closed

form Green's function. With the first tabulated Green's function, based on directly summing the derivative of the Legendre series with Cesaro regularization, the RMSE improves to 4.05 cm. With this tabulated Green's function, the difference in the degree-1 terms of the SAL zonal acceleration spectra also is much smaller. However, the approaches based on Kummer's transformation have a larger error. The Kummer remainder sum, regularized with Cesaro summation, results in an RMSE of 4.36 cm, while the Kummer remainder sum, regularized with adjacent term averaging, results in an RMSE of around 4.78 cm. The Green's function derived using finite differencing resulted in the model blowing up due to issues with applying finite differences to the singular point.

Comparing the approximated closed form Green's function and the Cesaro tabulated Green's function, the deep non-polar RMSE drops from 4.10 cm to 4.05 cm. Comparing this improvement to the difference in the M2 tidal error when properly accounting for tidal geocenter motion presented in Desai and Ray, 2014, which reports a 0.08 cm improvement. Thus, these two improvements appear to be of similar orders of magnitude. However, it is difficult to attribute the 0.05 cm improvement to a more correct value at any particular wavenumber, and it is also hard to say if the 0.05 cm is robust with respect to external factors, such as topography and different wave drag schemes.

Working only with the Cesaro tabular Green's function, we also ran experiments on the 1/12th degree grid as well. On this grid, the Cesaro tabular Green's function has a deep non-polar RMSE of 2.32 cm, compared to 2.36 cm with the closed form Green's function. The results with the Cesaro tabular Green's function are now incorporated into the paper.

RC: Minor point: At line 96, remove the citation to Chen et al. That paper discussed how measurements of GRACE alias the S2 tide; it has nothing to do with aliasing of spherical harmonics.

AR: We have removed this citation.

RC: Perhaps not so minor: In their Reply to my review, they write "it is our understanding that the M2 tide shouldn't have a wavenumber 1 signal." That understanding is incorrect. It definitely does. In a center-of-mass reference frame, the whole earth system has no degree-1 terms, but the ocean tide itself does. And in any event, MOM6 will not be using a C-M reference frame. When the authors' fix their Green's function method (as above), perhaps the degree-1 terms in Figure 6 will more closely agree.

AR: We have found that using the tabular Green's function produces much better agreement in the degree-1 terms in Figure 6. This figure has been updated with the Tabular Green's function.

2. Reviewer 2

RC: Coming in for the second round of reviews, I have only a few minor points and suggestions. The manuscript is already well-crafted in both scientific content and presentation, benefiting from a carefully designed set of experiments. I think the convolution-based SAL approach presented here will be very valuable for regional tide modeling efforts.

AR: We thank Referee 2 in advance for their comments and suggestions. Following their comments, we have made a number of changes through the paper.

RC: Figure 6, following up on a comment by Reviewer #1: The degree-1 terms in the SAL zonal acceleration from the convolution and SHT approaches differ by almost an order of magnitude. In their response, the authors attribute the mismatch to errors in the LLNs and argue that degree-1 terms of M2 should be zero anyway. I think the authors should address the point with greater rigor. Notably, degree-1 terns relate to tidal geocenter motion, which is not zero for M2 (https://doi.org/10.1002/2014GL059614). The mismatch evident from Figure 6 may indeed stem from approximation errors in the calculation of the LLNs, which are then magnified by the gradient computation. Please investigate the issue and include a proper discussion in the manuscript.

AR: Thank you for this reference. Testing with a tabulated Green's function, as suggested by Reviewer 1, where the LLNs are used exactly, does result in much closer agreement in the degree-1 terms of the SAL zonal acceleration spectra between the convolution and the SHT approaches. This suggests that the previous mismatch was due to the approximations used in the derivation of the closed form Green's function.

RC: L43/L77: I don't need the citation but it would be natural to point readers to the original paper on SHT-based inline SAL computation in an ocean tide model (https://doi.org/10.1029/2018JC013959).

L48: Untrue. Stepanov and Hughes (2004) incorporated SAL in their barotropic model using the spherical convolution integral, so please acknowledge. You could use this citation to your advantage by pointing out the computational overhead mentioned in that paper (factor 9).

L93-94: I suggest to end the sentence after '… N = 40 (Brus et al. 2023)' and move the clause on error behavior to the subsequent discussion. This is an important point that should not be introduced in passing.

L166: A short explanation or definition of the interpolation degree n_0 would be useful.

AR: Thank you for these suggestions. We have made these changes.

RC: L8: 'against tidal datasets based on satellite altimetry' simplify, for example 'against altimetry-based tides'

L78: '… decomposition of the sea surface height η (i.e., the tidal mass anomaly) is computed, …'.

Please check whether equations are referenced according to the style guide throughout the manuscript (Eq. or Equation? With parentheses?).

L201: 'of' missing after 'approximation'?

L210f: Inconsistent use of tenses.

L226: Maybe you can use this passage to define the term 'non-polar' in preparation for Section 4.

AR: Thank you for these suggestions. We have made these changes.