Supplementary material to "A minimal model for soil-erosion connectivity in drylands"

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Table S1. Observational field data: monitored rainfall events on grassland $\mathbb{G}$ and shrubland $\mathbb{S}$.

empirical landscape	date	duration *(mn)*	average rainfall intensity *(mm/hr)*	monitored sediment yield *(kg)*	MAHLERAN-modelled sediment yield *(kg)*
grassland	07.09.05	128	22.46	4.1	27.1
	05.07.06	25	31.06	0.9	0.6
	29.08.06	68	20.39	2	2.6
	07.09.06	36	22.86	0.9	1.7
shrubland	07.09.05	31	23.11	10.82	12
	05.07.06	31	27.29	5.84	5.8
	28.07.06	126	13.3	4.8	4.5
	11.08.06	27	15.8	0.81	0.57
	23.08.06	31	19.17	2.6	1.92

Table S2. Average SC-FC correlations, scale exponents and cut-off parameters for the avalanche size distributions observed on generated grasslands and shrublands for different vegetation clustering probabilities.

generated landscape	observed average	vegetation clustering probability		
		p_c=0.2	p_c=0.5	p_c=0.8
grassland	SC-FC correlation $\overline{\rho}^G$	0.63	0.64	0.62
	scale exponent $\overline{\alpha}^G$	1.39	1.37	1.38
	cut-off parameter $\overline{z}_0^G$	25.7 $\times 10^{-3}$	23.3 $\times 10^{-3}$	22.8 $\times 10^{-3}$
shrubland	SC-FC correlation $\overline{\rho}^S$	0.58	0.58	0.57
	scale exponent $\overline{\alpha}^S$	1.56	1.55	1.57
	cut-off parameter $\overline{z}_0^S$	17.5 $\times 10^{-3}$	15.6 $\times 10^{-3}$	14.1 $\times 10^{-3}$

Table S3. Standard deviation of SC-FC correlations, scale exponents and cut-off parameters for the avalanche size distributions observed on generated grasslands and shrublands for different vegetation clustering probabilities.

generated landscape	observed standard deviation	vegetation clustering probability		
		p_c=0.2	p_c=0.5	p_c=0.8
grassland	SC-FC correlation $\overline{\rho}^G$	0.10	0.11	0.11
	scale exponent $\overline{\alpha}^G$	0.083	0.087	0.086
	cut-off parameter $\overline{z}_0^G$	14.3 $\times 10^{-3}$	13.3 $\times 10^{-3}$	11.3 $\times 10^{-3}$
shrubland	SC-FC correlation $\overline{\rho}^S$	0.10	0.10	0.09
	scale exponent $\overline{\alpha}^S$	0.086	0.084	0.089
	cut-off parameter $\overline{z}_0^S$	10 $\times 10^{-3}$	9.8 $\times 10^{-3}$	7.3 $\times 10^{-3}$

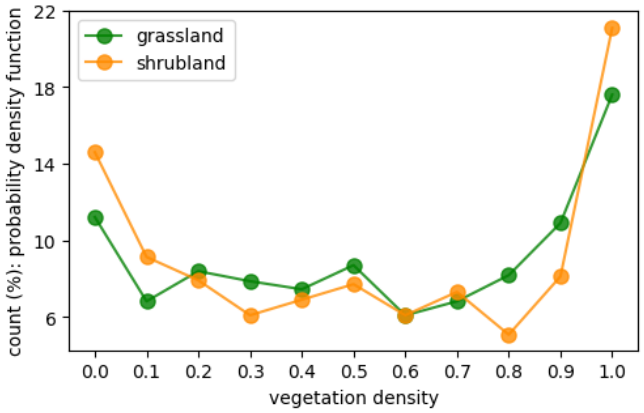


Figure S1. Distribution of vegetation densities on grassland and shrubland to be used as probability density functions from which vegetation densities are randomly sampled.

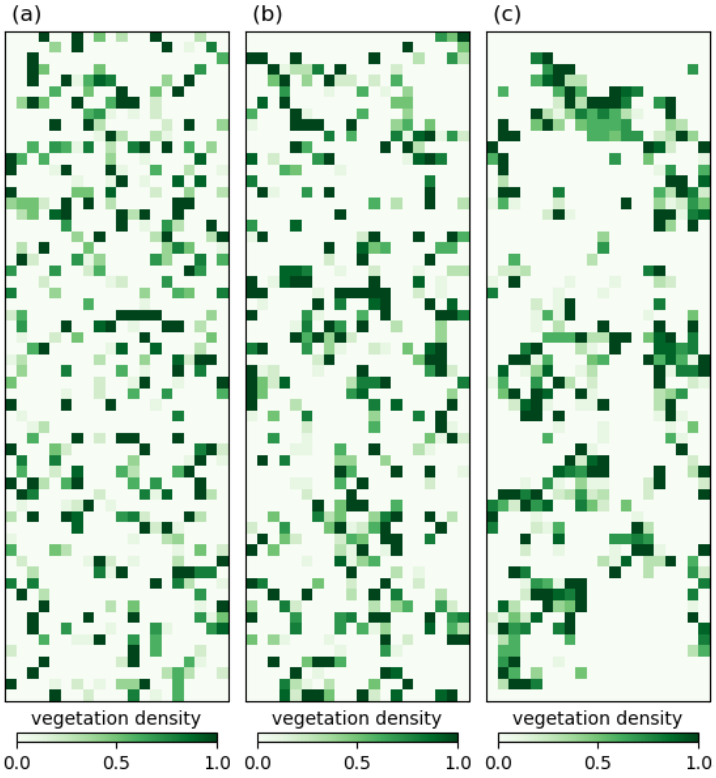


Figure S2. Vegetation clustering examples on a lattice of $n_x = 20$ horizontal and $n_y = 60$ vertical nodes for a percentage vegetation cover $c = 0.3$ distributed according to a clustering probability **(a)** $p = 0.2$, **(b)** $p = 0.5$ and **(c)** $p = 0.8$.

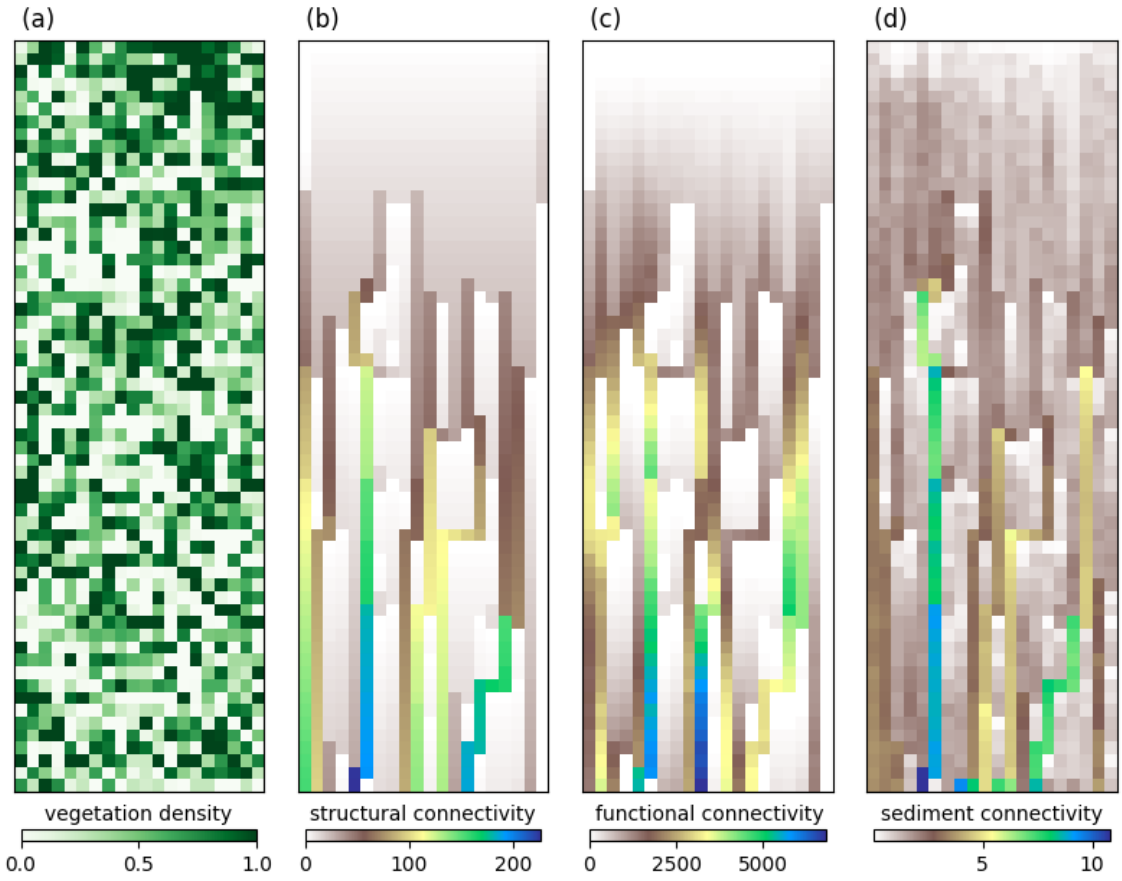


Figure S3. Comparison between the sandpile steady state and MAHLERAN sediment connectivity on grassland: **(a)** Vegetation density on grassland, **(b)** its *SC*, **(c)** the *FC* measured after running *deterministic* descent dynamics for 10^4 time steps, averaged over 100 runs, and **(d)** MAHLERAN sediment connectivity *FC** on this same plot.