



Geocenter sensitivity from satellite tracking

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Abstract. In this study, we revisit Kaula’s orbit perturbation theory to establish how satellite orbital geometry controls the sensitivity to degree-1 gravitational signals associated with geocenter motion along the z -axis and the (x,y) -plane, respectively. We explicitly derive the Kaula-based expressions for the degree-1 potential terms and provide complete tables of the corresponding inclination $F_{1mp}(i)$ and eccentricity functions $G_{1mp}(e)$ for all relevant degree-1 terms.

As a case study, we apply this framework to the upcoming ESA Genesis mission, which will be the first in-orbit geodetic observatory carrying all four geodetic techniques (GNSS, SLR, DORIS and VLBI) onboard.

We examine the dependence of the spacecraft orbital parameters through contributions at the spacecraft location of the degree-1 gravitational potential (V_{10} and V_{11}). Our analysis shows that the V_{10} term associated with the geocenter’s z -component scales with $\sin i$, implying that polar orbits maximize sensitivity to vertical geocenter variations, whereas equatorial orbits remain largely insensitive to them. Conversely, the two-term structure of the V_{11} expression enhances the detectability of the geocenter’s x - and y -components for equatorial or near-equatorial orbits, while retaining reduced but non-negligible sensitivity in polar configurations. For the Genesis orbital characteristics, we also assess the orbit inclinations providing the best trade-off to obtain the three components of the geocenter motion.

This work demonstrates how Kaula’s theory can guide the design and orbital configuration of future geodetic missions by identifying orbital parameters that optimize sensitivity to degree-1 signals. Such optimization can enhance the sensitivity to degree-1 signals and thereby contribute to improved geocenter motion recovery and strengthen the link between satellite-based gravimetry and terrestrial reference frame realization.

1 Introduction

Today, the Terrestrial Reference Frame (TRF) is realized from the positions and velocities of a globally distributed network of ground stations. It is established by combining observations from the four major space geodetic techniques: the Global Navigation Satellite System (GNSS), Satellite Laser Ranging (SLR), Doppler Orbitography and Radiopositioning Integrated by Satellite (DORIS), and Very Long Baseline Interferometry (VLBI). The resulting coordinates and velocities provide a realization of the terrestrial reference system, enabling precise positioning and the consistent combination of geodetic observations over space and time.



The origin of the International Terrestrial Reference Frame (ITRF) is intended to coincide, in the long term, with Earth's mean Centre of Mass (CM). In ITRF2020, as in previous realizations, the origin is determined exclusively from SLR observations provided by the International Laser Ranging Service (ILRS), by imposing zero translation and zero translation rate between the ITRF and the long-term SLR frame (Altamimi et al., 2023). ITRF2020 also reveals seasonal variations of the CM at the centimetre level. The consistency of the ITRF origin positions can be assessed by comparing it with previous realizations, namely ITRF2005, ITRF2008, and ITRF2014, whose origins were likewise determined from SLR observations (Altamimi et al., 2007, 2011, 2016). The translations between these realizations range from -1.4 to 3.3 mm at epoch 2015.0, while the corresponding translation rates vary between -0.1 and 0.3 mm yr⁻¹. Thus, successive ITRF origins agree at the few-millimetre level and exhibit long-term stability better than about 0.3 mm yr⁻¹. A conservative estimate is therefore 5 mm for the origin and 0.5 mm yr⁻¹ for its long-term stability.

The term geocenter motion refers to the displacement of the instantaneous CM relative to the long-term origin of the ITRF (Ray, 1999). The geocenter plays a fundamental role in describing Earth's motion in inertial space and serves as the dynamical centre of all Earth satellites. Because the ITRF origin represents a long-term average of satellite CM realizations (Altamimi et al., 2007, 2011, 2016, 2023), any geodetic analysis performed in the ITRF requires corrections for instantaneous geocenter motion unless degree-1 gravity coefficients are explicitly estimated. Consequently, the accuracy and long-term stability of the ITRF origin are directly linked to the capability of the space-geodetic techniques (SLR, GNSS and DORIS) to observe geocenter motion.

Technique-independent estimates of geocenter motion can be combined with ground-station geodetic coordinate time series to express observations consistently in the instantaneous CM frame, ultimately providing a globally unified CM displacement time series. Current positioning accuracy is approximately 1–2 cm for individual satellite observations, improving about 4 mm accuracy after one week of observations (Delva et al., 2023).

Geocenter motion reflects global mass redistribution associated with sea-level change, atmospheric and oceanic circulations, present-day ice mass imbalance, continental hydrology, ocean tides, glacial isostatic adjustment (GIA), and deep-Earth processes involving the mantle and core. These phenomena are thus contributing to degree-1 gravitational potential coefficients. The response of the solid Earth depends strongly on the timescale of the mass redistributions. From sub-daily to decadal timescales, the Earth behaves predominantly elastically, whereas over timescales of 10^3 – 10^6 years it responds viscoelastically and is commonly approximated as a linear Maxwell body (Greff-Lefftz and Legros, 1997).

Typical geocenter displacements are of the order of 1 cm due to diurnal and semidiurnal ocean tides, a few millimetres due to seasonal atmospheric and hydrological mass redistribution, and less than 1 mm yr⁻¹ on secular timescales (e.g., Wu et al., 2012). Despite these relatively small amplitudes, reliable estimates remain difficult because of modelling errors, incomplete uncertainty quantification, systematic errors, and limitation in the precision with which the dynamics of geodetic satellites can be modelled. Although geocenter motion can in principle be determined directly from satellite tracking, detecting millimetre-level displacements requires extremely accurate and internally consistent observations, together with highly accurate force and



65 measurement models. As many geodetic applications now require millimetre accuracy, the effects of geocenter motion can no longer be neglected.

Achieving the target stability of 0.1 mm yr^{-1} required for global change monitoring will require further improvements in tracking accuracy, network geometry, data consistency, and modelling strategies. While advances in both observing technology and geophysical modelling have significantly improved our ability to model seasonal and long-term geocenter variations, 70 constraining instantaneous geocenter motion remains a major challenge. These limitations indicate that improving the observation of geocenter motion should become an explicit objective of future geodetic missions. In this context, studying the sensitivity of satellite orbits to geocenter motion provides a rational basis for selecting mission parameters that maximize geocenter observability.

The sensitivity of satellite orbits to degree-1 gravitational coefficients has been recognized in previous studies. In particular, 75 the degree-1 coefficients C_{10} , C_{11} , and S_{11} , which describe the three components of geocenter motion, can in principle be determined from satellite orbit observations when the satellite motion is expressed in the ITRF (e.g., Swenson et al., 2008; Bergmann-Wolf et al., 2014; Chanard et al., 2018; Razeghi et al., 2019; Kang et al., 2019; Bouillé et al., 2000). Degree-1 coefficients have subsequently also been estimated from satellite tracking data, including GNSS tracking of low-Earth-orbit satellites. More recently, Sośnica (2024) used Kaula's theory of gravitational perturbations to investigate the sensitivity of 80 future geodetic satellite orbits to low-degree gravity coefficients, including C_{10} , as a function of orbital altitude, inclination, and eccentricity. For degree 1, the gravitational signal is manifested mainly through short- and long-periodic orbital perturbations, while no secular contribution is expected. For C_{10} , the study showed that the sensitivity is particularly favourable for near-polar orbits and that the z -component of geocenter motion is determined considering only short-periodic orbital perturbations (Sośnica, 2024). However, these studies do not provide a systematic Kaula formulation of the complete degree- 85 1 terms and their dependence on the orbital elements.

Recent work has also investigated the dependence of satellite orbit perturbations on orbital altitude, inclination, and eccentricity for gravity-field recovery, including degree-1 terms associated with geocenter motion (Sośnica et al., 2025). However, to our knowledge, a systematic derivation and tabulation of the complete degree-1 Kaula inclination and eccentricity functions, together with an explicit analysis of the separate order-0 and order-1 sensitivities and their implications for geocenter 90 observability, has not yet been presented in the literature.

In particular, the forthcoming Genesis mission, which will co-locate GNSS, SLR, DORIS, and VLBI on a single satellite, aims to realize the terrestrial reference frame with millimetre-level accuracy, requiring geocenter motion to be recovered at a comparable level of accuracy.

To this end, we derive the contribution of geocenter motion to the gravity field observed by satellites and use Kaula's analytical 95 framework to investigate how the orbital elements control the sensitivity to the degree-1 gravity field associated with geocenter motion.

The paper is organized as follows. Section 2 introduces the degree-1 gravitational potential in the terrestrial reference frame and relates its coefficients to geocenter motion. Section 3 develops the corresponding Kaula formulation in terms of the



satellite's orbital elements and derives the degree-1 inclination and eccentricity functions. Section 4 applies these expressions to the Genesis orbit and analyses the dependence of the sensitivity of the spacecraft motion to the geocenter motion (through the degree-1 gravitational potential contributions) for different orbital inclination and eccentricity. The implications of these results for geocenter observability and the design of future space-geodetic missions are discussed in Section 5.

2 Degree-one gravity field when satellite motions are in the ITRF

We consider the case where a gravitational potential perturbs the orbit, affecting both the spacecraft's position and velocity expressed in an Earth-fixed geocentric reference frame. To begin, we provide the expression of the gravitational potential U developed in spherical harmonics:

$$U = \frac{GM_{\oplus}}{r} + V = \frac{GM_{\oplus}}{r} + \sum_{n=1}^{\infty} \sum_{m=0}^n V_{nm} = \frac{GM_{\oplus}}{r} + \frac{GM_{\oplus}}{r} \sum_{n=1}^{\infty} \left(\frac{a_{\oplus}}{r}\right)^n \left\{ \sum_{m=0}^n [C_{nm} \cos m\lambda + S_{nm} \sin m\lambda] P_{nm}(\sin \phi) \right\} \quad (1)$$

The first term in Eq. (1) corresponds to the Keplerian orbit of the satellite around the Earth considered as a point-mass with the total mass of the Earth; the second term being the perturbing gravitational potential:

$$V_{nm} = \frac{GM_{\oplus}}{r} \left(\frac{a_{\oplus}}{r}\right)^n [C_{nm} \cos m\lambda + S_{nm} \sin m\lambda] P_{nm}(\sin \phi) \quad (2)$$

where $M_{\oplus}$ is the Earth mass, $a_{\oplus}$ is the mean equatorial radius, C_{nm} and S_{nm} are the so-called Stokes coefficients (unnormalized), P_{nm} are the unnormalized associated Legendre polynomials, and (r, ϕ, λ) are the distance, latitude, and longitude of the satellite on which the potential is applied. The subscripts n and m represent the degree and the order of the potential.

As the Earth is not homogeneous, there are other degrees and orders in the gravitational potential acting on the satellite. The degree-1 terms correspond to geocenter motion effects and is expressed as:

$$V_{1m} = \frac{GM_{\oplus}}{r} \left(\frac{a_{\oplus}}{r}\right) [C_{1m} \cos m\lambda + S_{1m} \sin m\lambda] P_{1m}(\sin \phi) \quad (3)$$

3 Expression of the gravitational potential in terms of the satellite's orbital elements

The orbital elements, also called Keplerian elements in absence of perturbations, are used to describe the orbital plane, and the satellite's trajectory within it. The orbital elements referred to as the metric orbital elements are the semi-major axis a , the eccentricity e , and the inclination i , while the elements known as the angular orbital elements are the argument of the perigee ω , the longitude of the node Ω , and the true anomaly v .

The longitude of the spacecraft is expressed as

$$\lambda = \alpha - GMST = (\alpha - \Omega) + (\Omega - GMST) \quad (4)$$



where α is the right ascension, $GMST$ is the Greenwich Mean Sidereal Time, and Ω is the longitude of the orbital node (see Figure 1).

Applying trigonometric identities, $\cos m\lambda$ and $\sin m\lambda$, appearing in Eqs. (1) and (2), can be written as

$$\cos m\lambda = \cos m(\alpha - \Omega) \cos m(\Omega - GMST) - \sin m(\alpha - \Omega) \sin m(\Omega - GMST) \quad (5)$$

$$\sin m\lambda = \sin m(\alpha - \Omega) \cos m(\Omega - GMST) + \cos m(\alpha - \Omega) \sin m(\Omega - GMST) \quad (6)$$

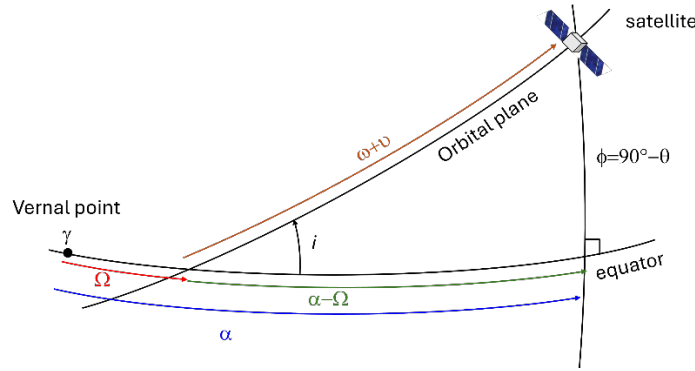


Figure 1: Orbit-equator-meridian triangle, where γ denotes the vernal point, α the right ascension, Ω the longitude of the orbital node, ϕ the latitude (θ the colatitude) of the spacecraft, ω the argument of perigee, i the inclination of the orbit, and v the true anomaly of the spacecraft.

Subsequently, and using classical spherical-triangle relations, illustrated in Figure 1:

$$\cos(\alpha - \Omega) = \frac{\cos(\omega + v)}{\cos \phi} \quad (7)$$

$$\sin(\alpha - \Omega) = \frac{\sin(\omega + v) \cos i}{\cos \phi} \quad (8)$$

$$\sin \phi = \sin(\omega + v) \sin i \quad (9)$$

After some algebraic manipulations and application of trigonometric identities, following the development of Kaula (1966), V_{lm} becomes:

$$V_{lm} = \frac{GM_{\oplus}}{r} \left(\frac{a_{\oplus}}{r} \right)^l \sum_{p=0}^l F_{mp}(i) \left\{ \begin{bmatrix} C_{lm} \\ -S_{lm} \end{bmatrix} \begin{matrix} l-m \text{ even} \\ l-m \text{ odd} \end{matrix} \cos((l-2p)(\omega + v) + m(\Omega - GMST)) \right. \\ \left. + \begin{bmatrix} S_{lm} \\ C_{lm} \end{bmatrix} \begin{matrix} l-m \text{ even} \\ l-m \text{ odd} \end{matrix} \sin((l-2p)(\omega + v) + m(\Omega - GMST)) \right\} \quad (10)$$

where the inclination functions $F_{nmp}(i)$ are given by:



$$F_{lmp}(i) = \sum_{t=0}^{\min(p,k)} \frac{(2l-2t)!}{t!(l-t)!(l-m-2t)! 2^{2l-2t}} \sin^{l-m-2t} i \\
\times \sum_{s=0}^m \binom{m}{s} \cos^s i \sum_{c=0}^{\max \text{ possible}} \binom{l-m-2t+s}{c} \binom{m-s}{p-t-c} (-1)^{c-k} \quad (11)$$

where k is equal to $\max(\mathbb{Z} \leq \frac{l-m}{2})$, and the binomial coefficient is:

$$\binom{l}{j} = \frac{l!}{j!(l-j)!} \quad (12).$$

The bounds for the variable c are limited to ensure that both binomial coefficients involved are defined and nonzero. Specifically, for the first binomial term, c must satisfy $0 \leq c \leq l-m-2t+s$, while for the second binomial term, the constraint is $0 \leq p-t-c \leq m-s$. Regarding the parameters $[l, m, p]$, the degree $l \in \mathbb{N}$, the order m , which represents the longitudinal variation in the potential, must be $0 \leq m \leq l$, and finally the index p must also lie within $0 \leq p \leq l$.

The values of the inclination functions for degree-1 are given in Table 1.

It remains to express $\left(\frac{1}{r}\right)^{l+1}$ of Eq. (10) and the arguments involving the true anomaly v in terms of the classical orbital elements, in particular the mean anomaly M . For this purpose, we use the relations between r , M , E , v , where E is the eccentric anomaly, and expand the resulting expressions in terms of Bessel functions to obtain:

$$\left(\frac{1}{r^{l+1}}\right) \left[\frac{\cos}{\sin}\right] ((l-2p)(\omega+v) + m(\Omega - GMST)) \\
= \left(\frac{1}{a^{l+1}}\right) \sum_t^{\pm l} G_{lpt}(e) \left[\frac{\cos}{\sin}\right] ((l-2p)\omega + (l-2p+t)M + m(\Omega - GMST)) \quad (15)$$

The $G_{lpt}(e)$ functions are also provided in Table 1. These functions can be divided into two parts: long-period terms, for which M is absent ($t = 2p - l$), and short-period terms, for which $l - 2p + t \neq 0$.

For the long periods ($t = 2p - l$):

$$G_{lp(2p-l)}(e) = \frac{1}{(1-e^2)^{l-\frac{1}{2}}} \sum_{d=0}^{p'-1} \binom{l-1}{2d+l-2p'} \binom{2d+l-2p'}{d} \left(\frac{e}{2}\right)^{2d+l-2p'} \quad (16a)$$

in which

$$p' = \begin{cases} p, & \text{for } p \leq \frac{l}{2} \\ l-p, & \text{for } p \geq \frac{l}{2} \end{cases}$$



170 For the short periods ($l - 2p + t \neq 0$):

$$G_{lpt}(e) = (-1)^{|t|} (1 + q^2)^l q^{|t|} \sum_{k=0}^{\infty} P_{lptk} Q_{lptk} q^{2k} \quad (16b)$$

where $q = \frac{e}{1 + \sqrt{1 - e^2}}$, and

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$$P_{lptk} = \sum_{s=0}^h \frac{(-1)^s}{s!} \binom{2p' - 2l}{h - s} \left(\frac{(l - 2p' + t') e}{2q} \right)^s$$

with

$$h = \begin{cases} k + t', & \text{for } t' > 0 \\ k, & \text{for } t' < 0 \end{cases}$$

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and

$$p' = \begin{cases} p, & t' = t, & \text{for } p \leq \frac{l}{2} \\ l - p, & t' = -t & \text{for } p > \frac{l}{2} \end{cases}$$

In parallel,

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$$Q_{lptk} = \sum_{s=0}^h \frac{1}{s!} \binom{-2p'}{h - s} \left(\frac{(l - 2p' + t') e}{2q} \right)^s.$$

Note that a binomial coefficient with a negative upper argument is defined using the generalized formula $\binom{-n}{k} = (-1)^k \binom{n+k-1}{k}$, for $k, n \in \mathbb{N}$; thus:

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$$\binom{-2p'}{h - s} = (-1)^{h-s} \binom{2p' + h - s - 1}{h - s}$$

with

$$h = \begin{cases} k, & \text{for } t' > 0 \\ k - t', & \text{for } t' < 0 \end{cases}$$

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and the same values for p' under the same conditions as described earlier.

Equation (10) can be written as

$$V_{lm} = \frac{GM_{\oplus}}{a} \left(\frac{a_{\oplus}}{a}\right)^l \sum_{p=0}^l F_{lmp}(i) \sum_{q=-\infty}^{\infty} G_{lpq}(e) S_{lmpq}(\omega, M, \Omega, GMST) \quad (17)$$

with

$$\begin{aligned} S_{lmpq}(\omega, M, \Omega, GMST) &= \begin{bmatrix} C_{lm} \\ -S_{lm} \end{bmatrix} \begin{matrix} l-m \text{ even} \\ l-m \text{ odd} \end{matrix} \cos((l-2p)\omega + (l-2p+q)M + m(\Omega - GMST)) \\ &+ \begin{bmatrix} S_{lm} \\ C_{lm} \end{bmatrix} \begin{matrix} l-m \text{ even} \\ l-m \text{ odd} \end{matrix} \sin((l-2p)\omega + (l-2p+q)M + m(\Omega - GMST)) \end{aligned}$$

The functions $F_{lmp}(i)$ and $G_{lpq}(e)$ are provided by Kaula (1966), except for degree 1, for which we have derived the corresponding functions.

For degree 1,

$$V_{1m} = \frac{GM_{\oplus}}{r} \left(\frac{a_{\oplus}}{a}\right) \sum_{p=0}^1 F_{1mp}(i) \sum_{q=-1}^1 G_{1pq}(e) S_{1mpq}(\omega, M, \Omega, GMST). \quad (18)$$

We have evaluated the G -functions to second order in eccentricity, as well as the F -functions and the S_{lmpq} -terms. The resulting expressions are given in Table 1. See Table 2 for more detailed expressions of the G -functions, up to eccentricities of the tenth order.

l	m	p	q	$F_{lmp}(i)$	$G_{lpq}(e)$	$S_{lmpq}(\omega, M, \Omega, GMST)$
1	0	0	-1	$F_{100}(i) = \frac{1}{2} \sin i$	$G_{10-1}(e) = 0$	$S_{100-1}(\omega, M, \Omega, GMST) = C_{10} \cos(\omega)$
1	0	1	-1	$F_{101}(i) = \frac{-1}{2} \sin i$	$G_{11-1}(e) = 2e$	$S_{101-1}(\omega, M, \Omega, GMST) = C_{10} \cos(-\omega - 2M)$
1	0	0	0	$F_{100}(i) = \frac{1}{2} \sin i$	$G_{100}(e) = 1 - \frac{e^2}{2}$	$S_{1000}(\omega, M, \Omega, GMST) = C_{10} \cos(\omega + M)$
1	0	1	0	$F_{101}(i) = \frac{-1}{2} \sin i$	$G_{110}(e) = 1 - \frac{e^2}{2}$	$S_{1010}(\omega, M, \Omega, GMST) = C_{10} \cos(-\omega - M)$
1	0	0	1	$F_{100}(i) = \frac{1}{2} \sin i$	$G_{101}(e) = 2e$	$S_{1001}(\omega, M, \Omega, GMST) = C_{10} \cos(\omega + 2M)$
1	0	1	1	$F_{101}(i) = \frac{-1}{2} \sin i$	$G_{111}(e) = 0$	$S_{1011}(\omega, M, \Omega, GMST) = C_{10} \cos(-\omega)$
1	1	0	-1	$F_{110}(i) = \left(\frac{1}{2} + \frac{1}{2} \cos i\right)$	$G_{10-1}(e) = 0$	$S_{110-1}(\omega, M, \Omega, GMST)$ $= C_{11} \cos(\omega + (\Omega - GMST))$ $+ S_{11} \sin(\omega + (\Omega - GMST))$



1	1	1	-1	$F_{111}(i) = \left(\frac{1}{2} + \frac{-1}{2} \cos i\right)$	$G_{11-1}(e) = 2e$	$S_{111-1}(\omega, M, \Omega, GMST)$ $= C_{11} \cos(-\omega - 2M + (\Omega - GMST))$ $+ S_{11} \sin(-\omega - 2M + (\Omega - GMST))$
1	1	0	0	$F_{110}(i) = \left(\frac{1}{2} + \frac{1}{2} \cos i\right)$	$G_{100}(e) = 1 - \frac{e^2}{2}$	$S_{1100}(\omega, M, \Omega, GMST)$ $= C_{11} \cos(\omega + M + (\Omega - GMST))$ $+ S_{11} \sin(\omega + M + (\Omega - GMST))$
1	1	1	0	$F_{111}(i) = \left(\frac{1}{2} + \frac{-1}{2} \cos i\right)$	$G_{110}(e) = 1 - \frac{e^2}{2}$	$S_{1110}(\omega, M, \Omega, GMST)$ $= C_{11} \cos(-\omega - M + (\Omega - GMST))$ $+ S_{11} \sin(-\omega - M + (\Omega - GMST))$
1	1	0	1	$F_{110}(i) = \left(\frac{1}{2} + \frac{1}{2} \cos i\right)$	$G_{101}(e) = 2e$	$S_{1101}(\omega, M, \Omega, GMST)$ $= C_{11} \cos(\omega + 2M + (\Omega - GMST))$ $+ S_{11} \sin(\omega + 2M + (\Omega - GMST))$
1	1	1	1	$F_{111}(i) = \left(\frac{1}{2} + \frac{-1}{2} \cos i\right)$	$G_{111}(e) = 0$	$S_{1111}(\omega, M, \Omega, GMST)$ $= C_{11} \cos(-\omega + (\Omega - GMST))$ $+ S_{11} \sin(-\omega + (\Omega - GMST))$

Table 1. Expressions of $F_{lmp}(i)$, $G_{lpq}(e)$, and $S_{lmpq}(\omega, M, \Omega, GMST)$ for degree $l=1$, $m=0$ or 1 , $p=0$ or 1 , and $q=-1$, 0 or 1 .

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l	p	q	$G_{lpq}(e)$
1	0	-1	0
1	0	0	$1 - \frac{e^2}{2} - \frac{e^4}{64} - \frac{29e^6}{1152} - \frac{2347e^8}{147456} - 0.0113869e^{10} + O[e^{11}]$
1	0	1	$2e - \frac{3e^3}{2} + \frac{5e^5}{24} - \frac{e^7}{18} - \frac{41e^9}{1920} + O[e^{11}]$
1	0	2	$\frac{27e^2}{8} - \frac{27e^4}{8} + \frac{999e^6}{1024} - \frac{513e^8}{2560} - 0.00992889e^{10} + O[e^{11}]$
1	1	-1	$2e - \frac{3e^3}{2} + \frac{5e^5}{24} - \frac{e^7}{18} - \frac{41e^9}{1920} + O[e^{11}]$
1	1	0	$1 - \frac{e^2}{2} - \frac{e^4}{64} - \frac{29e^6}{1152} - \frac{2347e^8}{147456} - 0.0113869e^{10} + O[e^{11}]$
1	1	1	0
1	1	2	$\frac{e^2}{8} + \frac{e^4}{24} + \frac{25e^6}{1024} + \frac{367e^8}{23040} + \frac{40297e^{10}}{3538944} + O[e^{11}]$
1	2	-1	$4e - 24e^3 + \frac{241e^5}{6} - \frac{455e^7}{18} + \frac{493e^9}{64} + O[e^{11}]$
1	2	0	$1 - \frac{17e^2}{2} + \frac{951e^4}{64} - \frac{973e^6}{128} + \frac{25757e^8}{16384} - 0.254191e^{10} + O[e^{11}]$
1	2	1	$-2e + \frac{9e^3}{2} - \frac{41e^5}{24} + \frac{43e^7}{288} - \frac{101e^9}{1920} + O[e^{11}]$
1	2	2	$\frac{7e^2}{8} - \frac{5e^4}{24} - \frac{29e^6}{1024} - \frac{581e^8}{23040} - 0.0195917e^{10} + O[e^{11}]$



Table 2. Eccentricity functions $G_{lpq}(e)$ for degree $l=1, p=0, 1$ or 2 , and $q=-1, 0, 1$ or 2 , up to order 10 in eccentricity.

Finally, the full expression of degree-1 gravitational potential is:

$$V_{1tot} = V_{10} + V_{11} \quad (19)$$

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$$V_{10} = \frac{GM_{\oplus}}{a} \left(\frac{a_{\oplus}}{a} \right) C_{10} \sin i \left[\left(1 - \frac{e^2}{2} \right) \{ \sin(\omega + M) \} + 2e \{ \sin(\omega + 2M) \} \right]$$

and

$$\begin{aligned} V_{11} = & \frac{GM_{\oplus}}{a} \left(\frac{a_{\oplus}}{a} \right) \left[\left(\frac{1}{2} + \frac{1}{2} \cos i \right) \left\{ \left(1 - \frac{e^2}{2} \right) [C_{11} (\cos(\omega + M + (\Omega - GMST))) + S_{11} (\sin(\omega + M + (\Omega - GMST)))] \right. \right. \\ & + 2e [C_{11} \cos(\omega + 2M + (\Omega - GMST)) + S_{11} \sin(\omega + 2M + (\Omega - GMST))] \Big\} \\ & + \left(\frac{1}{2} + \frac{-1}{2} \cos i \right) \left\{ 2e [C_{11} \cos(-\omega - 2M + (\Omega - GMST)) + S_{11} \sin(-\omega - 2M + (\Omega - GMST))] \right. \\ & \left. \left. + \left(1 - \frac{e^2}{2} \right) [C_{11} (\cos(-\omega - M + (\Omega - GMST))) + S_{11} (\sin(-\omega - M + (\Omega - GMST)))] \right\} \right]. \end{aligned}$$

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4. Analysis of the results

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Using Genesis orbital parameters such as semi-major axis $a = 12378 \text{ km}$ value, and considering the eccentricity $e \sim 0$, we have computed the two terms of degree-1 gravitational potential acting on the satellite:

$$\begin{aligned} V_{10} &= \frac{GM_{\oplus}}{a} \left(\frac{a_{\oplus}}{a} \right) C_{10} \sin i \{ \sin(\omega + M) \}, \\ V_{11} &= \frac{GM_{\oplus}}{a} \left(\frac{a_{\oplus}}{a} \right) [(\cos i) \sin(\omega + M) \{ -C_{11} \sin(\Omega - GMST) + S_{11} \cos(\Omega - GMST) \} \\ &+ \cos(\omega + M) \{ C_{11} \cos(\Omega - GMST) + S_{11} \sin(\Omega - GMST) \}]. \end{aligned}$$

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Regarding the Stokes coefficients, following authors such as Blewitt (2003), Sun et al. (2016), and recently Yu et al. (2026), the geocenter coordinates (X, Y, Z) can be related to degree-1 coefficients in ITRF frame as:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \sqrt{3} a_{\oplus} \begin{bmatrix} C_{11} \\ S_{11} \\ C_{10} \end{bmatrix}$$

240 Note that in this case $a_{\oplus}$ represents the Earth radius as used previously.



Using the estimated annual geocenter motion amplitudes from ITRF2020 (Altamimi et al. 2023) and the previous equation, we have calculated C_{11} , S_{11} , C_{10} values, which will be used for the gravitational potential computation in the rest of the paper. The corresponding values are shown in Table 3.

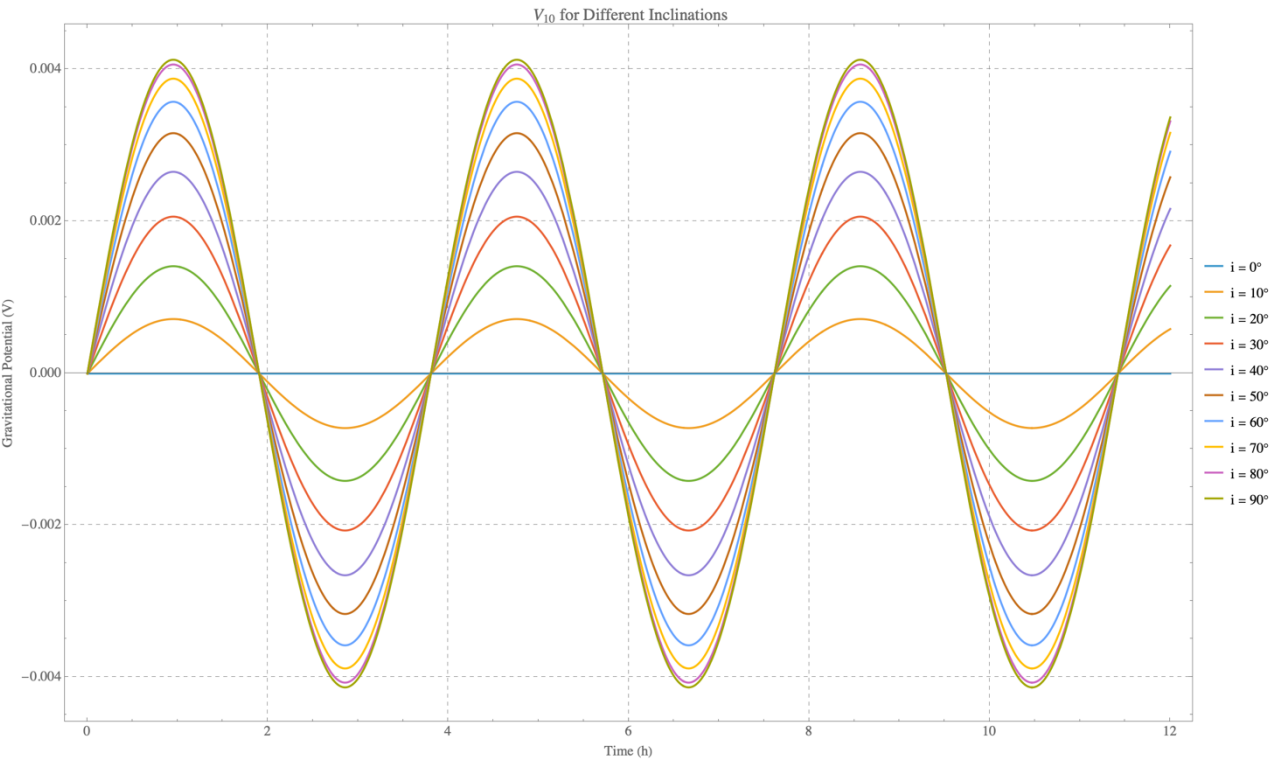
245

Geocenter coordinate [mm]	Degree-1 coefficient
$X = 1.23 \pm 0.16$	$C_{11} = 1.11 \times 10^{-10}$
$Y = 3.48 \pm 0.15$	$S_{11} = 3.15 \times 10^{-10}$
$Z = 2.76 \pm 0.33$	$C_{10} = 2.49 \times 10^{-10}$

Table 3. Annual geocenter motion amplitudes and corresponding unnormalized Stokes degree-1 spherical harmonics coefficients of the gravitational potential.

Figure 2 shows the sensitivity of the spacecraft orbital motion through the first component of degree-1 gravitational potential V_{10} contribution (assuming $\omega = \Omega = 0$) for different inclinations of the orbit (from 0° to 90°). The plot covers a 12-hour interval. Since the curve has the same period as the satellite’s orbital period of approximately 3.8 hours, three complete periods are shown. Note that in the V_{10} expression only the Stoke-coefficient C_{10} is present.

250





255 **Figure 2. Sensitivity of the spacecraft orbital motion through the contribution of degree-1-order-0 gravitational potential (related to C_{10}) for different orbit inclinations (from 0° to 90°), during 12 hours.**

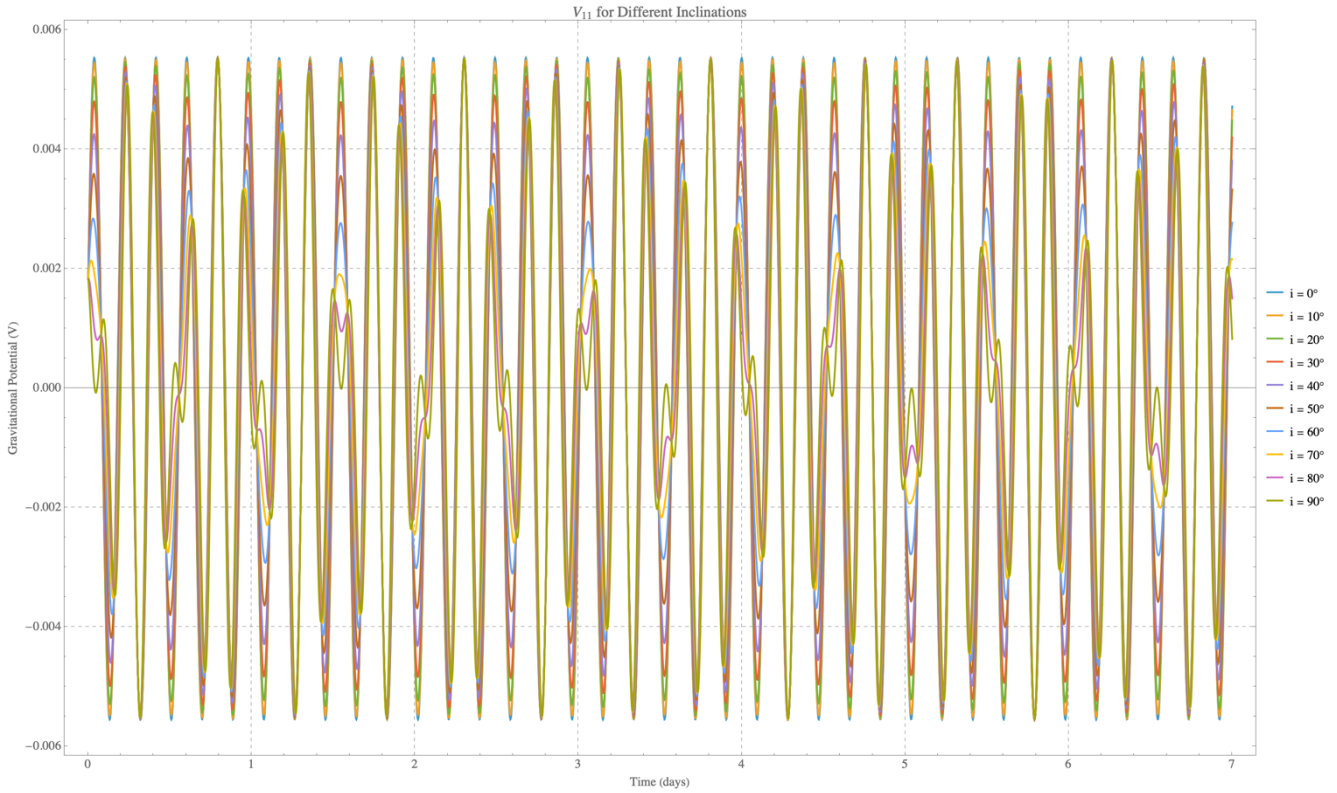


Figure 3. Sensitivity of the spacecraft orbital motion through the contribution of degree-1–order-1 gravitational potential (related to C_{11} and S_{11}) for different orbit inclinations (from 0° to 90°) for 7 days. This interval represents the ground track repetition of the orbit perturbation associated with V_{11} contribution for a semi-major axis like Genesis of 12378 km.

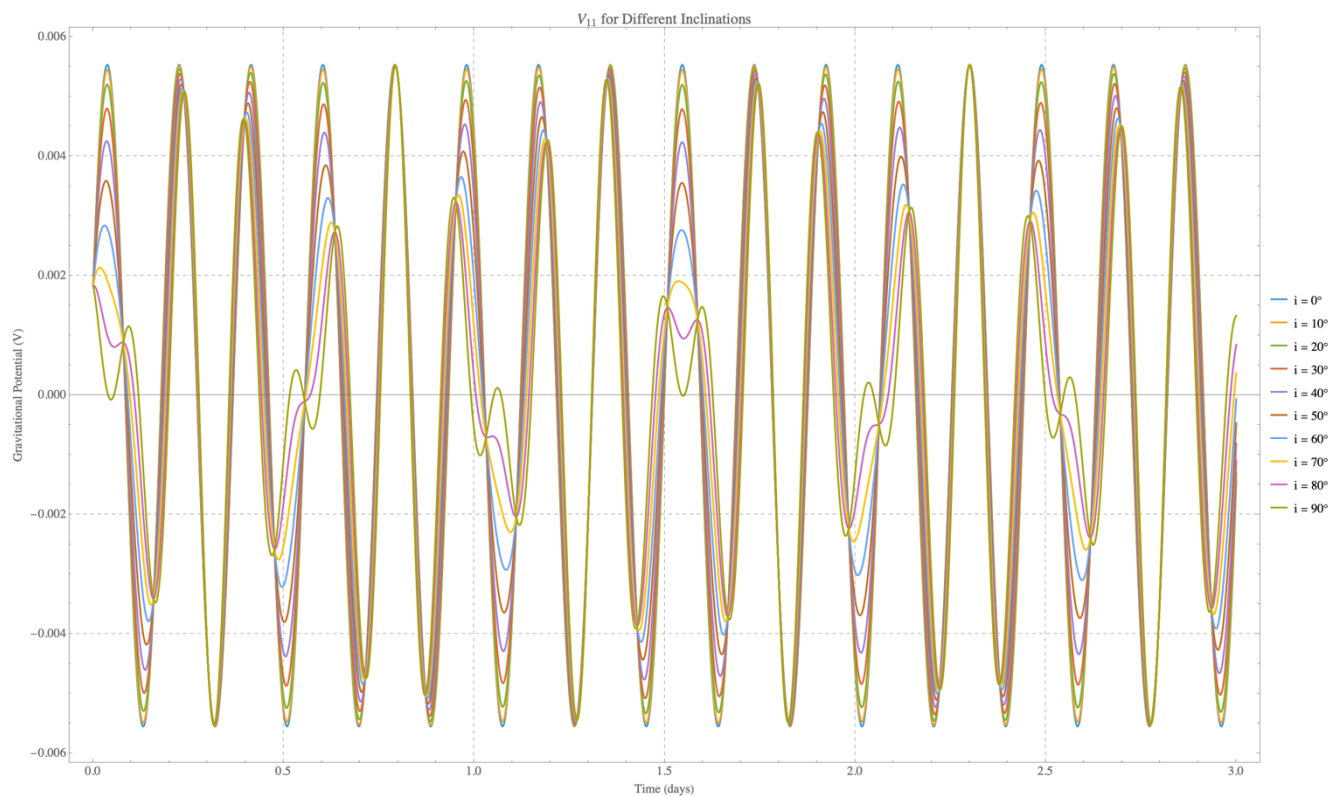
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In **Figure 3**, we show the sensitivity of the spacecraft orbital motion through degree-1, order-1 gravitational potential (V_{11}) contribution for different orbit inclinations from 0° to 90° , for 7 days. This time interval was chosen since the ground track repetition of the orbit perturbation associated with V_{11} contribution is 6.98 days. Here, only C_{11} and S_{11} are present. To better see the differences in V_{11} contributions for the specific orbits, depending on the inclination, a zoom of this plot for 3 days can

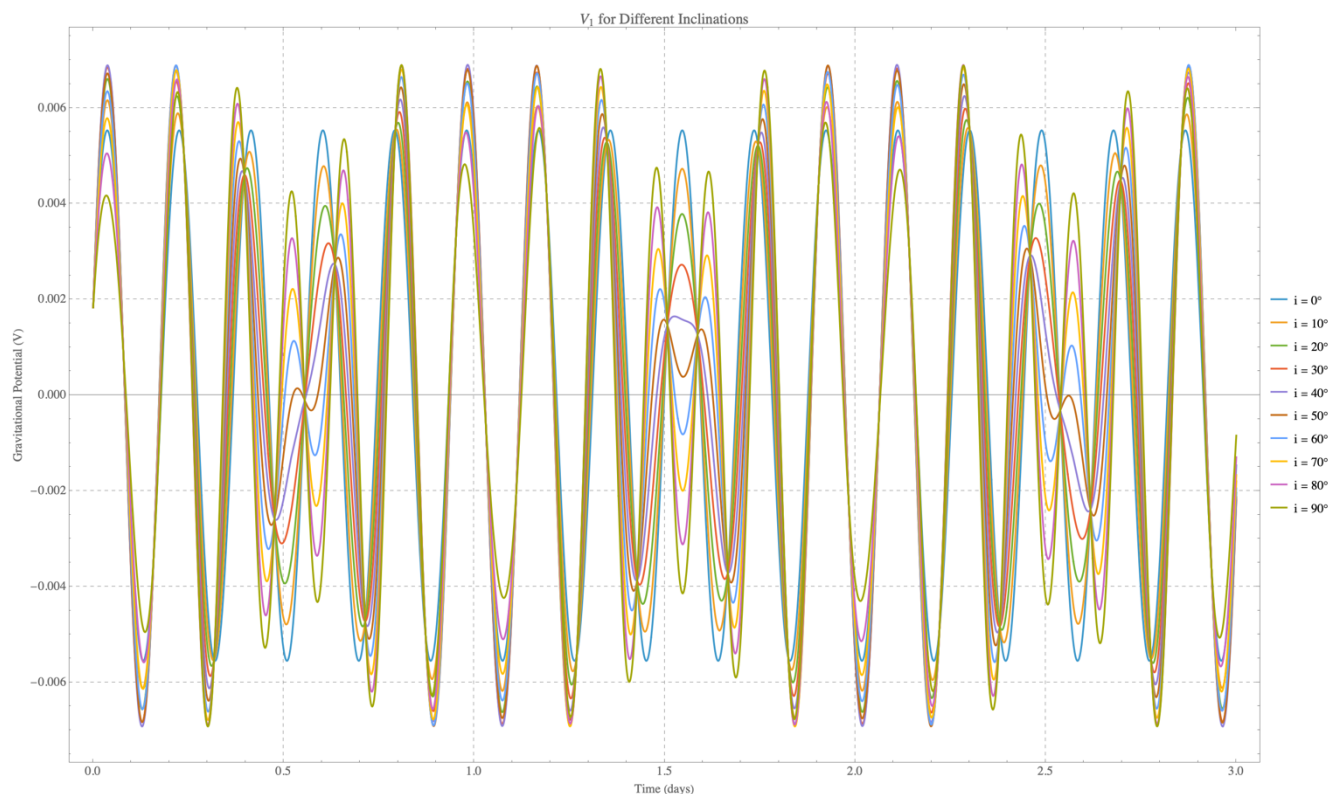
265

be seen in **Figure 4**.

Figure 5 shows the sensitivity of the orbital motion to the whole degree-1 gravity potential V_{1tot} effects on the specific orbits for orbit inclinations from 0° to 90° and for 3 days.



270 **Figure 4. Sensitivity of the orbital motion through degree-1, order-1 gravitational potential (V_{11}) contributions for different orbit inclinations, ranging from 0° to 90° , over a 3-day period. This interval was chosen to improve the visibility of the differences between the curves.**



275 **Figure 5. Sensitivity of the orbital motion through the total degree-1 gravitational potential V_{1tot} contribution for different orbit inclinations (from 0° to 90°), for 3 days.**

Another way to assess the orbit sensitivity to degree-1 gravitational contributions is by computing the integral of the absolute value of each gravitational potential term, normalized by the mean of the total degree-1 potential contribution over seven days. This provides a cumulative measure of the sensitivity over time, which is equivalent to the instruments acquiring measurements
 280 continuously throughout a certain observation window.

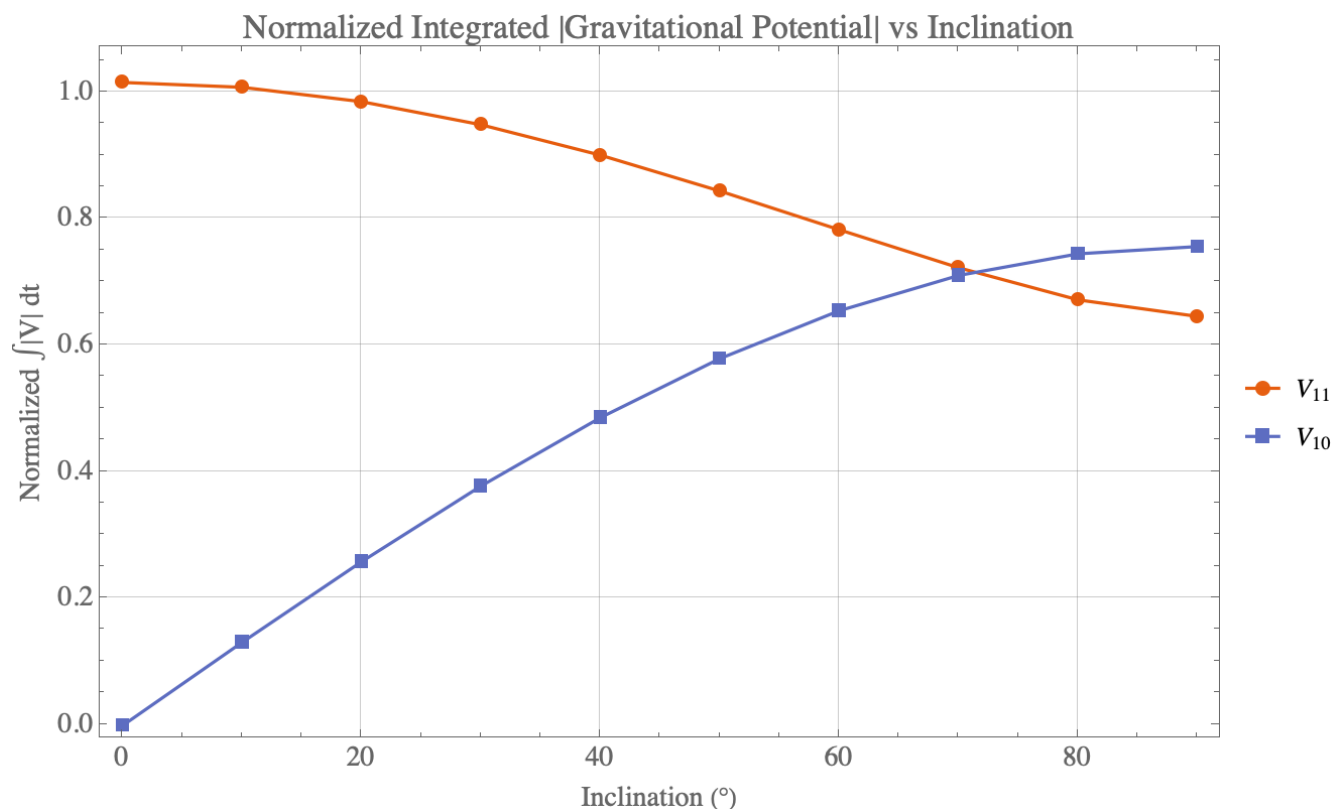
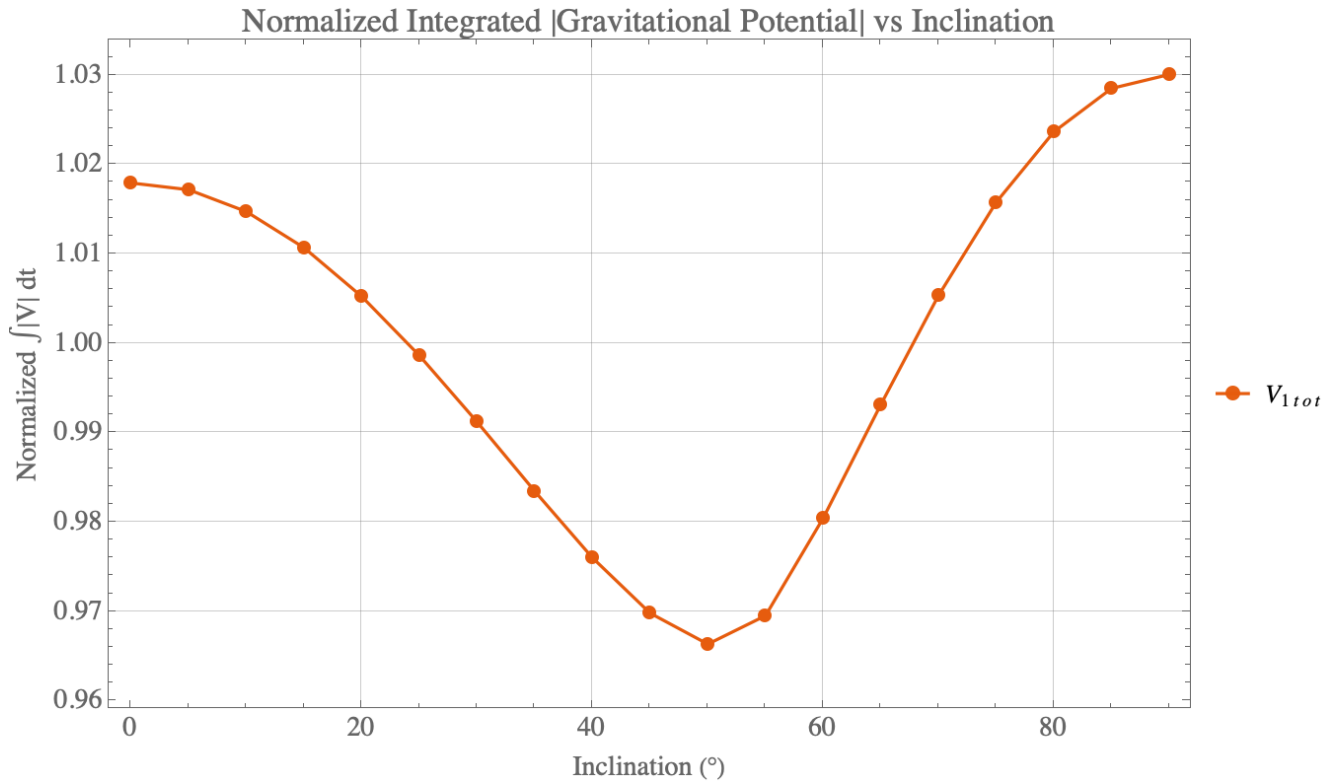


Figure 6. Integral of the absolute value of the sensitivity for different inclinations over seven days, equivalent to the ground track repetition of the potential. Degree-1, order-0 contribution is shown in blue, while orange represents degree-1, order-1.

285

Figure 6 illustrates the cumulative sensitivity of the spacecraft orbital motion through the integral of the absolute value of the gravitational potential terms for different inclinations over seven days, separately for degree-1, order-0 V_{10} (blue) and degree-1, order-1 V_{11} (orange). It is important to mention that these curves are symmetric around 90° . For the order-0 contribution, our computations show that the sensitivity is maximized for polar orbits. This result is consistent with the independent analysis of Kur et al. (2025), who found that the proposed Genesis orbit, with an inclination of 95° and an altitude of about 6000 km, provides particularly good observability of the Z-component of geocenter motion, with the potential to reduce its uncertainty below 1 mm when combined with GNSS observations. For the order-1 contribution, in contrast, the sensitivity is maximized for equatorial orbits.

290



295 **Figure 7. Integral of the absolute value of the Degree-1 sensitivity for different inclinations over seven days.**

Figure 7 confirms that, when orders 0 and 1 are considered together V_{1tot} , the optimal orbit for geocenter motion retrieval is polar, while nearly equatorial and nearly polar orbits also provide high sensitivity.

Conclusion

300 In this study, we revisited Kaula's orbit perturbation theory to investigate the sensitivity of satellite orbits to degree-1 gravitational potential terms associated with geocenter motion. Building on previous studies of orbit perturbations and gravity-field recovery, we derived the Kaula expressions specifically for degree 1 and provided, to our knowledge for the first time, a systematic tabulation of the corresponding degree-1 inclination and eccentricity functions. The eccentricity functions were evaluated to tenth order in eccentricity, providing the analytical framework required to investigate the sensitivity of different
 305 orbital configurations to degree-1 gravitational signals.

Using the orbital characteristics of the forthcoming ESA Genesis mission as a reference, we investigated how the sensitivity of the orbital spacecraft motion to the degree-1 gravity field contribution depends on orbital inclination. The results reveal a clear distinction between the order-0 and order-1 contributions. The order-0 term, associated with the z -component of geocenter motion, reaches its maximum sensitivity for polar orbits and decreases towards equatorial configurations. In contrast, the order-



310 1 terms, associated with the x - and y -components, are most sensitive to equatorial and near-equatorial orbits, while retaining non-negligible sensitivity for polar configurations. When the order-0 and order-1 contributions are considered together and for the estimated values of the annual variations of the geocenter, polar orbits provide the highest overall sensitivity, although nearly equatorial and nearly polar configurations also exhibit substantial sensitivity. An orbit of 50° is the least sensitive to geocenter motion.

315 These results demonstrate that orbital inclination plays a fundamental role in determining the observability of the different components of geocenter motion. In particular, no single orbital configuration maximizes the sensitivity to all three components simultaneously. The choice of orbital parameters should therefore take into account the specific geocenter components and temporal signals that a mission is intended to constrain or to estimate. This provides a useful analytical criterion for assessing and optimizing orbital configurations for future space-geodetic missions.

320 For Genesis, the results demonstrate that its orbital configuration provides significant sensitivity to degree-1 gravitational signals and therefore offers the potential to contribute to annual geocenter motion determination. This is particularly relevant given the mission's unique co-location of GNSS, SLR, DORIS, and VLBI on a single platform, which is intended to improve the consistency and stability of the terrestrial reference frame. A quantitative assessment of the actual geocenter recovery capability, however, requires the analytical sensitivity derived here to be combined with realistic observation errors, force and
325 measurement modelling errors, correlations with other gravity-field parameters, and temporal characteristics of geophysical degree-1 signals.

We have extended this study into a deeper evaluation of the variations of the Stokes coefficients in function of the orbital elements in our companion paper (López Rosson et al., 2026, in preparation).

330 More generally, this study shows how Kaula's analytical framework can be used to identify orbital configurations that enhance sensitivity to weak degree-1 gravitational signatures. The degree-1 inclination and eccentricity functions derived here provide a basis for such analyses and can be extended to different mission concepts and orbital regimes. They therefore offer a useful tool for future mission design aimed at improving geocenter monitoring and, ultimately, strengthening the realization and long-term stability of the terrestrial reference frame.

Code and data availability

335 All computations presented in this paper were performed using Wolfram Mathematica. The codes were specifically developed by the authors for this study. Upon acceptance, the code for the tabulation of degree-1 inclination and eccentricity functions will be made available in a Zenodo repository.

Author contributions

VD and MF made the first step in the computations and GLR recomputed in full details, incorporating sensitivity studies.



340 Competing interests

The authors declare no conflict of interest.

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