



Nonlinear feedback and memory in an Ising model of sea ice

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Abstract. Sea ice retreat depends on how ice and open water are arranged and on the heat available for melting. We extend a kinetic Ising model of ice and water by allowing neighbour coupling to respond to the surrounding pattern and retain its history. This coupling sets the resistance to changing an interface. We keep the average coupling fixed to isolate the effect of making some regions easier to melt and others harder. We derive the finite-block reversal energy and show why changing the coupling affects corners and straight fronts differently. In the global mean-field closure near ordering, coupling memory can destabilise a water-rich state through a subcritical Hopf bifurcation. In the strongly ordered seasonal regime, memory delays changes in interface barriers. Calculations driven by ERA5 reanalysis for the Pacific Arctic, initialised from satellite concentration in June 2022–2024, illustrate the seasonal interface response through September. Feedback increases front activity even under spatially uniform forcing. Additional melting occurs at exposed sites as the ice pattern evolves. A field shift matches the threshold for a front with equal incident bonds but not the full retreat pattern. With their early coupling amplitudes matched, memory and reduced instantaneous gain give similar seasonal transition counts but different histories. The model connects the geometry of retreat to environmental feedback and its response time.

1 Introduction

Coupled sea ice, ocean and atmosphere models increasingly resolve thermodynamics, thickness distributions and surface melt (Rousset et al., 2015; Flocco et al., 2010). Machine learning provides another route to seasonal prediction, through probabilistic concentration forecasts from IceNet (Andersson et al., 2021) and theory-guided real-time forecasts of September pan-Arctic sea ice extent (Kondrashov et al., 2026). Conceptual models connect thresholds and timescales to mechanisms of change in an ice cover.

Arctic energy balance models show how stabilising ice growth competes with ice–albedo feedback (Eisenman and Wettlaufer, 2009). Their bifurcations depend on the treatment of seasonal forcing, heat transport and the albedo transition (Eisenman, 2012; Wagner and Eisenman, 2015; Hill et al., 2016). Spatial coupling also matters: Müller-Stoffels and Wackerbauer (2012) studied the reversibility of sea ice decay on a two-dimensional energy balance network. Stochastic extensions describe how seasonal stability shapes fluctuations and their persistence (Moon and Wettlaufer, 2017). A Fokker–Planck description



combines thermodynamic growth and mechanical redistribution to explain the seasonal ice-thickness distribution (Toppal-
 25 adoddi et al., 2023).

Melt pond models describe spatial organisation on the ice pack surface. Sudakov et al. (2015) linked stochastic pond growth
 and changes in pond geometry to albedo and climate bifurcations. Models based on contours of random surfaces reproduce
 pond boundaries (Bowen et al., 2018), while the void model represents ponds as spaces between overlapping random circles
 (Popović et al., 2018). Percolation theory constrains late-summer pond coverage (Popović et al., 2020). Cellular-automaton
 30 models describe melting and water redistribution over evolving surface topography (Lüthje et al., 2006; Scott and Feltham,
 2010), and Flocco et al. (2010) incorporated pond physics into a climate-model sea ice component. An Ising description
 separates the local preference for ice or water from the coupling between neighbouring sites (Ising, 1925). Ma et al. (2019)
 used this approach with a random field representing topography to reproduce melt pond geometry through metastable ice–water
 configurations. At the regional scale, Wang (2024) used continuous local spin variables and a penalty that discourages changes
 35 in the local ice state between successive updates. Here, memory acts on the coupling strength itself, which adjusts gradually to
 the evolving ice–water surroundings.

In a feedback Ising model, the coupling between neighbouring sites depends on the evolving configuration of the system.
 For sea ice, this closes a feedback loop: melting changes the ice–water arrangement, which changes the coupling and hence
 the energy cost of subsequent ice–water transitions. Ma et al. (2025) analysed coupling that depends on the mean spin state,
 40 deriving the reversal energy and the corresponding mean-field stationary equation. Ma et al. (2026) showed that feedback can
 stabilise mixed states containing both spin orientations, even at zero temperature. They also formulated a spatial extension
 in which each bond coupling depends on the average spin state in its surrounding neighbourhood, and derived the associated
 energy correction.

Here, we study how environmental contrasts and a delayed coupling response affect sea ice melting. In the global mean-
 45 field model near its ordering transition, coupling memory can destabilise a water-rich equilibrium through a subcritical Hopf
 bifurcation. We then introduce a spatial coupling law for finite blocks of lattice sites. Its instantaneous target depends on
 the difference between the surrounding open-water fraction and the regional average: coupling is lower in relatively open
 surroundings and higher in compact ice. Keeping its spatial mean fixed isolates the effect of these contrasts from uniform
 weakening of the coupling. We derive the exact reversal energy for the instantaneous law and allow the coupling to adjust
 50 towards its target over a finite response time. We examine the spatial law in simulations of the Pacific Arctic melt season in
 2022–2024, initialised from observed ice concentration and driven by prescribed ERA5 surface temperatures and heat fluxes.
 Matched experiments for 2024 show that instantaneous feedback increases melting along locally flat ice fronts and reduces
 melting at boundary corners compared with constant coupling. The additional melting occurs mainly at isolated ice sites or
 sites with only one ice neighbour. A finite response time delays the development of spatial differences in coupling and reduces
 55 early-season ice loss relative to instantaneous feedback.

The paper is organised as follows. Section 2 introduces the two-phase energy, its reduction to an ice–water lattice and the
 albedo feedback. Section 3 examines global feedback and how coupling memory changes stability in the mean-field model.
 Section 4 develops the mean-centred spatial coupling law, derives its reversal energy and relates local melting thresholds



to interface geometry. Section 5 presents the driven Pacific Arctic computational experiments and matched controls used to distinguish the effects of spatial feedback and memory. Section 6 discusses the physical interpretation and limitations of the model. Section 7 summarises the principal results and their implications for conceptual sea ice modelling.

2 Physical construction: phases, interfaces and albedo

2.1 A two-phase energy and its binary reduction

We begin with an energy description that separates the local preference for ice or water from the cost of an ice–water interface. Retaining only the two phase states gives the lattice model used below to describe local melting and freezing.

Let $u(\mathbf{x}, t) \geq 0$ distinguish water, $u = 0$, from an ice phase with $u > 0$. We assign an energy to each phase configuration through the Ginzburg–Landau functional

$$\mathcal{F}[u, T] = \int_{\Omega_A} \left[\frac{\epsilon^2}{2} |\nabla u|^2 + \Phi(u, T) \right] dA, \quad \Phi = \frac{a_1(T)}{2} u^2 - \frac{a_2}{3} u^3 + \frac{a_3}{4} u^4, \quad (1)$$

Here T is surface temperature, Ω_A the horizontal domain, $\epsilon^2 > 0$ weights spatial gradients, and $a_1(T)$ controls local phase preference; $a_2, a_3 > 0$. The potential favours a phase and the gradient term penalises interfaces (Cahn and Hilliard, 1958; Allen and Cahn, 1979; Thoms et al., 2014). Its non-zero uniform stationary states and the ice spinodal, where the ice minimum disappears, are

$$u_{\pm} = \frac{a_2 \pm \sqrt{a_2^2 - 4a_1a_3}}{2a_3}, \quad a_{\text{sp}} = \frac{a_2^2}{4a_3}. \quad (2)$$

For $0 < a_1 < a_{\text{sp}}$, u_+ is an ice minimum, while u_- is the intervening maximum. The ice and water minima have equal energies at

$$a_c = \frac{2a_2^2}{9a_3}, \quad u_c = \frac{2a_2}{3a_3}. \quad (3)$$

Here a_c is the coexistence value of a_1 and u_c the corresponding ice state. Water loses local stability at $a_1 = 0$. Coexistence and loss of stability are distinct (Fig. 1a).

Choose a reference temperature in the two-phase range and let u_{ice} be its ice minimum u_+ . Retain these two phase values by writing $u_i = U_0(1 - s_i)$, with $U_0 = u_{\text{ice}}/2$ and a binary variable (spin) $s_i = \pm 1$. For a neighbouring pair on discretisation elements of side a in d dimensions,

$$\frac{\epsilon^2}{2} a^d \left(\frac{u_i - u_j}{a} \right)^2 = \epsilon^2 U_0^2 a^{d-2} (1 - s_i s_j). \quad (4)$$

Removing the additive constant leaves an Ising bond with $J \propto \epsilon^2 U_0^2 a^{d-2}$. For $d = 2$ the power of a cancels. The two-state potential supplies a local field, giving the conditional energy (Hamiltonian)

$$\mathcal{H}(\mathbf{s} | J, h) = - \sum_{\langle p, q \rangle} J_{pq} s_p s_q + \sum_p h_p s_p, \quad (5)$$

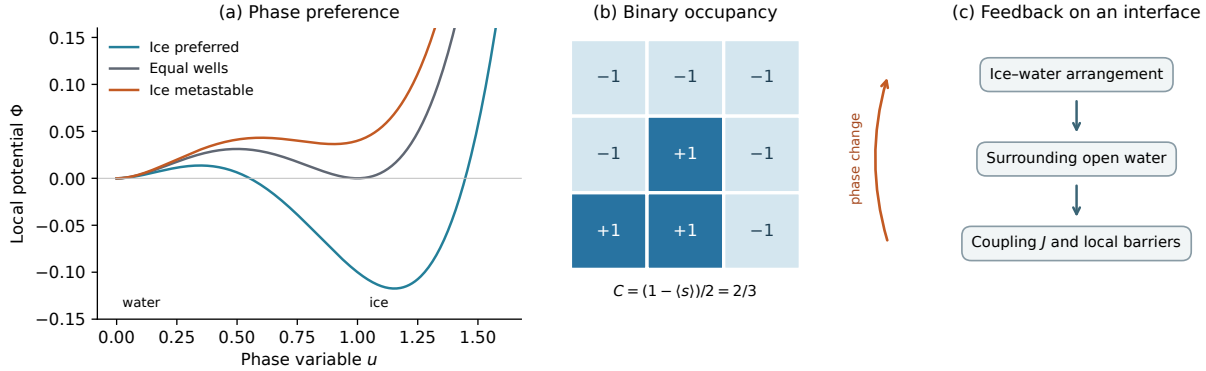


Figure 1. Physical construction of the model. (a) The quartic potential separates coexistence from metastability; the illustrative coefficients are $a_2 = 3$, $a_3 = 2$ and $a_1 = 0.8, 1, 1.08$, with Maxwell value $a_c = 1$ and ice spinodal $a_{sp} = 1.125$. (b) Six ice spins in a nine-spin cell give occupancy $C = 2/3$. (c) The coupling responds to the surrounding phase arrangement and changes the barriers experienced during subsequent evolution.

Each nearest-neighbour bond $\langle p, q \rangle$ is counted once; $q \sim p$ denotes a neighbour of p . Ice is $s = -1$ and water is $s = +1$; positive h favours ice and positive J favours neighbour agreement.

With J, h held fixed, the energy difference for a reversal $s_p \rightarrow -s_p$ is

$$\Delta \mathcal{H}_p = 2s_p \sum_{q \sim p} J_{pq} s_q - 2h_p s_p. \quad (6)$$

90 Metropolis updating accepts $\Delta \mathcal{H} \leq 0$ and accepts an uphill move with probability $e^{-\beta \Delta \mathcal{H}}$ (Metropolis et al., 1953). Here β is the inverse energy scale, which sets stochastic activation. Only βJ and βh enter the rule; setting $\beta = 1$ fixes an energy convention. With d_0 denoting the background coupling, the seasonal case has $\beta d_0 \simeq 5.75$, compared with the zero-field square-lattice reference $\beta J_c \simeq 0.4407$ (Onsager, 1944). The seasonal coupling is about 13 times this exact critical value; in mean-field notation, $K = 4\beta d_0 \simeq 23$ versus $K_c = 1$. Both indicate strong ordering.

95 In this lattice description, h favours ice or water, J sets the cost of unlike neighbouring states, and β controls the probability of crossing a positive energy barrier. The next subsection allows the coupling to respond to the surrounding ice–water arrangement and connects the local ice fraction to absorbed radiation.

2.2 Conditional environmental coupling and surface feedback

To describe a coupling that responds to its environment, introduce a coarse environmental state v and write

$$100 \quad \mathcal{F}[u | v, T] = \int_{\Omega_A} \left[\frac{\kappa[v](\mathbf{x})}{2} |\nabla u|^2 + \Phi(u, T) \right] dA. \quad (7)$$

During a conditional update, v is held fixed. Define $\kappa_i[v]$ so that, in the chosen energy units, $\kappa_i[v] U_0^2 a^{d-2} = J_i[v]$, and average adjacent coefficients on each bond. The binary discretisation gives the symmetric bond law $J_{pq} = (J_{c(p)} + J_{c(q)})/2$, where $c(p)$



is the concentration cell containing spin p . Choosing $J_i[v]$ to decrease where the neighbourhood is relatively water-rich gives the nonlocal law in Sect. 4.

105 Changes in the ice–water arrangement modify the coupling, which in turn changes the energy cost of subsequent local melting and freezing (Fig. 1c).

Substituting $v = u$ adds $\frac{1}{2} \int_{\Omega_A} [\delta\kappa[u](\mathbf{x})/\delta u(\mathbf{y})] |\nabla u(\mathbf{x})|^2 dA_{\mathbf{x}}$ to the variational derivative at $\mathbf{y}$, where the nonlocal functional derivative is a kernel. This term accounts for the changed interfacial cost when the environment is perturbed; a conditional update omits it. Sections 3 and 4 derive the global and spatial lattice counterparts.

110 The ice occupancy of a cell is $C = (1 - \langle s \rangle)/2$, where $\langle s \rangle$ is the mean of its spins. For example, six ice sites in a nine-site cell give $C = 2/3$ (Fig. 1b). The corresponding binary-mixture albedo is

$$\alpha(C) = \alpha_w + (\alpha_{\text{ice}} - \alpha_w)C. \quad (8)$$

Here α_w and α_{ice} are water and ice albedos. The mixture connects phase organisation to absorbed radiation (Budyko, 1969; Sellers, 1969). This thermal influence enters the seasonal field; coupling controls the resistance associated with neighbour
 115 geometry.

We first consider a global coupling that depends only on the mean ice–water state. This simpler case makes the distinction between held and state-dependent reversal energies explicit.

3 Nonlinear consequences of the feedback definition

3.1 Global state-dependent Hamiltonian

120 For N spins, let $M = N^{-1} \sum_p s_p$ and prescribe $J(M) = J_0 + J_1 M$, where J_0 is the reference coupling and J_1 its response to the mean spin. The field h is uniform. For $J_1 < 0$, increasing open water weakens the coupling. Including this coupling law in the Hamiltonian gives

$$\mathcal{H}_M(\mathbf{s}) = -[J_0 + J_1 M(\mathbf{s})] \sum_{\langle p,q \rangle} s_p s_q + h \sum_p s_p. \quad (9)$$

A reversal changes the coupling on every bond, so its energy difference contains a global contribution as well as the local
 125 bond change. The global coupling correction and its mean-field consequence were derived by Ma et al. (2025). We state the finite-lattice form here to establish the connection with the spatial closure. Writing $\mathcal{B} = \sum_{\langle p,q \rangle} s_p s_q$ and $S_p = \sum_{q \sim p} s_q$, the exact finite-size energy difference is

$$\Delta \mathcal{H}_{M,p} = 2J(M) s_p S_p - 2h s_p + \frac{2J_1 s_p}{N} \mathcal{B} - \frac{4J_1}{N} S_p. \quad (10)$$

Holding coupling during reversal omits the last two terms. Since $\mathcal{B} = \mathcal{O}(N)$, the first correction generally survives increasing
 130 system size. The last is $\mathcal{O}(N^{-1})$ for bounded coordination. The order-one correction is exactly a uniform state-dependent field shift, $h_{\text{eff}} = h - J_1 \mathcal{B}/N$; the last term remains a finite-size correction.



For coordination number z , let m denote the homogeneous mean spin and define $K = \beta z J_0$, $\Lambda = \beta z J_1$ and $H = \beta h$. With $\mathcal{B}/N \simeq z m^2/2$, the shift becomes $\beta \delta h = -\Lambda m^2/2$. It explains the stationary coefficients 3/2 and 1 below. The Bragg–Williams approximation replaces neighbour correlations by products of this mean. Its free energy per spin is

$$\beta f(m) = -\frac{K}{2}m^2 - \frac{\Lambda}{2}m^3 + Hm + S(m), \quad S(m) = \frac{1+m}{2} \ln \frac{1+m}{2} + \frac{1-m}{2} \ln \frac{1-m}{2}. \quad (11)$$

Here $S(m)$ is minus the dimensionless mixing entropy of the two spin states. Consequently,

$$H = Km + \frac{3}{2}\Lambda m^2 - \text{atanh } m, \quad K + 3\Lambda m = \frac{1}{1-m^2} \quad (12)$$

give the stationary branches and their folds. Non-zero Λ breaks ice–water reflection symmetry and shifts the metastable branches. At a non-degenerate fold, a stable and unstable branch meet in a saddle-node bifurcation (Guckenheimer and Holmes, 1983).

The preceding energy calculation includes the coupling change caused by a reversal. We now consider dynamics in which the coupling is held during each reversal and adjusts towards a state-dependent target over time.

3.2 Conditional feedback and memory

A different dynamics forms a coupling target from the state and then evolves the spins conditionally on that coupling. Write $k(t) = \beta z J(t)$ for the dimensionless relaxing coupling. With time and τ_J measured in attempted reversals per spin, a homogeneous relaxation model is

$$\dot{m} = \Gamma(\ell)[\tanh \ell - m], \quad \ell = km - H, \quad \tau_J \dot{k} = K + \Lambda m - k. \quad (13)$$

Replacing the fluctuating neighbour sum by its mean gives the Metropolis mobility $\Gamma(\ell) = 1 + \exp(-2|\ell|) > 0$, which sets the adjustment rate. Choosing $\Gamma = 1$ changes that rate but not stationarity. The mobility is non-smooth at $\ell = 0$, but smooth near the Hopf point $\ell_* = \text{atanh}(0.4) > 0$.

At stationarity, $k_* = K + \Lambda m_*$, and hence

$$H = Km + \Lambda m^2 - \text{atanh } m, \quad K + 2\Lambda m = \frac{1}{1-m^2}. \quad (14)$$

The factor differs from Eq. (12) because the spin feels a held coupling, not the derivative of the global state-dependent energy. The definitions produce different homogeneous branches (Fig. 2a); $\tau_J \rightarrow 0$ recovers instantaneous conditional feedback. Finite-rate lattice cycles also distinguish these definitions; their switching intervals are kinetic (Supplement Sect. S2).

In the Landau–Ising example of De Martino (2019), negative feedback through the applied field replaces the ordering transition with a supercritical Hopf bifurcation. Here the relaxing variable is the coupling. Its response time changes stability without moving the stationary branch. With $\chi = 1 - m_*^2$ and $\Gamma_* = \Gamma(k_* m_* - H)$, define $a = \Gamma_*[\chi k_* - 1]$ and $c = \Gamma_* \chi m_*$. The Jacobian governs small disturbances of the mean phase and coupling; they decay when both eigenvalues have negative real parts. The matrix and its invariants are

$$L = \begin{pmatrix} a & c \\ \Lambda/\tau_J & -1/\tau_J \end{pmatrix}, \quad \text{tr } L = a - \tau_J^{-1}, \quad \det L = -\frac{a + c\Lambda}{\tau_J}. \quad (15)$$

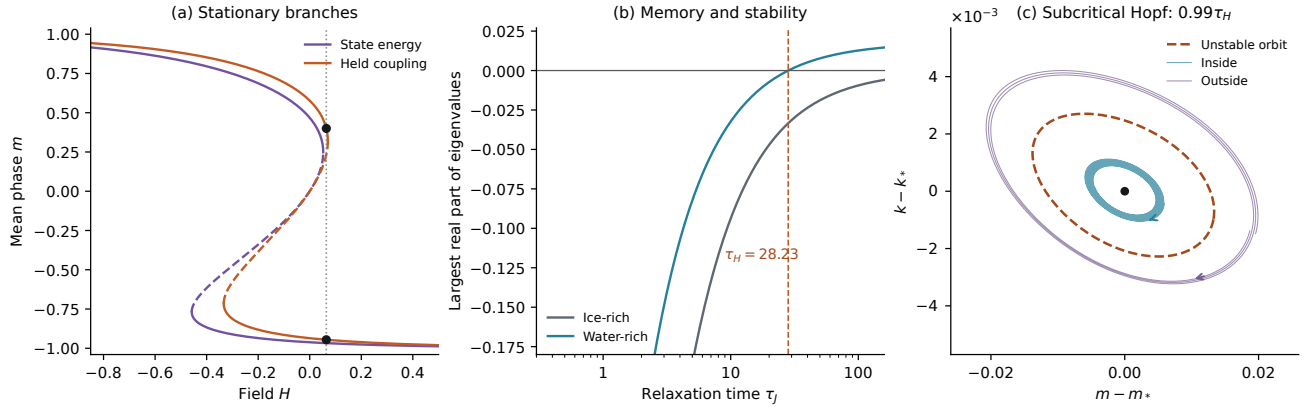


Figure 2. Homogeneous feedback and memory at $K = 1.4$, $\Lambda = -0.45$. (a) State-energy and conditional branches; solid and dashed lines denote scalar stability and instability. Conditional scalar stability assumes instantaneous feedback. Black points mark conditional equilibria at $H = 0.0643511$. (b) Largest eigenvalue real part versus response time in attempted reversals per spin (Eq. 13). Only the water-rich branch loses stability. (c) Unstable orbit below the subcritical Hopf, at $\tau_J = 0.99\tau_H$; arrows show forward motion inside and outside the dashed orbit. Coordinates are deviations from the water-rich equilibrium.

Here $a > 0$ means instability at fixed coupling, whereas $a + c\Lambda < 0$ means stability under instantaneous feedback. The identity $a + c\Lambda = \Gamma_* \{ \chi(K + 2\Lambda m_*) - 1 \}$ places the zero eigenvalue at the conditional fold for every τ_J . The two sides of the ice–water system nevertheless respond differently to a delay. For $\Lambda < 0$ on an ice-rich branch, $c\Lambda > 0$; positive determinant then requires $a < 0$, and stability is retained at every positive relaxation time. On a water-rich branch, $c\Lambda < 0$. If $0 < a < -c\Lambda$, the instantaneous response is stable but the trace changes sign at

$$\tau_H = 1/a, \quad \omega_H^2 = -a(a + c\Lambda) > 0. \quad (16)$$

Here ω_H is the angular frequency at onset. At $K = 1.4$, $\Lambda = -0.45$, $H = 0.0643511$, the water-rich equilibrium $m_* = 0.4$ loses stability at $\tau_H = 28.23$ kinetic units; the ice-rich equilibrium remains stable (Fig. 2b). This near-ordering example is separate from seasonal $K \simeq 23$. The positive cubic radial coefficient $a_r = 0.94288$ establishes a subcritical Hopf (Supplement Sect. S2.2). At $0.99\tau_H$, an unstable periodic orbit bounds the perturbations that return to the water-rich state (Fig. 2c). Its Floquet multiplier 1.02885 measures transverse growth over one circuit. Above threshold, tested perturbations approach the coexisting ice-rich equilibrium. Thus memory can trigger switching without creating a stable small-amplitude cycle.

The homogeneous analysis describes phase stability without resolving an ice boundary. We now introduce spatial contrasts to examine how feedback changes melting barriers along that boundary.



4 Spatial coupling and the motion of phase boundaries

4.1 A mean-centred coupling law

Let each concentration cell contain a block of binary microspins. For block mean m_i , define

$$C_i = \frac{1 - m_i}{2}, \quad F_{w,i} = \frac{1 + m_i}{2}, \quad J_{pq} = \frac{J_{c(p)} + J_{c(q)}}{2}, \quad (17)$$

180 where $c(p)$ is the cell containing spin p . The cell values J_i, h_i are shared by its microspins. Symmetric bond averaging defines the conditional energy across cell boundaries.

Let $F_{w,R(i)}$ average the water fraction over a square neighbourhood of half-width R cells. Its spatial mean is $\overline{F_{w,R}} = N_c^{-1} \sum_i F_{w,R(i)}$, where N_c is the number of concentration cells. The preliminary coupling is

$$J_{*,i} = d_0 + g(\overline{F_{w,R}} - F_{w,R(i)}), \quad g > 0, \quad (18)$$

185 where $d_0 > 0$ is a reference coupling and g sets the contrast response. Here J is a phenomenological coefficient in the transition barrier, not a material stiffness. Wave penetration and floe breakup provide physical reasons for exposed cover to rearrange more readily than compact pack (Dumont et al., 2011; Horvat and Tziperman, 2015). A regional water-fraction contrast can summarise differences in exposure and pack connectedness without resolving those processes. Equation (18) is a lowest-order representation of that contrast. Its window is truncated and normalised at open boundaries, and the overbar averages the
 190 resulting environments over the crop. Holding the mean fixed separates the contrast response from any overall weakening. The scale R describes the environmental average; it is not an inferred wave-attenuation length.

For the linear averaging matrix W_R , define $u_j = N_c^{-1} \sum_i (W_R)_{ij}$ and $A_{ij} = (W_R)_{ij} - u_j$. Then

$$\mathbf{J}_* = d_0 \mathbf{1} - \frac{g}{2} \mathbf{A} \mathbf{m}, \quad \mathbf{A} \mathbf{1} = 0, \quad \mathbf{1}^\top \mathbf{A} = 0. \quad (19)$$

Here $\mathbf{1}$ is the vector of N_c ones. Adding the same amount to the water fraction in every cell leaves the target coupling un-
 195 changed. For any spatial ice–water pattern, the arithmetic mean of the target values $J_{*,i}$ is d_0 . Unlike the bond-centred neighbourhoods of Ma et al. (2026), we calculate feedback from neighbourhood averages of cell water fractions, measured relative to the mean of these averages over the study region. Each bond takes the average coupling of the cells containing its two endpoints. If any preliminary cell coupling falls below the prescribed positive lower bound, truncation and rescaling enforce this bound while retaining the cell mean d_0 . No correction is applied when all values already satisfy the bound (Supplement
 200 Sect. S1.2).

The evolving conditional dynamics generally lacks detailed balance, the balance of forward and reverse probability fluxes at equilibrium. A finite response time separates local reversals from environmental adjustment. The preliminary memory update is

$$\tilde{\mathbf{J}}^n = \rho \mathbf{J}^{n-1} + (1 - \rho) \mathbf{J}_*(\mathbf{m}^n), \quad \rho = e^{-1/\tau_J}. \quad (20)$$



205 Here n indexes daily updates and τ_J is measured in days. The positivity correction gives $\mathbf{J}^n$. Starting the coupling history from $d_0\mathbf{1}$ preserves the spatial mean d_0 . Each feedback simulation forms its target from its own evolving ice pattern. The correction is inactive on the seasonal memory trajectories with $\tau_J = 14$ d described in Sect. 5.1.

Specifying the coupling law does not yet specify the reversal energy. For instantaneous feedback, we distinguish holding the coupling fixed during a reversal from including the coupling changes caused by the proposed configuration.

210 4.2 The energy derivative survives spatial mean centring

For the instantaneous spatial law, assign half of each incident spin product to its endpoint cell:

$$Q_i = \frac{1}{2} \sum_{p:c(p)=i} \sum_{q \sim p} s_p s_q, \quad \mathcal{H}_J = - \sum_i J_i(\mathbf{m}) Q_i. \quad (21)$$

This reproduces the symmetric bond energy. For a reversal of s_p in cell i , write $\delta \mathbf{J}$ and $\delta \mathbf{Q}$ for the finite changes in these fields, and let $\Delta \mathcal{H}_{\text{held}}$ denote Eq. (6) with the pre-reversal couplings. At fixed h , the full energy change is

$$215 \quad \Delta \mathcal{H}_{\text{state}} = \Delta \mathcal{H}_{\text{held}} - \delta \mathbf{J} \cdot \mathbf{Q} - \delta \mathbf{J} \cdot \delta \mathbf{Q}. \quad (22)$$

The environmental derivative corresponds to that of Ma et al. (2026); finite blocks and overlapping bond environments also retain $-\delta \mathbf{J} \cdot \delta \mathbf{Q}$. Equation (22) remains exact when $\delta \mathbf{J}$ is calculated from the corrected couplings. For a square block with b spins on each side, the linear coupling law gives $\delta J_j = g s_p A_{ji}/b^2$, provided the positivity correction is inactive in both configurations. The leading additional energy term is therefore $-g s_p (A^T \mathbf{Q})_i/b^2$. Mean centring removes the response to a
 220 uniform change in water fraction. At interfaces, spatial variations in the cell bond sums $\mathbf{Q}$ retain an energy correction. Its leading term is equivalent to the cell-dependent field shift $g(A^T \mathbf{Q})_i/(2b^2)$; the full increment also includes the simultaneous changes in coupling and bond order. Unlike the global shift, this spatial shift depends on the neighbour configuration. A field substitution based only on the local coupling can thus match a particular interface threshold without reproducing the thresholds for other geometries.

225 The two reversal energies define distinct kinetic prescriptions. At fixed external field, Metropolis updating with the full state-dependent energy satisfies detailed balance. The held rule treats coupling as an environmental variable fixed during a local reversal and generally does not satisfy detailed balance with that full energy. In the seasonal calculations, coupling and field are held within each day and updated between days. To isolate the effect of the energy definition, we compare the two rules with coupling refreshed before every proposal (Sect. 5.5).

230 The reversal energy determines the cost of a local transition, while coupling memory determines how the environment adjusts over time. We next examine how spatial averaging and this adjustment affect the growth or decay of disturbances in the ice–water pattern.



4.3 Spatial scales and memory

Spatial averaging retains broad contrasts between ice-rich and water-rich surroundings while smoothing variations over a few cells. Mean centring makes the coupling response to a uniform phase change zero. The remaining eigenvalues describe how strongly non-uniform spatial patterns pass through the averaging operator.

For the 60×60 grid of 25-km concentration cells used later in the Pacific Arctic computational experiments (Sect. 5.1), $R = 12$ gives an interior averaging window containing 25×25 cells, with a nominal width of 625 km. The largest non-uniform eigenvalue is 0.91435: one application of the operator retains about 91.4 % of the amplitude of the corresponding spatial contrast. This pattern therefore remains strongly represented in the coupling target. Whether the associated phase disturbance grows or decays under the coupled dynamics also depends on the feedback gain and the spin response to changes in coupling.

To analyse the daily feedback loop, we use a periodic one-spin mean-field reduction, in which a Fourier mode represents a perturbation with wavevector q and a single wavelength. Write its held spin response as $\delta \dot{m} = a\delta m + c\delta J$ and its coupling target as $\delta J_* = \ell_q \delta m$. Here Δt is one day expressed in kinetic units. If $e = e^{a\Delta t}$ and $f = c(e^{a\Delta t} - 1)/a$, with $f = c\Delta t$ at $a = 0$, the daily update of $(\delta m_n, \delta J_{n-1})$ has the matrix

$$M_q = \begin{pmatrix} e + f(1 - \rho)\ell_q & f\rho \\ (1 - \rho)\ell_q & \rho \end{pmatrix}. \quad (23)$$

Its eigenvalues are daily multipliers: decay requires both magnitudes below one; growth occurs if either magnitude exceeds one. Its determinant is ρe and its trace is $e + \rho + f(1 - \rho)\ell_q$, including feedback from phase to coupling. Supplement Sect. S2.3 gives the mode-dependent coefficients and stability conditions on both branches. Near the ordering transition, long-wavelength perturbations can grow on the ice-rich branch while corresponding water-rich modes decay; memory changes those growth and decay rates. Here the coupling target changes with the evolving ice–water state, and the coupling in turn affects subsequent melting and freezing. The multipliers therefore describe the stability of this complete feedback loop.

At seasonal coupling, a small uniform disturbance of an ordered interior decays rapidly. The zero-field mean-field reference has $4\beta d_0 = 23.0079$, ordered mean spin m_0 and susceptibility factor $1 - m_0^2 \simeq 4.15 \times 10^{-20}$. The uniform spin mode has daily multiplier 0.2240. Separately, the 14-day coupling filter retains a fraction $\rho = 0.9311$ of a departure from a fixed target after one day (Supplement Sect. S2.3 defines this reference). Seasonal change occurs at interfaces, where ice and water neighbours partly cancel and smaller fields suffice.

When the positivity correction is inactive, the linear filter has unit steady gain and, for a prescribed harmonic target with angular frequency ω in radians per daily update, frequency response

$$G(\omega) = \frac{1 - \rho}{1 - \rho e^{-i\omega}}. \quad (24)$$

A smaller instantaneous gain reduces every frequency equally. Memory attenuates rapid changes and introduces a phase lag while retaining the response to a sustained target. An amplitude-matched seasonal experiment tests this distinction below.

This stability analysis concerns small disturbances of a uniform state. To understand retreat in the strongly ordered regime, we turn to local melting and freezing at an ice–water boundary.



265 4.4 Corner erosion, front pinning and nucleation

Consider an ice spin with z_p neighbours, of which n_p are ice. If its incident bonds have a common ferromagnetic value $J > 0$, Eq. (6) becomes

$$\Delta\mathcal{H}_{-\rightarrow+} = 2J(2n_p - z_p) + 2h. \quad (25)$$

For an interior spin of the square lattice, $z_p = 4$, this is $4J(n_p - 2) + 2h$. A convex corner of an axis-aligned ice region has two
 270 ice neighbours and two water neighbours; a flat edge has three ice neighbours; a site surrounded by ice has four. A staircase or irregular interface may share these counts. Recall that negative h favours water under the sign convention in Eq. (5). The downhill conditions are

$$\begin{aligned} n_p = 2: \quad & h < 0, \quad \text{corner erosion,} \\ n_p = 3: \quad & h < -2J, \quad \text{advance of a melt front,} \\ n_p = 4: \quad & h < -4J, \quad \text{bulk nucleation.} \end{aligned} \quad (26)$$

These neighbour-count barriers are classical features of kinetic Ising models, underlying field-driven metastable decay and
 275 zero-temperature blocking (Rikvold et al., 1994; Spirin et al., 2001). Environmental feedback changes their local coupling, not their geometrical origin. Neutral moves are also accepted under the Metropolis convention. The interval $-2J < h < 0$ permits corner erosion while pinning a flat front. Decreasing J at fixed negative h moves a site across the front threshold $J = -h/2$; nucleation requires the stronger reduction $J = -h/4$. This is a concrete mechanism by which spatial organisation changes subsequent phase change.

280 For non-uniform bond couplings held during a reversal, the exact melt-direction energy difference is

$$\Delta\mathcal{H}_{-\rightarrow+} = 2 \left[h + \sum_{q \in \mathcal{I}_p} J_{pq} - \sum_{q \in \mathcal{W}_p} J_{pq} \right], \quad (27)$$

where $\mathcal{I}_p$ and $\mathcal{W}_p$ are the sets of ice and water neighbours of site p . For an interior ice site with $n_p = 2$, the coupling contribution
 285 on the bonds connecting the selected site to its neighbours. The resulting energy change determines the Metropolis acceptance probability. The state-energy control additionally includes the coupling changes in Eq. (22).

For uniform incident bonds, a coupling change δJ is equivalent to a field shift that depends on neighbour geometry:

$$\delta h = (2n_p - z_p)\delta J. \quad (28)$$

For interior sites ($z_p = 4$), the equivalent shift is zero when $n_p = 2$, $2\delta J$ when $n_p = 3$ and $4\delta J$ when $n_p = 4$. A field shift
 290 chosen to match the flat-front threshold therefore does not generally match the corner and bulk thresholds. Coupling changes affect these local transition geometries differently.



Keeping $\bar{J} = d_0$ fixes the average coupling while allowing local melting barriers to vary. At fixed field and with equal incident bonds, smaller coupling increases the probability of crossing a positive front or bulk barrier exponentially. The distribution of coupling values therefore matters for transition rates even when its mean is unchanged (Supplement Sect. S3).

295 These interface thresholds identify which local changes coupling feedback can favour. We now examine how these changes accumulate during a melt season under prescribed surface forcing.

5 Sea ice as a driven spatial realisation

5.1 Surface forcing and the seasonal experiment

We now examine how changes in local melting barriers affect the retreat of an evolving ice cover. The Pacific Arctic calculations
 300 provide a seasonal illustration, initialised from observed ice concentration and driven by prescribed ERA5 surface temperatures and heat fluxes. The three principal cases use constant coupling (CONST), instantaneous adjustment to the environmental target (INST), or relaxation towards that target with a 14-day response time (MEM). We use a 60×60 Pacific-sector crop of the NOAA/NSIDC 25-km concentration grid (Meier et al., 2026). Each cell contains 3×3 microspins, giving an open 180×180 lattice. The 3×3 block is the smallest square block with both an interior site and boundary sites. At the calibrated coupling,
 305 mixed cells ($0 < C_i < 1$) fall from 33.5 % to 6.9 % within two days in the eight 2024 INST paths. Persistent interfaces then mostly follow the 25-km cell boundaries. The internal sites represent transition geometry, not persistent partial concentrations. Geographical maps are in Supplement Sect. S1.

All cases share thermal parameters and forcing. In cell i on day n , heat input q_i^n feeds the dimensionless reservoir E_i^n and phase-preference field h_i^n :

$$310 \quad q_i^n = [1 - \alpha(C_i^n)]S_{\downarrow,i}^n + L_{\text{net},i}^n + H_{s,i}^n + H_{l,i}^n, \quad (29)$$

$$E_i^{n+1} = e^{-1/\tau_E} E_i^n + q_i^n / Q_0 - \lambda_L \Delta F_{w,i}^n, \quad (30)$$

$$h_i^n = a_0 - b_T(\Theta_i^n - T_m) - c_E E_i^n. \quad (31)$$

Here a_0 is the field offset, b_T and c_E weight temperature and stored thermal influence, τ_E is the reservoir decay time in days, and λ_L sets the reservoir change per unit increase in open water. The daily temperature filter is controlled by τ_T ; parameter
 315 meanings and values are listed in Supplement Table S3.

ERA5 supplies downward short-wave radiation $S_{\downarrow}$, net long-wave radiation L_{net} , signed sensible and latent heat fluxes H_s , H_l , and skin temperature T_s (Hersbach et al., 2020; Copernicus Climate Change Service, 2023). Fluxes are daily means in W m^{-2} , $Q_0 = 100 \text{ W m}^{-2}$, and $T_m = 273.15 \text{ K}$. The filtered temperature obeys $\Theta_i^n = \Theta_i^{n-1} + \tau_T^{-1}(T_{s,i}^n - \Theta_i^{n-1})$. The albedo is $\alpha(C) = \alpha_w + (\alpha_{\text{ice}} - \alpha_w)C$, and $\Delta F_{w,i}^n = C_i^n - C_i^{n+1}$ is the daily increase in open water. Heat input raises E and favours
 320 water. Melting consumes stored influence; freezing releases latent heat and increases E .

The temperature and reservoir recursions are local to each cell. Cells communicate through nearest-neighbour spins and the regional target J_* . Each day, coupling and field are evaluated from the current state and held during the spin updates.



A proposed reversal selects a site uniformly at random. Downhill moves are accepted; an uphill move is accepted when an independent uniform draw is below $e^{-\beta\Delta\mathcal{H}}$. The reservoir is then updated using the day's fluxes, albedo at the start of the day and realised change in open water.

Each 2022–2024 season runs from observed concentration on 16 June to 30 September. Initial spin counts use stochastic rounding and random placement; thermal variables use forcing history from 1 May (Supplement Sect. S4.1). Later observed concentration does not enter the principal updates. Figure 3 uses the observations to locate the seasonal envelope. The protocol fixes 64 members to sample variability from initial allocation and spin updating, with common forcing and parameters. Matched mechanism experiments use the first eight seeds and share the proposal and acceptance random streams. The kinetic interval gives 48,468 proposals per day, approximately 1.50 per spin.

The seasonal equations contain ten effective backbone parameters (Supplement Table S3), together with g , R and τ_J . Their provenance spans a 2012/2019/2020 fit, thermal-architecture selection in 2022 and spatial development through 2023. Thus $d_0 \simeq 5.75$, the field scale and the strongly ordered regime are calibration outcomes. The retained gain $g = 10d_0/9$ sets the linear law's zero at a water-fraction contrast of 0.9; it is a development scale, not a measured critical concentration. The radius $R = 12$ was chosen from 2022–2023 concentration errors and gives a nominal 625-km window. These parameters were fixed before the 2024 illustration. Fourteen-day memory is illustrative; the 2024 tests at 3 and 7 d were added subsequently. Supplement Sect. S1 gives the calibration history, and Sect. S4 the memory scan.

5.2 Seasonal timing and threshold accessibility

Under the same prescribed ERA5 forcing, instantaneous feedback (INST) advances the seasonal decline in ice occupancy relative to constant coupling (CONST) in all three seasons (Fig. 3). The 14-day coupling response in MEM reduces early-season ice loss relative to INST. The lower panels show occupancy differences from CONST for INST and MEM, comparing ensemble members with the same random seed. Maps of the 2024 ice fields are shown in Supplement Fig. S2.

Figure 4 relates the seasonal evolution to the local melting thresholds. The upper panels compare the domain-mean field h with the reference thresholds for corner erosion, flat-front advance and bulk melting: 0, $-2d_0$ and $-4d_0$. These reference values are common to all three cases, whose arithmetic mean cell coupling remains $\bar{J} = N_c^{-1} \sum_i J_i = d_0$. Local front and bulk thresholds nevertheless vary with the cell coupling J_i .

The lower panels show the fractions of cells satisfying $h < 0$, $h < -2J_i$ and $h < -4J_i$. For this diagnostic, each cell coupling is assigned to four equal bonds around an interior site. The fractions describe where the local field and coupling would favour downhill melting for each neighbour geometry. They include open-water cells and do not require an ice site with that geometry to be present. Actual reversal energies use the individual incident bond couplings (Eq. 27).

Across the eight 2024 paths, the equal-bond front condition is satisfied on 16.5 % of cell-days for CONST, 31.6 % for INST and 23.6 % for MEM. A cell-day represents one cell on one simulation day. Averaged over these cell-days, the heat reservoir contributes -7.86 , -8.97 and -8.20 model field units, respectively, through the term $-c_E E$. This contribution favours melting and brings the total field towards the reference front threshold $-2d_0 \simeq -11.5$. Although the prescribed forcing is identical, each evolving ice pattern determines its own albedo and heat storage, so the cases develop different local fields.

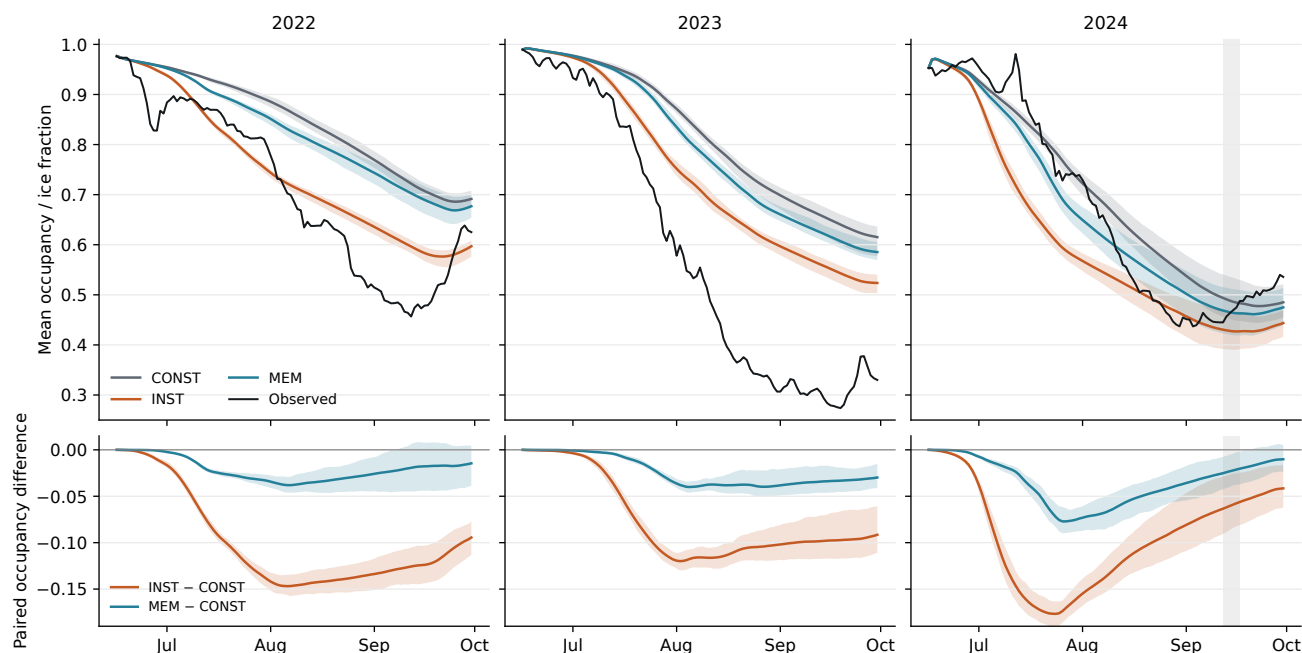


Figure 3. Pacific Arctic seasonal evolution. Upper panels: ensemble-mean ice occupancy over the model domain and domain-mean observed concentration. Lower panels: occupancy differences from CONST, calculated between paired members. Bands span the 10th–90th member percentiles across 64 realisations sharing forcing and parameters. MEM uses a 14-day response time. Grey shading marks interpolated dates in the NSIDC concentration product in September 2024.

The threshold fractions describe where the local field and coupling favour melting for each neighbour geometry. Actual melting also depends on the ice sites present and their acceptance probabilities. On the same configurations visited by INST, the constant-coupling rule accepts 11,401 of 486,364 attempts to melt interior ice sites with three ice neighbours ($n_p = 3$; Sect. 5.3). Under INST, none of the 23,753,491 attempts to melt sites surrounded by four ice neighbours ($n_p = 4$) is accepted. Across these attempts, the mean coupling of the four incident bonds is at least 2.8997, and the smallest energy change is 5.104. These sites therefore retain positive melting barriers, even when the cell-based bulk condition is satisfied elsewhere in the domain.

We next examine the neighbour configurations encountered by the simulations and how their local barriers determine accepted melting and freezing events.

5.3 Melting pathways and stochastic activation

Figure 5a–c shows the neighbour arrangements associated with corner erosion, melting along a flat ice front and melting within an ice-covered interior.

We isolate the effect of coupling using eight INST simulations for 2024. At every attempted ice–water change, we evaluate the acceptance decision with both the INST coupling and constant coupling d_0 . The two calculations use the same ice–water

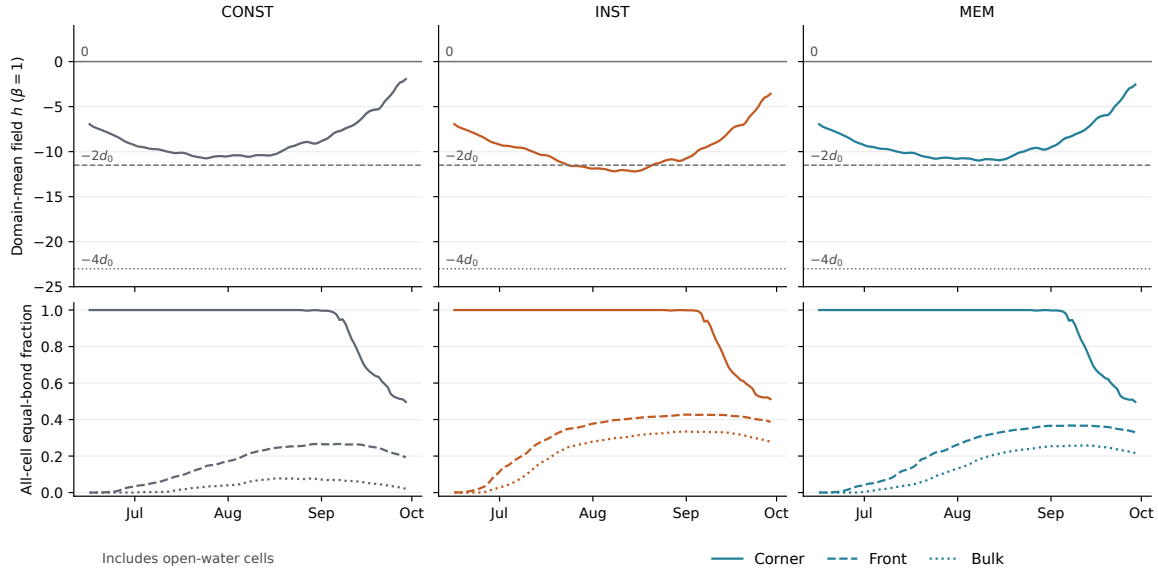


Figure 4. Seasonal field and melting thresholds, 2024. Upper panels: domain-mean field h and its 10th–90th member percentiles, compared with the reference thresholds 0, $-2d_0$ and $-4d_0$. Lower panels: fractions of all cells satisfying the corner, front and bulk conditions under the four-equal-bond approximation using daily J_i , averaged over eight members. Open-water cells are included; these fractions describe local field and coupling conditions, rather than melt-event probabilities. Actual proposal energies use the individual bond couplings in Eq. (27).

configuration, local field, selected site and random number for Metropolis acceptance. INST determines how the configuration evolves. Evaluating constant coupling on these recorded configurations isolates its immediate effect on acceptance.

We call a change enabled by feedback when INST accepts it and the constant-coupling rule rejects it. Almost all melting changes enabled in this way occur at interior ice sites with three ice neighbours ($n_p = 3$). Of 33,642 such changes, feedback
 375 removes the positive energy barrier for 21,598 (64.2 %). The remaining 12,044 (35.8 %) retain positive barriers but are accepted because feedback increases their Metropolis acceptance probabilities. Melting along fronts therefore combines barrier removal and stochastic crossing of positive barriers.

To examine the role of stochastic acceptance, we repeat these calculations at $\beta = 0.5, 1, 2$, retaining the recorded configurations, fields, couplings and random numbers. The fraction of feedback-enabled front changes that cross positive barriers is 53.5,
 380 35.8 and 20.1 %, respectively. At $\beta = 0.5$, 138 attempts to melt fully surrounded ice sites ($n_p = 4$) are accepted across positive barriers, compared with none at $\beta = 1$ and 2. These calculations isolate the effect of changing the acceptance probability on the same recorded ice patterns (Supplement Sect. S3.1). Table 1 summarises the melting counts for interior sites.

Figure 5d shows the local field and bond configuration for all attempts at interior $n_p = 3$ sites whose acceptance decision changes under feedback. To express the melting threshold for unequal incident bonds, define

$$385 \quad J_{\text{geom},p} = \frac{1}{2} \left(\sum_{q \in \mathcal{I}_p} J_{pq} - \sum_{q \in \mathcal{W}_p} J_{pq} \right), \quad n_p = 3, z_p = 4. \quad (32)$$

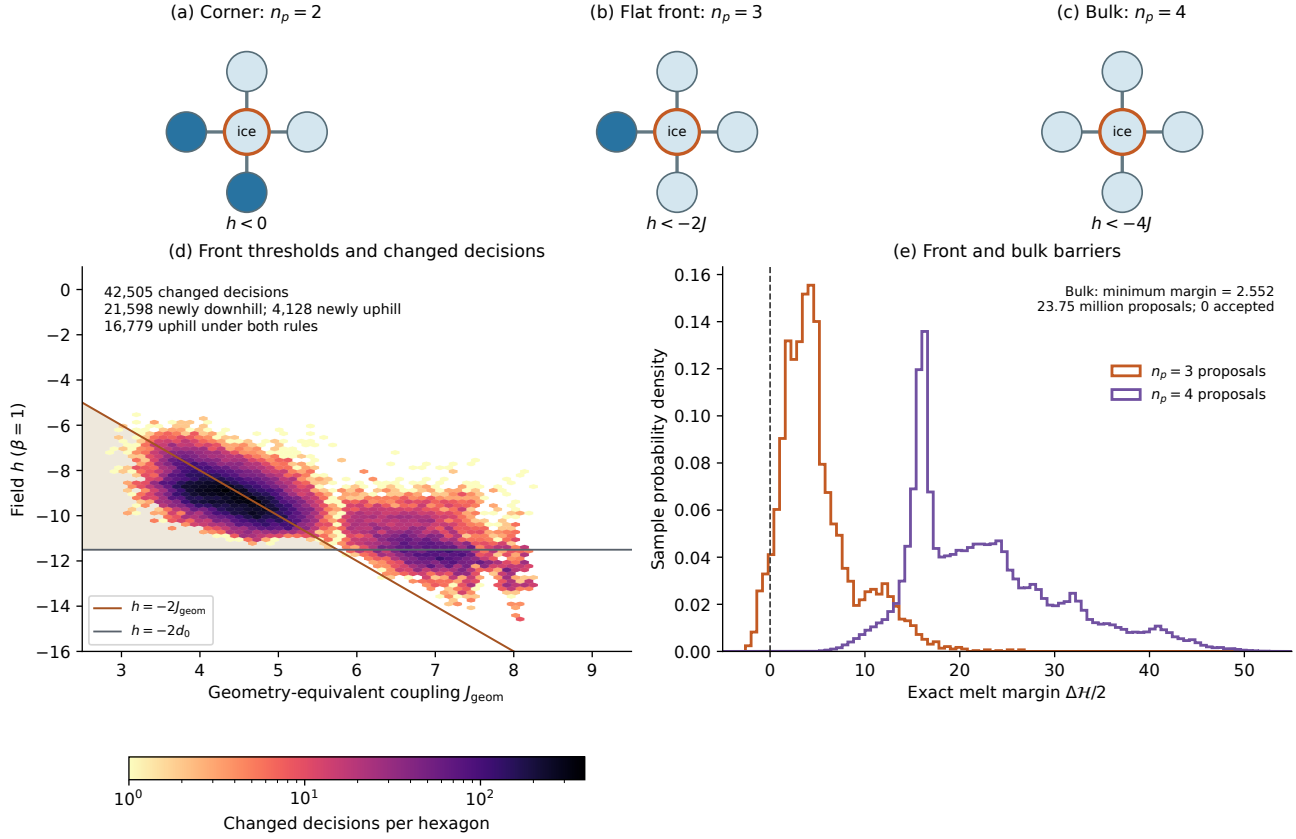


Figure 5. Interface geometry, acceptance decisions and melting barriers. (a–c) Ice is light and water dark; the selected ice site is outlined. The conditions for energy-lowering melting with equal bonds are shown below each arrangement. (d) Attempts at interior sites with three ice neighbours whose acceptance decisions differ between INST and constant coupling, including both melting and freezing. The threshold lines use d_0 and J_{geom} from Eq. (32); counts are annotated. (e) Distributions of $\Delta\mathcal{H}/2$ for attempts to melt interior sites with three or four ice neighbours, sampled every 101st attempted spin change on each day. The annotated bulk minimum and attempt count use all recorded attempts.

The ice-to-water energy change is then exactly $\Delta\mathcal{H}_{-\rightarrow+} = 4J_{\text{geom},p} + 2h$. When all four incident bonds have coupling J_i , $J_{\text{geom},p} = J_i$. For unequal bonds, this effective coefficient combines the couplings to ice and water neighbours with opposite signs. Feedback-enabled melting changes whose positive barrier is removed lie between the threshold lines $h = -2d_0$ and $h = -2J_{\text{geom},p}$. Changes accepted across positive barriers can occur outside this region.

390 Figure 5e shows half the reversal energy, $\Delta\mathcal{H}/2$, for attempts to melt sites with three and four ice neighbours. Negative values indicate energy-lowering changes; positive values indicate barriers that require stochastic activation for acceptance.

At $\beta = 1$, the acceptance probabilities for attempts to melt fully surrounded ice sites sum to 0.342 over the eight simulations. A Poisson approximation for these rare events, using the recorded environments, gives a probability of no accepted events of $e^{-0.342} \simeq 0.71$. Ordered interiors can therefore persist while corners erode and fronts advance intermittently.



Table 1. Attempts to melt interior ice sites on eight 2024 INST simulations. Enabled events are accepted by INST but rejected by the constant-coupling rule evaluated on the same configuration with the same acceptance random number. The two enabled columns distinguish events with non-positive and positive INST reversal energies. Suppressed events are accepted only by the constant-coupling rule. Both rules are evaluated on the configurations visited by INST.

Ice neighbours	Attempts	Accepted	Enabled: $\Delta\mathcal{H} \leq 0$	Enabled: $\Delta\mathcal{H} > 0$	Suppressed
0	7,963	7,963	0	0	0
1	16,088	16,088	0	0	0
2 (corner)	110,070	100,448	0	0	405
3 (front)	486,364	36,180	21,598	12,044	8,863
4 (bulk)	23,753,491	0	0	0	0

395 The effect on freezing also depends on neighbour geometry. At a site with three ice neighbours, lowering a common incident
 coupling J favours melting and opposes freezing: the respective downhill conditions are $h < -2J$ and $h > -2J$. Freezing
 onto a flat ice edge instead involves a water site with one ice neighbour ($n_p = 1$) and requires $h > 2J$, so weaker coupling
 favours freezing there. Summing over all recorded attempts, INST accepts 17.0 % more melting changes and 9.49 % fewer
 freezing changes than the constant-coupling rule evaluated on the same configurations. Across melting and freezing, 10.68 %
 400 of accepted INST changes cross positive energy barriers.

These comparisons measure the immediate effect of coupling at a fixed configuration. We next compare runs in which each
 model evolves its own ice pattern, changing the sites available for subsequent melting and freezing.

5.4 How local transitions change the path

We now compare the eight matched 2024 CONST and INST simulations, in which each model evolves its own ice pattern.
 405 The coupling-target maps show lower values near open-water margins and higher values in the compact pack (Supplement
 Fig. S2d,h).

On the INST configurations analysed above, melting at a front site removes one ice neighbour from each adjacent ice site.
 Of the enabled interior front melts, 69.0 % create at least one new interior site with two ice neighbours ($n_p = 2$), with a mean
 of 0.900 new sites per event. Front advance can therefore initiate further boundary erosion.

410 Over the season, however, increased front melting partly replaces melting in the corner class. The ratio of accepted corner
 ($n_p = 2$) to front ($n_p = 3$) melts falls from 13.22 for CONST to 2.78 for INST. Their combined interior melt count decreases by
 1,833, while melting at exposed interior sites ($n_p = 0, 1$) rises from 7,913 to 24,051. The additional melting is thus concentrated
 at isolated ice sites and sites with only one ice neighbour. These totals describe the evolving populations of sites; they do not
 trace the exposed sites back to particular front events.

415 INST also accepts 10.7 % more freezing events than CONST (31,707 versus 28,647), whereas the same-state comparison
 gives 9.49 % fewer. An immediate change in acceptance therefore does not determine the seasonal outcome: each model



subsequently encounters different ice configurations. The complete budget, including sites at the crop boundary, gives 10,571 additional net ice losses under INST and exactly reproduces the final occupancy difference (Supplement Sect. S3).

These differences develop through the interaction of coupling, interface geometry and the albedo–reservoir feedback. The following matched experiments change individual modelling choices while retaining common inputs and random numbers within each pair.

5.5 Matched spatial feedback experiments

We first compare the held rule, which keeps coupling fixed during a local reversal, with the state-energy rule, which includes the coupling changes caused by that reversal (Sect. 4.2). In both cases, coupling is recomputed from the current configuration before every attempted reversal. Unless otherwise stated, paired differences below are reported as the mean $\pm$ sample standard deviation (SD) across eight matched 2024 pairs.

The state-energy rule produces 193 ± 111 additional interior front melts per member and lowers the individual minimum occupancy by 0.0166 ± 0.0083 . Both changes have the same sign in all eight pairs. The minima of the ensemble-mean occupancy curves are 0.4171 for the held rule and 0.4004 for the state-energy rule (Fig. 6a). For the eight-member held case, changing from daily coupling updates to updates before every attempted reversal changes this minimum from 0.4188 to 0.4171. The refresh interval therefore has a small effect on this statistic. The corresponding minimum for the separate 64-member daily INST ensemble is 0.4269.

FIELD replaces the spatial coupling by constant coupling d_0 and adjusts the local field according to $h'_i = h_i + 2(J_{*,i} - d_0)$. Here $J_{*,i}$ is calculated from FIELD's own evolving ice pattern. Under the equal-bond approximation, this shift matches the thresholds for melting an ice site with three ice neighbours and for freezing a water site with three ice neighbours, thereby filling a notch. Freezing onto a flat ice edge involves a water site with one ice neighbour and requires the opposite field shift (Eq. 28).

FIELD gives similar front melt counts to INST but produces 840 ± 682 fewer corner melts per member, with fewer in all eight pairs (Fig. 6b,c). Its final occupancy is higher in every pair (Table 2). Once the accepted changes differ, the two simulations develop different interface geometries, albedos and heat storage. Supplement Sect. S3 separates these seasonal counts from the field substitution evaluated on the same recorded INST configurations.

To separate coupling feedback from changes in albedo, we hold albedo at its initial spatial distribution in both CONST and INST. Feedback still increases front melting and lowers final occupancy in all eight pairs; the individual minimum occupancy is lower in six pairs (Fig. 6d). The coupling effect therefore persists without evolving albedo. The reservoir still responds to melting and freezing, and rapid ordering of initially mixed cells affects this heat budget (Supplement Sect. S3).

We next vary the environmental averaging scale and compare spatial feedback with simpler coupling prescriptions. We also compare delayed coupling adjustment with a smaller instantaneous feedback gain.

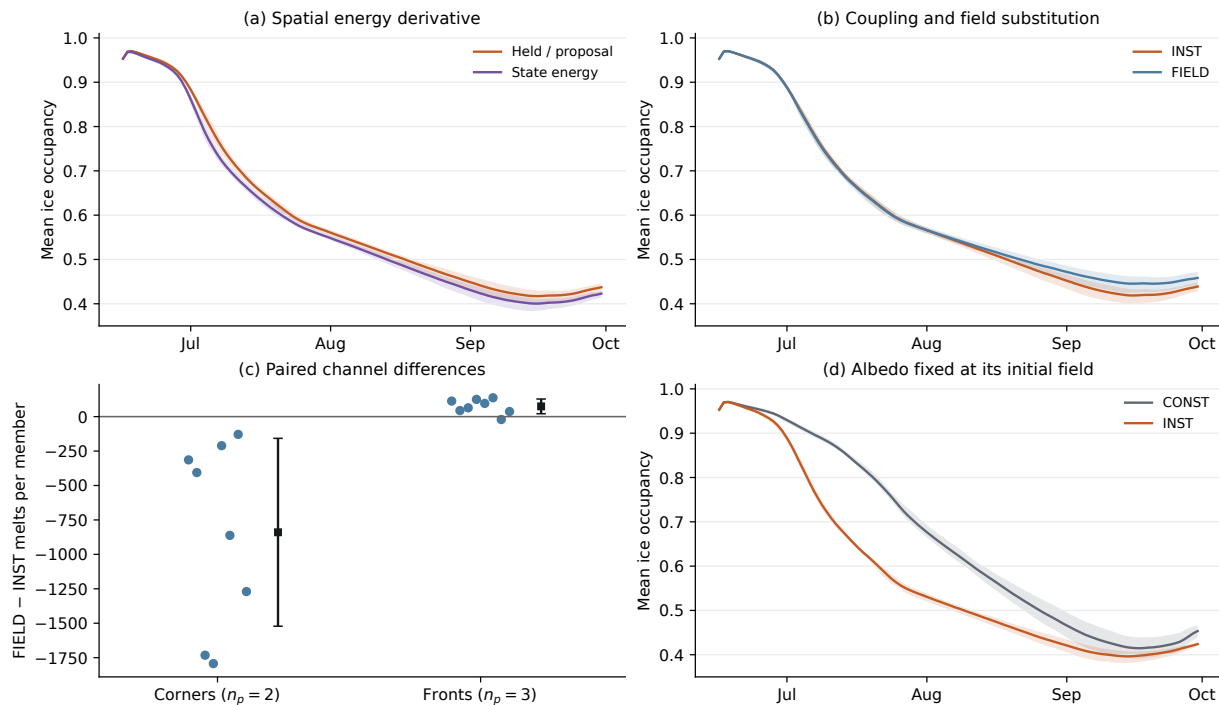


Figure 6. Spatial feedback in eight matched 2024 realisations. (a) Held and state-energy rules, with coupling recomputed before every attempted reversal. (b) Daily INST and FIELD, which uses constant coupling and the adjusted field $h' = h + 2(J_* - d_0)$. (c) FIELD minus INST accepted melt counts for each paired member. Dots show the eight differences; squares and bars show their mean and sample SD. (d) CONST and INST with albedo fixed at its initial spatial distribution. Curves in (a,b,d) show ensemble-mean domain occupancy; bands span the 10th–90th member percentiles. Paired simulations share inputs and random numbers, while each evolves its own ice pattern.

5.6 Environmental scale and coupling memory

Increasing the width of the environmental average strengthens front melting under INST. At $R = 2, 6, 12$, corresponding to
 450 nominal window widths of 125, 325 and 625 km, the eight-member totals are 29,068, 35,007 and 36,180 interior front melts, respectively. The common CONST reference has 9,736. Across these radii, FIELD produces 11.6, 3.00 and 1.64 % more front melts than INST, but 3.47, 4.50 and 6.69 % fewer corner melts. Broader averaging therefore improves FIELD's agreement with INST in total front melting while increasing their difference in corner melting (Table 2; Supplement Fig. S4).

A geometric alternative defines the coupling target from signed distance to the current boundary of cells with $C_i \geq 0.15$,
 455 with positive distance inside the ice-covered region. Each day, this distance field is rescaled to have the same spatial mean and SD as the water-fraction coupling target calculated from that simulation's own ice pattern. In 64-member runs, its seasonal mean absolute concentration errors are close to INST in 2023 but higher in 2022 and 2024 (Supplement Table S5). The two descriptions therefore give different seasonal behaviour. This comparison concerns concentration fields and does not determine whether their corner and front melting pathways differ.



Table 2. Matched controls in eight paired 2024 realisations. Entries give the mean $\pm$ sample SD of differences between paired members. Counts are accepted interior melts over 106 daily updates; final occupancy is evaluated on 30 September. Each simulation evolves its own ice pattern. Uniform forcing replaces each prescribed ERA5 field by its daily domain mean.

Comparison	Corner melts	Front melts	Final occupancy
FIELD – INST, $R = 2$	-464 ± 401	420 ± 49	-0.0029 ± 0.0104
FIELD – INST, $R = 6$	-556 ± 548	132 ± 82	0.0084 ± 0.0089
FIELD – INST, $R = 12$	-840 ± 682	74 ± 53	0.0194 ± 0.0123
Uniform: INST – CONST	-4157 ± 770	3547 ± 165	-0.0499 ± 0.0222
MEM – reduced-gain INST	-237 ± 274	-29 ± 87	0.0067 ± 0.0061

460 We also test whether a spatially uniform, weaker coupling can reproduce the INST response. At $\beta = 1$, we set

$$J_w = -\frac{1}{4} \log \langle e^{-4J_{\text{INST}}} \rangle = 2.35805,$$

where the average pools all cell couplings over 106 daily updates and eight INST simulations. This value matches the pooled coupling-dependent factor in the uphill, equal-bond front acceptance probability; the thermal parameters are unchanged. All eight uniformly weakened simulations lose their ice cover and exhibit bulk melting, which is absent under INST. Matching this
 465 coupling factor alone therefore does not reproduce the spatial-feedback trajectory (Supplement Sect. S3).

To test dependence on spatial variations in the prescribed forcing, we replace each ERA5 temperature and flux field by its domain mean on every date, including the preceding thermal history. The initial ice geometry, local albedo and reservoir evolution are retained. INST still increases front melting and reduces corner melting relative to CONST in every pair (Table 2). This redistribution of melting therefore persists without spatial contrasts in the prescribed ERA5 forcing.

470 Finally, we compare MEM with instantaneous feedback at the reduced gain $g' = 0.52156g$. The factor g'/g is the ratio of the root-mean-square departures from d_0 of the applied MEM coupling and its instantaneous target, evaluated over all cells and the first 30 daily updates of the eight MEM simulations (Supplement Sect. S2.3). This choice uses the early coupling history rather than a concentration score.

The season-total corner and front melt counts are similar (Table 2). MEM's final occupancy exceeds that of reduced-gain
 475 INST by 0.0067 ± 0.0061 , with positive differences in seven of eight pairs. Their accumulated effects are therefore close in this comparison, although their coupling histories differ: memory delays adjustment, whereas a smaller gain reduces the instantaneous response throughout the season. The amplitude matching applies to the first 30 updates and does not establish equivalence over the full season (Supplement Sect. S2).

Together, these experiments show how changes in local acceptance combine with evolving geometry and heat storage to alter
 480 seasonal ice loss. We discuss this strongly ordered lattice response separately from the near-ordering mean-field instability.



6 Discussion

The seasonal experiments show how differences in the surrounding ice cover can change the route of retreat, even when the average coupling remains fixed. Here, coupling represents an effective resistance to local changes in the ice–water arrangement. With equal incident couplings, their contributions cancel at a boundary site with two ice and two water neighbours, whereas
 485 a locally flat front with three ice neighbours retains a coupling-dependent melting barrier. Lower coupling near relatively open surroundings therefore favours front advance. The matched seasonal experiments follow this prediction, including when albedo is fixed and when the prescribed forcing is spatially uniform. Front motion combines removal of these barriers with stochastic crossing of barriers that remain positive; the latter accounts for about one-third of the newly enabled front melts in the recorded-state comparison.

490 The seasonal response also depends on the changing geometry of the cover. In the independently evolving 2024 simulations, the increase in front melting largely offsets the decrease in the two-ice-neighbour class, while the additional melting is concentrated at isolated ice sites and sites with only one ice neighbour. The budget thus links the changing distribution of local ice configurations to the accumulated ice loss. As melting and freezing alter the boundary, they also change the configurations available for subsequent transitions. Their seasonal balance need not have the same sign as an immediate acceptance
 495 change evaluated on a fixed configuration, as illustrated by the opposite freezing responses in the same-state and independently evolving comparisons. Albedo and heat storage develop alongside these geometrical changes and contribute to the timing of retreat.

This interpretation extends the geometrical perspective of melt pond models (Bowen et al., 2018; Popović et al., 2018; Ma et al., 2019) to a regional ice–water cover whose evolving surroundings modify its local coupling. Mean centring provides
 500 a controlled way to examine this effect: relatively open surroundings lower the target coupling, while compact surroundings raise it, without changing its arithmetic mean. The construction isolates the redistribution of local resistance from a uniform weakening of the cover. For uphill melting at a flat front or within the ice, the equal-bond acceptance probability weights the lower-coupling part of the distribution exponentially. The arithmetic mean alone therefore does not determine this response. The controls show the importance of retaining the spatial context of this response. FIELD matches the equal-bond front threshold
 505 but treats other melting and freezing geometries differently, while the proxy-matched uniform coupling permits bulk melting and produces a different seasonal evolution. The reference environment is the mean over the chosen regional domain, so the crop and averaging scale specify the spatial context of the contrast response (Supplement Sect. S1.2).

Coupling memory introduces the history of the environment into this geometrical picture. In the global mean-field model near ordering, delayed adjustment can destabilise the water-rich equilibrium through a subcritical Hopf bifurcation. Below the
 510 bifurcation threshold, an unstable periodic orbit bounds its local basin of attraction; at the parameters examined, trajectories escaping this basin approach the ice-rich equilibrium. This supplies a reduced-model route to abrupt switching between phase states, alongside other nonlinear descriptions of sea ice stability (Eisenman and Wettlaufer, 2009; Moon and Wettlaufer, 2017). Its connection to regional sea ice depends on the processes represented: spatial heat transport and seasonal forcing can stabilise sea ice in energy-balance models (Wagner and Eisenman, 2015).



515 In the strongly ordered seasonal lattice, memory instead acts through the timing of local interface barriers. The coupling
 adjusts progressively to changes in open-water exposure, delaying the development of spatial contrasts and moderating early-
 season ice loss. Matching the coupling amplitudes over the first 30 daily updates brings the season-total transition counts close
 to those of reduced-gain instantaneous feedback. Their histories nevertheless retain a physical distinction: memory delays
 adjustment to a changing environment, whereas reduced gain decreases the instantaneous response throughout the season.
 520 The comparison therefore identifies an early-season amplitude effect without equating the two prescriptions over the complete
 seasonal cycle.

The energy definition and the response time describe different aspects of environmental coupling. Holding the environment
 fixed during a local reversal treats it as a separate variable that adjusts between transitions. The state-energy rule instead
 includes the coupling changes caused by the reversal throughout the affected neighbourhood and satisfies detailed balance with
 525 its stated energy at fixed external field. Sending $\tau_J \rightarrow 0$ makes the conditional coupling respond instantaneously, but does not
 introduce this energy correction. The distinction remains relevant when connecting an equilibrium energy description to driven
 regional kinetics.

The seasonal calculations provide a driven illustration of these mechanisms. Ice transport, thickness and ocean heat ex-
 change offer natural extensions, particularly for the larger 2023 retreat and autumn recovery. Agreement with observations
 530 varies by season and diagnostic: climatology has smaller concentration error in 2023, whereas INST has a higher Brier score
 than CONST in 2024. Prescribed ERA5 fields carry information about ice conditions (Hirahara et al., 2016; Uhlíková et al.,
 2024), and summer concentration retrievals are affected by surface melt (Kern et al., 2016). The persistence of the channel
 redistribution under spatially uniform forcing helps separate it from the local prescribed forcing pattern. The concentration and
 contour comparisons, recovery diagnostics and calibrated thermal scales are documented in Supplement Sect. S4.

535 7 Conclusions

We have examined how an evolving environment modifies local melting and freezing in a conceptual model of regional sea ice.
 Within the feedback-Ising framework, we define a mean-centred coupling law on finite blocks, derive its exact reversal energy
 and introduce a finite coupling response time. Holding the average coupling fixed provides a controlled comparison of spatially
 different melting barriers. Relatively open surroundings reduce the target coupling, while compact surroundings increase it,
 540 allowing the effect of environmental contrasts to be examined separately from uniform weakening.

The Pacific Arctic experiments connect these contrasts to identifiable routes of retreat. In the matched 2024 simulations, in-
 stantaneous feedback increases melting along locally flat fronts and reduces melting in the two-ice-neighbour class. Additional
 melting is concentrated at exposed sites with zero or one ice neighbour, while stochastic activation contributes substantially
 to front motion. These results describe the joint evolution of local acceptance probabilities and the populations of ice configu-
 545 rations encountered during the season. The redistribution persists with fixed albedo and spatially uniform prescribed forcing.
 Matching a front threshold through FIELD, or matching a pooled coupling factor through uniform weakening, gives different
 seasonal budgets, demonstrating the importance of the spatial arrangement of local resistance.



Memory has distinct consequences in the two regimes studied. Near ordering, the global mean-field model admits a subcritical Hopf bifurcation that can destabilise a homogeneous water-rich equilibrium. In the strongly ordered seasonal lattice, finite response time delays the development of coupling contrasts and reduces early-season ice loss relative to instantaneous feedback. A smaller instantaneous gain gives similar accumulated transition counts after matching the early coupling amplitudes, although it does not reproduce the same coupling history.

The resulting picture connects regional environmental contrasts, local interface geometry and response time. Average ice concentration and average coupling describe the state of the cover, while its spatial arrangement and environmental history help determine how that cover changes. The model provides a dynamical framework for studying these connections and for incorporating further physical descriptions of the processes that govern sea ice retreat.

Code and data availability. The simulation code, processed model inputs, parameter files, random seeds and numerical outputs supporting this study are available in the Zenodo reproducibility archive (<https://doi.org/10.5281/zenodo.23147089>; Sudakow et al., 2026). The deposit includes the temperature, surface heat-flux and ice-concentration arrays used in the calculations, together with analysis and reproduction scripts. The original NOAA/NSIDC and ERA5 datasets are cited separately (Meier et al., 2026; Copernicus Climate Change Service, 2023). First-party code is released under the MIT licence and author-generated derived data under CC BY 4.0; upstream data retain their provider terms.

Author contributions. I.S. and S.V. developed the continuum and feedback model theory. I.S. and D.K. collected data and designed and verified the numerical experiments. I.S., S.V. and D.K. contributed to the physical interpretation, manuscript writing and editing.

Competing interests. The authors declare that they have no conflict of interest.

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References

- Allen, S. M. and Cahn, J. W.: A microscopic theory for antiphase boundary motion and its application to antiphase domain coarsening, *Acta Metall.*, 27, 1085–1095, [https://doi.org/10.1016/0001-6160\(79\)90196-2](https://doi.org/10.1016/0001-6160(79)90196-2), 1979.
- 575 Andersson, T. R., Hosking, J. S., Pérez-Ortiz, M., Paige, B., Elliott, A., Russell, C., Law, S., Jones, D. C., Wilkinson, J., Phillips, T., Byrne, J., Tietsche, S., Sarojini, B. B., Blanchard-Wrigglesworth, E., Aksenov, Y., Downie, R., and Shuckburgh, E.: Seasonal Arctic sea ice forecasting with probabilistic deep learning, *Nat. Commun.*, 12, 5124, <https://doi.org/10.1038/s41467-021-25257-4>, 2021.
- Bowen, B., Strong, C., and Golden, K. M.: Modeling the fractal geometry of Arctic melt ponds using the level sets of random surfaces, *J. Fractal Geom.*, 5, 121–142, <https://doi.org/10.4171/JFG/58>, 2018.
- 580 Budyko, M. I.: The effect of solar radiation variations on the climate of the Earth, *Tellus*, 21, 611–619, <https://doi.org/10.1111/j.2153-3490.1969.tb00466.x>, 1969.
- Cahn, J. W. and Hilliard, J. E.: Free energy of a nonuniform system. I. Interfacial free energy, *J. Chem. Phys.*, 28, 258–267, <https://doi.org/10.1063/1.1744102>, 1958.
- Copernicus Climate Change Service: ERA5 hourly data on single levels from 1940 to present, Climate Data Store (CDS) [data set], <https://doi.org/10.24381/cds.adbb2d47>, 2023.
- 585 De Martino, D.: Feedback-induced self-oscillations in large interacting systems subjected to phase transitions, *J. Phys. A-Math. Theor.*, 52, 045002, <https://doi.org/10.1088/1751-8121/aaf2dd>, 2019.
- Dumont, D., Kohout, A., and Bertino, L.: A wave-based model for the marginal ice zone including a floe breaking parameterization, *J. Geophys. Res.-Oceans*, 116, C04001, <https://doi.org/10.1029/2010JC006682>, 2011.
- 590 Eisenman, I.: Factors controlling the bifurcation structure of sea ice retreat, *J. Geophys. Res.*, 117, D01111, <https://doi.org/10.1029/2011JD016164>, 2012.
- Eisenman, I. and Wettlaufer, J. S.: Nonlinear threshold behavior during the loss of Arctic sea ice, *Proc. Natl. Acad. Sci. USA*, 106, 28–32, <https://doi.org/10.1073/pnas.0806887106>, 2009.
- Flocco, D., Feltham, D. L., and Turner, A. K.: Incorporation of a physically based melt pond scheme into the sea ice component of a climate model, *J. Geophys. Res.-Oceans*, 115, C08012, <https://doi.org/10.1029/2009JC005568>, 2010.
- 595 Guckenheimer, J. and Holmes, P.: *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, Springer, New York, USA, <https://doi.org/10.1007/978-1-4612-1140-2>, 1983.
- Hersbach, H., Bell, B., Berrisford, P., Hirahara, S., Horányi, A., Muñoz-Sabater, J., Nicolas, J., Peubey, C., Radu, R., Schepers, D., Simmons, A., Soci, C., Abdalla, S., Abellan, X., Balsamo, G., Bechtold, P., Biavati, G., Bidlot, J., Bonavita, M., De Chiara, G., Dahlgren, P., Dee, D., Diamantakis, M., Dragani, R., Flemming, J., Forbes, R., Fuentes, M., Geer, A., Haimberger, L., Healy, S., Hogan, R. J., Hólm, E., Janisková, M., Keeley, S., Laloyaux, P., Lopez, P., Lupu, C., Radnoti, G., de Rosnay, P., Rozum, I., Vamborg, F., Villaume, S., and Thépaut, J.-N.: The ERA5 global reanalysis, *Q. J. Roy. Meteor. Soc.*, 146, 1999–2049, <https://doi.org/10.1002/qj.3803>, 2020.
- Hill, K., Abbot, D. S., and Silber, M.: Analysis of an Arctic sea ice loss model in the limit of a discontinuous albedo, *SIAM J. Appl. Dyn. Syst.*, 15, 1163–1192, <https://doi.org/10.1137/15M1037718>, 2016.
- 605 Hirahara, S., Balmaseda, M. A., de Boisseson, E., and Hersbach, H.: Sea Surface Temperature and Sea Ice Concentration for ERA5, ERA Report Series, 26, ECMWF, <https://www.ecmwf.int/en/elibrary/79624-sea-surface-temperature-and-sea-ice-concentration-era5>, 2016.
- Horvat, C. and Tziperman, E.: A prognostic model of the sea ice floe size and thickness distribution, *The Cryosphere*, 9, 2119–2134, <https://doi.org/10.5194/tc-9-2119-2015>, 2015.
- Ising, E.: Beitrag zur Theorie des Ferromagnetismus, *Z. Phys.*, 31, 253–258, <https://doi.org/10.1007/BF02980577>, 1925.
- 610 Kern, S., Rösel, A., Pedersen, L. T., Ivanova, N., Saldo, R., and Tonboe, R. T.: The impact of melt ponds on summertime microwave brightness temperatures and sea ice concentrations, *The Cryosphere*, 10, 2217–2239, <https://doi.org/10.5194/tc-10-2217-2016>, 2016.
- Kondrashov, D., Sudakow, I., Livina, V., and Yang, Q.: Accurate and robust real-time prediction of September Arctic sea ice, *Chaos*, 36, 023110, <https://doi.org/10.1063/5.0295634>, 2026.
- Lüthje, M., Feltham, D. L., Taylor, P. D., and Worster, M. G.: Modeling the summertime evolution of sea-ice melt ponds, *J. Geophys. Res.-Oceans*, 111, C02001, <https://doi.org/10.1029/2004JC002818>, 2006.
- 615



- Ma, Y.-P., Sudakov, I., Strong, C., and Golden, K. M.: Ising model for melt ponds on Arctic sea ice, *New J. Phys.*, 21, 063029, <https://doi.org/10.1088/1367-2630/ab26db>, 2019.
- Ma, Y.-P., Sudakow, I., Krapivsky, P. L., and Vakulenko, S. A.: Dynamics of feedback Ising model, *arXiv [preprint]*, <https://doi.org/10.48550/arXiv.2510.07301>, 2025; version 2, 30 March 2026, <https://arxiv.org/abs/2510.07301v2>.
- 620 Ma, Y.-P., Sudakow, I., and Krapivsky, P. L.: Mixed phases in feedback Ising models, *Phys. Rev. Lett.*, 137, 027101, <https://doi.org/10.1103/3ck1-gzzf>, 2026.
- Meier, W. N., Fetterer, F., Windnagel, A. K., Stewart, J. S., and Stafford, T.: NOAA/NSIDC Climate Data Record of Passive Microwave Sea Ice Concentration, Version 6, NSIDC [data set], <https://doi.org/10.7265/b18j-z797>, 2026; data accessed 29 August 2026.
- Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N., Teller, A. H., and Teller, E.: Equation of state calculations by fast computing machines, 625 *J. Chem. Phys.*, 21, 1087–1092, <https://doi.org/10.1063/1.1699114>, 1953.
- Moon, W. and Wettlaufer, J. S.: A stochastic dynamical model of Arctic sea ice, *J. Climate*, 30, 5119–5140, <https://doi.org/10.1175/JCLI-D-16-0223.1>, 2017.
- Müller-Stoffels, M. and Wackerbauer, R.: Albedo parametrization and reversibility of sea ice decay, *Nonlin. Processes Geophys.*, 19, 81–94, <https://doi.org/10.5194/npg-19-81-2012>, 2012.
- 630 Onsager, L.: Crystal statistics. I. A two-dimensional model with an order-disorder transition, *Phys. Rev.*, 65, 117–149, <https://doi.org/10.1103/PhysRev.65.117>, 1944.
- Popović, P., Cael, B. B., Silber, M., and Abbot, D. S.: Simple rules govern the patterns of Arctic sea ice melt ponds, *Phys. Rev. Lett.*, 120, 148701, <https://doi.org/10.1103/PhysRevLett.120.148701>, 2018.
- Popović, P., Silber, M., and Abbot, D. S.: Critical percolation threshold restricts late-summer Arctic sea ice melt pond coverage, *J. Geophys. Res.-Oceans*, 125, e2019JC016029, <https://doi.org/10.1029/2019JC016029>, 2020.
- 635 Rikvold, P. A., Tomita, H., Miyashita, S., and Sides, S. W.: Metastable lifetimes in a kinetic Ising model: Dependence on field and system size, *Phys. Rev. E*, 49, 5080–5090, <https://doi.org/10.1103/PhysRevE.49.5080>, 1994.
- Rousset, C., Vancoppenolle, M., Madec, G., Fichefet, T., Flavoni, S., Barthélemy, A., Benshila, R., Chanut, J., Levy, C., Masson, S., and Vivier, F.: The Louvain-La-Neuve sea ice model LIM3.6: global and regional capabilities, *Geosci. Model Dev.*, 8, 2991–3005, 640 <https://doi.org/10.5194/gmd-8-2991-2015>, 2015.
- Scott, F. and Feltham, D. L.: A model of the three-dimensional evolution of Arctic melt ponds on first-year and multiyear sea ice, *J. Geophys. Res.-Oceans*, 115, C12064, <https://doi.org/10.1029/2010JC006156>, 2010.
- Sellers, W. D.: A global climatic model based on the energy balance of the Earth–atmosphere system, *J. Appl. Meteorol.*, 8, 392–400, [https://doi.org/10.1175/1520-0450\(1969\)008<0392:AGCMBO>2.0.CO;2](https://doi.org/10.1175/1520-0450(1969)008<0392:AGCMBO>2.0.CO;2), 1969.
- 645 Spirin, V., Krapivsky, P. L., and Redner, S.: Fate of zero-temperature Ising ferromagnets, *Phys. Rev. E*, 63, 036118, <https://doi.org/10.1103/PhysRevE.63.036118>, 2001.
- Sudakov, I., Vakulenko, S. A., and Golden, K. M.: Arctic melt ponds and bifurcations in the climate system, *Commun. Nonlinear Sci. Numer. Simul.*, 22, 70–81, <https://doi.org/10.1016/j.cnsns.2014.09.003>, 2015.
- Sudakow, I., Vakulenko, S. A., and Kondrashov, D.: Reproducibility archive for Nonlinear feedback and memory in an Ising model of sea 650 ice, R6.19, Zenodo [code and data set], <https://doi.org/10.5281/zenodo.23147089>, 2026.
- Thoms, S., Kutschan, B., and Morawetz, K.: Phase-field theory of brine entrapment in sea ice: short-time frozen microstructures, *arXiv [preprint]*, <https://doi.org/10.48550/arXiv.1405.0304>, 2014.
- Toppaladoddi, S., Moon, W., and Wettlaufer, J. S.: Seasonal evolution of the Arctic sea ice thickness distribution, *J. Geophys. Res.-Oceans*, 128, e2022JC019540, <https://doi.org/10.1029/2022JC019540>, 2023.
- 655 Uhlířková, T., Vihma, T., Karpechko, A. Y., and Uotila, P.: Effects of Arctic sea ice concentration on turbulent surface fluxes in four atmospheric reanalyses, *The Cryosphere*, 18, 957–976, <https://doi.org/10.5194/tc-18-957-2024>, 2024.
- Wagner, T. J. W. and Eisenman, I.: How climate model complexity influences sea ice stability, *J. Climate*, 28, 3998–4014, <https://doi.org/10.1175/JCLI-D-14-00654.1>, 2015.
- Wang, E.: A study on Arctic sea ice dynamics using the continuous spin Ising model, *J. Appl. Phys.*, 135, 194901, 660 <https://doi.org/10.1063/5.0202612>, 2024.