

S1 Realising the conceptual model

S1.1 States, parameters and experiments

CONST uses constant coupling, INST instantaneous spatial feedback, and MEM a 14-day relaxation towards the spatial target. Ice is $s = -1$ and water is $s = +1$; positive h favours ice. Each concentration cell contains $b^2 = 9$ spins with occupancy $C_i = (1 - m_i)/2$. The open 180×180 lattice represents a 60×60 crop of the 25-km NSIDC grid, with 32,400 spins. Blocks contain central and boundary sites and rapidly order, leaving persistent interfaces mostly at 25-km concentration-cell boundaries. Tables S1 and S2 give notation and experiments.

Table S1: Notation. Global mean-field variables and local lattice fields use distinct symbols.

Symbol	Meaning
u, Φ, κ	Continuum phase variable, local potential and conditional interfacial stiffness.
s_p, m_i, M	Spin, mean spin in cell i , and mean spin over the lattice.
$C_i, F_{w,R(i)}$	Cell ice occupancy and the neighbourhood average of open water.
$J_i, J_{*,i}, \tilde{J}_i$	Applied cell coupling, its instantaneous target and its preliminary memory update.
J_{pq}, d_0, g	Bond coupling, reference mean coupling and spatial feedback gain.
W_R, A	Row-normalised averaging matrix and its mean-centred form.
$Q_i, \mathcal{B}$	Cell incidence sum of spin products and global bond-product sum.
$J_{\text{geom}}, J_{\text{eff}}^{(\gamma)}$	Exact front-equivalent coefficient and exponential barrier proxy.
h_i, h'_i	Local phase-preference field and the field-substitution control.
K, Λ, H, k	Global scaled reference coupling, feedback, field and relaxing coupling.
a, c, Γ	Derivatives of the phase response with respect to state and coupling, and Metropolis mobility.
ℓ_q, M_q	Fourier coefficient of the contrast response and its coupled update matrix.
E_i, Θ_i	Reduced thermal reservoir and filtered skin temperature.
$\tau_E, \tau_T, \tau_J, \rho$	Reservoir and temperature parameters, coupling relaxation time and e^{-1/τ_J} .
n_p, z_p	Ice-neighbour count and coordination of a proposed site; distinct from the homogeneous coupling k .
$\Delta t_{\text{kin}}, \beta$	Attempted reversals per spin per day and inverse energy scale.

Each day, coupling and field are evaluated before 48,468 random sequential proposals with $p_{\text{acc}} = \min(1, e^{-\beta \Delta \mathcal{H}})$. Only existing neighbours count: $z_p = 4$ in the interior, 3 on an edge and 2 at a crop corner. Diagnostic seeds are 220001+997 r , $r = 0, \dots, 7$. From 16 June to 30 September, 107 daily states span 106 updates. Initial counts use stochastic rounding and random placement, preserving observed concentration in expectation. Members share parameters and forcing; spread describes configuration and update randomness conditional on these inputs.

Table S3 lists the ten active backbone parameters. The background coupling and kinetic/temperature scales descend from a fit to 2012, 2019 and 2020 concentration. The 2022 selection then compared three radiation–reservoir architectures using the mean absolute error (MAE) of an eight-member ensemble mean at seven post-initialisation dates, retaining closed short-wave absorption, prescribed net long-wave flux and symmetric reservoir leakage. Only the combined local-field offset a_0 enters the seasonal updates; the stored emissivity is inactive because net long-wave flux is prescribed.

Spatial development used 2022 and 2023. Radius-scan MAE was smallest at $R = 18$ in 2022 and $R = 12$ in 2023; $R = 12$ was retained. Gain and memory time are additional settings, with $\beta = 1$ fixing the energy convention. The retained thermal and spatial parameters precede the 2024 runs; the 2024 short-memory comparisons are subsequent sensitivities. The attempted-reversal time is 0.67 d, the temperature filter’s effective e-folding is 3.53 d, and the illustrated coupling and reservoir relax over 14

Table S2: Experiment inventory. All seasonal cases use the same thermal equations and initialisation unless stated. “Own” means calculated from that trajectory’s state. Evolving albedo uses its current occupancy; fixed albedo uses the initial observed field. Matched channel diagnostics use the first eight common seeds.

Case	Coupling	Field/forcing	Albedo	Members	Seasons
CONST	d_0	Own / ERA5	Evolving	64	2022–24
INST	Own spatial target	Own / ERA5	Evolving	64	2022–24
MEM	Relaxing own target	Own / ERA5	Evolving	64	2022–24
SCHED	Prescribed calendar map	Own / ERA5	Evolving	64	2022–24
O	Target driven by observed cover	Own / ERA5	Evolving	64	2022–24
FIELD	d_0	$h' = h + 2(J_* - d_0)$	Evolving	8	2024
Held / proposal	Own target, each proposal	Own / ERA5	Evolving	8	2024
State energy	Full own-state energy	Own / ERA5	Evolving	8	2024
CONST, fixed albedo	d_0	Own / ERA5	Fixed	8	2024
INST, fixed albedo	Own target	Own / ERA5	Fixed	8	2024
Uniform CONST/INST	As CONST/INST	Daily uniform ERA5	Evolving	8 each	2024
Radius scan	INST/FIELD, $R = 2, 6$	As above	Evolving	8 each	2024
Field replay	Same visited INST state	Candidate h'	Held state	8	2024
Reduced-gain INST	Own target, $g' = 0.52156g$	Own / ERA5	Evolving	8	2024
Uniformly weakened	$J_w = 2.35805$	Own / ERA5	Evolving	8	2024
Activation replay	Same visited INST state	Candidate $\beta = 0.5, 1, 2$	Held state	8	2024
Edge distance (archive)	Centred signed distance	Own / ERA5	Evolving	64	2022–24

and 84 d. SCHED uses a development-derived calendar map; O uses observed cover.

S1.2 Averaging, positivity and the geographical domain

For a cell water fraction $F_w = 1 - C$, the window operator gives $F_{w,R} = W_R F_w$. The square window is truncated and normalised at open boundaries. Both the crop mean and the truncated averaging windows depend on the chosen domain. At $R = 12$ it contains 625 cells in the interior, 169 at a corner, and 501.76 on average. Its nominal central width is 625 km. It summarises regional differences in open-water exposure and pack connectedness. The radius scan compares nominal widths of 125, 325 and 625 km; no individual wave or floe length is identified with this averaging scale. The difference $\overline{F_{w,R}} - \overline{F_w}$ ranges from -0.03786 to -0.00217 across the eight 2024 principal paths and update states.

The linear target $J_* = d_0 + g(\overline{F_{w,R}} - F_{w,R})$ can become negative for sufficiently large contrast. The value $g/d_0 = 10/9$ puts its sign change at contrast 0.9; 0.9 is not a general upper bound on admissible contrast. We retain this development setting and ensure positive coupling explicitly. If any preliminary value is below $\varepsilon = 10^{-6}d_0$, set

$$\eta_i = \max(\tilde{J}_i - \varepsilon, 0), \quad J_i = \varepsilon + (d_0 - \varepsilon)\eta_i/\bar{\eta}. \quad (\text{S1})$$

Here $\bar{\eta} = N_c^{-1} \sum_i \eta_i$. For INST, replace $\tilde{J}_i$ by $J_{*,i}$ in this correction; for MEM, apply it after the memory update. The corrected cell couplings satisfy $J_i \geq \varepsilon$ and $N_c^{-1} \sum_i J_i = d_0$. If all preliminary values already satisfy the lower bound, no correction is applied. It is inactive in 2024 and in the displayed 14-day memory runs. It is rarely invoked in 2022–2023; the retained principal anchor fields are unchanged. The minimum 2024 INST coupling is 0.432669 on the eight diagnostic paths and 0.236788 across all 64 members. The archive records every correction activation and the exact comparison with the retained anchor fields.

Table S3: Parameters of the seasonal realisation. The ten fitted backbone parameters are effective quantities retained from the calibration stages in Sect. S1.1. Coupling settings, conventions, block structure and the derived proposal count are identified separately. Values are rounded; full values and seeds are archived.

Parameter	Value	Status	Meaning
d_0	5.75197	Fitted	Reference mean coupling
a_0	-1.62989	Fitted	Combined local-field offset
b_T	2.06653	Fitted	Temperature coefficient
τ_T	4.05304	Fitted	Temperature recursion coefficient
Δt_{kin}	1.49594	Fitted	Attempts per spin per day
α_w	0.0651453	Fitted	Water albedo
α_{ice}	0.729866	Fitted	Ice albedo
τ_E	84.0081	Fitted	Reservoir relaxation time (days)
c_E	0.412344	Fitted	Reservoir coefficient
λ_L	9.92370	Fitted	Phase-change reservoir term
g	6.39108	Setting	Feedback gain, $10d_0/9$
R	12	Setting	Window half-width in cells
τ_J	14	Setting	Coupling response time (days)
Q_0	100	Convention	Flux scale (W m^{-2})
β	1	Convention	Inverse energy scale
b	3	Structure	Microspin block side
Proposals/day	48,468	Derived	Rounded number of reversals tried

The crop lies in the Pacific Arctic, approximately $69.24\text{--}86.65^\circ\text{N}$, crossing the dateline from 150.16°E to 114.23°W . Figure S1 locates it; Fig. S2 shows the observed and model fields with geographical graticules. Projected distances use the native 25-km grid.

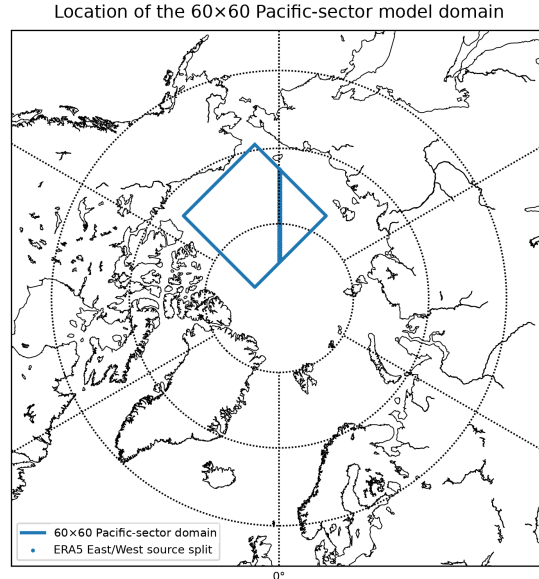


Figure S1: Location of the fixed Pacific-sector crop. The rectangle marks the concentration grid used in all seasonal experiments.

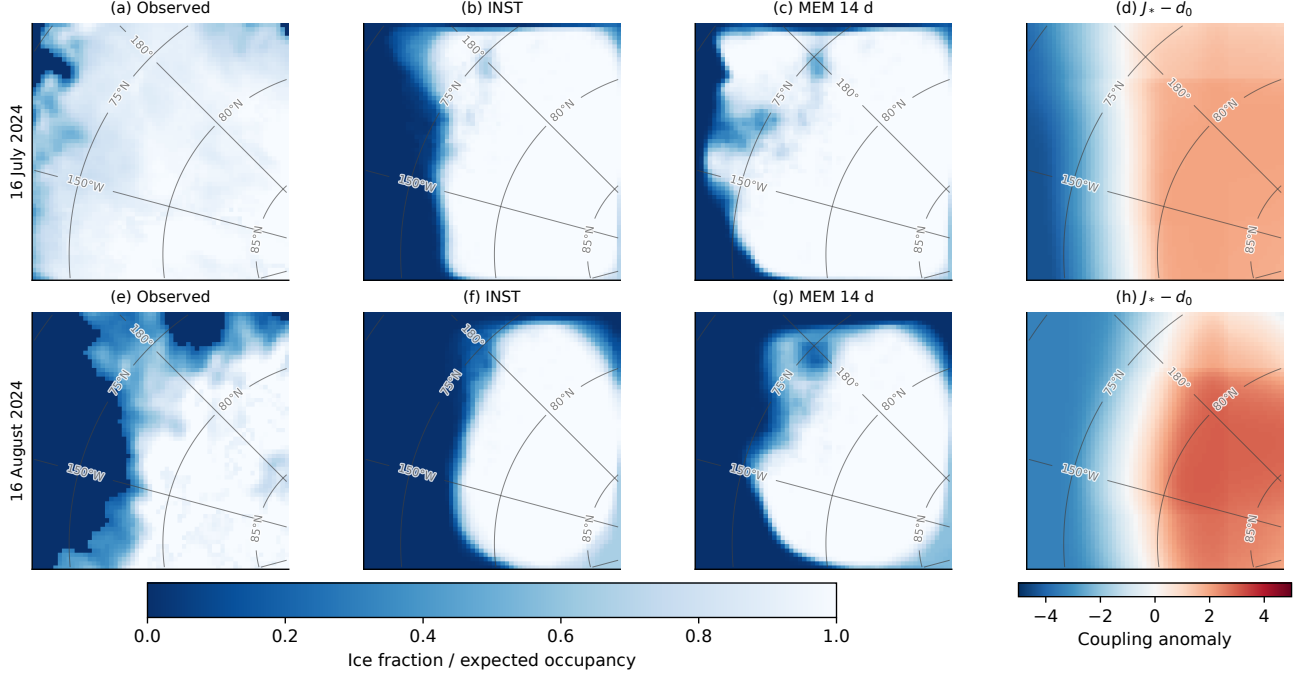


Figure S2: Spatial illustration on 16 July and 16 August 2024. Observed concentration is compared with ensemble-mean occupancy from 64 members each of INST and MEM. The final column is the linear instantaneous coupling-target anomaly $J_* - d_0$ computed from ensemble-mean INST occupancy. Meridians and parallels use the archived cell coordinates on the native polar stereographic grid. Both dates precede the September 2024 interpolation in the NSIDC product (Sect. S4.1).

S2 Energy and memory calculations

The global-feedback stability calculation in Sect. S2.2 and the mean-centred spatial calculation in Sect. S2.3 use distinct coupling laws (main-text Sects. 3–4).

S2.1 Global and spatial reversal energies

For the global law $J(M) = J_0 + J_1 M$ in main-text Eqs. (9)–(10), N is the number of spins and $S_p = \sum_{q \sim p} s_q$.

For global feedback, a reversal has $\delta M = -2s_p/N$, $\delta J = -2J_1 s_p/N$ and $\delta \mathcal{B} = -2s_p S_p$. Substitution into $-[J + \delta J][\mathcal{B} + \delta \mathcal{B}] + J\mathcal{B}$ gives the main-text global increment. Its extensive-bond term is exactly a field shift $-J_1 \mathcal{B}/N$; the remaining $-4J_1 S_p/N$ is a finite-size correction. The differentiated global correction and the 3/2 mean-field coefficient belong to the prior feedback-Ising theory (Ma et al., 2025).

For the spatial law, N_c is the number of concentration cells and $c(p)$ labels the cell containing spin p (main-text Eq. (17)). Put $u_j = N_c^{-1} \sum_i (W_R)_{ij}$ and $A = W_R - \mathbf{1}u^\top$. Both $A\mathbf{1} = 0$ and $\mathbf{1}^\top A = 0$. For

$$Q_i = \frac{1}{2} \sum_{p:c(p)=i} \sum_{q \sim p} s_p s_q, \quad \mathcal{H}_J = -\mathbf{J}(\mathbf{m}) \cdot \mathbf{Q}, \quad (\text{S2})$$

a reversal in cell i changes m_i by $-2s_p/b^2$. Under the linear target,

$$\delta J_j = \frac{g s_p}{b^2} A_{ji}, \quad \delta Q_j = -s_p \sum_{q \sim p} s_q [\mathbf{1}_{j=i} + \mathbf{1}_{j=c(q)}]. \quad (\text{S3})$$

Expanding the product gives

$$\Delta\mathcal{H}_{\text{state}} = \Delta\mathcal{H}_{\text{held}} - \delta\mathbf{J} \cdot \mathbf{Q} - \delta\mathbf{J} \cdot \delta\mathbf{Q}. \quad (\text{S4})$$

The leading correction $-gs_p(A^\top \mathbf{Q})_i/b^2$ vanishes when all cell bond sums Q_i are equal, since $A^\top \mathbf{1} = 0$. It generally survives at interfaces. On an open lattice, missing boundary bonds can make Q_i vary between cells even when all spins are aligned. With the positivity correction, Eq. (S4) remains exact when $\delta\mathbf{J}$ is the difference between the corrected couplings; the linear expression applies only if neither state requires this correction.

Direct finite-energy recomputation verifies 720 reversals (archived verification records). At fixed external field, full state-energy Metropolis rates satisfy detailed balance with this energy. The held state-dependent rule need not: a four-step closed-cycle test in the archive has non-zero affinity under the held rule and vanishing affinity under the exact rule. Affinity is the log ratio of forward and reverse transition-rate products around the cycle.

S2.2 Stability on both homogeneous branches

We use the homogeneous system and Metropolis mobility in main-text Eq. (13), with $\ell = km - H$ and $\Gamma_* = \Gamma(\ell_*)$. Time and τ_J are measured in attempted reversals per spin. The numerical normal-form calculation uses the water-rich point in main-text Fig. 2.

For $\dot{m} = \Gamma(km - H)[\tanh(km - H) - m]$ and $\tau_J \dot{k} = K + \Lambda m - k$, stationarity gives $k_* = K + \Lambda m_*$. The derivative of Γ multiplies a vanishing residual at equilibrium, so

$$a = \Gamma_*[(1 - m_*^2)k_* - 1], \quad c = \Gamma_*(1 - m_*^2)m_*. \quad (\text{S5})$$

The trace and determinant in main-text Eq. (15) imply a complete local classification. If $a + c\Lambda > 0$, the equilibrium is a saddle for all $\tau_J > 0$. If $a + c\Lambda < 0$ and $a \leq 0$, it is stable for all finite τ_J . If $a + c\Lambda < 0$ and $a > 0$, it is stable for $\tau_J < 1/a$ and unstable for $\tau_J > 1/a$. At $\tau_H = 1/a$ the frequency is $\sqrt{-a(a + c\Lambda)}$, and the trace derivative with respect to τ_J is $1/\tau_J^2 > 0$. Thus the imaginary-axis crossing is transverse. The eigenvalue discriminant is

$$(a - 1/\tau_J)^2 + 4(a + c\Lambda)/\tau_J. \quad (\text{S6})$$

Its sign separates real from complex eigenvalues. The mobility has a cusp at $\ell = 0$, but the water-rich Hopf point has $\ell_* = \text{atanh}(0.4) = 0.42365$. Its local derivatives are therefore well defined. Here $x = (m - m_*, k - k_*)^\top$ and F is the translated vector field. Its derivatives $L = DF(0)$, $B = D^2F(0)$ and $C = D^3F(0)$ give the linear, quadratic and cubic responses; B and C are bilinear and trilinear forms, and $\langle p, q \rangle = \bar{p}^\top q$. We expand $F(x) = Lx + B(x, x)/2 + C(x, x, x)/6 + \dots$ there. Let $Lq = i\omega q$, $L^\top p = -i\omega p$, $\|q\|_2 = 1$, $q_1 > 0$ real, and $\langle p, q \rangle = 1$. The cubic calculation uses

$$G_{21} = \langle p, C(q, q, \bar{q}) - 2B(q, L^{-1}B(q, \bar{q})) + B(\bar{q}, (2i\omega I - L)^{-1}B(q, q)) \rangle. \quad (\text{S7})$$

The first Lyapunov coefficient is $l_1 = \text{Re } G_{21}/(2\omega) = 47.15362 > 0$, giving a subcritical Hopf with an unstable small-amplitude cycle. In real coordinates $x = [\text{Re } q, -\text{Im } q]^\top(u, v)^\top$, the radial normal form has cubic coefficient $a_r = \omega l_1/4 = 0.94288$. Independent derivatives and small-radius Poincaré returns confirm the positive coefficient. At $0.99\tau_H$, reverse-time convergence recovers the unstable orbit with period 78.5242 kinetic units. The positive divergence integral gives multiplier 1.02885, so a transverse perturbation grows after one orbit.

The archived eight-member 64×64 lattice cycles use $K = 3.2$, $\Lambda = -1$ and 50 sweeps per field. On descent, the ensemble mean changes sign between $H = -1.4, -1.3$ for state-energy updating and between $-0.9, -0.8$ for held updating. These finite-rate switching intervals are not thermodynamic folds or seasonal thresholds.

S2.3 The coupled spatial map

For a periodic one-spin mean-field reduction, let $q = (q_x, q_y)$ be the lattice wavevector and define $a_q = 2 \cos q_x + 2 \cos q_y$ and

$$\widehat{W}_R(q) = \prod_{\nu=x,y} \frac{\sin[(2R+1)q_\nu/2]}{(2R+1)\sin(q_\nu/2)}, \quad \ell_q = -\frac{g}{2}\widehat{W}_R(q) \ (q \neq 0), \quad \ell_0 = 0. \quad (\text{S8})$$

The limits define zero arguments. For a uniform equilibrium with $\chi = 1 - m_0^2$, linearising the symmetric incident bonds gives

$$a = \Gamma_*[-1 + \beta\chi d_0 a_q], \quad c = \Gamma_*\beta\chi m_0(4 + a_q)/2. \quad (\text{S9})$$

Here Δt is one daily update in kinetic units and τ_J is measured in days. Let $e = e^{a\Delta t}$, $f = c(e^{a\Delta t} - 1)/a$, with $f = c\Delta t$ at $a = 0$, and $\rho = e^{-1/\tau_J}$. We take the state of the map to be the current spin perturbation and the previous held coupling, $(\delta m_n, \delta J_{n-1})$. Updating the target first and then integrating the held spin equation over Δt gives

$$\begin{pmatrix} \delta m_{n+1} \\ \delta J_n \end{pmatrix} = M_q \begin{pmatrix} \delta m_n \\ \delta J_{n-1} \end{pmatrix}, \quad M_q = \begin{pmatrix} e + f(1-\rho)\ell_q & f\rho \\ (1-\rho)\ell_q & \rho \end{pmatrix}. \quad (\text{S10})$$

Its trace T_q and determinant D_q are

$$T_q = e + \rho + f(1-\rho)\ell_q, \quad D_q = \rho e. \quad (\text{S11})$$

The multipliers give the amplification of a perturbation over one daily update. Both lie inside the unit circle, giving decay, precisely when

$$1 - D_q > 0, \quad 1 - T_q + D_q > 0, \quad 1 + T_q + D_q > 0. \quad (\text{S12})$$

The stationary boundary is especially informative:

$$1 - T_q + D_q = (1 - \rho)(1 - e - f\ell_q). \quad (\text{S13})$$

For every finite τ_J , the zero of this expression is $a + c\ell_q = 0$, interpreted by continuity at $a = 0$. Memory therefore leaves this stationary mode boundary unchanged while altering the multipliers. At $q = 0$, mean preservation restricts $\delta J_0 = 0$, so the admissible multiplier is $e^{a\Delta t}$ evaluated at $q = 0$; the additional filter eigenvalue in the unrestricted matrix is not a physical uniform-coupling perturbation. A unit-modulus complex pair also requires $D_q = 1$ and $|T_q| < 2$; a multiplier -1 is governed by the third inequality. For all 6262 matrices (two branches, 31 axis modes, 101 response times), direct eigenvalues and stability inequalities give the same classification. These conditions address the coupled feedback, not just the memory filter.

The uniform zero-field reference satisfies $m_0 = \tanh(4\beta d_0 m_0)$ on an ordered branch. At seasonal $4\beta d_0 = 23.0079$, $1 - m_0^2 \simeq 4.15 \times 10^{-20}$. For the admissible uniform mode, the daily spin multiplier is $e^{a\Delta t} = 0.2240$; the separate 14-day coupling filter has multiplier $\rho = e^{-1/14} = 0.9311$.

Figure S3a,b illustrates the long-wavelength modes near ordering. We choose $m_0 = \pm 0.4$, $h = 0$, $d_0 = \text{atanh}(0.4)/(4 \times 0.4) = 0.26478$ and $g = 10d_0/9$. The ice-rich branch admits a growing non-uniform contrast, whereas the water-rich modes decay.

To define the gain control in Fig. S3c,d, reconstruct the applied 14-day MEM coupling and its instantaneous target on the first eight retained memory paths. Over the first 30 updates set

$$\frac{g'}{g} = \left[\frac{\sum_{r,n,i} (J_{\text{MEM},r,i}^n - d_0)^2}{\sum_{r,n,i} (J_{*,r,i}^n - d_0)^2} \right]^{1/2} = 0.5215643. \quad (\text{S14})$$

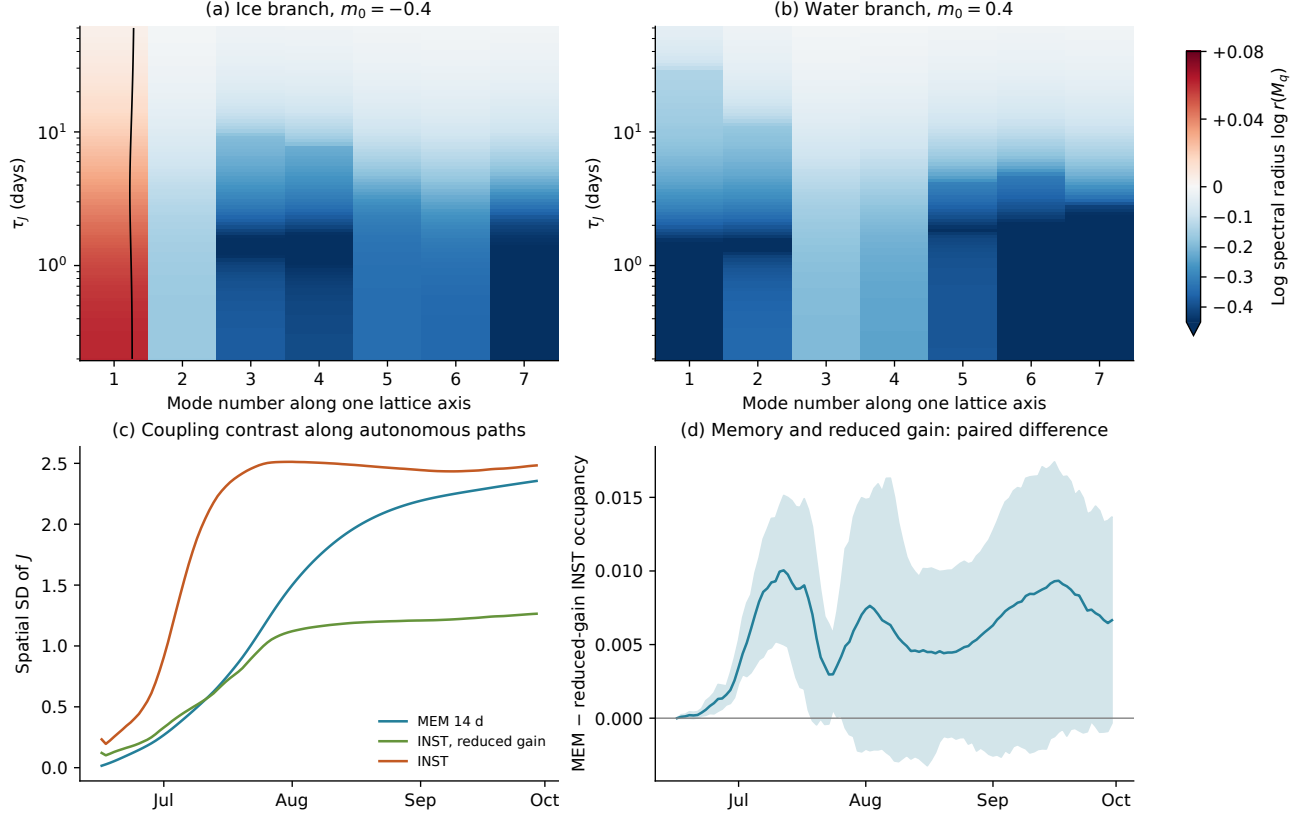


Figure S3: Memory in the coupled model. (a,b) Natural log of the spectral radius (largest multiplier magnitude) for the first seven non-uniform axis modes of a periodic 60-site one-spin reduction near ordering, at $m_0 = \mp 0.4$, $R = 12$, and $\Delta t = 1.49594$ kinetic units per day. Positive values denote growth; the black curve interpolates unit radius between displayed modes. Values below -0.45 share the darkest blue, marked by the colour-bar extension. The uniform mode is excluded. These map parameters are near ordering; the seasonal reference has $K \simeq 23$. (c) Mean across eight members of each member’s spatial coupling standard deviation (SD), along autonomous 2024 paths. (d) Paired occupancy difference between MEM and reduced-gain INST, with 10th–90th member spread.

The sum covers members r , dates 16 June–15 July and all cells. This matches the early coupling amplitudes on the retained MEM paths; the reduced-gain run then evolves from its own state. No fit to observed concentration or to the final channel counts enters the definition. The integrated counts are similar even though the late-season coupling amplitudes differ. This comparison separates the coupling history from its accumulated effect on the occupied channels.

S3 Physical controls and transition pathways

A same-state comparison asks how the acceptance decision changes at a visited configuration. An autonomous comparison asks how two configurations subsequently evolve. These are different questions. The main channel table uses the first definition; the paired controls use the second. The immediate melt gain is $24,374 - 438 = 23,936$: the interior contribution in main-text Table 1 is reduced by 438 net boundary suppressions (29 enabled and 467 suppressed melts). Together with 3,324 fewer freezes, this gives 27,260 net changed decisions, not an autonomous occupancy change. Table S4 instead gives the complete budget on each model’s own path.

Table S4: Complete autonomous phase-change budget, summed over the same eight 2024 members and 106 update intervals. Interior rows have coordination $z_p = 4$ and n_p ice neighbours. The boundary row sums all $z_p = 2, 3$ classes; the archive gives their separate counts. Each accepted reversal changes the ice count by one. These counts describe each case’s own path, not a same-state substitution.

z_p	n_p	CONST		INST	
		Melts	Freezes	Melts	Freezes
4	0	1,258	0	7,963	0
4	1	6,655	70	16,088	0
4	2	128,725	14,443	100,448	13,557
4	3	9,736	5,909	36,180	10,003
4	4	0	7,552	0	7,545
Boundary		4,955	673	4,281	602
Total		151,329	28,647	164,960	31,707
Net ice loss		122,682		133,253	

Both ensembles start with 247,029 ice spins. CONST ends with 124,347 and INST with 113,776, giving final occupancies 0.47973 and 0.43895. Additional net loss is $13,631 - 3,060 = 10,571$. Exposed-site melts rise from 7,913 to 24,051; corner-plus-front and boundary totals fall by 1,833 and 674. Autonomous freezing rises by 10.7 %, opposite to the 9.49 % same-state reduction.

For a common change of incident coupling, the equivalent field is $\delta h_p = -\sum_q \delta J_{pq} s_q = (2n_p - z_p)\delta J$. The identity holds in either direction. At $n_p = 3$ it matches front melting and notch-filling freezing. Flat-front freezing has $n_p = 1$ and requires $-2\delta J$; balanced $n_p = 2$ sites require no shift. FIELD uses $h' = h + 2(J_* - d_0)$ from its own state. In 2024 it produces 36,775 autonomous front melts; the same-state field replay produces 36,826. Main-text Table 2 gives the paired residuals. They combine the geometry mismatch with subsequent melting, freezing and storage; they do not isolate a corner-only effect.

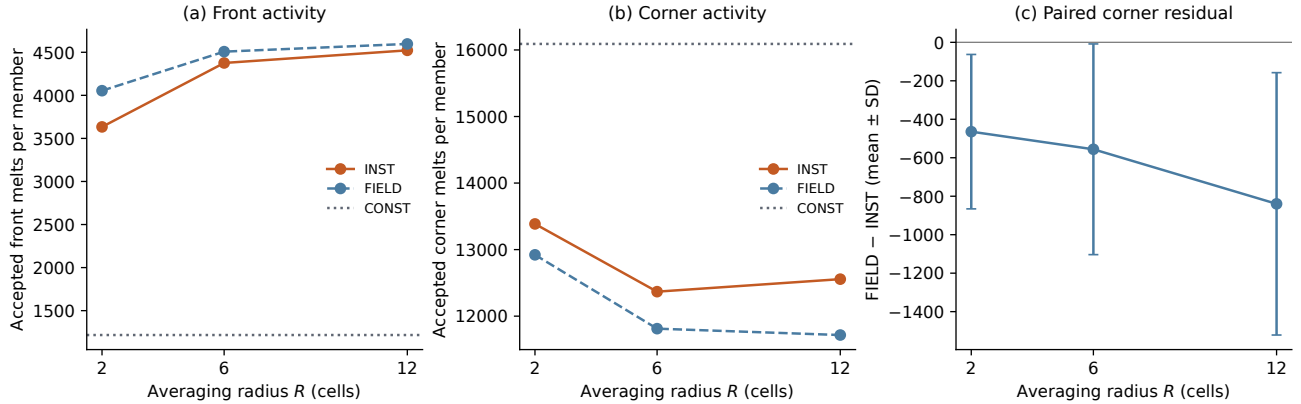


Figure S4: Environmental radius and interface response in eight matched 2024 members. Panels (a,b) show season-total accepted interior melts averaged across eight members. The common CONST reference is unchanged. Error bars in (c) are sample SDs of paired FIELD minus INST corner counts. Smaller radii reduce the aggregate corner residual while increasing the residual in front counts.

The radius scan in Fig. S4 changes only R , leaving all thermal and kinetic parameters unchanged. At $R = 2, 6, 12$, the eight-member INST front totals are 29,068, 35,007 and 36,180; the common CONST total is 9,736. At $R = 2, 6, 12$, FIELD has 3.47 %, 4.50 % and 6.69 % fewer corner melts than INST. Its excess front counts are 11.6 %, 3.00 % and 1.64 % of the corresponding INST totals. Smoothing improves the flat-front approximation but leaves a path-dependent corner difference. The geometric

control uses the Euclidean distance transform of the current $C \geq 0.15$ cell mask: distance inside ice minus distance inside water. It centres this signed distance and scales it to the mean and spatial SD of the instantaneous target calculated from its own state each day. Table S5 reports the 64-member seasonal errors. The surrogate is close to INST in 2023, slightly better on this score, similar to CONST in 2022 and worse than CONST in 2024. These fields do not provide a transition-channel decomposition, so the comparison does not identify the source of their channel differences.

Table S5: Seasonal ensemble-mean MAE on the seven common post-initialisation dates, using 64 members. The edge-distance target follows each realisation’s current ice mask; no channel counts were retained for this control.

Season	CONST	INST	Edge distance
2022	0.1913	0.1497	0.1927
2023	0.3075	0.2397	0.2359
2024	0.1633	0.1553	0.1834

The uniform-forcing control takes the spatial mean of each ERA5 temperature and flux field on each date, including the history from 1 May. It retains the observed concentration history used for reservoir initialisation, the 16 June initial spin allocation, and local state-dependent albedo thereafter. It therefore removes local ERA5 contrasts without removing the initial ice geometry or its subsequent feedback on heating. Both front enhancement and reduced corner counts have the same sign in all eight pairs. Main-text Table 2 gives the paired effects of these and the other controls.

The exact-energy comparison uses the same proposal cadence in both members of each pair. Front counts are 193 ± 111 larger per member and individual minima 0.0166 ± 0.0083 lower under state-energy dynamics. The eight-member minima of the ensemble means are 0.4171 (held/proposal) and 0.4004 (state energy). Daily INST gives 0.4188 for eight members and 0.4269 for 64. A mean of individual minima is not the minimum of an ensemble mean.

With albedo fixed at its initial observed field, INST produces 41,895 front melts versus 15,694 for CONST. Final occupancy is lower in every pair; the individual minimum is lower in six of eight. The early ordering of mixed cells matters here. Where a cell becomes more ice-rich than its initial state, fixed albedo is lower than evolving albedo. The mean absorbed short-wave flux nevertheless decreases from 48.812 to 35.938 W m^{-2} for CONST and from 56.053 to 35.938 W m^{-2} for INST. The corresponding ice-weighted reservoir increases from 14.232 to 14.618 and from 13.870 to 14.244. These means average 106 pre-update dates and eight members; the ice-weighted reservoir is $\sum_i C_i E_i / \sum_i C_i$ on each member-day. These budgets reconcile the lower mean absorption with greater melting in this control; they describe the initialisation and local storage of this model.

Mean coupling alone also cannot determine a transition rate. Convexity gives

$$J_{\text{eff}}^{(\gamma)} = -\gamma^{-1} \log \langle e^{-\gamma J} \rangle \leq \langle J \rangle. \quad (\text{S15})$$

The constant $\gamma = 4\beta$ represents the bond contribution to a uniform-bond front barrier, and 8β its bulk counterpart. On eight 2024 INST paths the arithmetic mean is 5.752, while the member-day mean proxies are 2.998 and 2.715. Pooling before taking the logarithm gives 2.358 and 1.683. These are coupling-distribution summaries; actual rates depend jointly on field, geometry and proposal state. Figure S5 shows the distribution and both averaging conventions.

S3.1 Uniform weakening and stochastic activation

The uniformly weakened control uses $J_w = -\log \langle e^{-4J} \rangle / 4 = 2.35804854$. The average pools $8 \times 106 \times 3600$ cell values reconstructed from the exact nine-spin block occupancies. It differs from averaging the member-day proxies, which gives 2.99837490. All thermal coefficients, forcing and random streams are

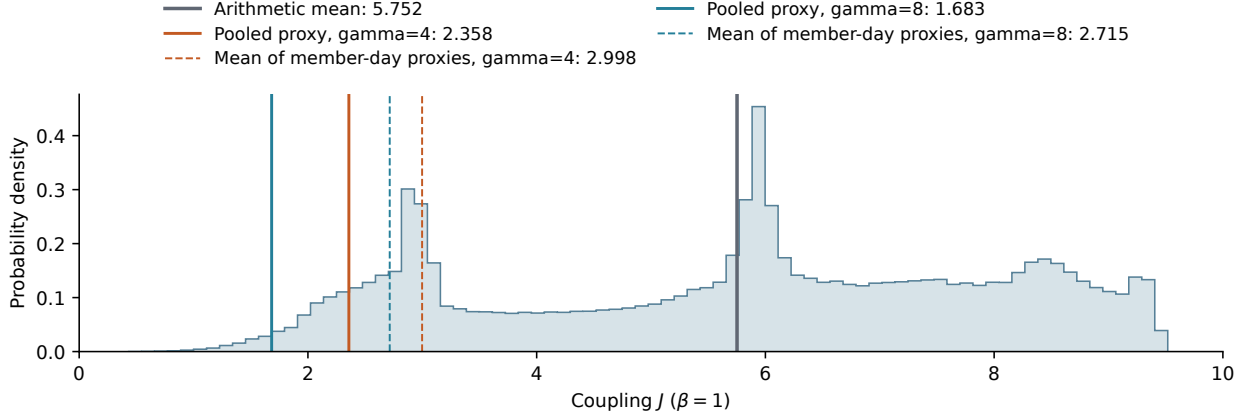


Figure S5: The mean coupling does not fix a barrier-weighted response. The density pools cell couplings on the eight INST paths over 106 daily updates. Solid coloured lines evaluate the exponential proxy after pooling; dashed lines average its separate member-day values. All use the same coupling fields and unit β . These summaries describe coupling contrasts, not complete transition rates.

retained, and the control evolves its own albedo and reservoir. Figure S6 shows its loss of cover and channel counts. Relative to INST, the paired increase in front melts is 7742 ± 119 per member and in bulk melts 478 ± 10 (mean $\pm$ sample SD; all eight differences positive). This matches the coupling factor alone, not its joint distribution with field and geometry. A uniform coupling fitted to final occupancy would answer a different question.

The activation diagnostic replays every proposal of the same eight INST trajectories, changing only the candidate acceptance scale to $\beta = 0.5, 1, 2$. The $\beta = 1$ decision alone advances each path; every daily field matches the archived trajectory exactly. Thus this is a conditional barrier test, not three autonomous seasonal integrations. The realised activated shares of enabled front melts are 53.5, 35.8 and 20.1% (Fig. S6c). The expectation-based values, 53.47, 35.88 and 20.11%, differ by at most 0.084 percentage points. Bulk acceptances are 138, 0 and 0; all 138 cross positive barriers, and 137 are enabled relative to the constant-coupling candidate. The expected counts are 126.90, 0.3424 and 0.000344. Absence of realised bulk melt at $\beta \geq 1$ is therefore not a structural prohibition.

S4 Seasonal forcing and observational context

S4.1 Heat storage and observational support

Daily ERA5 fluxes retain the provider sign convention after conversion to W m^{-2} (Copernicus Climate Change Service, 2023). The reservoir and temperature filter use forcing from 1 May. On that date, $E_i = 0$ and $\Theta_i = T_{s,i}$. The reservoir recursion then uses observed concentration through 16 June and simulated concentration thereafter (main-text Eqs. (29)–(31)).

The offset $a_0 = -1.63$ makes balanced corners favour water at $\Theta = T_m$ and $E = 0$. This fitted bias absorbs processes outside the reduced heat budget; it is not a thermodynamic freezing-point condition. Likewise $\alpha_{\text{ice}} = 0.73$ is an effective fitted albedo, not a prescribed value for melting summer ice. The reservoir stores thermal influence without resolving thickness. Its calibrated occupancy cost, $\lambda_L Q_0 \times 86400 = 8.5741 \times 10^7 \text{ J m}^{-2}$, is equivalent to 0.280 m using 917 kg m^{-3} and 334 kJ kg^{-1} . The 84-day e-folding acts together with signed fluxes and phase change; it does not impose a lower bound on freeze-up time. The weak recovery motivates an explicit autumn heat budget.

Recovery is the increase in domain-mean occupancy from its seasonal minimum to 30 September. It is 0–0.021 in the 64-member principal runs, compared with 0.056–0.169 in the observations.

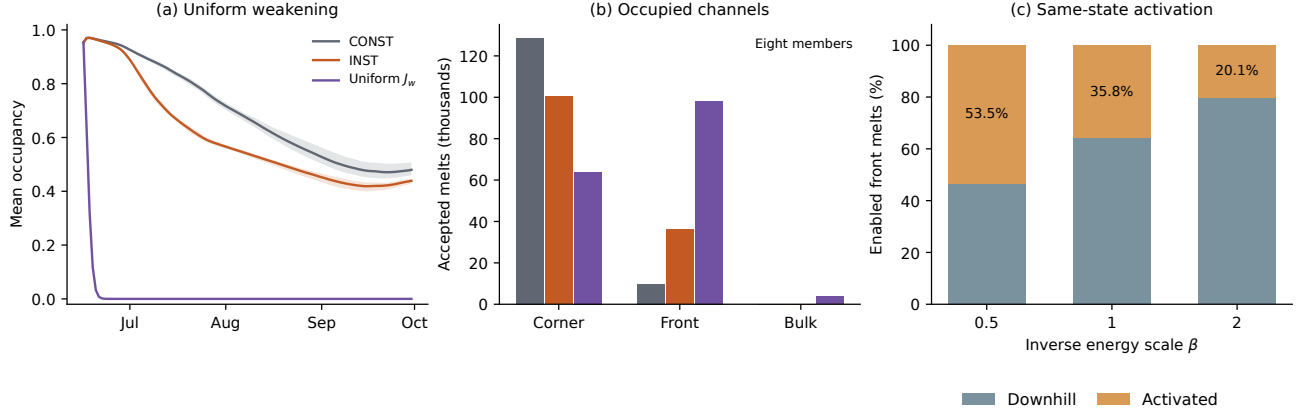


Figure S6: Two tests of the barrier interpretation, using eight matched 2024 paths. (a) Uniform weakening matches the pooled front-coupling proxy but changes the seasonal trajectory; bands span member 10th–90th percentiles. (b) Interior melt counts summed over members distinguish corner, front and bulk activity. (c) Realised downhill and activated shares of enabled front melts on unchanged INST states, fields and proposals. Expected shares agree within 0.1 percentage points. Only the candidate acceptance scale changes.

NOAA/NSIDC G02202 Version 6 interpolates 12–17 September 2024 (Meier et al., 2026); the common six-date comparison omits 16 September. Summer melt ponds and wet-surface changes affect retrieved concentration, with bias not uniformly negative in partially open cells (Kern et al., 2016). This uncertainty is separate from ensemble spread.

ERA5 skin temperature and turbulent fluxes depend on prescribed ice conditions (Hirahara et al., 2016; Uhlíková et al., 2024). On days containing both observed classes, mean water-minus-ice temperature differences are 2.46, 1.93 and 1.90 K in 2022–2024, equivalent to 3.9–5.1 field units. Uniform forcing removes this local pattern but retains regional seasonality. A periodic source grid handles the dateline. The ratio of mean absolute adjacent-cell differences across the dateline join to matched nearby differences reaches 1.0375 when averaged over days. At $R = 12$, the coupling forms a broad gradient, weaker near the marginal ice zone and stronger in the compact pack. Two empirical orthogonal functions (EOFs) carry 96–98 % of its temporal variance. This reflects broad smoothing; neither the EOF concentration nor the seam diagnostic establishes an attractor or zero seam projection.

S4.2 Concentration and memory comparisons

Scores use the same seven post-initialisation dates: 1 and 16 July, 1 and 16 August, and 1, 16 and 30 September. Mean absolute error (MAE) and root-mean-square error (RMSE) compare the ensemble-mean occupancy with observed concentration, pooling equally weighted cell–date pairs. The continuous ranked probability score (CRPS) evaluates the empirical distribution of member occupancies. Brier score is the mean squared difference between the fraction of members with $C \geq 0.15$ and observed ice presence at that threshold; binary disagreement is the fraction of cell–date pairs where the thresholded ensemble mean differs from observed ice presence. Table S6 includes day-of-year climatology from 2012, 2019 and 2020, treated as three members for CRPS. Persistence is deterministic, so its CRPS equals MAE. The prescribed-schedule errors are 0.1575, 0.2067 and 0.1526 in 2022–2024. The observed-cover-driven O diagnostic gives 0.1271, 0.2028 and 0.1342; it uses later observations and is not an autonomous control. The archive also retains the six-date 2024 comparison.

Table S6: Concentration context on the same seven dates in each season. The principal cases use 64 members. Climatology is the three reference years; persistence holds the initial 16 June field. The archive contains these four scores and binary disagreement, all controls and the common six-date 2024 comparison.

Year	Case	MAE	RMSE	CRPS	Brier
2022	CONST	0.1913	0.3651	0.1834	0.1268
2022	INST	0.1497	0.3084	0.1444	0.1338
2022	MEM 14 d	0.1649	0.3279	0.1573	0.1078
2022	Climatology	0.1682	0.2786	0.1300	0.1107
2022	Persistence	0.2941	0.4872	0.2941	0.2183
2023	CONST	0.3075	0.4748	0.2955	0.2096
2023	INST	0.2397	0.4016	0.2304	0.1634
2023	MEM 14 d	0.2813	0.4491	0.2713	0.1899
2023	Climatology	0.1238	0.2109	0.0927	0.0833
2023	Persistence	0.4675	0.6260	0.4675	0.3587
2024	CONST	0.1633	0.2916	0.1409	0.1113
2024	INST	0.1553	0.2915	0.1419	0.1545
2024	MEM 14 d	0.1385	0.2588	0.1227	0.1051
2024	Climatology	0.1592	0.2647	0.1237	0.1212
2024	Persistence	0.3166	0.4924	0.3166	0.2480

Table S7: Ensemble-mean MAE over seven dates. A dash denotes an unevaluated time scale in 2024.

τ_J (d)	2022	2023	2024
Instantaneous	0.14967	0.23967	0.15530
3	0.14844	0.25544	0.14580
7	0.15410	0.26902	0.13999
14	0.16494	0.28133	0.13846
28	0.17532	0.29262	–
60	0.18171	0.29997	–

Development MAE increases across the tested finite memory times. Three days is best in 2022; instantaneous response is best in 2023. In 2024, 14 d has the smallest evaluated MAE, with 3 and 7 d added after inspecting that result. Fourteen days remains an illustrative response time, not a generally optimal or prospectively skill-selected value.

INST improves MAE but worsens Brier score and binary disagreement in 2022 and 2024. The paired 2024 Brier difference is 0.043 with a 95 % paired leave-one-member-out jackknife interval [0.040, 0.047]. On observed support $0.05 < C < 0.95$, the mean of seven date-specific MAEs is 0.251 for CONST and 0.308 for INST. State-energy dynamics gives MAE 0.169 and Brier 0.180 in the common eight-member comparison, above the unfixed-albedo held cases; fixed-albedo INST is higher still.

S4.3 Apparent contour displacement

Crossings at $C = 0.15$ or 0.50 are linearly interpolated between cell centres. Daily displacement averages the two directed mean nearest-contour distances on the projected grid. The crop exterior is excluded; missing contours are not assigned zero. We omit the first two intervals and 2024 intervals ending 12–18 September that touch provider interpolation.

At $C = 0.15$ in 2024, means are 5.02, 7.34 and 5.89 km d^{−1} for CONST, INST and MEM, versus 17.90 observed; medians are 4.18, 4.29 and 4.56, versus 14.10. INST’s larger mean reflects its upper

tail. The per-member INST mean is 6.43, compared with 7.30 for the eight-member ensemble field and 7.34 for 64 members.

The kinetic interval times microspin spacing, $1.50 \times 8.33 \simeq 12.5 \text{ km d}^{-1}$, is a nominal single-layer front scale, not a hard ceiling for sequential or topology-changing contour motion. Observed displacement also includes transport, retrieval variability and connectivity changes. Gaussian smoothing by one cell lowers the observed mean from 17.90 to 16.43; half-cell smoothing gives 17.93. This is sensitivity to spatial scale, not a separation of retrieval noise from physical motion.

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