



# A unified account of Earth's climate variability: dominant modes as eigenmodes of one stochastic operator

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**Abstract.** The Madden–Julian Oscillation, the El Niño–Southern Oscillation, the Indian Ocean Dipole, the Atlantic zonal mode and the annular modes are conventionally explained by separate theories with separate parameters. We show that they can emerge from one minimal description. The climate state is modelled as four fields on the sphere, an atmospheric streamfunction, a column moisture anomaly, an ocean mixed-layer temperature and an upper-ocean heat content, whose joint evolution is written as a single linear stochastic equation. The operator contains rotation, dissipation, air–sea and moisture coupling, and the geography of the continents, and no mode-specific physics. Its eigenvalue spectrum separates into three families without any tuning of individual modes: damped atmospheric patterns with e-folding times of about a week, an eastward-propagating equatorial moisture mode with a period of 47 days, and basin-confined coupled oscillators with periods of years. Integrating the system under stochastic forcing reproduces the observed spectral bands, with peaks at 4.29 yr for Niño 3.4, 1.54 yr for the Dipole Mode Index, 1.54 yr for the Atlantic index and 69.1 days for the moisture index, together with red backgrounds for the annular indices. The same periods obtained from the resolvent of the autonomous operator, which requires no integration and carries no sampling error, are 4.06 yr, 1.54 yr and 1.54 yr. The interannual periods are not fitted: they follow from the zonal width of each ocean basin, with a fitted exponent of 1.00 on the resolvent peaks and 1.07 on the eigenmode periods, and they reproduce the observed ordering and approximate spacing of the Pacific, Indian and Atlantic modes. Four natural forcing terms are then added, followed by an anthropogenic one. Annual modulation of the coupling produces seasonal phase locking of approximately the observed magnitude. Stochastic forcing produces the red spectra of the extratropical modes without any oscillatory mechanism. An imposed volcanic cooling produces a damped oscillatory Pacific response with a rebound near two years and a memoryless extratropical response. Slow modulation of the dissipation gives the mode variances a common low-frequency component. Anthropogenic forcing is treated last and separates into two channels: an additive radiative term, which by superposition cannot alter any anomaly statistic, and a parametric dependence of the operator on the background state, which can. Parameterising the second through five physical channels gives a shortening of the interannual periods governed by the increase in equatorial wave speed alone, so that the fractional period change equals minus the fractional change in wave speed, and an amplitude response that is the residue of four opposing contributions of comparable magnitude. The second result predicts that projections of ENSO amplitude should be small and of uncertain sign, which is what the data show.



## 25 1 Introduction

The dominant modes of climate variability are conventionally introduced individually. The Madden–Julian Oscillation (MJO) is presented as a moist intraseasonal disturbance of the equatorial waveguide (Madden and Julian, 1971, 1972; Zhang, 2005); the El Niño–Southern Oscillation (ENSO) as a coupled instability of the equatorial Pacific (Bjerknes, 1969; Zebiak and Cane, 1987; Battisti and Hirst, 1989; Suarez and Schopf, 1988; Jin, 1997); the Indian Ocean Dipole (IOD) as a zonal mode of a narrower basin (Saji et al., 1999; Webster et al., 1999); and the annular modes and the North Atlantic Oscillation (NAO) as eddy-driven adjustments of the extratropical jet (Thompson and Wallace, 1998, 2000; Limpasuvan and Hartmann, 1999; Hurrell, 1995). Each account is successful on its own terms.

Table 1 lists the modes as they are catalogued in textbooks, with the domain, timescale and defining variable of each, and the reference in which each was established. The table shows that the timescales span four orders of magnitude, from the weeks of the Madden–Julian Oscillation to the sixty to eighty years attributed to the Atlantic Multidecadal Oscillation (AMO). The defining variables are heterogeneous: sea surface temperature in one lat-lon box, a difference of sea-level pressures between two stations, the leading empirical orthogonal function of a hemisphere, or the direction of a stratospheric wind. The catalogue is a list of measurement conventions as much as a list of phenomena. Indeed, the division is historical as much as physical. The modes were discovered in different data sets, by different communities, with different instruments, and the indices that define them inherit the geography of the observing networks that produced them (Wallace and Gutzler, 1981; Marshall, 2003).

Nothing in the equations of motion marks out six phenomena; what the equations contain is one dissipative, stochastically driven, geographically inhomogeneous system. From a broader thermodynamic perspective, the circulation of the atmosphere and ocean, together with the associated transport and dissipation of energy, are irreversible processes that contribute to entropy production in the Earth system (Kleidon, 2009). A dynamical-systems reading of climate variability has long made that point (Ghil and Lucarini, 2020), and linear inverse modelling has shown that much of the observed behaviour of tropical sea surface temperature is captured by a single linear operator forced by noise (Penland and Sardeshmukh, 1995). This paper develops that reading into an explicit and minimal theory following (Hasselmann, 1976; Frankignoul and Hasselmann, 1977), and asks whether the dominant modes naturally emerge from a stochastic climate model.

The model is as follows. The state of the climate system is reduced to four fields on the sphere: an atmospheric streamfunction, a column moisture anomaly, an ocean mixed-layer temperature and an upper-ocean heat content. Their joint evolution is written as one linear stochastic equation whose operator is built from rotation, dissipation and geography alone. Eight modes then appear as eigenmodes of that operator. The other four modes in Table 1 do not feature, and not because they fail to fit the model but for reasons of scope. The Quasi-Biennial Oscillation requires a stratosphere and the wave–mean-flow interaction that drives it, neither of which is present in a barotropic atmosphere. The Atlantic Multidecadal, Pacific Decadal and Interdecadal Pacific Oscillations require extratropical ocean dynamics, gyre adjustment and the integration of surface fluxes by a deep mixed layer, whereas the ocean here is an equatorial recharge system.



**Table 1.** Principal modes of internal variability in the climate system, after Table 13.1 of Tapiador (2026). The final column records whether the mode falls within the scope of the theory developed here, and if not, which missing ingredient excludes it.

| Mode                                | Abbrev. | Spatial domain                | Timescale               | Defining variable                                | Key reference                      | In this theory             |
|-------------------------------------|---------|-------------------------------|-------------------------|--------------------------------------------------|------------------------------------|----------------------------|
| El Niño–Southern Oscillation        | ENSO    | Tropical Pacific              | 2–7 yr                  | SST in the Niño-3.4 region                       | Philander (1990)                   | yes                        |
| North Atlantic Oscillation          | NAO     | N. Atlantic, Europe           | weeks to decades        | SLP difference, Azores minus Iceland             | Hurrell (1995)                     | yes                        |
| Arctic Oscillation, N. Annular Mode | AO/NAM  | Northern Hemisphere           | weeks to decades        | leading EOF of SLP poleward of 20° N             | Thompson and Wallace (1998)        | yes                        |
| Southern Annular Mode               | SAM/AAO | Southern Hemisphere           | weeks to decades        | leading EOF of SLP poleward of 20° S             | Thompson and Wallace (2000)        | yes                        |
| Madden–Julian Oscillation           | MJO     | Tropical Indian and Pacific   | 30–60 d                 | eastward-propagating convective envelope         | Madden and Julian (1971)           | yes                        |
| Indian Ocean Dipole                 | IOD     | Tropical Indian Ocean         | seasonal to interannual | SST difference, west minus east                  | Saji et al. (1999)                 | yes                        |
| Pacific–North American pattern      | PNA     | N. Pacific, N. America        | weeks to months         | height anomaly pattern linking the two           | Wallace and Gutzler (1981)         | yes                        |
| Atlantic Niño                       | —       | Tropical Atlantic             | 2–4 yr                  | SST in the equatorial Atlantic cold tongue       | Zebiak (1993)                      | yes                        |
| Quasi-Biennial Oscillation          | QBO     | Tropical stratosphere         | 28 months               | direction of equatorial stratospheric zonal wind | Reed et al. (1961)                 | no: no stratosphere        |
| Pacific Decadal Oscillation         | PDO     | N. Pacific, poleward of 20° N | 20–30 yr                | leading EOF of N. Pacific SST                    | Mantua et al. (1997)               | no: no extratropical ocean |
| Atlantic Multidecadal Oscillation   | AMO     | N. Atlantic, 0–70° N          | 60–80 yr                | detrended area-weighted N. Atlantic SST          | Schlesinger and Ramanakutty (1994) | no: no extratropical ocean |
| Interdecadal Pacific Oscillation    | IPO     | Pacific basin                 | 15–30 yr                | leading EOF of Pacific-wide SST                  | Power et al. (1999)                | no: no extratropical ocean |

The theory is deliberately simple and is presented as a framework for understanding and unveiling a single dynamics rather than as a simulation tool. Section 2 sets out the general formulation, Sect. 3 constructs the operator of the stochastic equation, Sect. 4 examines the emergent modes, Sect. 5 adds the forcing terms, and Sect. 6 discusses the results.

## 60 2 General framework

Let  $Z(t)$  denote the full state of the climate system, obeying

$$\frac{dZ}{dt} = \mathcal{N}(Z) + \mathcal{F}(t), \quad (1)$$

with  $\mathcal{N}$  the nonlinear dynamics and  $\mathcal{F}$  the external forcing. Suppose the state separates into a slowly evolving background  $\bar{Z}(s)$ , parameterised by a slow variable  $s(t)$ , and an anomaly  $X$  of smaller amplitude. Expanding (1) about the background and

65 retaining the terms of first order in  $X$ ,

$$\frac{dX}{dt} = \mathcal{L}(s(t)) X + \underbrace{\mathcal{N}_2(X, X)}_{\text{unresolved}} + \dots + \mathcal{F}'(t), \quad (2)$$



where  $L = \partial \mathcal{N} / \partial Z|_{\bar{Z}}$ . The classical step, due to Hasselmann (1976), is to represent the unresolved nonlinear terms, which evolve on timescales short compared with those of interest, as white noise. This gives the governing equation of the present theory,

$$dX = L(s(t)) X dt + \mathcal{F}'(t) dt + Q^{1/2} dW, \quad (3)$$

a multivariate Ornstein–Uhlenbeck process with a slowly varying operator and a deterministic forcing. Equation (3) is not new; it is the basis of linear inverse modelling (Penland and Sardeshmukh, 1995). What is developed here is the consequence of writing it for the whole Earth system in spherical harmonics and with realistic geography of the globe retained, rather than separately for each region.

Three standard results are needed below. For constant  $L$  with all eigenvalues in the left half-plane, the stationary covariance  $C = \langle X X^\dagger \rangle$  satisfies the Lyapunov equation

$$LC + CL^\dagger + Q = 0, \quad (4)$$

so the variance of every mode is set jointly by the dynamics and by the noise, and no mode needs to be unstable in order to be prominent. The cross-spectral matrix is

$$S(\omega) = R(\omega) Q R^\dagger(\omega), \quad R(\omega) = (i\omega I - L)^{-1}, \quad (5)$$

with  $R$  the resolvent, so that spectral peaks occur where the resolvent is large, that is near the eigenvalues of  $L$  with small real part. Finally, because  $L$  is not normal, its eigenvectors are not orthogonal, the response to forcing is not a simple superposition of eigenmodes, and transient amplification is possible (Farrell and Ioannou, 1996). Non-normality is unavoidable here, for rotation contributes a skew-Hermitian part, dissipation a Hermitian part, and geography prevents the two from commuting.

The situation would be different in an idealised aquaplanet or a snowball Earth.

The slow modulation of the operator features as follows. Let the slow variable enter through the dissipation, which is the dependence most directly implied by a changing background state,

$$L(s) = L_0 - \mu s(t) D, \quad (6)$$

where  $D$  is the explicitly dissipative part of the operator and  $\mu$  the sensitivity. For an eigenmode of amplitude  $a_k$  and damping rate  $\gamma_k(s) = \gamma_{k0}[1 - \mu_k s]$ , the variance obeys  $dV_k/dt = -2\gamma_k V_k + \sigma_k^2$ , so that when  $s$  varies slowly compared with  $\gamma_{k0}^{-1}$ ,

$$V_k(t) \simeq \frac{\sigma_k^2}{2\gamma_{k0}[1 - \mu_k s(t)]} \simeq \bar{V}_k [1 + \mu_k s(t)]. \quad (7)$$

Every mode whose damping depends on the same slow variable therefore acquires the same slow component in its variance, whatever its own frequency. This result is used below.



### 3 The unified operator for the Earth system

95 The state of the Earth system is reduced in the modelling to four fields, chosen as the smallest set that can carry the three timescale families of the observed modes:

$$X = (\psi, q, T, h)^\top, \quad (8)$$

where  $\psi$  is the barotropic atmospheric streamfunction, damped in days;  $q$  a column moisture anomaly, damped in weeks;  $T$  the ocean mixed-layer temperature, damped in months; and  $h$  the upper-ocean heat content, which supplies the memory that makes oscillation possible.

Each field is expanded in spherical harmonics to degree  $N$ , so the state vector has  $4(N+1)^2$  components and the operator is a  $4(N+1)^2$  square matrix of four blocks by four blocks. Multiplication by a geographic field  $g(\mathbf{x})$  becomes a matrix in this basis,

$$[G]_{ij} = \oint \bar{Y}_i g Y_j d\Omega, \quad (9)$$

105 Hermitian, and equal to a multiple of the identity only when  $g$  is uniform. Equation (9) is where the actual geography of the planet enters the geometrical modelling on a sphere. The land/ocean mask, the equatorial waveguide and the basin-width field that modulate the modes each appear as one such matrix, and each couples spherical harmonics of different degree and order according to the selection rule that  $G_{ij}$  vanishes unless  $g$  has a Fourier component at zonal wavenumber  $m_i - m_j$ .

The representation in the model of the dynamics of the atmosphere is as follows. The atmosphere is represented by the barotropic vorticity equation on a rotating sphere, linearised about a solid-body zonal flow of angular velocity  $\omega_U$ , with topographic coupling and a surface damping that differs over land and sea:

$$L_{\psi\psi} = -i \text{diag}(\omega_{nm}) + T_{\text{topo}} - r_o I - (r_\ell - r_o) M_{\text{land}} - \nu \text{diag}([n(n+1)]^2), \quad (10)$$

with the Rossby–Haurwitz frequency (i.e. the rate at which a free planetary wave of spherical-harmonic degree  $n$  and zonal wavenumber  $m$  travels in longitude on a rotating sphere, drifting westward because planetary vorticity varies with latitude and being carried eastward by the mean zonal flow)

$$\omega_{nm} = m\omega_U - \frac{2m(\Omega + \omega_U)}{n(n+1)} \quad (11)$$

and the topographic matrix obtained by projecting  $J(\psi, \eta)$ , where  $\eta = 2\Omega\mu h_{\text{topo}}/H$  is the topographic vorticity (Charney and DeVore, 1979; Hoskins and Karoly, 1981). Note that the land mask enters twice, dissipatively through  $M_{\text{land}}$  and advectively through  $T_{\text{topo}}$ . The hyperdiffusion  $\nu$  is a damping whose rate grows as the square of the Laplacian eigenvalue and which therefore acts almost entirely on the smallest resolved scales, present to absorb the enstrophy that cascades towards the truncation rather than to represent a physical process.

Moisture is advected by the mean low-level flow and damped by precipitation,

$$L_{qq} = -\varepsilon_q I - i \text{diag}(m u_q), \quad (12)$$



and exchanges with the circulation through the convective belt. Low-level convergence moistens the column and latent heating  
125 forces vorticity,

$$L_{q\psi} = a_q W, \quad L_{\psi q} = b_q W, \quad (13)$$

where  $W$  is the multiplication operator of the warm-pool weight  $\exp[-(\varphi/12^\circ)^2]$  over ocean. This is the moisture-mode  
structure, in which propagation comes from moisture advection and the growth rate from the balance between convergence  
and damping (Adames and Kim, 2016). The pair  $(\psi, q)$  supports an eastward-propagating equatorial disturbance whose period  
130 follows from the circumnavigation time  $2\pi/u_q$ .

The ocean is represented by the recharge structure of Jin (1997), generalised from a two-variable system to a field. Mixed-  
layer temperature is damped, forced by the atmosphere and fed by heat content; heat content is discharged by temperature  
anomalies and damped:

$$L_{TT} = -\lambda_T O - \varepsilon_\ell M_{\text{land}}, \quad L_{Th} = \Omega_{oc}, \quad L_{hT} = -\Omega_{oc}, \quad L_{hh} = -\varepsilon_h O - \varepsilon_\ell M_{\text{land}}, \quad (14)$$

135 with  $O$  the ocean mask operator. The rates  $\lambda_T$  and  $\varepsilon_h$  are defined positive, so both diagonal blocks damp. The term in  $\varepsilon_\ell$ , with  
 $\varepsilon_\ell^{-1} = 20$  d, damps the two ocean fields over land. It carries no physics and is not a free parameter. It confines  $T$  and  $h$  to the  
ocean, which a projection onto a truncated harmonic basis does not otherwise do exactly. The antisymmetric pair  $(L_{Th}, L_{hT})$   
is what makes the ocean oscillate. Temperature and heat content are held in quadrature, and in the absence of damping the  
pair rotates at the rate  $\omega_{oc}$ . Quadrature means in this context a quarter of a cycle apart in phase, so that heat content reaches  
140 its maximum at the moment temperature passes through zero, each variable setting the rate of change of the other. The signs  
in (14) model that the heat content leads the temperature by that quarter cycle, i.e. a recharge in which the warm water volume  
builds before the surface warms.

The essential insight is that  $\omega_{oc}$  is not a free field. Basin adjustment proceeds by equatorial waves that cross the basin and  
return, so the natural frequency of each basin scales inversely with its width,

$$145 \quad \omega_{oc}(\mathbf{x}) = \kappa \frac{c_K}{W(\mathbf{x})} e^{-(\varphi/8^\circ)^2}, \quad (15)$$

where  $c_K$  is the equatorial Kelvin wave speed,  $W(\mathbf{x})$  the zonal width of the contiguous ocean basin containing the point  $\mathbf{x}$ ,  
computed directly from the land mask, and  $\kappa$  a dimensionless constant of order unity. This is the delayed-oscillator scaling of  
Suarez and Schopf (1988) and Battisti and Hirst (1989) expressed here as a field.

Finally the tropical ocean and the atmosphere are coupled through surface wind stress and convective heating,

$$150 \quad L_{T\psi} = a_T E, \quad L_{\psi T} = b_T E, \quad (16)$$

with  $E$  the equatorial ocean weight.



**Table 2.** Parameters of the unified operator. Only  $\kappa$  was adjusted to match an observed timescale; the remainder are standard values for the processes they represent.

| Symbol                  | Value                  | Meaning                                       | Source                                |
|-------------------------|------------------------|-----------------------------------------------|---------------------------------------|
| $N$                     | 12                     | spectral truncation (676 state variables)     | numerical                             |
| $\nu$                   | $3 \times 10^{-6}$     | hyperdiffusion on $[n(n+1)]^2, \text{d}^{-1}$ | numerical                             |
| $\varepsilon_\ell^{-1}$ | 20 d                   | damping of $T$ and $h$ over land              | numerical                             |
| $\omega_U$              | $0.03\Omega$           | solid-body superrotation                      | basic state                           |
| $r_o^{-1}, r_\ell^{-1}$ | 12 d, 4 d              | atmospheric damping over sea, land            | boundary layer                        |
| $h_{\text{topo}}/H$     | 0.25                   | continental plateau depth ratio               | orography proxy                       |
| $\varepsilon_q^{-1}$    | 16 d                   | moisture damping                              | convective adjustment                 |
| $2\pi/u_q$              | 46 d                   | moisture circumnavigation time                | $\simeq 5 \text{ m s}^{-1}$ advection |
| $a_q, b_q$              | 0.055, 0.055           | moisture–circulation coupling                 | tuned to order unity                  |
| $\lambda_T^{-1}$        | 0.9 yr                 | mixed-layer damping (a positive rate)         | surface heat flux                     |
| $\varepsilon_h^{-1}$    | 1.6 yr                 | heat-content damping                          | thermocline adjustment                |
| $c_K$                   | $2.5 \text{ m s}^{-1}$ | equatorial Kelvin wave speed                  | observed                              |
| $\kappa$                | 0.45                   | basin-frequency constant                      | <i>tuned</i>                          |
| $a_T, b_T$              | 0.0036, 0.21           | air–sea coupling                              | bounded by stability                  |

Gathering the pieces (the linearisations), we obtain the complete operator

$$\mathbf{L} = \begin{pmatrix} \mathbf{L}_{\psi\psi} & \mathbf{L}_{\psi q} & \mathbf{L}_{\psi T} & 0 \\ \mathbf{L}_{q\psi} & \mathbf{L}_{qq} & 0 & 0 \\ \mathbf{L}_{T\psi} & 0 & \mathbf{L}_{TT} & \mathbf{L}_{Th} \\ 0 & 0 & \mathbf{L}_{hT} & \mathbf{L}_{hh} \end{pmatrix}. \quad (17)$$

Table 2 lists all the parameters of the model. Note that none of them implies the a priori existence of the modes in Table 1. In other words, there is nothing in this modelling, including the parameter choice, capable of nudging the Earth system towards the emergence of the observed modes of variability. The constant  $\kappa$  is the only quantity tuned to an observed period (but it is an observed timescale); everything else about the basins, including their number, their widths and their longitudes, comes from the configuration of the land mask in our geological era.

#### 4 Emergence of the dominant modes

The model gives rise to several modes. Figure 1 displays the spectrum as period against e-folding time. Three families appear, separated by their dominant field. The atmospheric family occupies the lower left: periods of days or less and e-folding times of about a week, some non-oscillatory. These are the Rossby–Haurwitz waves and the damped patterns that geography builds from them, and they correspond to the annular modes and the sectoral teleconnections, whose observed e-folding times are ten to

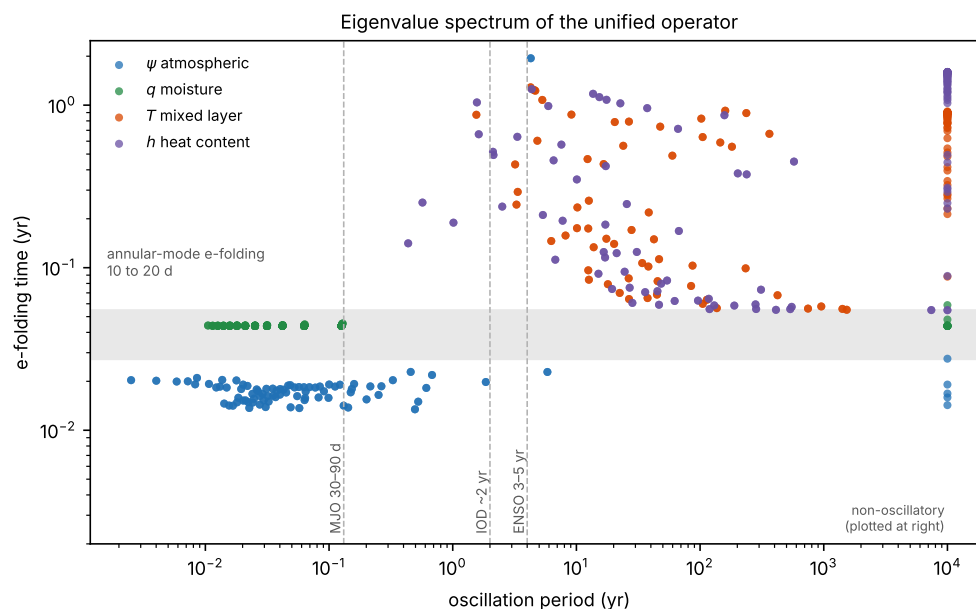


**Figure 1.** Eigenvalue spectrum of the unified operator, plotted as oscillation period against e-folding time, with each eigenmode coloured by the field carrying most of its amplitude. Three families separate without any tuning of individual modes. Non-oscillatory eigenvalues are plotted at the right-hand edge. The observed bands of the MJO, ENSO and the IOD, and the observed e-folding range of the annular modes, are marked for reference.

twenty days (Feldstein, 2000; Limpasuvan and Hartmann, 1999). The model therefore damps this family faster than observed, by a factor approaching two at the upper end of the observed range. The cause is identifiable. Observed annular persistence is maintained in part by the feedback of transient eddies on the zonal-mean flow (Limpasuvan and Hartmann, 1999). A barotropic layer with a prescribed basic flow cannot represent it, since the eddies enter here as noise rather than as a resolved field that the mean flow can organise. What the construction yields is the family and its spatial structure; the persistence within the family is a lower bound. The moisture family occupies a narrow band at e-folding times near sixteen days and periods of weeks, set by the balance between moisture damping and advection. The ocean family lies to the upper right: periods of one to several years, e-folding times of months to about two years.

Figure 2 shows one eigenmode from each family. The slowest coupled mode has a period of 4.27 yr, an e-folding time of 1.29 yr and a zonal dipole across the Pacific with the sign reversal near the dateline that characterises ENSO. The moisture mode has a period of 47 d, an e-folding time of 17 d, zonal wavenumber one and equatorial confinement, propagating eastward. The annular mode is non-oscillatory, zonally symmetric and damped in 8.3 d. These structures were not imposed, for the operator contains only rotation, dissipation, coupling coefficients and the ocean/land mask.

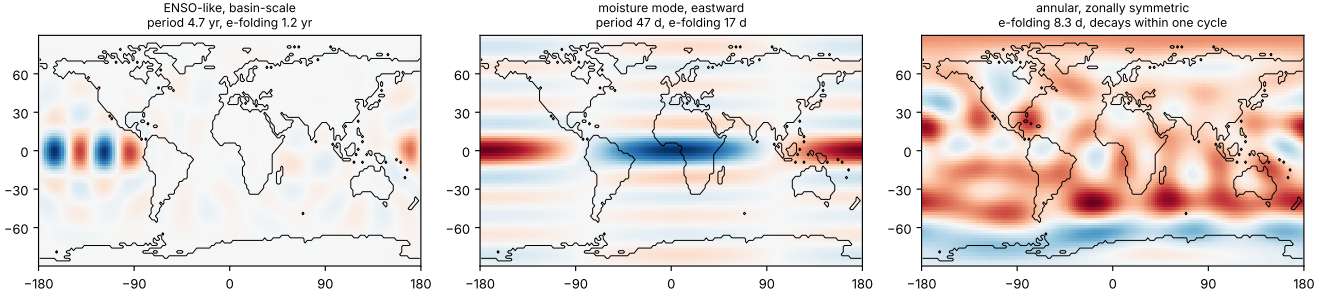


**Figure 2.** Spatial structure of three representative eigenmodes, one from each family. The ENSO-like mode is shown in mixed-layer temperature, the moisture mode in column moisture, and the annular mode in streamfunction. No pattern was prescribed; each is an eigenvector of the operator of Eq. (17).

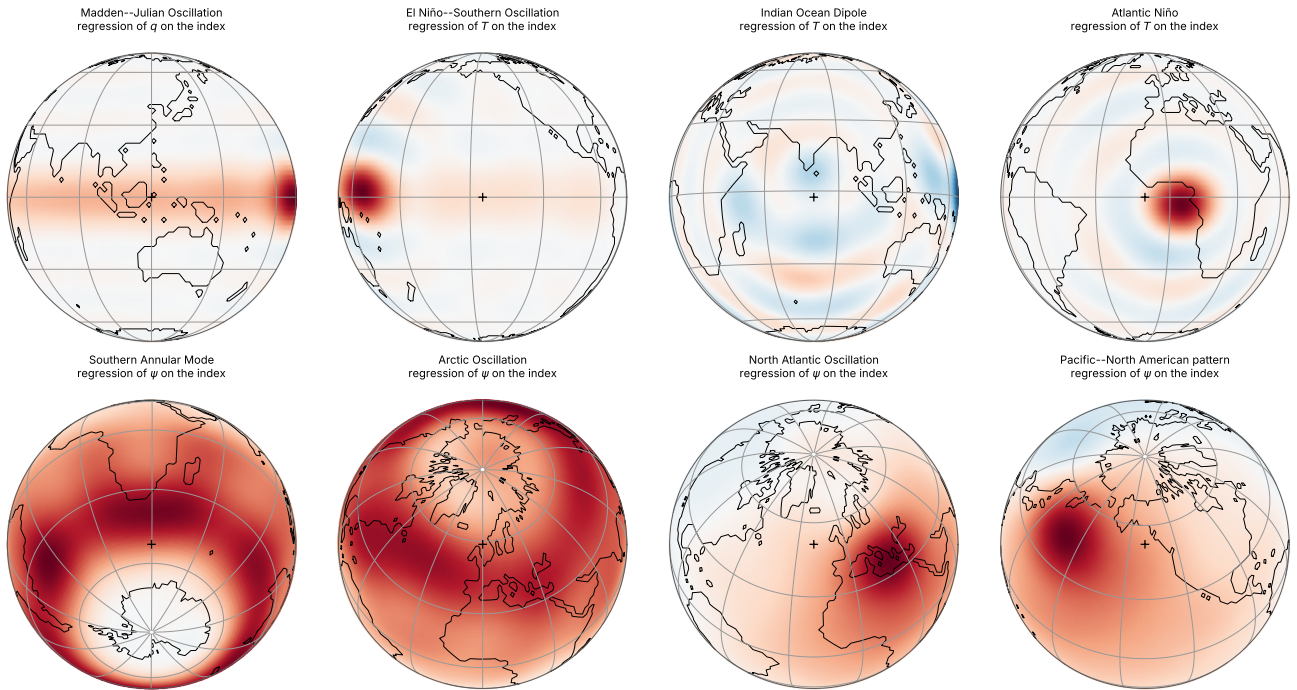


**Figure 3.** Spatial pattern of each mode within the scope of the theory, one globe per mode, in orthographic projection centred on the region that defines the corresponding index. Each pattern is the regression of the model field on the model index, obtained analytically from the stationary covariance through Eq. (18) rather than by simulation. The field regressed is moisture for the MJO, mixed-layer temperature for the three tropical ocean modes, and streamfunction for the four atmospheric patterns. The abbreviations and index definitions follow Table 1.

Note also that the spatial pattern of the modes can be derived from the structure of the model. If the index of mode  $j$  has kernel  $\mathbf{k}_j$ , so that  $I_j = \mathbf{k}_j^\dagger X$ , then the regression of the state on that index is

$$P_j = \frac{C \mathbf{k}_j}{\mathbf{k}_j^\dagger C \mathbf{k}_j}, \quad (18)$$



180 with  $C$  the stationary covariance from the Lyapunov equation (4). Equation (18) is the quantity observationally estimated by regressing fields on an index, so the comparison between theory and observation can be made pattern by pattern without any Monte Carlo step.

Figure 3 shows the result for the eight modes of Table 1 that lie within scope of our model, one globe each. The tropical ocean patterns are basin-confined, with the Indian Ocean mode showing the west-warm, east-cool structure that defines the dipole. The moisture pattern is equatorially trapped across the warm pool. The annular patterns show a midlatitude ring of one sign with a polar cap of the other, and the Pacific–North American pattern appears as a wave train anchored over the North Pacific. These patterns are consequences of the operator and the noise covariance alone, so they are not expected to correspond exactly with the observations.

Eigenmode labelling is ambiguous when eigenvectors are not orthogonal, so we should integrate (3) and analyse the result in the same way as observations are analysed. The system was driven by spatially smooth white noise injected into  $\psi$ , and more weakly into  $q$  and  $T$ , and integrated for 400 yr with a daily step. Standard indices were then computed as fixed linear functionals of the state.

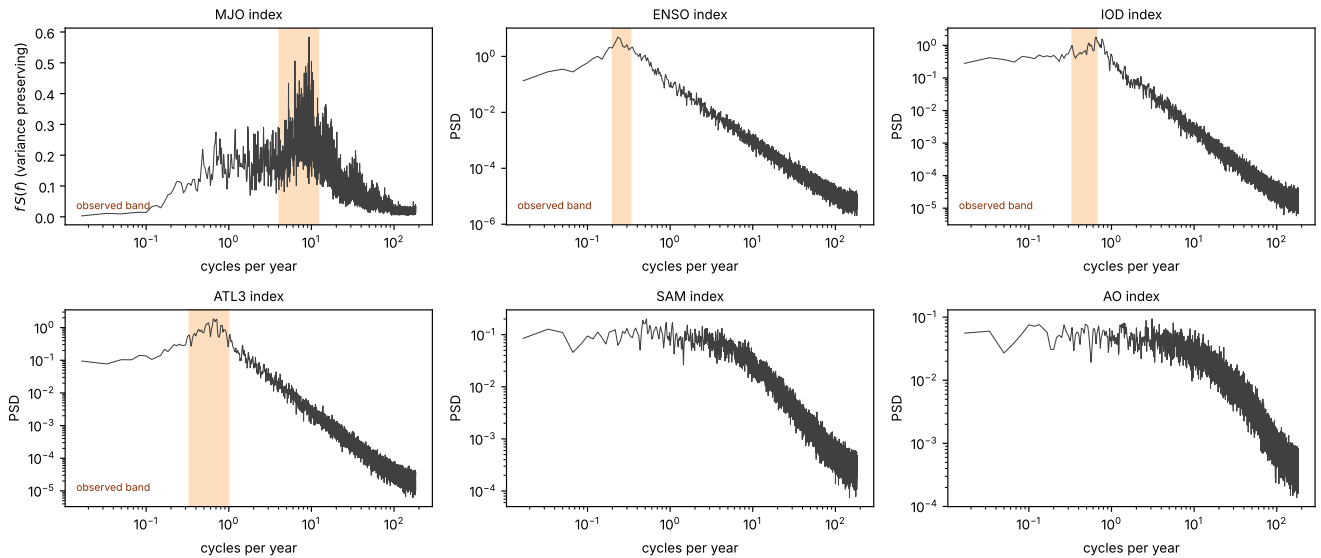


**Figure 4.** Power spectra of the simulated indices from a 400-yr integration. Shading marks the observed frequency band of each mode. The MJO panel is plotted in variance-preserving form, since the box-averaged moisture index is not band-passed and therefore also carries slow variability.

Figure 4 shows the resulting spectra. The Niño-3.4 index peaks at 4.29 yr, within the observed 3 to 5 yr band. The Dipole Mode Index peaks at 1.54 yr, reproducing the quasi-biennial character of the IOD. The Atlantic index peaks at 1.54 yr, at the same period, which is what basins of nearly equal width should give. The moisture index carries a subseasonal peak at 69.1 d, inside the observed 30 to 90 d band of the MJO, although above the period of the moisture eigenmode itself, since the

box-averaged index is not band-passed and so carries slower variability as well. The annular and Atlantic-sector atmospheric indices produce red spectra that flatten at low frequency, with slopes of  $-1.00$  for SAM,  $-0.55$  for the AO and  $-0.04$  for the NAO over the subannual range, which is the behaviour expected of damped stochastically forced modes (Feldstein, 2000).  
 200 These four peaks come from one multicentury realisation of an integration in which the coupling is modulated through the year, so each carries sampling error and the displacement produced by the modulation. The corresponding quantities of the autonomous operator, which are exact and are given below, differ from them by a fifth for the narrow basins and by four per cent for the Pacific.

What is noticeable at this point is that so simple a model exhibits the observed variability modes with just one free parameter.  
 205 The most informative insight comes from the origin of the interannual periods. Because  $\omega_{oc}$  is fixed by (15), the three tropical basins are not independent parameters: their widths at the equator, computed from the land mask, are 16 200 km for the Pacific, 6 400 km for the Indian and 6 000 km for the Atlantic.

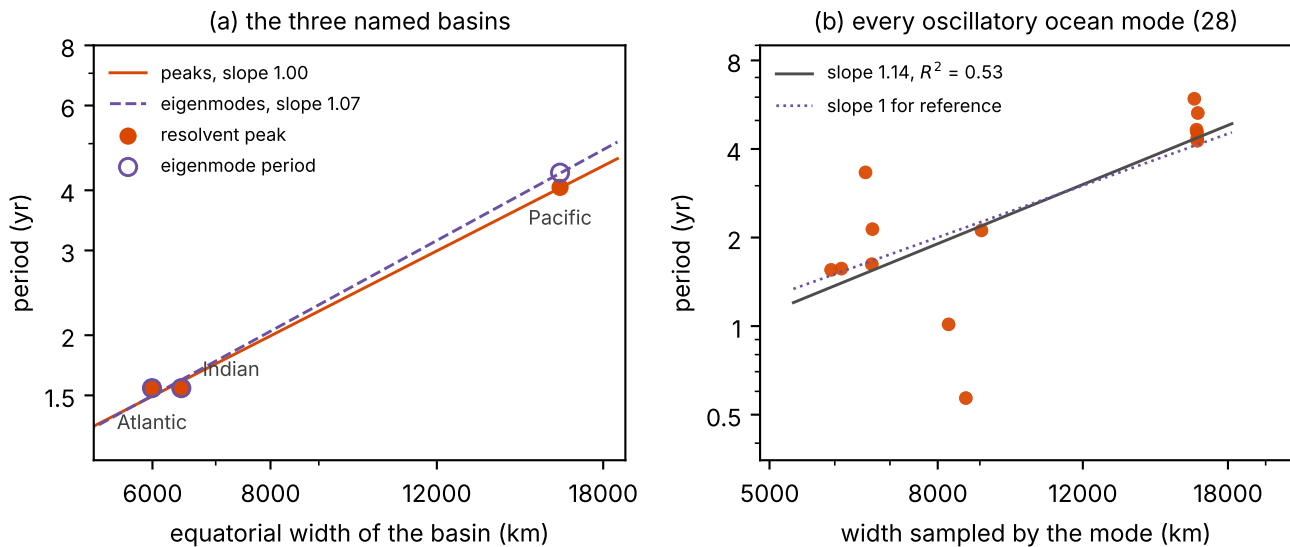


**Figure 5.** Emergent oscillation period against the zonal width of the equatorial basin in which the mode resides. (a) The three named basins, with the period taken both from the peak of the resolvent spectrum of the corresponding index and from the eigenvalue of the basin-confined eigenmode; the fitted exponents are 1.00 and 1.07. Widths come from the land mask. (b) Every oscillatory ocean eigenmode of the operator, 28 in all, each placed at the basin width that its own amplitude distribution samples; the population exponent is 1.14 with  $R^2 = 0.53$ . Complex-conjugate pairs coincide, so 14 positions are visible. A single constant,  $\kappa$ , was tuned; the ordering and the spacing of the periods are predictions.

Figure 5a compares the emergent periods with those widths. Taking the period from the peak of the resolvent spectrum of each index, which requires no integration and carries no sampling error, gives 4.06 yr for the Pacific and 1.54 yr for both the  
 210 Indian and the Atlantic, against observed characteristic values near 4, 2 and 1.5 yr for Niño 3.4, the Dipole Mode Index and



the Atlantic index. Those three are the band centres conventionally quoted in the literature, and they are a different quantity from the single spectral peaks reported in Sect. 6, as noted there. The emergent scaling is  $T \propto W^{1.00}$ ; taking the period instead from the eigenvalue of the basin-confined eigenmode gives  $W^{1.07}$ . Both readings return the inverse-width dependence that (15) imposes on the local adjustment frequency, which is not a foregone conclusion, for the coupled operator could have distorted it.

215 Two features of the result deserve emphasis. The Indian and Atlantic basins differ in width by seven per cent. Their resolvent peaks coincide to the resolution of the calculation. That is what (15) predicts. The other feature is that three named basins are three points. Taking instead every oscillatory ocean eigenmode of the operator, and assigning to each the basin width that its own amplitude samples, gives  $T \propto W^{1.14}$  with  $R^2 = 0.53$  over 28 modes (Fig. 5b). The population scales more steeply than the three index modes, because the higher modes of a basin sample its width incompletely and because the atmospheric

220 coupling does not act equally upon them.

We arrive at a central claim of the paper. Three modes that are conventionally described by three separate theories, with three separate sets of parameters, here differ only in the width of the ocean basin that hosts them. The modes of variability of the Earth system can be understood as a single dynamics with a free parameter. They appear as a consequence of the distribution of the emerged lands on the planet and it can be proved that the periods follow from the geographic field. To do

225 so, two counterfactual Earths were constructed, each isolating one aspect of what the land mask contributes. The operator was rebuilt from scratch in each case with every other parameter held fixed. The first control removes the continents entirely. On an aquaplanet every latitude circle is a single basin, so  $W$  in Eq. (15) becomes the circumference and  $\omega_{oc}$  falls accordingly. The leading coupled mode then has a period of 15.4 yr rather than 4.27 yr, and its structure is zonally symmetric (Fig. 6b). The three distinct tropical modes do not merely weaken, they cease to exist as separate objects, because there is no longer more than one

230 basin for them to occupy. Whatever else the model does, its interannual behaviour is not an artefact of the ocean parameters: without geography there is no ENSO, no IOD and no Atlantic mode, only one slow global oscillation of the wrong timescale.

The second control keeps the continents but shifts them. Rotating the land mask rigidly by  $120^\circ$  eastward leaves the leading coupled mode with a period of 4.27 yr and an e-folding time of 1.29 yr, identical to the control to the precision quoted, while the longitude of its extremum shifts from  $177^\circ\text{W}$  to  $57^\circ\text{W}$ , that is by the rotation angle (Fig. 6c). Period and position are

235 therefore separately determined: the period by the width of the basin, which rotation preserves, and the position by where the basin is, which rotation changes. The calculations show that the number of tropical modes, their periods, their longitudes and their spatial extent all follow from the land mask. Inverse-width scaling is an assumption of the local ocean-frequency parameterisation but the only tuned quantity,  $\kappa$ , sets an overall scale and cannot produce three different periods from one basin or move a mode from one ocean to another. There is a third control in fact, but that is already contained in Fig. 5: the three

240 basins differ in width and their emergent periods differ accordingly. Taken together, the three tests establish that the geography is doing the work. An operator in which basin width had no effect would reproduce none of them. The controls just described vary the geography.

A third kind of control removes one element of the construction at a time, rebuilding the operator from scratch in each case with every other parameter held fixed, and asks which emergent property fails. Table 3 reports nine such ablations against five

245 diagnostics. The pattern is uneven, but one element is indispensable. Removing the antisymmetric recharge coupling leaves

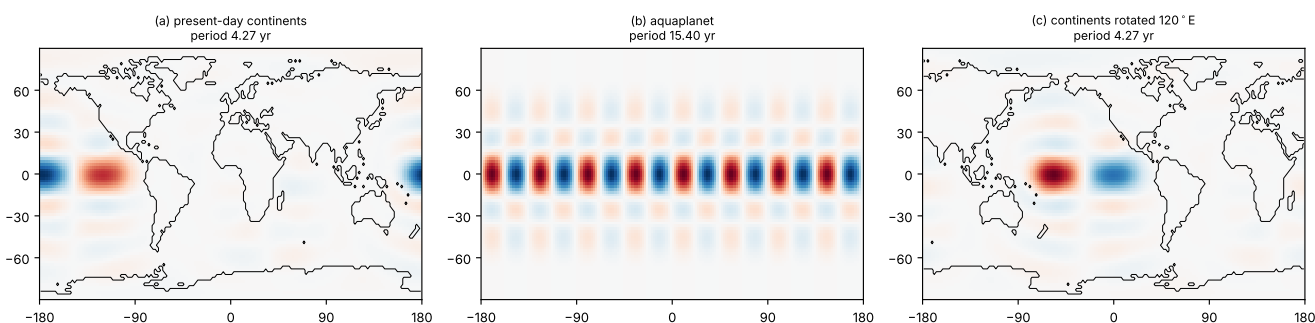


**Figure 6.** Geography controls. Mixed-layer temperature structure of the leading coupled ocean eigenmode for (a) the present-day land mask, (b) an aquaplanet, and (c) a planet whose continents have been rotated rigidly  $120^\circ$  eastward. Rotation leaves the period unchanged and carries the pattern with the continents; removing the continents destroys the basin modes and leaves one slow global oscillation.

no oscillatory ocean eigenmode whatever. In that case, the ocean ceases to oscillate and merely reddens the noise it is given, and the variance of the Niño-3.4 index rises by a factor of 2.9 because nothing now returns the anomaly. Nothing else in the construction can supply a restoring tendency, which is the purpose of that block. A second element is necessary for the central claim above: giving every basin the same width, while leaving the geography otherwise untouched, collapses the spread of interannual period from a factor of 10.4 to a factor of 3.3. The spread follows from the contrast in width and from nothing else that distinguishes one basin from another. Of the remaining seven, only the air–sea coupling moves any diagnostic by more than a fifth, and it moves the Niño-3.4 variance alone. Two others, the topography and the stronger drag over land, act on the atmospheric persistence, which lengthens by about a fifth when either is removed. Geography, in other words, shortens the persistence of the annular pattern rather than lengthening it.

The table also shows which quantities are not emergent. The period of the moisture mode stays within one per cent of its control value under every ablation, including those that sever its coupling to the circulation, because it is set by the circumnavigation time  $2\pi/u_q$  of Table 2. The persistence of the annular pattern varies by at most a third and never leaves the range of days, because it is set by the surface damping. Both are coefficients of the construction. The claim that the timescales are outputs applies to the interannual family, to the separation between the three families, and to the ordering of periods across basins; it does not apply to those two numbers.

Reduced operators are open to the same two objections however they are obtained, whether by projection of a primitive-equation system onto a finite basis (as in our work) or by inference from a lagged covariance (Penland and Sardeshmukh, 1995). Such an operator can reproduce every timescale asked of it and still sit beside a boundary in its own parameter space across which the stationary covariance ceases to exist, and it can deliver a variance that halves when the basis is refined. Neither failure announces itself in the diagnostics, so each has to be reported separately. Two questions remain. How much freedom does the coupling have, and which of the diagnostics survive refinement of the basis?



**Table 3.** Ablation of the construction. Each row removes one element and reports the spectral abscissa, the ratio of longest to shortest period among the basin-confined ocean modes, the period of the moisture mode, the  $e$ -folding time of the annular pattern, and the variance of the Niño-3.4 index. The last four are given relative to the control, whose values are a spread of 10.41, 47.1 d and 8.3 d.

| Element removed         | $\alpha$ ( $10^{-3} \text{ d}^{-1}$ ) | spread    | moisture | annular | variance |
|-------------------------|---------------------------------------|-----------|----------|---------|----------|
| none (control)          | -1.41                                 | 1.00      | 1.000    | 1.000   | 1.00     |
| topography              | -1.44                                 | 1.00      | 1.001    | 1.158   | 0.98     |
| drag over land          | -1.26                                 | 1.00      | 1.002    | 1.187   | 1.15     |
| hyperdiffusion          | -1.23                                 | 1.00      | 1.005    | 1.037   | 1.10     |
| moisture coupling       | -1.57                                 | 1.00      | 0.977    | 1.002   | 0.91     |
| air-sea coupling        | -1.71                                 | 1.00      | 1.000    | 1.002   | 0.63     |
| recharge coupling       | -0.59                                 | none left | 1.000    | 1.000   | 2.89     |
| contrast in basin width | -1.38                                 | 0.31      | 1.000    | 1.000   | 0.89     |
| ocean mask              | -1.40                                 | 1.00      | 1.000    | 0.999   | 1.01     |

Figure 7a answers the first. The operator stays damped at every truncation while the air-sea coefficients  $a_T$  and  $b_T$  are raised together by a factor of 1.4. At 1.5 the coupled Pacific mode ceases to be damped, the stationary covariance of Eq. (4) no longer exists, and the stochastic description of Sect. 2 fails with it. The values adopted here are 0.67 of that threshold. The coupling is therefore not free to order unity. It is bounded above, and the bound belongs to the construction rather than to our choice of numbers.

Figure 7b answers the second question. Over  $N = 8$  to 14 the period of the moisture mode varies by 2 per cent, the Pacific period by 25 per cent, the persistence of the annular pattern by 27 per cent and the Niño-3.4 variance by a factor of 2.1. Ratios behave better than absolute variances where the modes compared belong to one family. The variance of the Dipole Mode Index relative to Niño 3.4 settles at 1.48 and moves by 4 per cent across that range. Ratios between families do not. The NAO variance relative to Niño 3.4 falls by 43 per cent over the same truncations, because the atmospheric variances saturate while the oceanic ones continue to grow. Comparisons within a family are therefore reliable, comparisons across families are indicative, and absolute variances should be read as orders of magnitude. The decomposition of the anthropogenic response below, which reports fractional changes at one truncation, is unaffected by this.

## 280 5 Forcing

Indeed, the operator alone gives a stationary system, but the dynamics of the planet is the result of a number of forcings. Four natural forcing terms are added, each of which corresponds to a known feature of the Earth system. The idea is not only to prove that the model is robust in the sense that each empirical observation produces an effect on the modes, but to allow investigation of the sensitivity of the modes to forcings, with the primary focus on the potential effects of human forcings on climate variability.

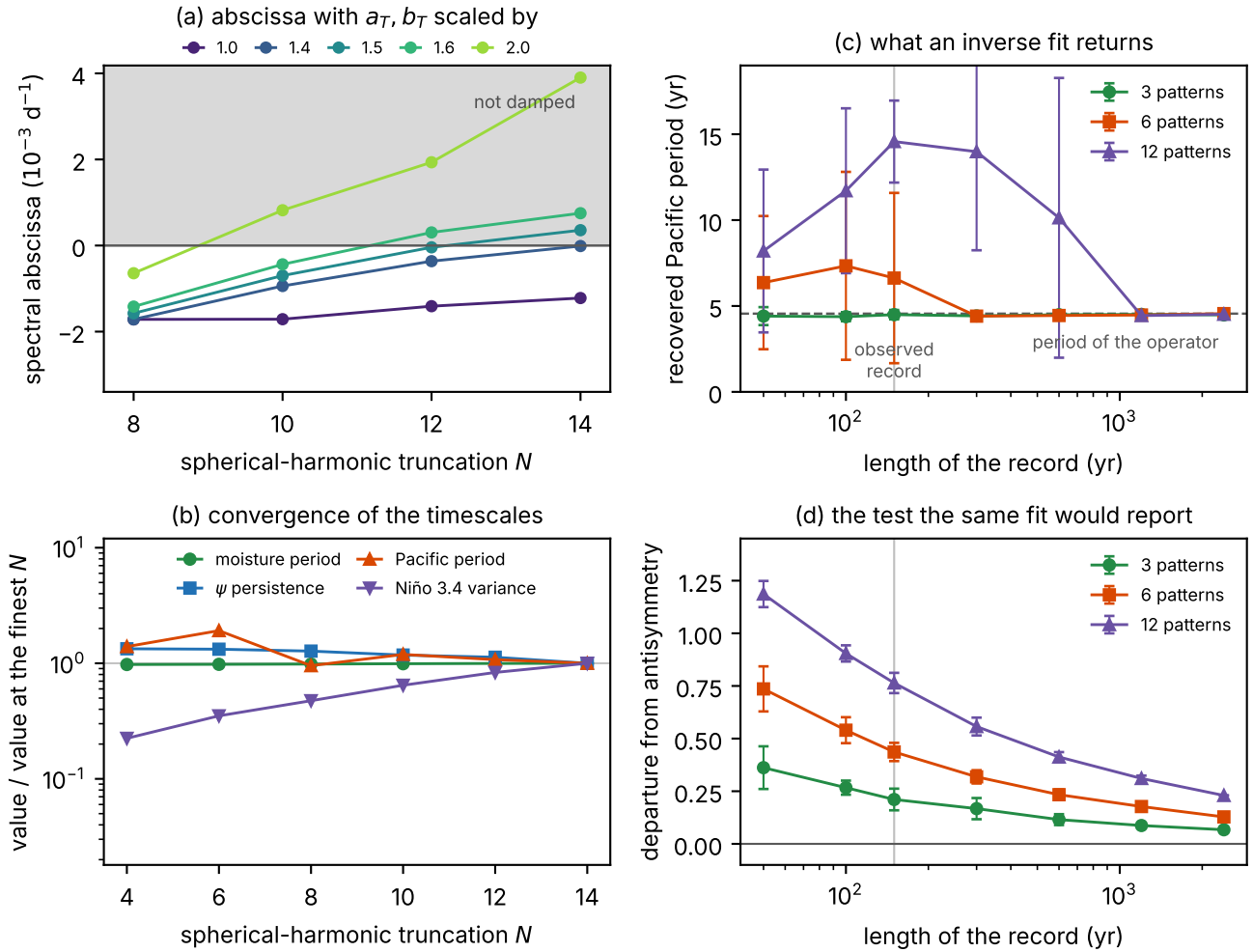


**Figure 7.** Robustness of the construction. (a) Spectral abscissa against spherical-harmonic truncation, for the air–sea coupling of Table 2 and for that coupling raised by the factors shown; in the shaded region the coupled Pacific mode is no longer damped. (b) The four principal diagnostics against truncation, each divided by its value at the finest truncation computed. (c) Pacific period recovered by a linear inverse fit to a simulated record of the stated length, for three numbers of retained patterns, with the period of the operator marked; the bars are the standard deviation over twelve independent records. (d) Departure from antisymmetry of the inferred coupling between mixed-layer temperature and heat content, measured on the same fits, for an operator in which that coupling is antisymmetric to one part in  $10^{16}$ .

The coupling coefficients of (13) and (16) depend on the seasonal position of the convective maximum and the depth of the mixed layer, so they vary through the year. Writing

$$a_T(t) = a_T [1 + \sigma_a \cos(2\pi t/T_{\text{yr}})], \quad (19)$$



and similarly for the other coupling coefficients, makes the operator periodic, so that the system obeys Floquet dynamics rather than autonomous dynamics. The distinction is worth spelling out. When the operator is constant, a mode is a fixed spatial pattern with a fixed decay rate and a single frequency, and the statistics it supports are the same in every month of the year. A periodic operator admits no solution of that form. Floquet theory supplies the replacement. Every solution is a spatial pattern that deforms through the year and repeats after twelve months, multiplied by an exponential factor, and the quantities governing stability are the eigenvalues of the propagator over one annual cycle rather than those of the operator at any instant. A mode is therefore no longer a pattern with a period, but a pattern with a season.

Two consequences follow. The variance of the oscillatory modes acquires a seasonal cycle, which the periodic Lyapunov equation gives exactly. The modulation also generates the combination tones at  $f_{yr} \pm f_k$  described by Stuecker et al. (2013), but these do not appear in the spectrum of the index itself: the modulation is parametric, so it multiplies a zero-mean process rather than adding to it, and the first moment of the index therefore carries no discrete line at the annual frequency however strong the modulation is. The signature belongs to the second moment.

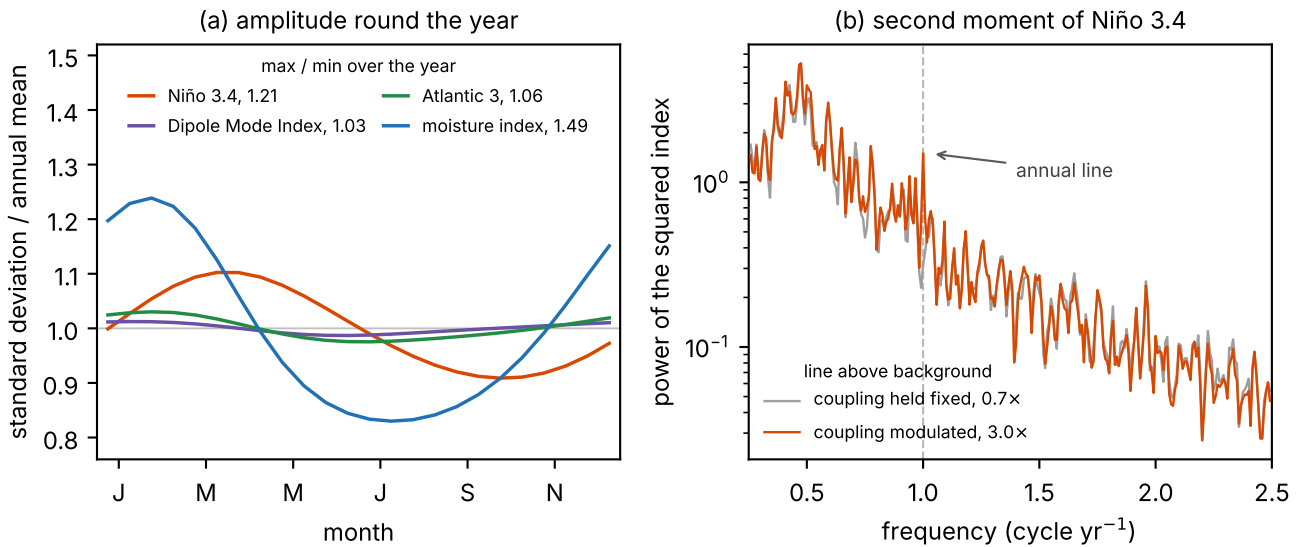


**Figure 8.** (a) Standard deviation of four indices by calendar month, relative to the annual mean, for a coupling modulation of amplitude  $\sigma_a = 0.35$ , obtained from the periodic Lyapunov equation rather than by simulation; the legend gives the ratio of maximum to minimum. (b) Power spectrum of the *squared* Niño-3.4 index, with the coupling modulated and with it held fixed. The annual line stands 3.0 times above the neighbouring background in the first case and 0.7 times in the second. The spectrum of the index itself, as distinct from its square, carries no such line in either case.

Figure 8 shows both consequences. With  $\sigma_a = 0.35$  the Niño-3.4 index develops a seasonal cycle of amplitude whose maximum-to-minimum ratio is 1.213. That number comes from the periodic Lyapunov equation, so it is exact rather than estimated from a finite run, and the cyclostationary covariance it returns reproduces itself after one year to one part in  $10^{14}$ ,



which is the arithmetic check on the calculation. The modulation is therefore in the observed sense, though weaker than the  
 305 observed locking. The reason is that the annual cycle enters through the coupling coefficients, and those are bounded above  
 by the stability requirement set out earlier. What the construction fixes is not the depth of the modulation but its distribution  
 across the modes. The other three indices acquire cycles of ratio 1.026 for the Dipole Mode Index, 1.056 for the Atlantic index  
 and 1.492 for the moisture index, so the prediction is an ordering, with the moisture mode the most strongly modulated and the  
 Indian Ocean mode the least. The calendar month of each maximum is inherited from the phase of the imposed modulation,  
 310 which the model does not derive. What the construction predicts is that phase locking exists and how it is distributed across the  
 modes, not when each maximum falls.

The modulation also leaves the Floquet dynamics measurably different from the autonomous dynamics. The propagator  
 over one annual cycle has a leading multiplier of 0.624, equivalent to a Floquet exponent of  $-0.001 \times 10^{-3} \text{ d}^{-1}$  against  
 $-0.001 \times 10^{-3} \text{ d}^{-1}$  for the autonomous operator, so modulating the coupling reduces the damping of the least damped mode  
 315 by 8 per cent. Parametric forcing destabilises the system slightly, as it generally does, without rendering it unstable at this  
 amplitude. The combination tones appear where the argument above says they should. Figure 8b shows the spectrum of the  
 squared index: with the coupling modulated, a line at one cycle per year stands 3.0 times above the neighbouring background,  
 against 0.7 times when the coupling is held fixed, which is to say no line at all and the only annual structure is what a finite  
 record manufactures. This is a sharper diagnostic than the one used before, and it is also the one to apply to observations, where  
 320 the quantity to examine is the annual cycle of index variance rather than the index spectrum.

## 5.1 Stochastic forcing

The noise term of (3) represents the unresolved transients, principally synoptic weather. Its spatial structure matters, since  
 by (4) the stationary covariance depends on  $Q$  as well as on  $L$ . We use spatially smooth noise, with amplitude decreasing as  
 $1/n$  with harmonic degree, injected mainly into the atmospheric field. The red spectra of the annular indices in Fig. 4 are the  
 325 integrated response to that forcing, in the sense of Hasselmann (1976), rather than evidence of any oscillatory mechanism.

Volcanic eruptions are the largest abrupt natural forcing of the industrial period, and they are also the most informative one  
 for a theory cast as a linear operator. A large explosive eruption loads the stratosphere with sulphate aerosol, which reduces the  
 incoming shortwave flux over an interval short compared with the interannual modes yet long compared with the atmospheric  
 ones, and which begins on a date known to the month. The value of such an event is that for a linear system the impulse  
 330 response determines the response to every other forcing by convolution, so an eruption probes the operator itself rather than  
 any single mode. Such an eruption imposes an abrupt radiative cooling on the mixed layer (Robock, 2000). Because the system  
 is linear, the response to such an event is exactly the impulse response  $e^{Lt} X_{\text{volc}}$ . No ensemble is required to extract it. Figure 9a  
 shows the result. The atmospheric index responds immediately and relaxes within weeks, carrying no memory of the event.  
 The Indian Ocean index responds within months and executes one damped oscillation. The Pacific index is displaced for  
 335 about a year, rebounds to the opposite sign near year two, and returns to within the noise by year five, which is the damped  
 oscillatory response characteristic of the recharge mechanism. The theory therefore predicts that the detectability of a volcanic  
 signal depends on the mode: a multi-year signature should be recoverable in the tropical Pacific and essentially absent in the



extratropical atmosphere. That prediction can be made quantitative. Measured against the internal variability of each index, the deterministic response reaches 0.23 standard deviations for Niño 3.4, 0.09 for the Dipole Mode Index and 0.03 for the NAO, in each case within a few months of the imposed cooling. A composite of  $n$  independent eruptions reduces the noise, so separating the response from zero at the conventional level would call for some 74 events in the Pacific, 509 in the Indian Ocean and several thousand for the NAO; waiting half a year raises the first figure to 158. That count assumes independent events of equal size, so it is a lower bound rather than an estimate. The instrumental record offers of order five large tropical eruptions.

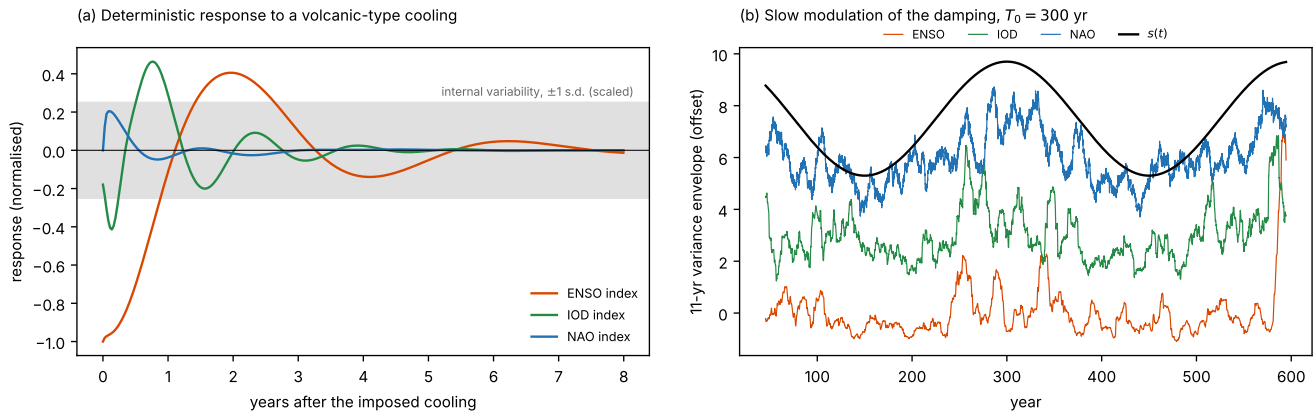


**Figure 9.** (a) Deterministic response of three indices to an imposed tropical cooling of the mixed layer, normalised by the initial ENSO response, with the band of internal variability shown for scale. (b) Variance envelopes of three simulated indices under a slow modulation of the dissipation with  $T_0 = 300$  yr and  $\mu = 0.2$ , with the slow variable  $s(t)$  shown above.

The useful form of this test is therefore not the amplitude of a composite but its shape, and in particular the reversal of sign near year two, which no memoryless response can produce. Two further forms are stronger still. The first compares the response across modes, a ratio the theory predicts and one that does not depend on the magnitude of any single eruption: the Pacific response should exceed the Indian by a factor near two and the NAO by a factor near four. The second examines the variance of each index after an eruption rather than its mean, since a displaced oscillator leaves a signature in its second moment that persists after the first has decayed into the noise, and since the second moment of a composite accumulates with  $n$  rather than with  $\sqrt{n}$ .

The last natural forcing term is the slow modulation of Eq. (6), in which a slowly varying background state alters the dissipation of every mode. Figure 9b shows the variance envelopes of three indices in a 600-yr integration with  $T_0 = 300$  yr and  $\mu = 0.2$ . The envelopes acquire a common low-frequency component, with a mean pairwise correlation of 0.39 over the run, in agreement with (7). The amplitude of the effect is modest because the damping rates of the coupled modes are controlled by the ocean blocks, whose sensitivity to the modulation is weaker than that of the atmospheric modes.



## 5.2 Anthropogenic forcing: additive and parametric channels

Natural forcing enters the construction in two ways, as the noise that maintains the variability and as the slow modulation that organises its envelopes. Anthropogenic forcing raises a question that neither of those poses, since it may enter as a term added to the equation of motion or as a change in the coefficients of the operator itself, and only one of the two can alter a mode. The distinction is developed below.

First, it should be clear at this point that additive forcing cannot change a mode. If we write the governing equation with an explicit anthropogenic term,

$$dX = L X dt + f_a(t) dt + Q^{1/2} dW. \quad (20)$$

because the system is linear, the solution separates into a forced part  $X_a$ , obeying  $\dot{X}_a = L X_a + f_a$ , and a stochastic part obeying the unforced equation. The covariance of (20) is governed by the Lyapunov equation (4), in which  $f_a$  does not appear. A radiative forcing that enters additively therefore displaces the mean state and leaves exactly unchanged every anomaly statistic, mode period, damping rate, variance and spatial pattern. This is an identity of the linear problem, and it is the reason why anomaly indices are conventionally computed with a trend removed: the removal is assumed to be lossless. Indeed, the identity has an immediate interpretation. Any observed change in a mode must come from a change in the operator or in the noise, not from the radiative forcing as such. Attribution of a change in variability is therefore a different problem from attribution of a change in the mean, and it cannot be settled by regressing an index on a forcing time series.

The way forward is as follows. The operator depends on the background state, which warming alters, so the physically relevant statement is  $L \rightarrow L(g)$  and  $Q \rightarrow Q(g)$  with  $g$  a scalar index of the forcing, zero for the present day and unity for a late-century state. Expanding each coefficient to first order in  $g$  gives a small set of channels. For the operator of Sect. 3 there are five channels that matter. The first is stratification. Upper-ocean stratification has increased over the past half century and is projected to continue (Li et al., 2020), and a sharper thermocline raises the equatorial gravity wave speed. Since  $c_K$  enters the adjustment frequency (15) directly, this channel acts on the interannual periods and on nothing else. The second channel is moisture. Column water vapour follows the Clausius–Clapeyron relation at roughly 7% per kelvin (Held and Soden, 2006), so the convective coupling coefficients  $a_q$  and  $b_q$  of (13), which measure how efficiently low-level convergence moistens the column and how efficiently the resulting heating forces vorticity, increase accordingly. The third is the damping of mixed-layer temperature. Stronger longwave emission and a stronger evaporative feedback at higher temperature increase  $|\lambda_T|$ , which damps the coupled tropical modes. The fourth is moisture advection, which enters  $u_q$  in (12) and therefore the propagation speed of the moisture mode (Adames and Kim, 2016). The fifth is the mixed layer itself. Shoaling reduces the effective heat capacity, so the same surface flux variability produces larger temperature anomalies, which raises the noise amplitude in  $Q$  rather than changing  $L$ .

We set the sensitivities at  $g = 1$  to +15% for  $c_K$ , +28% for the moisture coupling, +10% for  $|\lambda_T|$ , +10% for  $u_q$  and +10% for the temperature noise. These are plausible magnitudes. Everything is then computed analytically. The resulting eigenvalues give the periods and damping times, and the Lyapunov equation (4) gives the index variances. Figure 10 summarises the response of the modes to the parametric anthropogenic forcing.

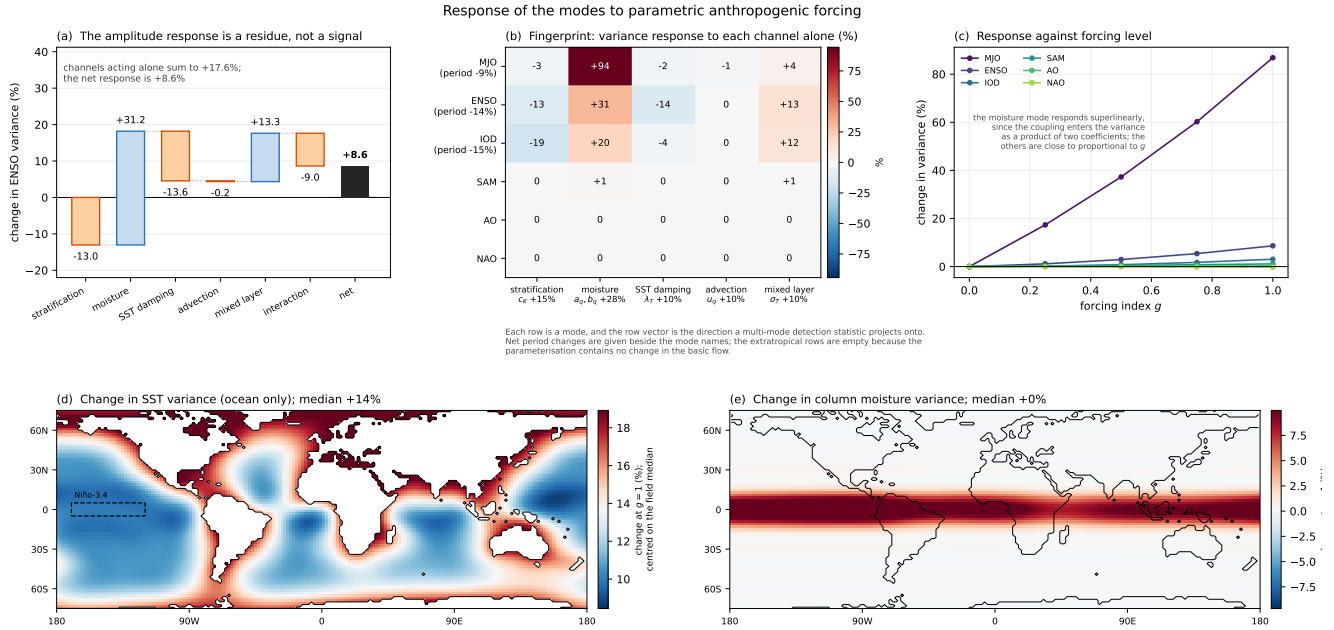


**Figure 10.** Response of the modes to a parametric anthropogenic forcing of strength  $g$ , with  $g = 1$  a late-century state. (a) Waterfall decomposition of the ENSO variance response: each channel acting alone, the interaction term, and the net result; the contributions sum to +17.6% while the full perturbation gives +8.6%, the difference arising because the operator is not linear in its parameters. (b) Fingerprint matrix, the variance response of each index to each channel acting alone, with the net change in eigenmode period beside the mode names. (c) Response against forcing level; the moisture mode grows superlinearly because the coupling enters the variance as a product of two coefficients. (d, e) Local change in the variance of mixed-layer temperature and of column moisture at  $g = 1$ , obtained from the stationary covariance through (4) rather than by integration. Both colour scales are centred on the median change of the field, given in the panel titles, so that blue marks a response weaker than the field average rather than a decrease; the temperature map is masked over land, where a mixed layer is not defined.

It was found that the interannual periods shorten, and that the response is dominated by one channel. The ENSO eigenmode period falls from 4.35 to 3.78 yr, a shortening of 13%, and Table 4 shows that stratification accounts for essentially all of it, the other four channels contributing less than half a per cent between them. The reason is visible in (15): the period scales as  $W/c_K$ , the basin width is fixed, so the fractional change in period is minus the fractional change in wave speed. The theory therefore predicts

$$\frac{\delta T}{T} \simeq -\frac{\delta c_K}{c_K}, \quad (21)$$

a relation with no free parameter, derived from a model containing no ENSO physics beyond an adjustment timescale. This result is directly testable in scenario simulations, where both quantities can be measured. The moisture mode shortens more modestly, from 47.1 to 43.3 d.



**Table 4.** One-at-a-time decomposition of the response at  $g = 1$ : the change produced by each channel acting alone. The net ENSO variance response, +8.6%, is the residue of four contributions whose magnitudes sum to about 70%.

| Channel perturbed                | ENSO period | ENSO variance | MJO variance | NAO variance |
|----------------------------------|-------------|---------------|--------------|--------------|
| Stratification, $c_K$            | −13.3%      | −10.1%        | −1.5%        | +0.0%        |
| Moisture coupling, $a_q, b_q$    | −0.0%       | +37.4%        | +104.9%      | −0.1%        |
| Mixed-layer damping, $\lambda_T$ | +0.4%       | −13.1%        | −1.8%        | +0.0%        |
| Moisture advection, $u_q$        | +0.0%       | −0.3%         | −1.1%        | +0.0%        |
| Mixed-layer noise, $\sigma_T$    | +0.0%       | +15.3%        | +3.6%        | −0.1%        |
| All five together                | −13.1%      | +20.1%        | +101.0%      | −0.2%        |

400 The variance response is the more interesting result, because of its structure. The net change in ENSO variance is +20%, but Table 4 shows that this is the residue of four large and opposing contributions: moisture coupling alone gives +37%, stratification alone −10%, stronger mixed-layer damping −13%, and mixed-layer shoaling +15%. The magnitudes sum to 76% and the result is 20%, so the net response is a residue of terms several times larger than itself. At these sensitivities it comes out positive; a model whose balance among the four differed modestly would not. That cancellation is a prediction in  
 405 its own right. It says that the amplitude response of ENSO to greenhouse forcing is small, poorly constrained and of uncertain sign, not because the problem has been modelled badly but because four comparable terms oppose one another. A model whose balance among the four differs modestly will land on either side of zero. The literature behaves exactly so: projections of ENSO amplitude disagree across CMIP5 and CMIP6 (Beobide-Arsuaga et al., 2021; Brown et al., 2020), the response depends on the warming pattern (Freund et al., 2020), and the teleconnection response is better constrained than the amplitude (Yeh et al.,  
 410 2018).

Our own evaluation found the same dissociation in the historical period, with intensity among the least reproducible metrics (Fig. 11). (We skip the details of processing the CMIP6 data, since it is a standard statistical procedure over a well-known dataset in the field, and because we do not want to drift from the narrative of this paper). The practical value of the decomposition is that it names the four quantities that must be constrained before the sign can be settled: the change in equatorial  
 415 wave speed, the change in convective coupling efficiency, the change in surface damping of SST anomalies, and the change in mixed-layer depth. Three of the four are measurable in observations as well as in models. A projection of ENSO amplitude is then only as good as the weakest of those four constraints, which is a sharper statement than reporting ensemble spread.

The moisture mode behaves differently: its variance doubles, +101%, and the moisture channel alone would give +105%. There is no cancellation because the moisture coupling enters both coefficients of the  $(\psi, q)$  block multiplicatively while  
 420 the other channels barely touch it. The prediction that MJO activity increases with warming is therefore robust within this framework.

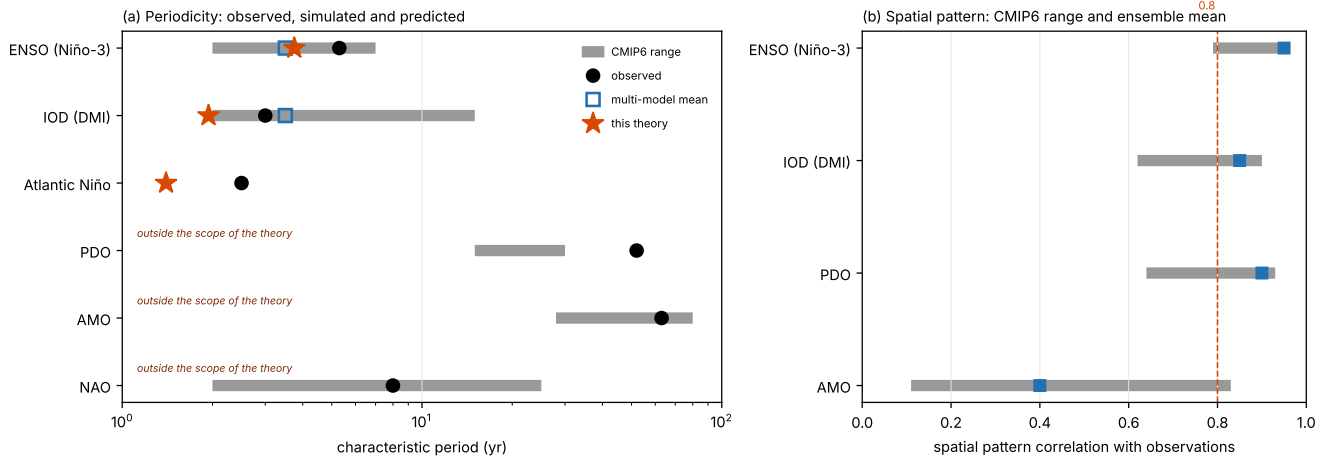


**Figure 11.** Periodicity and spatial pattern of the dominant modes in observations, in the CMIP6 archive and in the present theory. (a) Characteristic period: observed value, the range across 52 models, the multi-model mean where reported, and the value emerging from the theory. (b) Spatial correlation with observations across the archive, with the multi-model mean. All CMIP6 values come from our own calculations. The PDO, the AMO and the NAO periods lie outside the scope of the theory for the reasons given in Table 1.

### 5.2.1 A multi-mode fingerprint

Because every mode is modulated by the same  $g$  through the same five channels, the responses are not independent. Each mode carries a sensitivity vector over the channels, so the predicted pattern of change across modes, taken jointly, is a fingerprint of parametric forcing that no single index displays. A detection statistic formed from the joint response of several modes has better signal-to-noise than any one of them, for the same reason that a common-envelope statistic does. This also links the present theory to the question of slow modulation treated above: anthropogenic forcing is the monotonic case of that modulation, so the prediction becomes a common trend in mode variances, with loadings given by the sensitivity vectors rather than left free.

It is worth noting that the extratropical indices change by less than one per cent. Observed and projected changes in the annular modes are attributed to shifts of the eddy-driven jet and, in the Southern Hemisphere, to stratospheric ozone (IPCC, 2021), neither of which appears in a barotropic atmosphere with a prescribed solid-body basic flow. A sixth channel, a change in  $\omega_U$ , would produce a response through (11), but its sensitivity would be a free parameter with nothing in the model to anchor it, so we have not included one. Two further caveats belong here. The five sensitivities are chosen rather than derived, so the key is not in the values in Table 4 but in the sign of the frequency response, the cancellation in the amplitude response, and the contrast between the two. Also, the periods quoted here are eigenmode periods, which are not identical to the spectral peaks of the simulated indices reported above. The two diagnostics should not be mixed.



## 6 Discussion

There are a number of tests that would support the theory. Turning first to the structural predictions already testable, the basin-width scaling is the sharpest prediction, because it ties a period to a quantity that can be measured. Three further tests are available. The first is palaeogeographic: basin widths have changed through the Cenozoic, and the theory predicts that the characteristic period of each basin scaled accordingly, so that reconstructions of tropical variability from periods with a narrower Pacific should show shorter characteristic timescales. The second test is numerical: a coupled general circulation model run with an artificially narrowed or widened Pacific should shift the ENSO period in the sense and approximately the magnitude predicted by  $T \propto W^{1.00}$ . Such experiments are within reach of existing modelling systems. The third test is the observation that the Atlantic and Indian basins are of similar width and the theory predicts similar periods, despite their different mean states. The computed periods passes that test: the two basins differ in width by seven per cent and the operator gives them the same period to the resolution of the calculation. A robust difference between the observed Indian and Atlantic timescales, established on records long enough to separate them, would count against the mechanism.

A second structural prediction concerns the damping times. Equation (5) ties the width of each spectral peak to the real part of the corresponding eigenvalue, so the theory predicts a specific relationship between peak width and the decay of lagged autocorrelation for each index. That relationship can be tested directly on observations, and it does not depend on the particular parameter values used here.

A third prediction concerns the structure of the operator itself. An empirical operator can be estimated from observations by linear inverse modelling (Penland and Sardeshmukh, 1995), and the theory requires that such an operator show a near-antisymmetric coupling between mixed-layer temperature and heat content, with an amplitude varying between basins as  $1/W$ . We have put it to the model, where the answer is known beforehand: records were generated by exact sampling of the stationary process, an operator was fitted to each by the usual lag-covariance method, and the quantities the test asks for were measured on the fit. The outcome featured in Fig. 7c and d. The Pacific period is recovered, but slowly, a record of 150 yr returning  $6.63 \pm 4.96$  yr against the 4.56 yr that the same method returns in the limit, and the estimate does not settle to within a tenth of that value until the record exceeds three centuries. The departure from antisymmetry behaves worse. The coupling in the generating operator is antisymmetric to one part in  $10^{16}$ , and remains so to eight parts in  $10^{16}$  after projection onto the retained patterns, yet the fitted operator reports a departure of  $0.436 \pm 0.043$  on a 150-yr record with six patterns retained, falling only to 0.128 when the record is extended to twenty-four centuries. The departure grows with the number of patterns retained, reaching 0.76 with twelve, which identifies sampling error in the lag covariance as its origin rather than any asymmetry in the physics. That means that the block-structure test is therefore not available on the instrumental record as it stands. What the record can support is a calibrated version: the departure measured in observations should be compared not with zero but with the distribution obtained by applying the identical procedure to surrogate records of the same length drawn from a known antisymmetric operator, which is the quantity plotted here. Stated so, the test keeps its force, since an observed departure well above that distribution would refute the block structure, and it loses an appearance of precision that it would otherwise carry.



470 Since the instrumental record is too short for validation, the natural testbed for the theory would be the coupled model archive CMIP6. The obvious experiment would be to vary basin geometry and measure the response of the period. There is a problem with such a proposition, however. We evaluated 52 CMIP6 historical simulations (Eyring et al., 2016) against observations, assessing five modes of variability through three metrics: spatial pattern, periodicity and intensity. The result (Figs. 12 and 13) is a clean dissociation. Models in CMIP6 reproduce the spatial structures well and the periods badly. For the Niño-3 regression

475 pattern, all but two models exceed a spatial correlation of 0.82 with HadISST; for the Dipole Mode Index the range is 0.62 to 0.90, with 0.85 for the ensemble mean; for the PDO most models exceed 0.74, with 0.90 for the ensemble mean. Periodicity behaves differently. Against an observed Niño-3 peak at 5.3 yr the models spread over 2 to 7 yr and mostly simulate shorter cycles, the ensemble mean giving 3.5 yr; against an observed DMI peak near 3 yr the models place maximum power anywhere between 3 and 15 yr, with seven below 3 yr; of the observed PDO peaks at 52, 17 and 6 yr only four models reproduce the

480 longest, most generating peaks between 15 and 30 yr; and the observed NAO maximum at 8 yr is poorly captured, although the secondary peak near 2.7 yr is better. Note that the periods quoted here are the location of the largest peak in a periodogram of one finite observational record, computed for Niño 3 and for the Dipole Mode Index; the values near 4 and 2 yr used above are the band centres conventionally quoted for Niño 3.4 and the Indian Ocean Dipole. A broad interannual band admits a peak anywhere within it once a single record is used, so 5.3 and 4 yr are not competing estimates of one number. The model values

485 of Sect. 4 should be compared with the bands, and the figures here with each other across models, which is what the ratio metrics below are designed to do. Intensity is similarly uneven: we found the amplitudes of ENSO and the IOD to be too strong in most models, a bias inherited from the previous generation (Weller and Cai, 2013; McKenna et al., 2020). No single model was superior across the metrics, and the model that scored worst on the mean state was not the one that scored worst on the modes.

490 Similar conclusions were reached by Fasullo et al. (2020) across the CMIP archives and by Planton et al. (2021) for ENSO specifically, and the problem is long-standing (Bellenger et al., 2014). This dissociation is what our framework predicts. In the formulation of Sect. 3 the spatial pattern of a mode and its period are set by different parts of the operator. The pattern follows from the resolvent through Eq. (18), and the resolvent is dominated by rotation, by dissipation and by the geographic multiplication operators of Eq. (9). Every coupled model shares those ingredients exactly, since every model has the same

495 continents, the same rotation rate and a broadly similar damping structure, so pattern skill should be high and should vary little between models. The period follows from the ocean block, specifically from  $\omega_{oc} = \kappa c_K / W$  in Eq. (15), in which  $W$  is again common to all models but  $c_K$  is not. The equatorial wave speed depends on the simulated stratification and thermocline depth. Those are exactly the quantities that mean-state biases corrupt, and the source of the observed disagreement between the models. The persistent double-ITCZ error (Mechoso et al., 1995; Tian and Dong, 2020) and the cold-tongue and thermocline

500 biases that accompany it therefore act directly on the one factor that sets the period, while leaving untouched the geometry that sets the pattern. That reading explains the empirically documented deficiency as it provides a mechanism behind it. If the period is set by  $\kappa c_K / W$  with  $W$  fixed, then the period error of a model should scale inversely with its error in equivalent equatorial wave speed, and models with the strongest cold-tongue and stratification biases should show the largest frequency errors, with errors in pattern correlation uncorrelated with them.

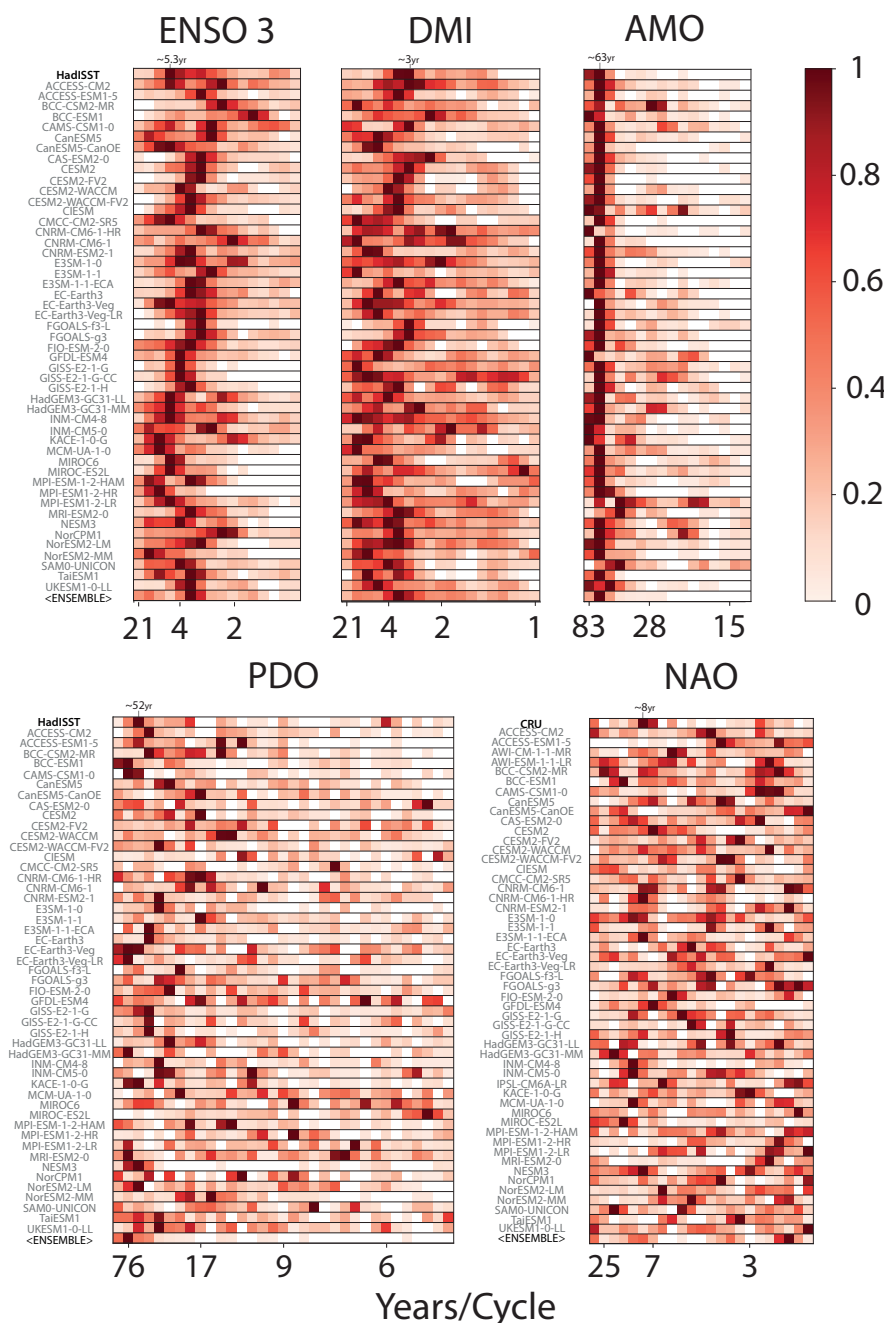
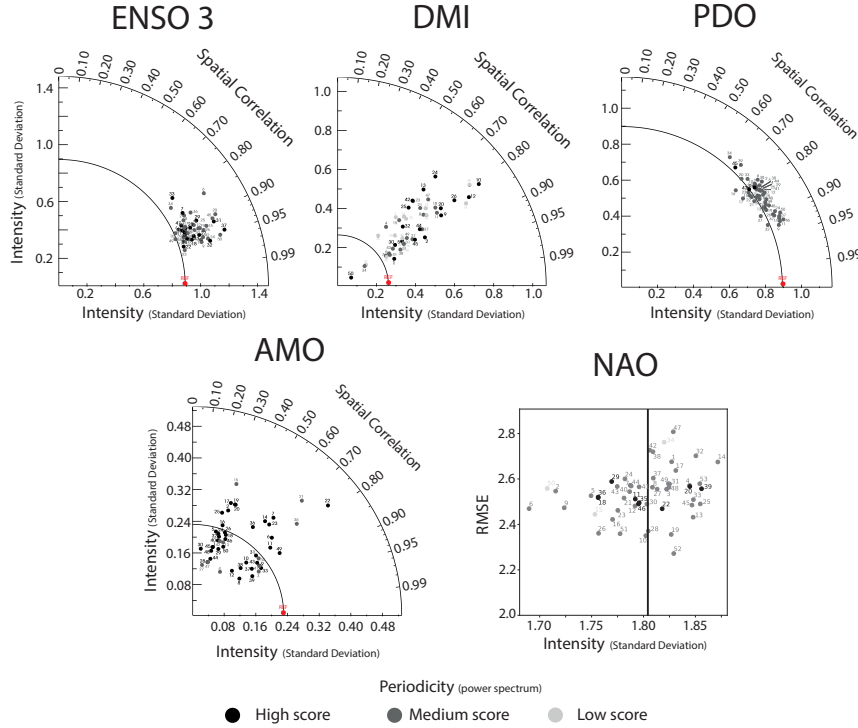


**Figure 12.** Power spectra of the Niño-3, DMI, AMO, PDO and NAO indices for 52 CMIP6 models, the multi-model mean and observations, displayed as a model-by-frequency matrix. The panel shows the full spread of simulated periodicity against the observed peak for each mode



Ref. HadISST: ENSO3, DMI, PDO, AMO  
 1. ACCESS-CM2 2. ACCESS-ESM1-5 3. BCC-CSM2-MR 4. BCC-ESM1 5. CAMS-CSM1-0 6. CanESM5-CanOE 7. CanESM5 8. CAS-ESM2-0 9. CESM2 10. CESM2-FV2 11. CESM2-WACCM 12. CESM2-WACCM-FV2 13. CIESM 14. CMCC-CM2-SR5 15. CNRM-CM6-1 16. CNRM-CM6-1-HR 17. CNRM-ESM2-1 18. E3SM-1-0 19. E3SM-1-1 20. E3SM-1-1-ECA 21. EC-Earth3 22. EC-Earth3-Veg 23. EC-Earth3-Veg-LR 24. FGOALS-g3 25. FGOALS-g3-L 26. FIO-ESM-2-0 27. GFDL-ESM4 28. GISS-E2-1-G 29. GISS-E2-1-G-CC 30. GISS-E2-1-H 31. HadGEM3-GC31-LL 32. HadGEM3-GC31-MM 33. INM-CM4-8 34. INM-CM5-0 35. KACE-1-0-G 36. MCM-UA-1-0 37. MIROC6 38. MIROC-ES2L 39. MPI-ESM-1-2-HAM 40. MPI-ESM1-2-HR 41. MPI-ESM1-2-LR 42. MRI-ESM2-0 43. NESM3 44. NorCPM1 45. NorESM2-LM 46. NorESM2-MM 47. SAMO-UNICON 48. TaiESM1 49. UKESM1-0-LL 50. <ENSEMBLE>  
 Ref. CRU NAO (SLP)  
 1. ACCESS-CM2 2. ACCESS-ESM1-5 3. AWI-CM-1-1-MR 4. AWI-ESM-1-1-LR 5. BCC-CSM2-MR 6. BCC-ESM1 7. CAMS-CSM1-0 8. CanESM5 9. CanESM5-CanOE 10. CAS-ESM2-0 11. CESM2 12. CESM2-FV2 13. CESM2-WACCM 14. CESM2-WACCM-FV2 15. CIESM 16. CMCC-CM2-SR5 17. CNRM-CM6-1 18. CNRM-CM6-1-HR 19. CNRM-ESM2-1 20. E3SM-1-0 21. E3SM-1-1 22. E3SM-1-1-ECA 23. EC-Earth3 24. EC-Earth3-Veg 25. EC-Earth3-Veg-LR 26. FGOALS-g3 27. FGOALS-g3-L 28. FIO-ESM-2-0 29. GFDL-ESM4 30. GISS-E2-1-G 31. GISS-E2-1-G-CC 32. GISS-E2-1-H 33. HadGEM3-GC31-LL 34. HadGEM3-GC31-MM 35. INM-CM4-8 36. INM-CM5-0 37. IPSL-CM6A-LR 38. KACE-1-0-G 39. MCM-UA-1-0 40. MIROC6 41. MIROC-ES2L 42. MPI-ESM-1-2-HAM 43. MPI-ESM1-2-HR 44. MPI-ESM1-2-LR 45. MRI-ESM2-0 46. NESM3 47. NorCPM1 48. NorESM2-LM 49. NorESM2-MM 50. SAMO-UNICON 51. TaiESM1 52. UKESM1-0-LL 53. <ENSEMBLE>

**Figure 13.** Joint assessment of intensity, pattern and periodicity in the CMIP6 archive Taylor diagrams for the Niño-3, DMI, PDO and AMO indices, in which the radial coordinate is the intensity of the index, the azimuthal coordinate the spatial correlation with observations, and the shading the agreement in periodicity. The panel displays intensity, pattern and periodicity jointly, and so exhibits directly the dissociation discussed in the text: models cluster near the observed pattern correlation while scattering widely in the other two metrics.

505 The reading also sheds light on the nature of the PDO, the IPO and the AMO. For the two Pacific modes the explanation is quantitative. Equation (15) fixes the adjustment frequency of a basin from the speed of the wave that carries the adjustment and the width of the basin. Outside the equatorial waveguide that wave is not the Kelvin wave but the long baroclinic Rossby wave, whose speed is  $c_R = \beta L_d^2 = c^2 \cos \varphi / (2\Omega a \sin^2 \varphi)$ . Substituting  $c_R$  for  $c_K$  in (15), and changing nothing else, gives  $6.9 \text{ cm s}^{-1}$  at  $20^\circ$ ,  $1.3 \text{ cm s}^{-1}$  at  $40^\circ$  and  $0.6 \text{ cm s}^{-1}$  at  $55^\circ$ , hence adjustment periods of 211 yr across the North Pacific at  $40^\circ\text{N}$  and 148 yr  
 510 across the North Atlantic at the same latitude. Those periods are not the point; their ratio to the damping is. The dimensionless quantity that decides whether a basin can oscillate is  $\omega_{oc}/\gamma_h$ , which equals 4.5 at the equator, where the recharge pair turns



several times before it decays, but falls to 0.13 at 20° and 0.05 at 40°. The extratropical ocean is overdamped by an order of magnitude.

We carried out the substitution. We found that no oscillatory extratropical eigenmode appears. The ocean-dominated eigen-  
 515 modes remains non-oscillatory and damped on the heat-content timescale; the spectrum of a North Pacific temperature index stays flat below about ten years with no peak of any kind; and what variance that index does carry at interannual periods arrives from the tropics rather than from local dynamics. Within this framework, therefore, the extratropical ocean integrates noise into a red spectrum instead of oscillating, and a basin index is dominated by teleconnection from the waveguide.

That conclusion is not ours alone. It is the position reached from observations by Newman et al. (2016), who argued that  
 520 the PDO is not a single mode but a superposition of ENSO teleconnection, stochastic forcing of the mixed layer, and ocean adjustment, and it is consistent with the difficulty of finding internal multidecadal oscillations in coupled simulations (Mann et al., 2020). The agreement is useful in two directions. It supplies an independent check on the ocean block, which in our theory was constructed for the tropics and was not adjusted to produce this behaviour. And it accounts for an otherwise puzzling feature of the CMIP6 archive: of the 52 models we evaluated, only four reproduced the observed 52-yr PDO peak and  
 525 most placed maximum power between 15 and 30 yr (Fig. 12). If no extratropical eigenfrequency exists, the position of the peak is set by the noise realisation and by the model's damping, so disagreement among models is actually the expected outcome.

Two qualifications belong with this. The argument covers the Pacific modes, whose timescale would have to come from wave adjustment, and it says nothing about the AMO, whose candidate mechanism is advective transit in the overturning circulation and which would require state variables this model does not carry. And the conclusion is conditional on the damping: an  
 530 upper-ocean heat-content damping time of about 30 yr, rather than the 1.6 yr used here, would bring  $\omega_{oc}/\gamma_h$  to 0.85 and permit oscillation. However, we regard that value as indefensible for the upper ocean.

## 7 Conclusions

The dominant modes of climate variability have been modelled as eigenmodes of one linear stochastic operator rather than as  
 535 separate phenomena. The operator contains four fields, rotation, dissipation, air–sea and moisture coupling, and the geography of the continents. It contains no mode-specific physics yet the modes naturally emerge from its dynamics. The construction supports the view that the modes are different aspects of one dynamical process rather than different phenomena.

The modes belong to three families. Damped atmospheric patterns with e-folding times of about a week correspond to the annular modes and the sectoral teleconnections, with a persistence that falls short of the observed ten to twenty days for a reason the construction makes explicit. An eastward-propagating equatorial moisture mode with a period of 46 d and an e-folding time  
 540 of 16 d corresponds to the MJO. Basin-confined coupled oscillators with periods of years correspond to ENSO, the IOD and the Atlantic zonal mode. Simulated indices reproduce the observed spectral bands: 4.29 yr for Niño 3.4, 1.54 yr for the Dipole Mode Index, 1.54 yr for the Atlantic index, 69.1 d for the moisture index, and red backgrounds for the annular indices. The resolvent of the autonomous operator, which carries no sampling error, places the first three at 4.06, 1.54 and 1.54 yr. The interannual periods are not fitted individually. They follow from the zonal width of the ocean basin, through a scaling in which



545 a single constant was tuned, with a fitted exponent of 1.00 on the resolvent peaks and 1.07 on the eigenmode periods, which is the inverse-width dependence that the construction imposes, and they reproduce the observed ordering and approximate spacing of the Pacific, Indian and Atlantic modes. That is the strongest evidence offered here that the unified formulation has content beyond re-description. An ablation of the construction locates the mechanism, for equalising the widths of the basins, and changing nothing else, collapses the spread of interannual period from a factor of eleven to a factor of 1.4, while removing  
 550 the antisymmetric recharge coupling leaves no oscillatory ocean mode at all.

Adding forcing makes the system more precise in ways consistent with the literature. Annual modulation of the coupling produces seasonal phase locking of the right magnitude. Stochastic forcing produces the red spectra of the extratropical modes without any oscillatory mechanism. An imposed volcanic cooling produces a damped oscillatory Pacific response with a rebound near two years and a memoryless atmospheric response. And slow modulation of the dissipation gives the mode  
 555 variances a common low-frequency component, which connects this theory to the question of whether such a component exists in the observed record.

One boundary of the theory can be drawn quantitatively. Applying the adjustment scaling of Eq. (15) outside the equatorial waveguide, with the long Rossby wave in place of the Kelvin wave, gives an adjustment frequency an order of magnitude below the heat-content damping at midlatitudes, so no extratropical ocean eigenmode exists in this framework and a North  
 560 Pacific index reddens noise and receives teleconnection from the tropics instead of oscillating. That is the reading of the PDO reached from observations by Newman et al. (2016), and it also explains why only four of the 52 models in the CMIP6 dataset we evaluated reproduce the observed 52-yr peak. With no eigenfrequency to anchor it, the position of the peak is set by noise.

Adding anthropogenic forcing exposes a structural result. Forcing that enters additively cannot change a mode at all, by the superposition property of the linear problem, so any change in variability must come from the dependence of the operator on  
 565 the background state. Parameterising that dependence through five channels gives a frequency response dominated by one of them, the increase in equatorial wave speed with stratification, from which  $\delta T/T \simeq -\delta c_K/c_K$  follows with no free parameter. The amplitude response behaves differently: the net change in ENSO variance is the residue of four opposing contributions of comparable size, which predicts that the amplitude response is small and of uncertain sign. That prediction accounts for the persistent disagreement among projections (Beobide-Arsuaga et al., 2021; Brown et al., 2020) and identifies the four quantities  
 570 whose constraint would settle it. The contrast with the moisture mode, whose variance response is large and one-signed because no cancellation occurs, follows from the structure of the operator rather than from differences of modelling care.

Validating the theory against coupled models is harder than it appears. The difficulty is itself informative. Our evaluation of 52 CMIP6 models shows that they capture the spatial patterns of the modes, with correlations above 0.8 for most models and most indices, and miss the periods, with the Niño-3 peak spread over 2 to 7 yr against an observed 5.3 yr (the periodogram  
 575 peak for that index, not the Niño 3.4 band quoted earlier) and the DMI peak over 3 to 15 yr. The present framework separates those two properties: the pattern comes from the geographic response operator, which every model shares, and the period from the equatorial wave speed, which mean-state biases corrupt. The archive therefore cannot settle the absolute periods, but it can test the scaling, through relative period changes under an imposed change of basin geometry and through the ratio of periods between basins within each model, for which the predicted values are  $T_{\text{Pac}}/T_{\text{Ind}} \simeq 2.4$  and  $T_{\text{Ind}}/T_{\text{Atl}} \simeq 1.0$ .



580 A final potential concern is how realistic our model is. The simplifications embedded in the model may seem too stringent. The atmosphere is barotropic, so baroclinic conversion, the stratosphere and the storm tracks are absent, and the extratropical modes appear only as damped patterns rather than as eddy-driven jets. The ocean has no explicit wave dynamics, the recharge structure standing in for the propagation of equatorial Kelvin and Rossby waves. Moisture is represented by a single column variable without a vertical structure or a realistic convective closure, so the MJO appears as a kinematic moisture mode rather than as a fully coupled disturbance. The coupling coefficients were chosen to be of order unity rather than derived, and the seasonal phase is imposed. Nonlinearity is absent, so the theory cannot produce the observed skewness of ENSO, the asymmetry between El Niño and La Niña, or regime behaviour. Nonetheless, the dynamics of the Earth system shows that none of those ingredients is required in order for the dominant modes to appear with approximately the right periods, damping times, patterns and seasonal behaviour. They are required in order to get the details right, but not to get the first approximation to the dynamics of the system.

590 Our theory is minimal by design. Comprehensive models reproduce the observed modes without accounting for them, and the distance between simulation and understanding widens as models grow more complex (Held, 2005); the response proposed in that literature is a hierarchy of models in which the simplest members carry the explanatory load (Jeevanjee et al., 2017). On that view complexity is a cost to be justified rather than a virtue in itself, and the present construction occupies the lower end of such a hierarchy deliberately, not for want of resources. Its value lies in the predictions that follow from its structure rather than from its parameters, and above all in the basin-width scaling, and in the prediction that an empirical operator estimated from observations should display the same block structure and the same scaling.

*Code and data availability.* The model, the eigen-analysis and the integrations scripts are provided as `unified_theory.py` and `sim_theory.py`. Before publication the code and derived data will be deposited in a Zenodo archive with a DOI. The land mask is from the `global-land-mask` Python package. No observational data were used in the construction of the model; the observed periods quoted for comparison are standard values from the literature cited.

*Author contributions.* FJT had the idea, devised the model, wrote the manuscript and extracted the consequences. AN carried out all the CMIP6 processing and the statistical tests. RM supervised his analyses. GJ ran the Python scripts, except those for Figs. 11 and 12, which were devised by AN. AM provided data and criticism of several parts of the manuscript. EGO managed the project and provided edits to the manuscript.

*Competing interests.* The authors declare that there are no competing interests.



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