



The Versatile Atmospheric Laser Doppler Instrument VALDI: A single Fizeau interferometer and imaging detector, for simultaneous wind speed, temperature and scattering ratio measurements from molecular and aerosol scattering

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Abstract. This feasibility study demonstrates, through modelling and simulation, that a Versatile Atmospheric Laser Doppler Instrument (VALDI), based on a single Fizeau interferometer and imaging detector, has the potential for precise, high-sensitivity, simultaneous measurements of wind speeds, temperature and atmospheric scattering ratio (ASR). In simple principle, the linear frequency dispersion of the Fizeau interferometer creates in the fringe plane an extended spectrum of atmospheric scattering.

5 This is made up of broadband molecular scattering resulting in a Rayleigh-Brillouin spectral component of ≈ 4 GHz full width at half maximum (at 355 nm wavelength), and a narrow band spectral component originating from aerosol (Mie) scattering with spectral characteristics defined by the laser source and Fizeau design parameters. For the present investigation, a 16 channel detector, similar to those employed in ESA's Aeolus spaceborne lidar is considered, together with an adapted Fizeau interferometer of specifications (plate separation, wedge angle, finesse, etc.) well within current practice. Extensive modelling and
10 simulation demonstrates that the 16 detector channels of spectral information provide versatile, high sensitivity operation, for widely varying atmospheric scattering and attenuation conditions throughout the troposphere and lower stratosphere. Drawing on operational data and experience of the ESA Aeolus spaceborne lidar systems, simulations indicate that a spaceborne VALDI could achieve wind speed uncertainties $\sigma_{v_{HLOS}}$ in the range of $\pm 1.4 \text{ ms}^{-1}$ to $\pm 3.2 \text{ ms}^{-1}$, from the aerosol signal in central detector channels (with ASR ≈ 2 and 12 km observational path). The outer channels, dominated by molecular signal, should give
15 wind speed uncertainties between $\pm 2.4 \text{ ms}^{-1}$ and $\pm 3.4 \text{ ms}^{-1}$ (for 87 km observation path length). It is further shown that fast analytical ratio techniques provide Doppler frequency analysis with accuracy approaching the Cramér-Rao ultimate quantum limit within 10% to 20%. Finally, questions of practical design, construction and operation are considered. In this regard the single interferometer and detector, with single optical delivery train, offers considerable advantages of optical simplicity, high signal throughput efficiency, robustness and good long term alignment control and stability, plus wide frequency/wind speed
20 capability extending well above 100 ms^{-1} .



1 Introduction and background

During the past 50 years, atmospheric Doppler lidars have become well established for wind measurements and the provision of meteorological information (see e.g. Weitkamp, 2005; Fujii and Fukuchi, 2005; Reitebuch and Hardesty, 2021). Indeed, many commercial systems are now available for rapid, reliable, stand-alone, remotely controlled atmospheric measurements.

25 Much of this work has been and is conducted with heterodyne detection systems at eye-safe wavelengths in the $10\ \mu\text{m}$, $2\ \mu\text{m}$ and $1.6\ \mu\text{m}$ regions. Nevertheless, direct detection systems are also employed and it is notable that some of the very first lidar measurements of atmospheric wind velocity and temperature were made with Fabry–Pérot interferometers and argon ion lasers in the visible green at $0.5\ \mu\text{m}$ (Fiocco et al., 1971; Benedetti-Michelangeli et al., 1972, 1974). In this early ferment of ideas, the ambitious possibility of global wind measurement from space was suggested by Huffaker and Hall Jr (1984) incorporating
30 a powerful $10\ \mu\text{m}$ heterodyne detection lidar.

In the following years, the merits of different systems, laser wavelengths and competing spectroscopic techniques were widely, and sometimes fiercely, discussed across the lidar community and the world’s space agencies. Two prime factors are immediately apparent for challenging applications. Firstly, heterodyne detection techniques are well suited to the narrow band Mie scattering from small particles and clouds in the atmosphere. Unfortunately, the strength of such scattering is extremely
35 variable, ranging over many orders of magnitude, and quite often below useful measurable levels for large regions of the mid and upper troposphere and lower stratosphere. Secondly, direct detection techniques are most effective for broad-band Rayleigh-Brillouin (RB) scattering from molecules, which varies predictably with atmospheric pressure and temperature. Unfortunately, such scattering is quite weak overall, but it is enhanced at shorter wavelengths by the strong λ^{-4} dependence of the Rayleigh scattering.

40 Against this background, in 1999, ESA committed to the detailed modelling, design, construction and testing of a global wind satellite with the status of a technical demonstrator (ESA, 1999). In August 2018, ESA launched Aeolus, the first ever spaceborne Doppler wind lidar mission, into a sun-synchronous orbit at about 320 km altitude with a repeat cycle of seven days. Aeolus carried the Atmospheric Laser Doppler Instrument (ALADIN) as a single payload and operated successfully until July 2023 (ESA, 2008; Reitebuch et al., 2024).

45 This was in fact a two-channel direct detection system operating at 355 nm wavelength. It incorporated a two-channel Fabry–Pérot interferometer, for analyzing the broad-band molecular scattering using the so-called double-edge technique (Chanin et al., 1989; Flesia and Korb, 1999). The narrow-band aerosol scattering was measured in the second channel comprising a Fizeau interferometer in the so-called fringe imaging configuration (McKay, 2002).

The complete system required separate detectors for each channel, and employed a quite complex optical arrangement to
50 direct the optical beam between the channels. In its four and a half years of operation, ESA’s Aeolus mission has provided an enormous volume of global wind lidar data. Initially intended primarily as a three-year technical demonstrator, the mission quickly proved that the volume and quality of its data could significantly enhance current numerical weather predictions. The Rayleigh-clear wind measurements from molecular backscatter provided the bulk of the data, with an area coverage of ≈ 8 to 10 times larger than Mie-cloudy winds (Lux et al., 2026). However, the Mie-cloudy winds from particulate backscatter



were shown to be of particular importance due to their higher precision and their availability in the planetary boundary layer region (PBL) (Rennie and Isaksen, 2020; Rennie et al., 2021; Rennie and Isaksen, 2024; Lux et al., 2026).

In another related investigation, two adjacent papers on the design analysis (Tucker et al., 2018) and airborne performance (Baidar et al., 2018) of the Optical Auto-covariance Wind Lidar (OAWL), based on a multi-wavelength, quadrature Mach-Zehnder interferometer (QMZ), were extensively discussed. For the airborne system, operating at 532 nm wavelength, impressive performance in a series of varied flight trials was established; the potential for extension to a spaceborne instrument was described and demonstrated. More recently Bruneau and Pelon (2021) considered the design of a compact, operational QMZ interferometer for deployment in space; extensive analysis and simulations established the potential for atmospheric wind and cloud/aerosol survey from space. More recently, the ground based lidar system VAHCOLI (Vertical And Horizontal COverage by LIgars) was introduced as a compact, mobile Doppler lidar equipped with a multi-field-of-view configuration for simultaneous horizontal and vertical wind measurements (Lübken and Höffner, 2021). With this system, Mense et al. (2024) demonstrated continuous aerosol-based wind profiling from 3 km to 25 km in unattended operation, with good agreement with ECMWF and Aeolus observations and sensitivity to small-scale wind-field asymmetries.

Against this background, the purpose of the present paper is to develop the concept of an atmospheric, single-channel, multi-purpose wind lidar, capable of spaceborne operation, thus, building on the proven space performance of the Fizeau interferometer aerosol channel used in ALADIN. This leads to the design considerations of the Versatile Atmospheric Laser Doppler Instrument VALDI, based around a single Fizeau interferometer. Obviously detailed analysis of the potential performance is required, and the achievements of ALADIN offer a most valuable and instructive example and guide. It should however be understood that, while the performance of this new VALDI system is herein discussed largely in the context of such possible operation from space, the concepts are readily applied to the generally less demanding requirements and specifications of ground and airborne operation. It should also be noted that Fizeau as well as Fabry–Pérot ground-based instruments have recently been demonstrated for measurement of tropospheric temperature from the molecular fringe profile (Witschas et al., 2014; Xu et al., 2021; Witschas et al., 2021). The VALDI system, as herein described primarily for Doppler shift measurement with its 16 channels of spectral information across the full profile is, as demonstrated, also immediately applicable to this task. It is also worth mentioning that the proposed VALDI receiver could principally be added to any kind of atmospheric lidar system that is using a narrow-band and seeded laser source.

In the construction of this paper, Sect. 2 summarises for reference the technology and performance achievements of ALADIN. Sect. 3 discusses the basic operating principles of VALDI, with an estimation of the required molecular and aerosol/cloudy signals to achieve wind measurements of useful accuracy. The following four Sections 4 to 7 present detailed analyses and extensive simulations for the four simultaneous tasks: Doppler winds from particulate backscattered referred to as Mie-cloudy conditions; Doppler winds from broad-band molecular backscatter referred to as Rayleigh-clear conditions; accurate evaluation of atmospheric scattering ratio (ASR), and finally atmospheric temperature. Sect. 8 presents the key points of a practical working design well within current interferometric practice, followed by summary and conclusions in Sect. 9. A short Appendix A presents in outline the two analytical algorithms, so called $R(4)$ and $R(A, B)$, that give versatile and speedy frequency shift analyses for the 16 channels of spectroscopic information from the interferometer. These offer valuable alternatives to the more



90 familiar spectral fitting techniques applied to fringe measurement and their performance in the presence of photo-detection shot noise is analysed in the sections of this Appendix. A further Appendix B outlines, at fundamental level, the Cramér–Rao lower bound for the statistical accuracy of VALDI Doppler measurements.

2 Outline of Aeolus technology, performance and signal strength estimation.

2.1 Aeolus technology and performance

95 The spaceborne ALADIN lidar has been extensively described (see e.g. ESA, 2008; Reitebuch et al., 2024; Lux et al., 2020, 2024). Detailed information about its global data coverage and wind quality are for instance given by Lux et al. (2026).

The frequency tripled Nd:YAG lasers operated at a wavelength of 355 nm with a pulse energy mainly in the range of 50 mJ to 80 mJ, and a pulse repetition rate of 50.5 Hz. The transmit-receive Cassegrain telescope had a primary mirror of 1.5 m diameter and was followed by an optical train that directed the scattered signal beam to the Mie spectrometer, which employed the Fizeau interferometer. The beam reflected from the Fizeau interferometer was then sequentially directed to the Rayleigh spectrometer which incorporated the two coupled Fabry–Pérot interferometers (FPI). The interference fringes from each of the interferometers were focused onto two separate high-performance accumulating charged coupled device (ACCD) detectors, each providing 16×16 , spatially resolved frequency pixel/channels. High speed control and switching of the detector outputs permitted signal integration with different horizontal and vertical resolution as required.

105 Aeolus data have been extensively investigated at many institutions worldwide. A particularly valuable, detailed evaluation of global wind measurement during the first 2 years of operation was given by Rennie et al. (2021). A brief summary of these results is shown in Fig 1 for both ALADIN channels and the two atmospheric conditions broadly categorised as Rayleigh-clear and Mie-cloudy. As noted, the Rayleigh-clear measurements are more numerous than the Mie-cloudy. However, the latter make a valuable meteorological contribution with data in the PBL.

	Planetary boundary layer	Free troposphere	Lower stratosphere
Altitude	below 2 km	2 km – 16 km	above ~16 km
Pressure	1100 – 800 hPa	800 – 100 hPa	~100 hPa
Aeolus Rayleigh channel (Double Fabry Perot interferometer)		Rayleigh-clear conditions	
		Signal collection: 87 km hor. res., 0.75 to 1.25 km vertical res..	Signal collection: 87km hor. res., 1.0 to 2.0 km vertical res.
		Range of v_{HLOS} error (st. dev.): 4.0 – 7.0 ms⁻¹	
Aeolus Mie channel (Fizeau interferometer)	Mie-cloudy conditions		Occasional rare events: e.g. stratospheric cirrus, volcanic aerosol etc.
	Signal collection: 12 km hor. res., 0.25 to 0.75 km vertical res..	Signal collection: 12 km hor. res. 0.75 to 1.25km vertical res.	
	Range of v_{HLOS} error (st. dev.): 2.8 – 3.6 ms⁻¹		

Figure 1. Outline of the Aeolus wind data performance, mainly for the first two years of operation (Sept. 2018 to Oct. 2020) as detailed by Rennie et al. (2021); Rennie and Isaksen (2024); Lux et al. (2026).



- 110 Inspection of Fig 1 reveals wind velocity errors $\sigma_{v_{\text{HLOS}}}$ along the horizontal line-of-sight (HLOS) in the range of 4.0 ms^{-1} to 7.0 ms^{-1} for Rayleigh-clear measurements, with signal accumulation in the free troposphere with a horizontal resolution of 87 km and a vertical resolution of 0.75 km to 1.25 km, increasing to 1.0 km to 2.0 km in the lower stratosphere. This compares to the signal accumulation for Mie-cloudy measurements with 12 km horizontal resolution and similar 0.75 km to 1.25 km vertical resolution in the free troposphere. Among other significant findings it became clear that:
- 115 1. A significant fraction of the valid Mie-cloudy wind data resulted from strong backscatter from ice and water clouds, including cloud top and more diffuse thin clouds.
2. There were few useful Mie-cloudy wind speed measurements that may be attributed unequivocally to purely aerosol backscattering, and these were almost entirely due to rare high-backscatter events caused, for example, by incipient cirrus cloud formation, volcanic eruption, dust plumes, and wildfire smoke swept high into the atmosphere.
- 120 3. There were almost no observations of Mie winds with errors $\sigma_{v_{\text{HLOS}}} < 1 \text{ ms}^{-1}$, in contrast to pre-launch expectations (ESA, 2016; Lux et al., 2026).

These findings, specifically for the Fizeau channel, appear to point to a loss of radiometric efficiency of signal throughput. They are in fact in accordance with early findings for the Fabry–Pérot channel that suggested possible initial loss factors of 2 to 4 (Witschas et al., 2022) with a generally accepted value ≈ 2.5 with respect to pre-launch expectations.

- 125 Besides the lower-than-foreseen laser energy (e.g., 60 mJ instead of 80 mJ at the beginning of the mission), other factors affecting the radiometric efficiency have been identified. These include quality and deformation issues of the primary telescope mirror, which modified the beam focus and introduced aberrations at the instrument field-stop level. This hypothesis is supported by the unexpectedly large apparent angles of incidence (AOI), of a few hundred μrad , reported for both interferometer channels (Witschas et al., 2022; Vaughan et al., 2025). In addition, laser-induced contamination in the optical transmission
- 130 chain of one of the two lasers (FM-B) caused a significant and progressive signal loss The Pierre Auger Collaboration et al. (2024). Consequently, the Aeolus signal levels varied by approximately a factor of 3 over the course of the mission. Nevertheless, they provide a valuable reference for representative signal levels in a spaceborne lidar system.

2.2 An estimation of molecular signal strength from ALADIN operation

- For analysis and discussion of the VALDI instrumental performance, it is desirable to have an authoritative reference value
- 135 for the expected molecular signal; the Aeolus findings provide an obvious source. With a similar optical receiving system to Aeolus, the atmospheric backscattered signal presented at the input is expected to be of similar order of magnitude for VALDI. The use of the measured ALADIN signal level provides a deliberately conservative reference for the VALDI performance assessment, since it is based on an actual in-orbit wind-lidar measurement rather than on projected performance for a future instrument. The resulting photon numbers as discussed in the next sections can readily be scaled to other instrument concepts
- 140 or improved performance assumptions; for example, the higher signal levels anticipated for future Aeolus generations would directly translate into correspondingly lower statistical uncertainties.

Compared to Aeolus, the proposed VALDI interferometer, with somewhat smaller reflectivity finesse $N_{\text{R}} \approx 22$ will require a 4-times larger free spectral range (FSR) of $\Gamma_{\text{FSR}} = 8.8 \text{ GHz}$ to fully cover the RB spectrum, with fringe full width half



maximum (FWHM) of $\Gamma_{\text{FWHM}} = 0.4$ GHz, and a pixel width also of $P_w = 0.4$ GHz. These values compare with $\Gamma_{\text{FSR}} =$
145 2.2 GHz, $N_R \approx 24.5$, $\Gamma_{\text{FWHM}} \approx 95$ MHz and pixel width ≈ 95 MHz in Aeolus. Note that the VALDI Fizeau spectrometer
specifications are discussed in detail in the following Section 2.3.

The molecular background signal in the Aeolus Mie channel offers useful evidence and can provide a baseline estimate as
follows. Data from ≈ 5 km to 8 km altitude (mid troposphere) for generally Rayleigh-clear conditions and low attenuation,
showed quite consistently a background level of ≈ 50 LSB (least significant bit) per pixel at most (see Fig. 15 in Vaughan et al.,
150 2025). This 50 LSB signal level is equivalent to ≈ 73 ph.el. (photo electrons) per pixel, considering the conversion rate of this
ALADIN detection chain which provides a radiometric gain of 0.684 LSB per ph. el. for the ACCD detector (Lux et al., 2024),
and thus provides a measure of the molecular signal passing through each pixel window (95 MHz) of the full 2.2 GHz FSR.
The total molecular signal across the full FSR would thus amount to $73 \text{ ph.el.} \times 2.2 \text{ GHz} / 0.095 \text{ GHz} \approx 1700$ ph.el. Note that
this also includes the overlap of second and third orders of the 2.2 GHz FSR, and thus includes the contributions from the other
155 parts of the wider molecular spectrum of ≈ 4 GHz FWHM; it is indeed the full profile molecular signal. It has to be noted that
this value is derived from a late operational period in June 2022, where the reached signal levels were significantly affected by
the aforementioned laser induced contamination (Lux et al., 2026). Taking this into account, the signal value calculated above
of 1700 ph.el. is proportionately increased by a factor of 2.5 to $\approx 4.2 \times 10^3$ ph.el.

In summary, these reference values are calculated to be 4.2×10^3 ph.el. as representative for 12 km path length, and
160 3.05×10^4 ph.el. for 87 km path length, and are employed in the following analyses.

Finally, the issues regarding the loss of radiometric efficiency should be noted. Considering an initial loss of a factor of
the order of 2 with respect to pre-launch expectations, together with a progressive signal loss of a factor of up to 3 due to
laser induced contamination (FM-B laser), an overall loss factor of approximately 2 to 6 could potentially be recovered, with a
significant impact on performance. In VALDI the statistical, signal-related errors as calculated in the following sections would
165 typically reduce by the usual square root factor, i.e. in the range 1.4 to 2.4.

2.3 Design Parameters of the VALDI Fizeau interferometer and detector

During the course of this study a wide range of instrumental Fizeau parameters have been investigated and their relative
performances simulated and assessed. In broad terms, high finesse, narrow instrumental fringes favour aerosol measurement
accuracy, while lower finesse, wider bandwidth give increased light collection and favour molecular measurements. The set of
170 instrumental values selected in Table 1 is thus presented as a versatile compromise that offers optimized levels of performance
for all four main tasks, namely measuring wind from aerosol and molecular backscatter, the ASR and temperature. Accordingly,
they are employed in the extensive VALDI simulations and performance calculations throughout the following sections. It
should be noted that a few of the parameters are rather similar to those of the Fizeau interferometer used in Aeolus (Reitebuch
et al., 2009), whereas others had to be adapted significantly to match the intended application of VALDI, namely exploiting
175 the signal from both aerosol and molecular backscatter with a single interferometer.

In the Fizeau instrument, the interference fringes are localized at a plane within the optical cavity. These fringes need to
be focused onto the square-cross-section imaging detector. According to the instrumental parameters in Table 1, with a useful



Table 1. Primary and derived instrumental parameters of the Fizeau interferometer as simulated for VALDI. The formulae used can be found in the textbook by Vaughan (1989).

Parameter	Value
Primary parameters:	
Plate spacing	$d = 17.05$ mm, vacuum gap
Wedge angle	$\alpha = 4.8$ μ rad
Plate reflectivity	$R = 0.867$
Plate absorption	$A = 0.006$
Reflective coating plate diameter	$d_R = 38.1$ mm
Number of imaging detector channels	$N_P = 16$
Spectral pixel width	$P_w = 0.4$ GHz
Derived parameters:	
Free spectral range	$\Gamma_{FSR} = c/2d = 8.8$ GHz
Reflectivity finesse	$N_R = R^{1/2}/(1 - R) \approx 22$
Fringe FWHM	$\Gamma_{FWHM} = FSR/N_R = 0.4$ GHz
Relative peak intensity	$I/I_0 = (1 - R - A)^2/(1 - R)^2 \approx 0.91$
Spatial FSR	$S_{FSR} = \lambda/2\alpha = 37$ mm
Frequency dispersion	$F_D = \Gamma_{FSR}/S_{FSR} = 0.23$ GHz/mm
Spatial pixel width	$P_w/F_D = 1.68$ mm
Useful spectral range (USR)	$\Gamma_{USR} = N_P \times P_w = 6.4$ GHz
Spatial USR	$S_{USR} = \Gamma_{USR}/F_D = 26.9$ mm
Aperture diameter	$D = S_{USR} \times 2^{1/2} = 38.1$ mm

spectral range (USR) $\Gamma_{USR} = 6.4$ GHz, the linearly dispersed fringes extend spatially over $S_{USR} = 26.9$ mm. For optimum light collection the primary telescope aperture should be imaged to fill such a square section of 26.9×26.9 mm at the Fizeau

180 with a minimum of spill-over. For circular plates this requires a usable, reflectively coated, diameter D greater than 38.1 mm.

For practical operation, the laser frequency may be precisely tuned to the centre of the spatial range, i.e. the junction of pixels 8 and 9. This provides an immediately accessible frequency range of ± 1 channel on either side, equal to ± 400 MHz. In velocity terms, at 355 nm wavelength, this corresponds to a direct LOS component $v_{LOS} = \pm 71$ ms^{-1} ($1 \text{ ms}^{-1} = 5.63$ MHz). For a laser off-nadir angle of 35° , this is equivalent to horizontal LOS $v_{HLOS} = v_{LOS}/\sin\theta = \pm 116$ ms^{-1} . At larger off-nadir angles

185 (e.g. at 42°), $v_{HLOS} = \pm 106$ ms^{-1} and well in excess of the generally required meteorological specification of 100 ms^{-1} . Further, the range can be extended to ± 1.5 to 2 detector pixels. Considered purely as an HLOS laser velocimeter, with laser frequency tuned to either end (channels 1 or 16), the frequency range of ± 6 GHz gives a velocity of greater than 1 kms^{-1} .



3 Operating principles of the multi-purpose Fizeau lidar VALDI

The purpose of this Section is to introduce, by way of simple diagrams, the basic principles of VALDI for its four main tasks, namely measuring wind from aerosol and molecular returns, as well as retrieving ASR and temperature profiles. These are then explored with extensive simulations in the following Sections (Sect. 4 to Sect. 7) which constitute the bulk of the paper.

3.1 Basic experimental techniques

The Fizeau interferometer with its two optical plates, set at a carefully chosen small wedge angle, has the vitally important property of providing a linear dispersion of frequency in its fringe plane. Combined with suitable spectroscopic parameters, this linear dispersion is particularly valuable for simultaneously measuring both narrowband aerosol Mie scattering and broadband molecular RB scattering.

The atmospheric scattering components for VALDI measurements are shown schematically in Fig. 2, with two contrasting RB spectra; near sea level ($T = 288$ K, $p = 1000$ hPa, light blue) and in the lower stratosphere at ≈ 20 km ($T = 217$ K, $p = 55$ hPa, dark blue). Both spectra were simulated for a wavelength of 355 nm and backscattering configuration by means of the spectral line shape model by Witschas (2011a, b), which is a simple analytical representation of the best-known RB line shape model in air by Tenti et al. (1974). At the lower altitude (light blue), the broadening due to the Brillouin components is obvious, compared with the near-Gaussian profile of the low-pressure high-altitude scattering (dark blue). The width of the narrow-band aerosol scattering at the centre is dominated by the Fizeau spectral shape (of near-Lorentzian profile) with $\Gamma_{\text{FWHM}} = 0.4$ GHz (Fig. 2, dark red line). In this schematic figure, a possible Fizeau detector arrangement is indicated below, with 16 pixels with $P_w = 0.4$ GHz, giving $\Gamma_{\text{USR}} = 6.4$ GHz in relation to a possible full Fizeau FSR of $\Gamma_{\text{FSR}} = 8.8$ GHz. Note also that, as drawn, the ASR, defined by the sum of molecular and aerosol scattering contribution divided by the pure molecular scattering contribution, is ≈ 1.13 .

The basic spectral information in VALDI is thus provided by the signal contents of these 16 pixels. Analysis of this information may be conducted by established techniques of fitting a lineshape model to the data. However, it has been shown that the formation of calibration curves, by direct ratioing of suitably chosen pixel content can provide rapid versatile analysis, with accurate measurement of spectral content and parameters of shift and width (Witschas et al., 2023). Furthermore, such analysis provides straightforward calculation of statistical Poisson and quantum-limited accuracy and system comparison and is more robust than fitting techniques as it does not suffer from non-convergence issues. The construction and analysis of such calibration curves are thus extensively investigated and are presented as a valuable operating principle for VALDI. It should be noted that such specifically constructed ratioing and calibration techniques have potentially very wide general applications for analyses across a wide range of optical and physical problems and measurements, but do not appear to have been extensively developed. Jacobsen and Kootsookos (2007) introduced a fast algorithm that is based on an intensity ratio calculation by using three pixels for the accurate analysis of periodograms to determine the frequency of tones, and Witschas et al. (2023) introduced a 4-pixel ratio approach, $R(4)$, to accurately determine the position of a Fizeau interferometer fringe. The latter one is used in this work and extended to further pixel ratios.

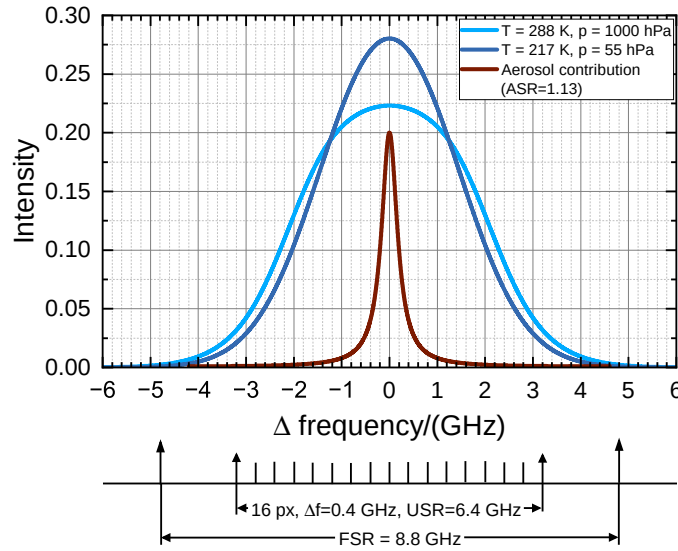


Figure 2. Schematic of molecular and aerosol scattering profiles linearly dispersed across the fringe plane of the Fizeau interferometer, shown with possible 8.8 GHz FSR and detector pixels 0.4 GHz wide (bottom) leading to 6.4 GHz USR, considering 16 detector pixels. The molecular spectra, calculated for $T = 288$ K, $p = 1000$ hPa (light blue) and $T = 217$ K, $p = 55$ hPa (dark blue) have been normalised to equal area and, as drawn, the aerosol line (dark red) represents an ASR of ≈ 1.13 .

For the illustration of the VALDI signal contents and their usage in the respective retrievals, Fig. 3 shows examples of backscattered light spectra ($ASR = 2.0$, $T = 270$ K, $p = 500$ hPa, $P_w = 0.4$ GHz), employing the Fizeau parameters from Table 1. These are depicted for zero Doppler shift and $\Gamma_{FWHM} = 0.4$ GHz. Note that in this particular fringe configuration, there are 2 central pixels of equal energy and this may be described in an obvious nomenclature, as a T2-type fringe. With frequency shifts of ± 200 MHz, ± 600 MHz, etc., the fringes would have a single dominant pixel (and with equal content on either side), and may be described as T1-type fringes. The distinction between T1 and T2 fringes becomes significant when considering measurement accuracy. This is because there are a relatively small number of pixels and their width is large compared to the desired accuracy of frequency measurement. As shall be seen in later sections, this can result in the measurement accuracy varying depending on the positioning of the fringe relative to the pixels. Of course, this situation would not arise if there were significantly more than 16 detector channels available. However, it was thought prudent to base the analysis on proven technology, that is, the ALADIN detector. If a better detector becomes available, with more, smaller pixels, then that would clearly be a better option.

3.2 Measurement of atmospheric parameters

In the following four subsections 3.2.1 to 3.2.4, the four diagrams of Fig. 3 illustrate via the different pixel content how the respective atmospheric measurements (wind, ASR and temperature) are derived.

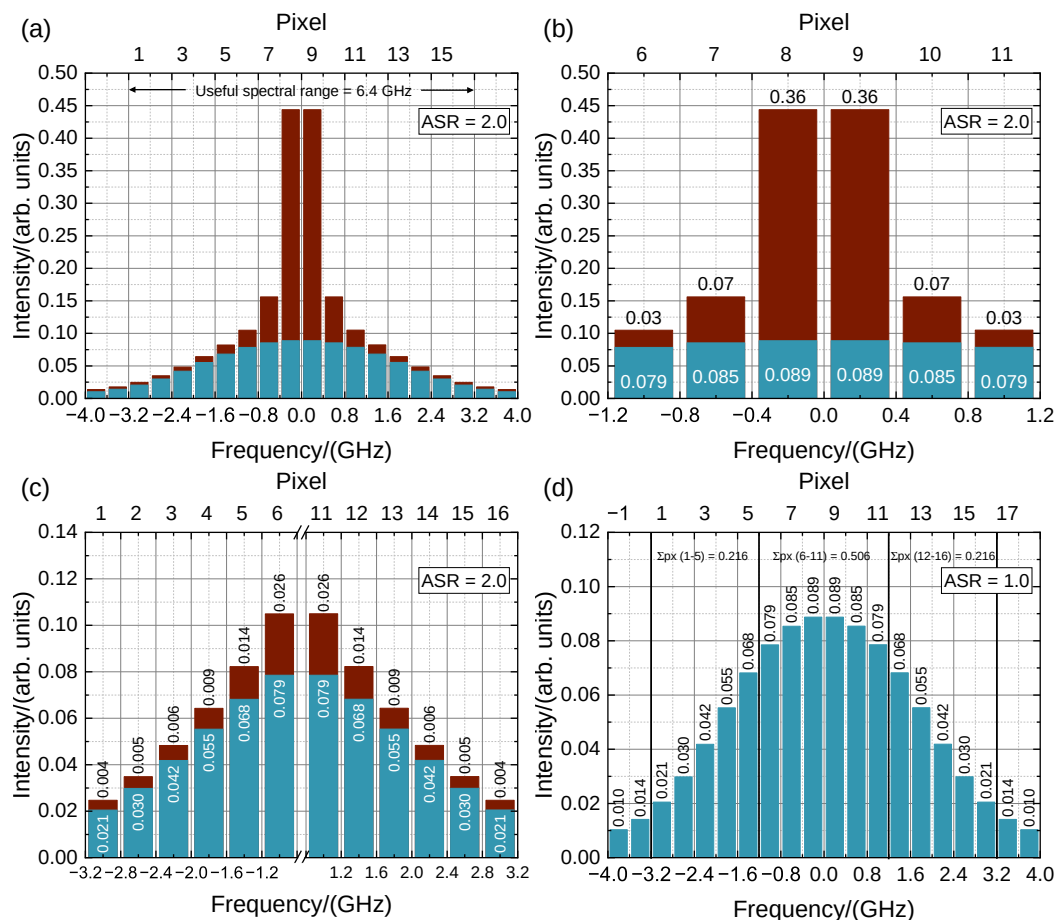


Figure 3. (a): Backscattered light spectra (ASR = 2.0, $T = 270$ K, $p = 500$ hPa, $P_w = 0.4$ GHz, Fizeau parameters from Table 1), illustrated for zero Doppler shift in the T2 fringe configuration, i.e. 2 central pixels of equal energy content. The RB spectrum is illustrated by the blue bars and the aerosol fringe by the red bars, respectively. (b) Same as panel (a), but a zoom to the inner six pixels. The numbers at each bar denote the fractional signal content of the respective channel. (c) Same as panel (a), but zoom to the outer six pixels on each side. (d) Pure RB spectrum (ASR = 1) for same conditions as shown in panels (a) to (c). The numbers indicate the fractional content of different pixel groups.



3.2.1 Doppler measurements from aerosol Mie scattering

In Fig. 3 (a), the blue bars indicate the RB-spectrum, and the red bars the near-Lorentzian profile of the aerosol fringe with $\Gamma_{\text{FWHM}} = 0.4$ GHz. A zoom to the inner six pixels is displayed in Fig. 3 (b). The spectrum is shown for zero Doppler shift, hence, pixels 8 and 9 each contain $\approx 36\%$ of the aerosol fringe energy (numbers above respective bars). The two adjacent pixels
240 7 and 10 each contain $\approx 7\%$. In comparison for the RB profile of the molecular fringe ($\Gamma_{\text{FWHM}} \approx 4.0$ GHz at $T = 270$ K), the two central pixels 8 and 9 each contain $\approx 8.9\%$ and the adjacent pixels $\approx 8.5\%$.

As outlined in Appendix A, the analytic ratios $R(4)$ and $R(A, B)$, that may be formed from such pixel content (see e.g. Witschas et al., 2023) are essentially normalised measures of fringe asymmetry. With appropriate simulation and experimental validation, they provide calibration curves of ratio versus frequency shift for immediate experimental reference. In practice,
245 the content of each pixel is made up of the sum of aerosol and molecular signal. For the central channels, this sum is heavily weighted to the aerosol component, e.g., 36% to 8.9% molecular contribution.

The potential accuracy σ_f is evaluated for the measured frequency shift. As outlined in Appendix B, the Cramér–Rao relationship according to Eq. (B2) provides a vital guide. In experimental practice, accuracies are typically attainable within $\approx 10\%$ of the ultimate quantum limit. With appreciable molecular signal, the situation is more complex, and the balance of
250 aerosol scattering levels and atmospheric attenuation are paramount. These are explored and detailed in Sect. 4.

3.2.2 Doppler measurements from molecular Rayleigh-Brillouin scattering

The principle of molecular Doppler shift measurement is illustrated with reference to Fig. 3 (c), which shows the spectrum plotted in Fig. 3 (a), but with a zoom to the outer six pixels to emphasize that in this region, the molecular scattering dominates the aerosol scattering with relative fractional contents per pixel, typically 3 to 8 times larger.

It is possible to select different sets of pixels and calculate various $R(4)$ and $R(A, B)$ calibration curves for a fast and reliable
255 wind retrieval algorithm. In terms of statistical accuracy, the Cramér–Rao coefficient that varies with the actual shape of the analysed spectrum was assessed as $C_{\text{CR}} \approx 0.53$ in the earlier work (Vaughan et al., 2025) for the near–Gaussian RB profile. Thus, the required signal, for a useful accuracy of $\sigma_f \approx 12$ MHz (equivalent to $v_{\text{HLOS}} = 3.5 \text{ ms}^{-1}$) and for a nominal molecular fringe of $\Gamma_{\text{FWHM}} \approx 4$ GHz, would be $N_{\text{mol}} = [(0.53 \times 4 \times 10^3)/(12)]^2 \approx 3.1 \times 10^4$ ph.el. This is very close to the standard
260 signal value (87 km horizontal accumulation length) estimated in Sect.2.2. Extensive simulations for different pixel groups and varying ASR, as well as regarding the optimum performance are detailed in Sect. 5.

3.2.3 Measurement of Atmospheric Scattering Ratio (ASR)

For VALDI, the principle of measuring ASR is to construct an appropriate pixel ratio $R(\text{ASR})$ that will provide a calibration curve for the measurement of the actual input ASR. Ratios of the form $R(\text{ASR}) = (\text{sum of pixels dominated by aerosol scattering})/(\text{sum of pixels dominated by molecular scattering})$ are an obvious solution and readily simulated for comparison with
265 experimental data. Inspection of the 4 central pixels of Fig.3 (b) offer an immediate first candidate with the ratio $R(\text{ASR})(2 : 2)$

$$= \sum_{i=8}^9 \text{px}(i) / \sum_{i=7}^8 \text{px}(i).$$



Other calibration ratios are readily evaluated and are explored in Sect. 6. The statistical accuracy is also examined and indicates errors for ASR of $\approx 5\%$ are achieved over a wide range of atmospheric scattering ratio.

270 3.2.4 Measurement of atmospheric temperature

The molecular RB spectrum changes quite markedly with temperature and pressure as illustrated in Fig. 2 for the two normalised extreme cases. The spectral width changes from $\Gamma_{\text{FWHM}} \approx 3.4$ GHz to $\Gamma_{\text{FWHM}} \approx 4.3$ GHz, but perhaps more obviously the shape changes from a near-Gaussian at the lower temperature to a flattened profile at the higher temperatures. Previous measurement techniques (Xu et al., 2021) have applied fitting techniques to such profiles to extract a corresponding
275 temperature.

In the present study the possibility of incorporating the shape changes more directly has been examined. The spectra of Fig. 2 indicate the relative proportions in the inner and outer regions. For VALDI this may be quantified, from the respective pixel content, by formation of a calibration ratio $R(T) = \sum \text{inner pixels} / \sum \text{outer pixels}$. As an example, a pure (ASR = 1) molecular profile is illustrated in Fig. 3 (d). This would immediately provide the calibration ratio $R(T) = \sum_{i=6}^{11} \text{px}(i) / (\sum_{i=1}^5 \text{px}(i) +$
280 $\sum_{i=12}^{16} \text{px}(i))$. This ratio and others are discussed in Sect. 7 and typical temperature errors approaching ≈ 3 K are indicated.

In final summary, it should be noted that all the errors indicated above would be improved (i.e. reduced) by a factor ≈ 2 or more if the potential enhancements of signal levels suggested at the end of Sect. 2 were achieved in practice.

4 Doppler Wind measurement from aerosol Mie scattering

It is worth re-emphasising the basic measurement principle: experimental data are analysed by comparing a suitably chosen
285 ratio of pixel signals with the corresponding ratio obtained from a model simulation. The Fizeau instrument and imaging detector, with successive channels sampling the spectral fringe across multiple pixels, are well suited to this procedure.

As a first step, the measured signal level in each pixel is used to calculate the experimental values of the ratio function. Second, based on the Fizeau parameters, the aerosol (Mie) and molecular RB lineshapes are modelled, and the corresponding signal levels are computed as a function of frequency shift. Third, these simulated signal levels are used to calculate the same
290 ratio function as a function of frequency shift, thereby providing the calibration curve. Finally, the measured ratio values are used to retrieve the corresponding frequency shift from the calibration curve. This procedure was introduced, described, and applied to the analysis of measurements from the ALADIN airborne demonstrator A2D in Witschas et al. (2023). For Doppler-shift measurements, two calibration ratios, $R(4)$ and $R(A, B)$, together with their statistical accuracy, are discussed in the Appendix. Ratios used for other measurements, namely ASR and temperature, are discussed in Sects. 6 and 7.

295 4.1 Fizeau fringe simulation and investigation

Fizeau fringe profiles for VALDI have been modelled from the opto-physical parameters of Table 1 using rigorous wave-optic techniques. These methods were extensively employed in a recent investigation of the Fizeau aerosol channel in ALADIN (Vaughan et al., 2025), which also presented a semi-tutorial Appendix on the basic wave-optic technique (Jakeman and Ridley,



2006). The following Figs. 4 to 6 illustrate the successive steps in the procedure. Of particular interest for this study was the investigation of the impact of a varying angle of incidence (AOI) and field of view (FOV) on the instrumental performance, as Aeolus-related studies revealed significant impact of these quantities (Witschas et al., 2022; Vaughan et al., 2025).

Fig. 4 shows three orders of interference extending over $3 \times \Gamma_{\text{FSR}} = 26.4$ GHz for the two fringe systems, the narrow-band aerosol Mie fringe of $\Gamma_{\text{FWHM}} = 0.4$ GHz (red) and a representative RB molecular fringe of input width $\Gamma_{\text{FWHM}} = 3.8$ GHz, modelled for $T = 250$ K and $p = 300$ hPa (blue). For clarity, the following figures show only a single order of interference typically within the usable spectral range $\Gamma_{\text{USR}} = 6.4$ GHz. However, the signal from multiple orders is considered for the performed investigations.

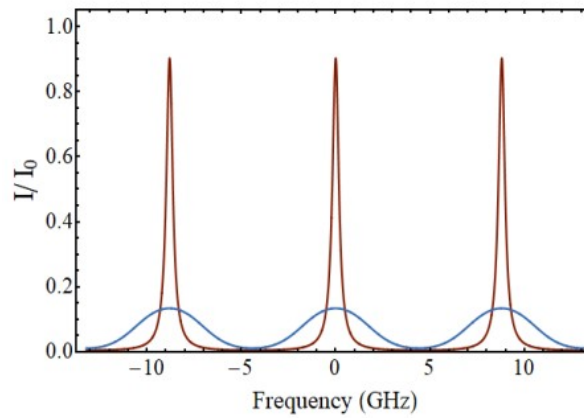


Figure 4. Simulated wave-optic fringes for the narrow band aerosol signal with $\Gamma_{\text{FWHM}} = 0.4$ GHz (red), and broad band molecular signal of intermediate width of $\Gamma_{\text{FWHM}} = 3.8$ GHz (blue). For this plot $\text{ASR} = 2$, i.e. the aerosol and molecular signal energies are equal.

Fig. 5 shows the impact of variation of AOI for an incident optical plane wave beam. Note that the fringe intensity, width and shift are closely similar for $\text{AOI} = 0$ mrad and ± 0.5 mrad, but deteriorate sharply for the other values of ± 1.0 mrad and ± 1.5 mrad. Furthermore, it can be seen that a changing AOI additionally leads to a shift of the center frequency.

Fig. 6 shows the variation of the FOV as a representative of the beam divergence for a Gaussian angular spectrum, at $\text{AOI} = -0.11$ mrad. With the given Fizeau design parameters, an AOI of -0.11 mrad represents the sweet-spot angle that yields the narrowest and most symmetric fringe as first noted for Fizeau fringes in reflection by Langenbeck (1970). The modelled fringes for $\text{FOV} = 0$ mrad and 0.5 mrad are very similar with relatively small changes up to 1 mrad. However, for a FOV of 1.5 mrad and 2.0 mrad, a significant line shape broadening and deviation from the original near-Lorentzian spectral profile are obvious.

These figures provide clear experimental/practical guidance for the VALDI optical signal delivery system. For optimum performance the alignment stability throughout life should provide an AOI controlled within the range of about -0.1 ± 0.3 mrad (Fig. 5). Similarly, per Fig. 6, the FOV should be close to 0.5 mrad angular divergence and less than 1 mrad. For a more extended analysis of this topic, the reader is referred to Vaughan et al. (2025).

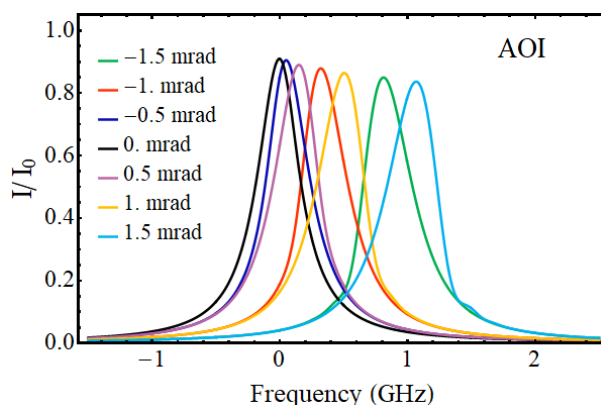


Figure 5. Wave-optic simulation for the instrumental Fizeau fringe with an incident plane wave beam at the different AOI. Note that the fringe is relatively unchanged for $\text{AOI} \pm 0.5$ mrad, but is increasingly distorted and asymmetric for larger values.

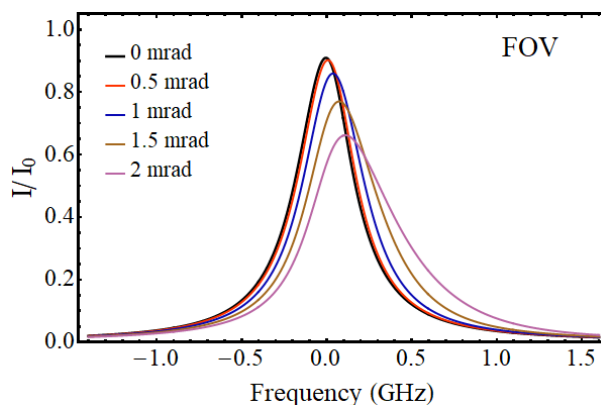


Figure 6. Simulated instrumental Fizeau fringes for $\text{AOI} = -0.11$ mrad with angular size, FOV set as indicated. Note that the fringes at $\text{FOV} = 0$ mrad and 0.5 mrad have very similar widths with relatively small changes up to 1 mrad, but increasing rapidly at larger values.

320 4.2 Derivation of aerosol Mie Doppler shift and wind speed

This subsection outlines topics required in the derivation of aerosol-dominated wind speed with typically $\text{ASR} \geq 2$. These include frequency calibration curves, fine details of the VALDI instrumental fringe, impact and relative weighting of the molecular scattering and comparative analysis of calibration curves for different ASR. The following Sect. 4.3 considers the investigation of systematic errors (bias) and the random statistical errors for different atmospheric levels broadly categorized as Rayleigh-clear and Mie-cloudy conditions.

325



4.2.1 Frequency calibration curves

The 16 channels of spectral data illustrated in Fig.3 (a) provide a wealth of information and demonstrate the relative and changing strengths of the two components. Analysis of this data may be tackled in many ways. For the present work, formation of frequency calibration curves derived from ratio of signal levels of different pixels is particularly effective as outlined in Appendix A. The so-called $R(A, B)$ ratio provided such a calibration curve using the outputs of the two Fabry–Pérot channels A and B of the ALADIN instrument. For the present work, the $R(4)$ four channel ratio, introduced by Witschas et al. (2023), is largely employed. For experimental data of small frequency shift (T2 fringes), the statistical uncertainties are identical for both ratios (see section A2).

For the aerosol Doppler measurement the large energy content of the 4 central pixels provides the obvious candidate with $R(4) = [\sum \text{px}(7, 8) - \sum \text{px}(9, 10)] / [\sum \text{px}(8, 9) - \sum \text{px}(7, 10)]$ which is equal to zero for the symmetric T2 fringe at the centre frequency $\Delta f = 0$. Note that $\text{px}(7, 10)$ denotes the individual pixels 7 and 10, rather than all pixels between 7 and 10. With pixel width 400 MHz, over the range -200 MHz to $+200$ MHz, the value of $R(4)$ ranges from $+1$ to -1 .

4.2.2 Simulation of frequency calibration curves

For a composite aerosol molecular fringe, as in Fig. 3 (a,b), numerical values of fractional pixel content need to be inserted in the $R(4)$ ratio. The mid-point for the ratio, with $\Delta f = \pm 100$ MHz, provides an illustrative example. Expressing the total fringe aerosol scattering as $N_{\text{aer}} = N_{\text{mol}} \times (\text{ASR} - 1)$ for the $R(4)$ fringe, for the 4 central pixels above gives:

$$R(4)(7, 8, 9, 10; \Delta f = -100 \text{ MHz}) = \frac{N_{\text{mol}} [0.0038 + 0.276(\text{ASR} - 1)]}{N_{\text{mol}} [0.0076 + 0.5414(\text{ASR} - 1)]}, \quad (1)$$

where the numerical values in Eq. (1) are derived from the sum of pixel content as indicated in Fig. 3. In this equation with $\text{ASR} = 1$, $R(4) \approx 0.50$, whereas for large ASR , $R(4) \approx 0.51$, indicating that the calibration curves from pure molecular to pure aerosol are very similar, and also close to linear across the ± 200 MHz range. Note also that the ratio of aerosol and molecular coefficients for both numerator and denominator in Eq. (1) amounts to a weighting factor of ≈ 70 , showing the dominance of the aerosol scattering in the calibration curve down to small ASR values.

This is well illustrated in the two following Figs. 7 and 8. Sets of $R(4)$ calibration curves over the range ± 400 MHz are shown in Fig. 7 for $\text{ASR} = 1.0, 1.1, 1.3, 1.6$ and 4 for an intermediate RB spectrum ($T = 250$ K, $p = 300$ hPa, ≈ 5.8 km altitude). In the central region, the differences between these calibration curves cannot be seen; for this reason, differences with respect to an $\text{ASR} = 2$ calibration curve are shown for a ± 200 MHz frequency range in Fig. 8. $\text{ASR} = 2$ was chosen as it is near the centre of relevant ASR values.

4.2.3 Systematic errors (bias)

Examination of Figs. 7 and 8 permits assessment of potential systematic errors (bias) inherent in possible uncertainties of the precise knowledge of the RB spectrum as well as the ASR for a set of experimental data. If required, frequency shift correction curves for intermediate altitude and ASR may be created from such simulated material, and show bias errors smaller than

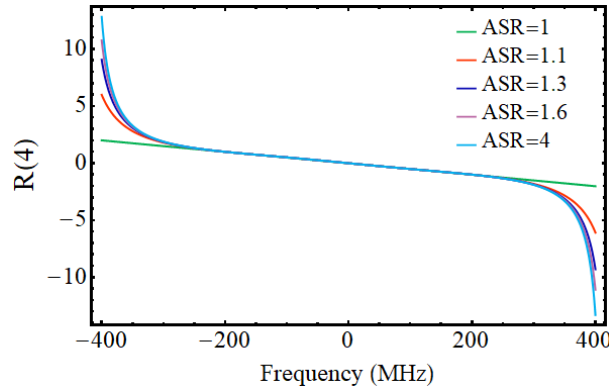


Figure 7. Set of $R(4)$ calibration curves for different ASR as indicated by the label and shown for a frequency range of ± 400 MHz, equivalent to a shift of ± 1 pixel.

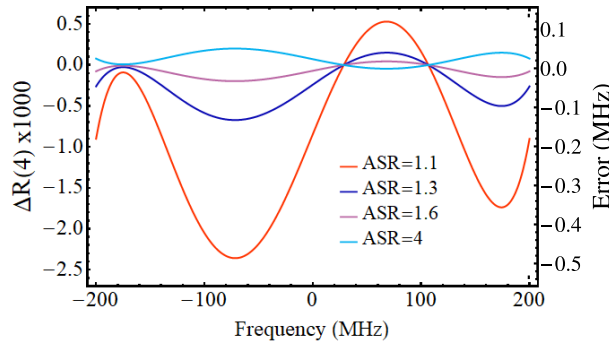


Figure 8. The corresponding difference of $R(4)$ calibration curves for different ASR as shown in Fig. 7, with respect to the $R(4)$ calibration curve for $ASR = 2$, shown for a frequency range of ± 200 MHz, equivalent to a shift of ± 0.5 pixel.

0.2 MHz corresponding to $\sigma_{\text{VHLOS}} = 0.06 \text{ ms}^{-1}$. It should be noted that an off-nadir angle of 35° similar to Aeolus is considered to calculate the HLOS error here and in the following. It should also be noted that the Mie fringe $R(4)$ dependence on fringe position is not quite symmetric. This arises because the fringe itself is not completely symmetric, due to the underlying asymmetry of the Fizeau wedge geometry. However, the asymmetry is nowhere near large enough to introduce any ambiguity when using $R(4)$ ratios to determine the frequency shifts, via suitable, experimentally measured calibration curves. For perpendicular incidence ($AOI = 0$), for example, the maximum change in $R(4)$ due to asymmetry is around 0.01 when the frequency is scanned between -200 MHz and $+200$ MHz (0 MHz being taken as the centre of the detector array). This asymmetry is a function of AOI, particularly in the plane of the Fizeau wedge and we find that it is minimised at an AOI of -0.11 mrad in this plane. For convenience, we have used this value of AOI for our modelling results, but this is purely because we think that the various features of interest are shown more clearly with the most symmetrical fringes; there would be no critical requirement to set the AOI this precisely in a practical instrument. When the instrument response is convolved with the (symmetric) RB



spectrum, we find that this small asymmetry is smoothed out, leading to an RB fringe with an $R(4)$ asymmetry more than an order of magnitude smaller than that of the Mie fringe.

370 As an example, Fig. 8 shows a comparison of data for $ASR = 1.3$ with $ASR = 1.6$ maximum errors of $\Delta R(4) < 0.001$, i.e. equivalent to a systematic error of $< 0.2 \text{ MHz}$ ($\sigma_{\text{VHLOS}} = 0.06 \text{ m s}^{-1}$). This compares with the large signal statistical (quantum limited) errors as calculated in the following subsection. We investigated the effect of fringe asymmetry on the $R(4)$ ratio by generating two further versions of the curves in Fig. 8 for AOIs of -0.5 mrad and $+0.5 \text{ mrad}$, i.e. one full FOV either side of $AOI = 0$. It was found that the results had a greater degree of asymmetry, but not substantially greater. In both cases, the
375 maximum differences between $ASR = 1.3$ and $ASR = 1.6$ values are 0.002, as opposed to 0.001 for the $AOI = -0.11 \text{ mrad}$ case, so the greatest frequency error is only increased by a factor of 2.

4.3 Accuracy assessment for aerosol Doppler shift and wind speed data

4.3.1 Introduction

This section considers the random statistical accuracy due to Poisson quantum fluctuations in the signal photo-detection. It
380 draws extensively on the investigation of ALADIN Fizeau data (Vaughan et al., 2025) and the more recent studies outlined in Appendices A and B. Of these, Appendix A considers the frequency calibration curves $R(A, B)$ and $R(4)$ derived from fringe simulation, and develops general results that may be fitted to the VALDI parameters (Table 1). These results thus provide accuracy values that might be expected in experimental practice.

Appendix B is more theoretically grounded and draws on recent investigations of the Cramér–Rao relationship for the
385 quantum limited, lower bound, statistical frequency accuracy. These investigations have been extended to the impact of fringe pixelation, relative pixel and centre frequency position, and broad band and parasitic background for both Lorentzian and Gaussian fringe profiles.

This section thus introduces the accuracy attainable from the aerosol fringe, considered as very close to Lorentzian, for the small central group of pixels which contain the bulk of the aerosol scattered signal. It may be noted that a broader discussion
390 (of aerosol and molecular signals) from the full, available USR is given in Sect. 5.1.

Inspection of Fig. 3 (b) establishes that the average molecular signal, over the 4 central pixels, provides a near uniform background given by $\sim 0.087 \times N_{\text{mol}}$ per pixel (at 250 K); this value remains within ± 0.005 over the temperature range 220 K to 280 K. With ASR being proportional to $(N_{\text{mol}} + N_{\text{aer}})/N_{\text{mol}}$ leading to $N_{\text{mol}} = N_{\text{aer}}/(ASR - 1)$ (see also Sect. 6), the aerosol signal of total energy N_{aer} thus has a background molecular, per pixel, given by $0.087 N_{\text{aer}}/(ASR - 1)$.

395 This situation has been examined in Appendices A1 and A2 for a wide range of relative pixel size and background signal. For present purposes Fig. 9, with effective value of the Cramér–Rao coefficient C_{CR} (y -scale) plotted ASR for different relative pixel width, provides useful information. The two solid lines are for the VALDI parameters of pixel width and Mie fringe width.

It can be seen that there is a significant difference, here, between the T1 and T2 fringes, with the T1 fringe giving a larger
400 coefficient: some 50% larger for $ASR=2$, for example (see also, Fig. B2 (a)). However, this reduction in performance may not

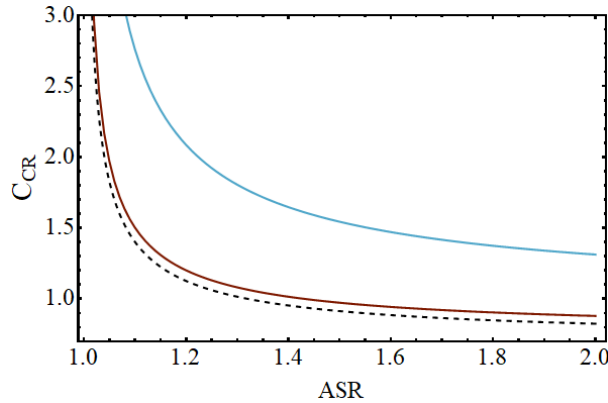


Figure 9. The quantum-limited Cramér–Rao coefficient C_{CR} plotted against ASR. Note these curves are for the special case where the molecular scattering is considered to provide a quasi-uniform broad background for the narrow band aerosol fringe under investigation. The lower dotted line (black) is for a large number of pixels (i.e. many pixels per FWHM) and derived directly, with change of scale, from Fig. B1. The red and blue curves show the relationship specifically for the VALDI parameters with FWHM/pixel width = 1 and T2 and T1 fringe positions, respectively. For the T2 fringe with ASR = 2.0 the resulting value of $C_{CR} \approx 0.88$.

be of great significance because, if the system is stabilised such that zero Doppler corresponds to the T2 position, fringes near T1 will represent very high wind speeds and these will not only occur infrequently but their relative error will be small, i.e. the error expressed as a fraction of the true value.

4.3.2 Mie-Cloudy Conditions

As noted in Sect. 2.2, the aerosol channel for ALADIN provided, for the 12 km observation path length, data of particular value in the lower troposphere down to the PBL with this high spatial resolution. At such lower altitudes atmospheric aerosol and cloud scattering is likely to be stronger, but the atmospheric attenuation is also typically higher. Based on the general experience of ALADIN (Vaughan et al., 2025), an attenuation factor $F_{att} \approx 5$ (two-way transmission) offers a reasonable working value for considering the attenuation due to aerosols and clouds. Here, $F_{att} = 5$ corresponds to a two-way transmission of $T^2 = 1/F_{att} = 0.2$, or, equivalently, to an optical depth of $OD = -\ln(T^2)/2 \approx 0.8$. Thus, for the 12 km observation path, the expected N_{mol} would be $(4.2 \times 10^3)/5 \approx 840$ ph.el. Consider now an ASR = 2, i.e. $N_{aer} = 840$ ph.el. also; per the above analysis $C_{CR} = 0.88$ and the expected limiting accuracy would be $\sigma_f = (0.88 \times 400)/\sqrt{840} \approx 12.1$ MHz, equivalent to $\sigma_{v_{HLOS}} \approx 3.3$ ms⁻¹.

In practice, in the lower troposphere, aerosol/cloud scattering and ASR values are likely to be higher, so that useful wind-speed measurements may be possible for a substantial fraction of the time.

4.3.3 Rayleigh-clear conditions

In the mid and upper troposphere, the bulk of the ALADIN measurements were made via the molecular Fabry–Pérot channel with observation path length 87 km. In such Rayleigh-clear conditions, ASR values considerably smaller than 2, and low



atmospheric attenuation with F_{att} close to 1 would be expected with very low attenuation due to aerosols and particles above the measurement volume. It is instructive to assess the potential performance of VALDI in such conditions.

420 As a starting point consider $\text{ASR} = 2$ and 12 km path. In conditions of low attenuation N_{mol} and N_{aer} would have the value 4.2×10^3 ph.el. and as a consequence $\sigma_f = (0.88 \times 400) / \sqrt{4.2 \times 10^3} \approx 5.4$ MHz. However a more representative value for such clear conditions would be $\text{ASR} = 1.25$ (i.e. $N_{\text{aer}} = 1.05 \times 10^3$ ph.el.). In consequence, from Figs. 9, for $\text{ASR} = 1.25$ and $C_{\text{CR}} \approx 1.13$, so that $\sigma_f = (1.13 \times 400) / \sqrt{1.05 \times 10^3} = 13.9$ MHz, which is still quite a useful level of accuracy.

Now consider performance for the 87 km operational path length as for ALADIN, i.e. the factor $87/12 = 7.25$ greater. The
425 C_{CR} values for $\text{ASR} = 2$ and 1.25 would remain the same as in the calculations above, but the aerosol N_{aer} is increased by 7.25. Thus the frequency errors are reduced by $(7.25)^{1/2} \approx 2.7$ and become respectively ~ 2.0 MHz (for $\text{ASR} = 2$) and ~ 5.1 MHz (for $\text{ASR} = 1.25$) – i.e. this latter value is equivalent to a $\sigma_{v_{\text{HLOS}}} \sim 1.4 \text{ ms}^{-1}$.

4.4 Summary and Conclusion

Those results outlined in Sect. 4.3.2 and 4.3.3 and the estimated Doppler shift and wind speed accuracies appear to be significant
430 and require detailed consideration. However such discussion and potential implications are deferred until the end of Sect. 5, and the investigation of VALDI fringe measurements in the region dominated by molecular scattering.

5 Doppler wind measurements from molecular scattering

Inspection of the composite molecular and aerosol fringe in the simulation of Figure 3 (a) illustrates that the central channels can be dominated by aerosol scattering. Thus, in practice, analysis drawn from the outer channels is likely to be more useful for
435 derivation of a Doppler shift largely attributable to the molecular scattering. As a starting point it is informative to simulate all 16 channels with the assumption of zero aerosol contribution, i.e. $\text{ASR} = 1$ as indicated in Fig. 3 (d). This situation is examined in the following Sect. 5.1. The investigation is then extended in Sect. 5.2 with different levels of aerosol scattering.

5.1 Doppler measurement with dominant molecular scattering

Application of the ratios $R(A, B)$ and $R(4)$ to the 16 channels of molecular scattering offers a very large selection of possible
440 pixel groupings. In broad terms one might expect that incorporation of a large number of pixels would increase signal level and improve statistical accuracy, and selection of pixels from the more strongly sloping sections of the profile should give increased sensitivity. During this study, many such groupings have been examined and some of the obtained results have been counterintuitive.

5.1.1 Investigation for small frequency shift

445 For swift initial analysis the calibration ratio $R(A, B)$ is particularly useful and gives for a T2 fringe with zero spurious background the same statistical accuracy as for $R(4)$ (see Appendix A). A set of $R(A, B)$ calibration curves (for the upper left



quadrant) over the frequency-shift range 0 MHz to 400 MHz is shown in Fig. 10. The individual curves are labeled (a) to (g) successively, and the pixel groupings are given in the first column of Table 2, using sums of 2 pixels, in (a), to 8 pixels in (g).

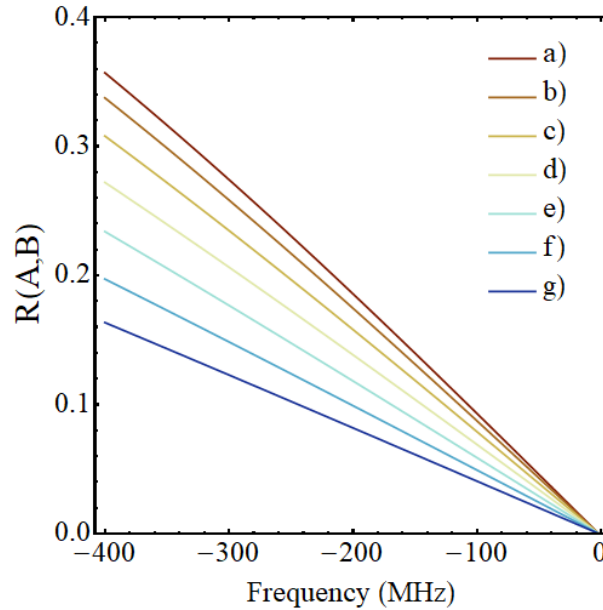


Figure 10. $R(A,B)$ calibration curves, shown for the left-hand upper quadrant of the full curve over ± 400 MHz, with 7 successive pixel groupings (a) to (g) as per text and Table 2. These curves are very close to linear and the slope is readily derived.

The Poissonian statistical fluctuation $\sigma_{R(A,B)}$ at the central T2 fringe position ($\Delta f = 0$) is derived as equal to $1/(A + B)^{1/2}$ (see Appendix A). The statistical errors in the frequency estimates are given by

$$\sigma_f = \frac{1}{(A+B)^{1/2} \times |\partial R(A,B)/\partial f|} \quad (2)$$

where $|\partial R(A,B)/\partial f|$ denotes the absolute value of the slope of the calibration curve. The $R(A,B)$ curves are close to linear functions of the frequency, but rather than using an average slope for the errors we use the actual slopes at a given frequency. In the case of the data presented in Table 2, the slopes at 0 MHz frequency were used and the corresponding values are given in the 4th column. The resultant errors are given in the final column of the Table and also plotted in Fig. 11. For this particular set of pixel groupings, a weak minimum for σ_f occurs for pixel totals 8 (c) to 14 (f), further illustrated in Table 2 (column 2), which also incorporates a few additional pixel groupings labeled (k), (m), and (n).

Examination of this table is quite instructive. Group (e) with $\sigma_f = 12.56$ MHz has minimally smaller error than group (d) with $\sigma_f = 12.58$ MHz. Clearly the addition of the considerable fringe energy of pixels 6 and 11 into the calculation for (e) compared with (d) has barely improved the accuracy. In contrast compare groupings (k) and (n). The exclusion of pixels 1 and 16, with much smaller fringe energy, has degraded the accuracies, compared with (d) and (e), by greater than 1 MHz. The important issue is clearly the useful spectral information contained within the respective pixels, and how it may be analysed, and not the crude measure of fractional spectral energy.

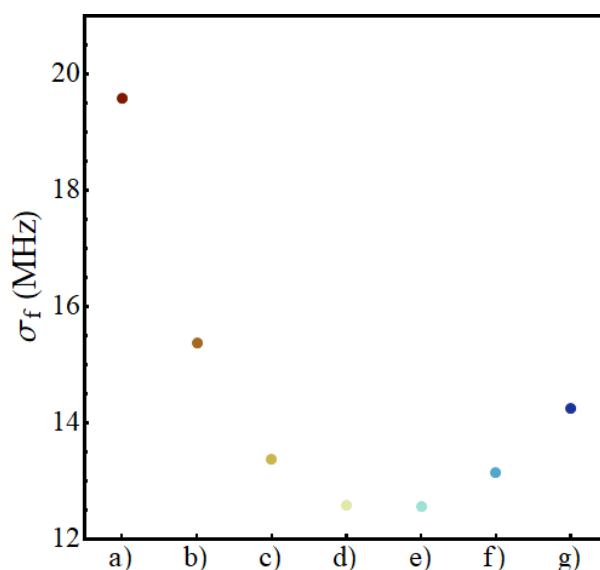


Figure 11. Statistical standard deviation error for the set of pixel groupings (a)-(g) and for a total molecular signal $N_{\text{mol}} = 3.05 \times 10^4$ ph.el., with minimum at (d)-(e) derived for a T2 fringe.

Table 2. $R(A, B)$ parameters for the T2 molecular fringe ($\Delta f \approx 0$ MHz) and for different pixel groupings. Column 3 is the fractional signal content $(A + B) \div N_{\text{mol}}$ and column 4 the slope of the corresponding calibration curve. The final column shows the standard deviation Doppler error for $N_{\text{mol}} = 3.05 \times 10^4$ ph.el. as plotted in Fig. 11. Comparison of the resulting accuracies in the final column is physically quite instructive - notably the changes of accuracy ($\pm$) with the addition/subtraction of pixels from the different groupings.

	$R(A, B)$	Tot. px	Fractional signal content	Slope $\times 1000$	σ_f
(a)	$\Sigma(1 \text{ to } 2) : \Sigma(15 \text{ to } 16)$	4	0.09418	0.9523	19.59 MHz
(b)	$\Sigma(1 \text{ to } 3) : \Sigma(14 \text{ to } 16)$	6	0.17402	0.8929	15.37 MHz
(c)	$\Sigma(1 \text{ to } 4) : \Sigma(13 \text{ to } 16)$	8	0.28167	0.8063	13.38 MHz
(d)	$\Sigma(1 \text{ to } 5) : \Sigma(12 \text{ to } 16)$	10	0.41742	0.7044	12.58 MHz
(e)	$\Sigma(1 \text{ to } 6) : \Sigma(11 \text{ to } 16)$	12	0.57728	0.5999	12.56 MHz
(f)	$\Sigma(1 \text{ to } 7) : \Sigma(10 \text{ to } 16)$	14	0.75414	0.5019	13.14 MHz
(g)	$\Sigma(1 \text{ to } 8) : \Sigma(9 \text{ to } 16)$	16	0.93960	0.4143	14.26 MHz
(k)	$\Sigma(2 \text{ to } 5) : (12 \text{ to } 15)$	8	0.37929	0.6771	13.73 MHz
(m)	$\Sigma(3 \text{ to } 6) : (11 \text{ to } 14)$	8	0.48309	0.5312	15.51 MHz
(n)	$\Sigma(2 \text{ to } 6) : (11 \text{ to } 15)$	10	0.53915	0.5733	13.60 MHz



These simulated values are in reasonable accord with the previous ALADIN Fizeau data (Vaughan et al., 2025), where a
465 median-derived estimate of $C_{CR} = 0.53$ provided an expression for the errors with the same functional form as the Cramér–
Rao expression of Eq. (B2) in Appendix B. This value thus gives a frequency measurement error $\sigma_f = (0.53 \times (4 \times 10^3)) /$
 $((3.05 \times 10^4)^{1/2}) = 12.2 \text{ MHz}$, very much in line with the above values. It is worth comparing these simulated values, using
calibration ratio techniques, with the Cramér–Rao lower bound (see Appendix B). The RB fringe is close to Gaussian in shape.
The ratio of fringe FWHM to pixel width is $3.8/0.4 = 9.5$, so the impact of pixellation is quite small. According to Fig. B2,
470 the C_{CR} value for a pure Gaussian fringe is $C_{CR} = 0.425$. However this would be a slight underestimate because the VALDI
molecular fringe, although close to Gaussian, has additionally a small Lorentzian component due to the Fizeau instrumental
FWHM of 400 MHz which slightly increases C_{CR} to about 0.47. In consequence $\sigma_f \approx [0.47 \times (4 \times 10^3)] / (3.05 \times 10^4)^{1/2} \approx$
10.8 MHz provides a quantum-limited performance value which is $\sim 14\%$ smaller than the calculated/simulated values shown
in Fig. 11 and Table 2.

475 It may be noted that on considering the full FSR, the spectral region outside the preset USR contains $\sim 6\%$ of the total
molecular fringe energy – quite a significant amount of spectral information. Estimates from calibration ratios incorporating
those outer regions (1.2 GHz wide on either side $\approx \pm 3$ pixel widths) give $\sigma_f \approx 11.6 \text{ MHz}$. For this simulated accuracy value
the limiting σ_f of 10.8 MHz as calculated above, is only $\sim 6\%$ smaller.

5.1.2 Extension to the full frequency range $\Delta f \pm 400 \text{ MHz}$

480 Error analysis is extended to the required full frequency range $\pm 400 \text{ MHz}$, using the error formulae for both calibration curves
 $R(4)$ and $R(A, B)$ as outlined in Appendix A and defined in Eqs. (A1) and (A2). In order to apply the $R(4)$ ratio to the same
groupings used for $R(A, B)$ we subdivide the groupings to give 4 groups for $R(4)$, as opposed to 2 groups for $R(A, B)$. The
results are illustrated in Figs. 12 and 13 for two of the pixel groupings (c) and (e) as shown in Table 2, where the respective σ_f
are 13.38 MHz and 12.56 MHz for the T2 fringe (frequency = 0 MHz).

485 The upturning, quadratic form of all the error curves is immediately apparent. The quadratic-type form means the increase is
greater over the 200 MHz to 400 MHz range than over the 0 MHz to 200 MHz range. This establishes the value of switching
at the $\pm 200 \text{ MHz}$ position, which is the half-pixel position, to the immediately adjacent set of pixels. The error curves when
this switch is made are indicated as dotted lines.

The second notable feature is the difference of the change of error σ_f between the calibration curves of $R(A, B)$ and $R(4)$,
490 over the frequency range, which is greater for the $R(4)$ results. This is not a universal finding and quite different from what is
seen when the sharp, central aerosol-dominated fringe is used. The degree to which the error changes with fringe position is
obviously a function of fringe profile and relative pixel size. In the present case, σ_f for $R(A, B)$ varies by less than $\approx 1 \text{ MHz}$
for $\Delta f \pm 400 \text{ MHz}$, compared with the largest change being $\approx 4 \text{ MHz}$ for the $R(4)$ technique. This offers a modest plus point
for the former in this case. It is worth noting that, here, the pixel width is much smaller than the fringe FWHM. For this reason,
495 the CRLB does not depend significantly on the fringe position relative to the pixels (see B2 (b)). However the ratio methods
do show noticeable variation going between the T2 and T1 positions, which means that they get closer to the CRLB for the T2
position than for the T1 position.

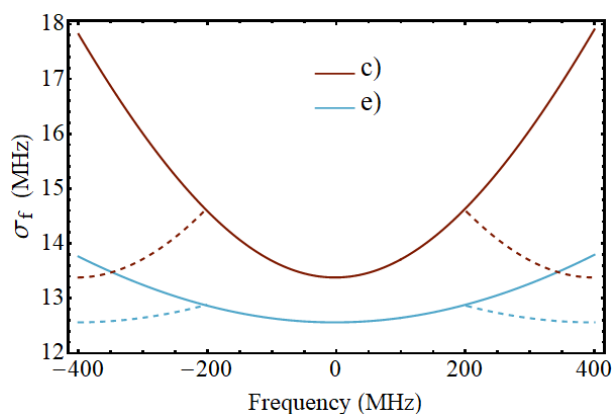


Figure 12. Simulated accuracy σ_f versus frequency shift for the $R(4)$ calibration ratio and pixel groupings (c) and (e). Note the increase of σ_f as the frequency increases successively from the T2 position (0 MHz) to the T1 position (± 200 MHz). Note also the reduction of error shown by the dotted lines for the range ± 200 MHz to ± 400 MHz on switching the ratios to the adjacent set of pixels at ± 200 MHz.

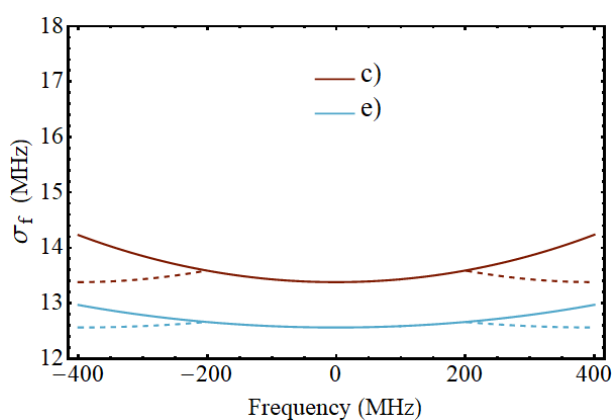


Figure 13. Simulated accuracy σ_f for the $R(A, B)$ calibration ratio and pixel groupings (c) and (e). Note the similar reduction in error shown by the dotted lines, as in Fig. 12, on switching pixels.



The final question is the error assessment for the changing molecular scattering profiles at different altitude, temperature and pressure. In broad terms, the FWHM of the RB line profiles will range between ≈ 3.5 GHz and 4.5 GHz, i.e. about $\pm 12\%$ around the presently selected 4.0 GHz for ≈ 5.8 km altitude. Per the Cramér–Rao analysis above, there are minor changes due to precise pixel content, Lorentzian fraction and pixel ratio which introduce small corrections and appear to reduce the variations to about $\pm 10\%$.

In summary the value for σ_f tends to increase by about 10% for low altitude and decrease by about 10% for high altitudes. In experimental practice in the real atmosphere the largest determining factor is more likely to be the level of atmospheric attenuation, and the level of molecular signal delivered to the instrument, coupled with the strong impact of aerosol scattering as discussed in the following subsection.

5.2 Molecular Doppler measurement with associated aerosol scattering

The impact of molecular scattering on the Doppler measurement of aerosol scattering in the 4 central pixels has been evaluated in Sect. 4. This demonstrates that useful measurements from 12 km path length can be made for ASR larger than about 2. The previous section 5.1 has demonstrated that, for zero aerosol scattering ($ASR = 1$), a molecular Doppler measurement can be achieved from the outer channels of spectral information with an 87 km path and larger probe depth. It is worth noting that the great bulk of the ALADIN Rayleigh-clear measurements were made in conditions with $ASR < 1.3$ with such 87 km path length. In the following subsections the ratios $R(A, B)$ and $R(4)$ are simulated from composite groupings of pixels that incorporate both aerosol and molecular scattering as delivered in the experimental detector output.

5.2.1 Incorporation of molecular and aerosol scattering in the calibration ratios

Two sets of groupings have been formed, one from the outer 12 channels (i.e. molecular dominated) giving:

$$R(A, B) \text{ where } A = \sum \text{px}(1 \text{ to } 6) \text{ and } B = \sum \text{px}(11 \text{ to } 16) \quad (3)$$

and an $R(4)$ composed of

$$\sum \text{px}(1, 2, 3); \sum \text{px}(4, 5, 6), \sum \text{px}(11, 12, 13), \sum \text{px}(14, 15, 16) \quad (4)$$

The second grouping of the inner 4 pixels (i.e. aerosol dominated) has:

$$R(A, B) \text{ where } A = \sum \text{px}(7, 8) \text{ and } B = \sum \text{px}(9, 10) \quad (5)$$

with $R(4)$ made up of the individual pixels:

$$\text{px}(7); \text{px}(8); \text{px}(9); \text{px}(10) \quad (6)$$

As in earlier sections, the composite groups are constructed with the individual values N_{mol} and $N_{\text{aer}} = (ASR - 1)N_{\text{mol}}$ as appropriate, where $N_{\text{mol}} = 3.05 \times 10^4$ ph.el. for 87 km path. The real atmosphere, with widely varying conditions, aerosol scattering levels and attenuation, provides an infinitude of situations. For simplicity and some initial understanding, two broad scenarios are considered in the following subsections for low and high atmospheric attenuation.



5.2.2 Low aerosol scattering and atmospheric attenuation

For generally clear conditions, low aerosol scattering with $ASR < 2$ and low attenuation factor $F_{att} \approx 1$ have been simulated.

530 The resultant frequency-shift errors are plotted in Figure 14 for the two sets of inner and outermost pixel groupings according to Eq. (3) to Eq. (6). These are for the T2 fringe position, where both $R(4)$ and $R(A, B)$ have the same frequency errors. Results for T1 fringes are found to give errors that are generally larger, with the $R(A, B)$ method having larger errors than the $R(4)$ (see also, Fig. A1). Note that these aerosol and molecular measurements, that may be derived from a single experimental fringe, have errors, represented by lines (a) and (b), that are statistically completely independent, as are the frequency-shift
535 measurements themselves.

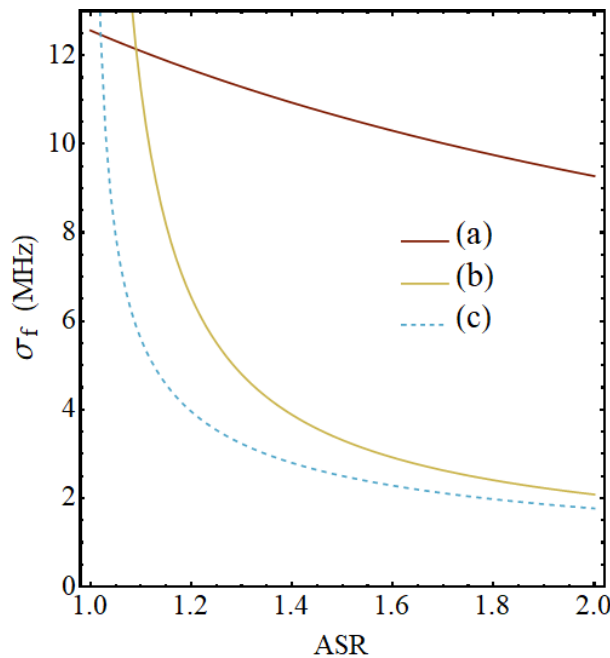


Figure 14. Doppler shift measurement errors versus ASR for composite aerosol and molecular fringes (plots (a) and (b)). The upper curve (a) is derived from the outermost 12 pixels and is dominated by molecular scattering. The middle curve (b) is derived from the central 4 pixels and is dominated by aerosol scattering. Curve (c) shows, for comparison with (b), the corresponding result for pure aerosol fringes of energy levels N_{aer} given by $(ASR - 1) \times N_{mol}$, with $N_{mol} = 3.05 \times 10^4$ ph.el.

Fig. 14 (a), derived from the outer 12 pixels, shows a statistical error $\sigma_f = 12.6$ MHz (pure molecular scattering, $ASR = 1$) that reduces steadily (quasi-linearly) to 9.3 MHz (for $ASR = 2$) when degraded by an equal amount of aerosol scattering; i.e. an improved level of accuracy. However, line (b) for the inner 4 pixels shows at $ASR = 2$ an error $\sigma_f = 2.1$ MHz. The lower plot (c) shows the calculated error for the exact same aerosol scattering, but without any molecular contribution; note that, for
540 this curve, the x-axis should be interpreted as signal level, e.g. $ASR=2$ corresponds to a signal level of $N_{aer} = 3.05 \times 10^4$ ph.el., which has σ_f of 1.8 MHz. Thus, for the outer pixels (per curve (a)) the addition of aerosol scattering reduces the error of the



molecular-dominated grouping, whereas for the aerosol-dominated pixels of curve (b) the addition of molecular scattering increases the error.

The physical explanation of this apparent anomaly is quite straightforward: the relative change of aerosol pixel content is still quite strong in this outer region and further augments the changes in the molecular pixel content, i.e. it contains useful spectral information. However, at the centre, the relative change in the molecular pixel content is quite small with little spectral information. As such it effectively increases the background noise and resultant error.

It is worth extending this discussion to the Rayleigh-clear region noted above. With $ASR = 1.25$, Figure 14 (a) provides a $\sigma_f = 11.5$ MHz and is obviously molecular-dominated. However, the inner 4 pixels give a composite $\sigma_f = 5.5$ MHz (per curve (b)), largely aerosol-dominated but increased from the $\sigma_f = 3.5$ MHz of the pure aerosol scattering of curve (c).

Finally, it is important to appreciate that these two measurements, made from the different sections of a single experimental fringe, are statistically completely independent as it was recently demonstrated for Aeolus Rayleigh winds in cloudy conditions (Marseille et al., 2023). This raises interesting questions. For example, inter-comparison of the two measured Doppler shifts and wind speeds — could this provide a meteorological interpretation in terms of the movement of discrete clouds, or narrow layers/bands of strong aerosol scattering, within the overall probe volume and its molecular element?

5.2.3 High atmospheric attenuation, $F_{att} \sim 5$

With attenuation factor $F_{att} \sim 5$ the σ_f in Fig. 14 will typically be increased by $5^{1/2} \approx 2.4$. Examination of the data indicates that the molecular-dominated results of upper curve (a), derived from the outer pixels, would not provide useful wind accuracy. However curve (b) would permit measurements with $\sigma_f \lesssim 13$ MHz for $ASR \gtrsim 1.25$ and be useful. As noted at the end of Sect. 4 this is due to the extended accumulation path of 87 km and the potential for VALDI in such conditions, and also to the very occasional low levels of aerosol scattering in the stratosphere.

6 Measurement of Atmospheric Scattering Ratio (ASR)

6.1 Introduction and basic technique

For this investigation precise, detailed knowledge of the Fizeau fringe profiles as derived with the VALDI parameters is essential. As primary input the atmospheric profiles derived from the US Standard Atmosphere (1976) have been employed. Extensive tables of fractional pixel content have been compiled for the required atmospheric temperatures, pressures and altitudes and for appropriate Doppler frequency shifts.

The atmospheric scattering ratio ASR is defined as (Eloranta, 2005)

$$ASR = \frac{\beta_{mol} + \beta_{aer}}{\beta_{mol}} = 1 + \frac{\beta_{aer}}{\beta_{mol}}, \quad (7)$$

where β_{mol} and β_{aer} are the molecular and aerosol backscatter coefficients, respectively.

The ASR is particularly relevant in the low-ASR regime, where it provides a measure of the aerosol contribution relative to the molecular (Rayleigh) background. In particular, $ASR = 1$ corresponds to purely molecular backscatter, while values



only moderately greater than unity indicate an increasing aerosol contribution. Consequently, for weak aerosol and cloud contributions against the molecular background, good sensitivity in approximately the $ASR = 1$ to 2 range is particularly important. For this reason, the performance of the VALDI ASR retrieval is considered primarily in this regime. In addition to the absolute statistical uncertainty σ_{ASR} , it is useful to express the uncertainty relative to the aerosol contribution, $ASR - 1$, as

$$\epsilon_{aer} = \frac{\sigma_{ASR}}{ASR - 1}. \quad (8)$$

This quantity directly indicates the relative uncertainty with which the aerosol contribution to the total backscatter can be determined. The behaviour at substantially larger ASR is considered subsequently as a secondary capability of the calibration ratios.

Considering the atmospheric two-way transmission to be equal for the aerosol and molecular backscattered light, Eq. (7) can be related to the measured photoelectron numbers according to

$$ASR = \frac{N_{mol} + N_{aer}}{N_{mol}} = 1 + \frac{N_{aer}}{N_{mol}}, \quad \text{from which} \quad \frac{N_{aer}}{N_{mol}} = ASR - 1, \quad (9)$$

where the terms N_{aer} and N_{mol} are the number of photoelectrons associated with aerosol and molecular scattering, respectively, integrated over one complete fringe of one FSR (amounting to 8.8 GHz, which is effectively equivalent to 22 channels' width).

Suitable calibration curves for estimating ASR are provided by ratios of the form: (sum of pixels dominated by aerosol scattering) / (sum of pixels dominated by molecular scattering). The 16 channels of VALDI offer a large and varied choice of pixel groupings and many potential candidates have been examined. Relevant criteria include high sensitivity and precision in the low-ASR regime, particularly $1 \lesssim ASR \lesssim 2$; adequate coverage of larger ASR values; relative insensitivity to temperature and the corresponding changes of the molecular scattering fringe profile; and relative insensitivity to Doppler frequency shift between T2 and T1 fringe positions. The bulk of the simulations have employed the molecular profile at the intermediate $T = 250$ K, $p = 480$ hPa, corresponding to ~ 5.8 km altitude, representative of the mid-troposphere. This has then been extended to the profiles at $T = 280$ K, $p = 850$ hPa and $T = 220$ K, $p = 220$ hPa as required.

6.2 Investigation of a simple calibration ratio

One of the most immediately obvious calibration ratios may be derived for the 8 central pixels and is given by:

$$R(ASR, 4:4) = \frac{\Sigma_{px}(7, 8, 9, 10)}{\Sigma_{px}(5, 6, 11, 12)}. \quad (10)$$

This incorporates over $\sim 90\%$ of the aerosol signal and $\sim 66\%$ of the molecular signal as shown numerically for the T2, 250 K fringe, by the pixel groupings:

$$R(ASR, T2, 250K, 4:4) = \frac{N_{mol} [0.3582 + (ASR - 1)(0.8484)]}{N_{mol} [0.2980 + (ASR - 1)(0.0812)]}, \quad (11)$$

where the numerical values are derived from profile simulation as in Fig. 3. In Fig. 15, the upper group of 4 curves are related calibration ratios $R(ASR)$ per the left-hand y -scale, for $ASR = 1.0$ to 1.5. The two central curves are for 250 K, T2 fringe



(Doppler shift $\Delta f = 0$ MHz, brown) and T1 fringe ($\Delta f = \pm 200$ MHz, dark blue), with $R(\text{ASR})$ increasing from 1.202 to 2.309 (T2 fringe) and to 2.274 (T1 fringe). The outer curves are both for T2 fringes at 220 K (yellow) and 280 K (light blue), respectively, starting at 1.267 and 1.147 for $\text{ASR} = 1.0$. Note the near parallelism of the three T2 fringe curves over this small range of ASR, which will run together for large ASR as discussed below.

The lower curve (black) represents the corresponding random quantum statistical error σ_{ASR} (right-hand y -scale) derived from $[\sigma_R(\text{ASR}, \text{T2}, 250 \text{ K}) / (N_{\text{mol}})^{1/2}] / [(dR/d\text{ASR})]$, where $dR/d\text{ASR}$ is the slope of the calibration curve, and $N_{\text{mol}} = 3.05 \times 10^4$. The derivation of the statistical uncertainty σ_{ASR} is additionally presented in Sect. A3.

The errors for all the other sets of fringes are very similar within a few percent. Note that the mean slope for the T2 fringes in Fig. 15 (for $\text{ASR} = 1.0$ to 1.5) is $\sim (2.3 - 1.2) / 0.5 = 2.2$, and the random error σ_{ASR} (lower curve) increases from $\sim 0.7 \times 10^{-2}$ to 1.4×10^{-2} . Expressed relative to the aerosol contribution, $\text{ASR} - 1$, these uncertainties correspond to $\epsilon_{\text{aer}} \approx 4.7\%$ at $\text{ASR} = 1.15$ and $\epsilon_{\text{aer}} \approx 2.8\%$ at $\text{ASR} = 1.5$.

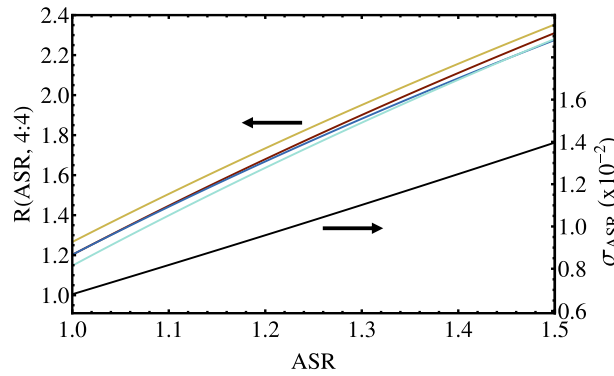


Figure 15. The $R(\text{ASR}, 4:4)$ ratio for small ASR of 1.0 to 1.5. The upper group of 4 curves are calibration ratios referred to the left-hand y -scale: the two central curves are for 250 K, T2 fringe (brown) and T1 fringe (dark blue). The outer curves are both for T2 fringes at 220 K (yellow) and 280 K (light blue), respectively. Note the near parallelism of the three T2 fringe curves which will meet together at large ASR. The single lower curve (black) is the statistical standard deviation error σ_{ASR} referred to the right-hand scale, calculated for $N_{\text{mol}} = 3.05 \times 10^4$ ph.el. and for the 250 K, T2 fringe. Note that the errors for all the other fringes are very similar.

The potential bias errors, due to uncertainties of temperature and Doppler shift, may also be assessed from the values above. For temperature the curve for T2 fringes $R(\text{ASR})$ differs by about ± 0.065 for 220 K and 280 K relative to 250 K. With a mean slope of ~ 2.2 , the potential bias error (for an uncertainty of ± 30 K) for ASR would thus amount to $\pm 0.065 / 2.2 \approx 0.03$ compared with the mean statistical error of ≈ 0.0105 (at $\text{ASR} = 1.25$). For a temperature error/uncertainty of 10 K, the random and bias errors would thus be quite close in value, namely 0.01.

In consideration of the Doppler shift uncertainty, from the T2 and T1 calibration curves for 250 K, the difference in $R(\text{ASR})$ is ≈ 0.015 at $\text{ASR} = 1.25$, corresponding to a potential bias error (for a 200 MHz Doppler shift) of $0.015 / 2.2 \approx 0.007$ – i.e. somewhat smaller than the random error (per Fig. 15) of $\sim 1.1 \times 10^{-2}$. This bias error is considerably smaller for a Doppler shift uncertainty of ~ 20 MHz, and indeed almost insignificant for small $\text{ASR} \lesssim 2$.



However, this situation changes very rapidly for large ASR. All the T2 curves in Fig. 15 will extrapolate at large ASR to the ratio of the aerosol fractions $R(\text{ASR} \rightarrow \infty; 4:4) = 0.8492/0.0820 = 10.4$. In comparison the T1 fringes would extrapolate to $R(\text{ASR} \rightarrow \infty; 4:4) = 0.8400/0.0900 = 9.3$, i.e. a considerable difference at large ASR of $1.1/9.3 \approx 12\%$. These issues are considered in the following Sect. 6.3.

6.3 Calibration ratios of varied sensitivity

As outlined there is a very large selection of possible pixel groupings to form an $R(\text{ASR})$. While $R(\text{ASR}, 4:4)$ provides particularly useful sensitivity in the low-ASR regime, alternative groupings can be investigated to extend the dynamic range of the ASR retrieval. Tables of fractional pixel content reveal several interesting properties. For example, pixel content $\Sigma \text{px}(5+12)$ over the range 220 K to 280 K and $\Delta f \pm 200$ MHz lies between 0.1370 ± 0.0006 , i.e. a variation of less than $\pm 0.4\%$.

On extension to different groupings, Table 3 below shows in the top row the grouping of 10 pixels (innermost 4 plus outermost 6), compared with the remaining middle 6 pixels in the second row. The corresponding $R(\text{ASR}, 10:6)$ in the third row, for $\text{ASR} = 1$, ranges from 1.296 to 1.333, corresponding approximately to 1.31 ± 0.02 , i.e. a total variation of $\pm \sim 1.5\%$ over the 60 K and ± 200 MHz ranges. This compares with the $\pm 5\%$ for $R(\text{ASR}, 4:4)$ as illustrated in Fig. 15.

Table 3. Sum (fractional content) of pixel groups of T2 and T1 fringes for the molecular scattering profiles 220 K, 250 K and 280 K. The upper row, of 10 pixels, sums the inner 4 and outer 6 pixels; the second row sums the remaining middle 6 pixels, and the third row shows the ratios $R(\text{ASR} = 1.0; 10:6)$. Finally, the bottom row shows the $R(\text{ASR})$ between the columns for the T2 and T1 fringes for $\text{ASR} = 10$, i.e. the composite fringe with addition of aerosol scattering.

Pixel Group	T2 Fringe, $\Delta f = 0$ MHz			T1 Fringe, $\Delta f = \pm 200$ MHz		
	220 K	250 K	280 K	220 K	250 K	280 K
10 pixels: $\Sigma [\text{px}(7 \text{ to } 10) + \text{px}(1 \text{ to } 3) + \text{px}(14 \text{ to } 16)]$	0.5416	0.5332	0.5266	0.5414	0.5334	0.5270
6 pixels: $\Sigma [\text{px}(4 \text{ to } 6) + \text{px}(11 \text{ to } 13)]$	0.4072	0.4076	0.4064	0.4061	0.4059	0.4043
Ratio $R(\text{ASR}, 10:6)$ for $\text{ASR} = 1.0$	1.330	1.308	1.296	1.333	1.314	1.296
Ratio $R(\text{ASR}, 10:6)$ for $\text{ASR} = 10$	6.504	6.496	6.497	6.091	6.086	6.089

The full numerical expressions for $R(\text{ASR}, 10:6)$ at 250 K are thus given by

$$R(\text{ASR}, \text{T2}, 250\text{K}, 10:6) = \frac{N_{\text{mol}} [0.5332 + (\text{ASR} - 1)(0.8800)]}{N_{\text{mol}} [0.4076 + (\text{ASR} - 1)(0.0993)]} \quad (12)$$

and

$$R(\text{ASR}, \text{T1}, 250\text{K}, 10:6) = \frac{N_{\text{mol}} [0.5334 + (\text{ASR} - 1)(0.8713)]}{N_{\text{mol}} [0.4059 + (\text{ASR} - 1)(0.1078)]}. \quad (13)$$

These calibration curves are shown in Fig.16, on a linear-log scale, up to $\text{ASR} = 100$. The behaviour at larger ASR is examined separately to assess the potential dynamic range of the retrieval.

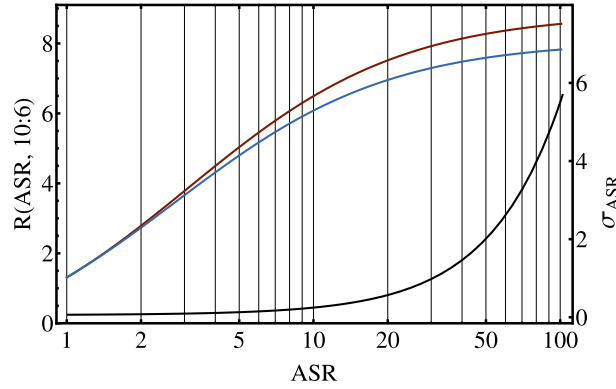


Figure 16. Calibration ratios $R(\text{ASR}, 250\text{K}, 10:6)$ plotted for the T2 (red) and T1 (blue) fringes [Eqs (12) and (13)] and molecular profile, for $\text{ASR} = 1$ to $\text{ASR} = 100$ (log-linear scale). The comparable ratios for profiles at 220 K and 280 K are almost identical (referred to left-hand y -scale). The statistical error σ_{ASR} from $R(\text{ASR}, 250\text{K}, \text{T2}, 10:6)$ (referenced to right-hand y -scale), is indicated by the lower curve (black).

The quantum-statistical random error for the T2 fringe is shown in the lower black curve and referenced to the right-hand y -axis for σ_{ASR} . At low $\text{ASR} \lesssim 2$ the calculated random errors derived from $R(\text{ASR}, 10:6)$ are $\approx 10\%$ greater than for $R(\text{ASR}, 4:4)$ shown in Fig. 15. However it should be noted that larger ASR values are likely to be associated with large atmospheric attenuation factors, possibly greater than the $F_{\text{att}} \sim 5$ discussed in Sect. 4. Such attenuation would increase the random errors σ_{ASR} by greater than $(5)^{1/2} \approx 2.3$.

The material of Table 3 and Fig. 16 allows the impact of temperature- and Doppler-shift-dependent bias errors to be assessed. Dealing with temperature dependence, per the final row of Table 3, for the temperature range $T = (250 \pm 30)$ K and $\text{ASR} \sim 10$, the $R(\text{ASR} = 10, \text{T2}, 10:6) = 6.50 \pm 0.004$ for the T2 fringe, and for T1 is 6.09 ± 0.003 . At $\text{ASR} = 10$ the slope of the calibration curve $R(\text{ASR})$, may be calculated as ~ 0.19 . Thus the potential bias error corresponding to $T = \pm 30$ K temperature is given by $\sim 0.004/0.19 \approx 0.021$. This is notably smaller than σ_{ASR} (random error) ≈ 0.20 evident on the right-hand axis of Fig. 16 for $\text{ASR} \approx 10$. Furthermore, this random error, as noted above, may be considerably increased by increased atmospheric attenuation. Thus, for temperature uncertainty of $\sim \pm 10$ K, the potential temperature bias error is likely to be insignificant at large ASR .

Turning to the potential Doppler shift bias, the T2 and T1 curves of Fig. 16 extrapolate to the limiting values of Eq. (14) and (15) that differ by $\approx 10\%$:

$$R(\text{ASR} \rightarrow \infty, \text{T2}, 10:6) = 0.8800/0.0993 = 8.86, \text{ and} \quad (14)$$

$$R(\text{ASR} \rightarrow \infty, \text{T1}, 10:6) = 0.8713/0.1078 = 8.08. \quad (15)$$



At higher ASRs the two curves separate and their slopes reduce. These two changes result in considerable increases in bias errors, i.e. errors in the ASR estimates arising from uncertainty in the value of the Doppler shift. Relating the bias errors to the value of the measured $R(\text{ASR})$ ratio, it is found that they become large for $R(\text{ASR})$ values greater than 6, as shown in the following examples. For a measured $R(\text{ASR}, 10:6) \approx 6.1$ (y -scale), the corresponding ASR (x -scale) lies between ~ 8 and 10, i.e. a potential bias uncertainty of ~ 2 . For a Doppler shift measured with random error of ± 12.5 MHz, the mean bias over the range would be approximately $\pm 2 \times (12.5/200) \approx \pm 0.125$, i.e. smaller than the corresponding σ_{ASR} . However, consider a measured $R(\text{ASR}, 10:6) \approx 7.5$; per Fig. 16, the corresponding T2 and T1 values for ASR are ~ 20 and ~ 42 . With a Doppler uncertainty of ± 12.5 MHz, the mean bias error would be $\sim 22 \times (12.5/200) \approx \pm 3$ and notably larger than the σ_{ASR} error over this range of ~ 0.5 to ~ 1.6 . Note that these examples used simple linear interpolation between the T1 and T2 calibration curves: if a more accurate estimate is needed, it could be obtained by generating additional curves at intermediate Doppler frequencies to account for the nonlinearity of R as a function of Doppler shift.

6.4 Summary and discussion

In summary, the simulations show that the choice of calibration ratio should be guided by the specific ASR range of interest. For the primary atmospheric application, namely discrimination and quantification of aerosol and cloud scattering against the molecular background, the $R(\text{ASR}, 4:4)$ ratio provides good sensitivity in the low-ASR regime ($1 \lesssim \text{ASR} \lesssim 2$). In this range, the predicted random uncertainty in ASR is of order 10^{-2} , corresponding to a relative uncertainty of a few percent in the aerosol contribution, $\epsilon_{\text{aer}} = \sigma_{\text{ASR}}/(\text{ASR} - 1)$.

At high ASR, alternative pixel groupings can extend the dynamic range, with values of several tens indicated by the simulations. However, the calibration curves become flatter and more sensitive to Doppler shift, making this regime less robust. Thus, high-ASR retrieval should be regarded as a secondary capability rather than the primary atmospheric application.

7 VALDI measurement of atmospheric temperature

The changing shapes of the molecular scattering fringe profiles, each normalised to unit energy within a full FSR, are illustrated in Fig. 17 for the temperatures indicated. As noted, the raw input profiles were constructed from the US Standard Atmosphere (1976) (T and p), and have then been used as input to the VALDI Fizeau interferometer, as modelled with $\Gamma_{\text{FSR}} = 8.8$ GHz. Fig. 17 immediately indicates the increasing energy redistribution from the central Rayleigh peak at higher altitude (lower temperature and pressure) into the two shifted Brillouin components (centred at $\sim \pm 2$ GHz) at lower altitude and higher temperature and pressure. These changes of shape suggest that the profiles may be analysed with calibration curves based on the relative distribution of fringe energy across the profiles, as recorded in the pixel contents.

7.1 Basic Temperature Estimation

Fizeau fringes incorporating the standard instrumental profile (for $\text{AOI} = -0.11$ mrad, $\text{FOV} = 0.5$ mrad and $\Gamma_{\text{FWHM}} = 0.4$ GHz) have been simulated for 9 molecular profiles at temperature intervals of 10 K in the range 210 K to 290 K with

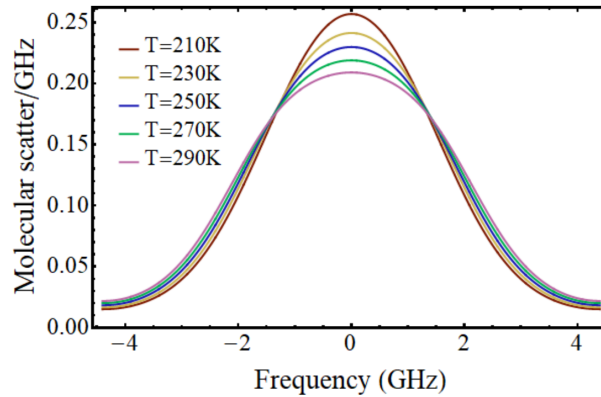


Figure 17. RB spectra (standard atmosphere) for 5 selected temperature/pressure pairs convolved with the Fizeau instrumental profile of 0.4 GHz FWHM. The actual values used are $T = 210$ K, 100 hPa (16200 m), $T = 230$ K, 312 hPa (8900 m), $T = 250$ K, 478 hPa (5900 m), $T = 270$ K, 719 hPa (2800 m), $T = 290$ K, 1013 hPa (0 m).

corresponding pressures taken from the US Standard Atmosphere (1976). Inspection of Fig. 17 shows that all the profiles have very similar intensity in the region of ± 1.2 GHz. For the 16 channels of VALDI, each 0.4 GHz wide, the most obvious temperature calibration ratio is the comparison of the inner 6 pixels (i.e. extending across ± 1.2 GHz), with the outermost 10 pixels, to form the ratio:

$$R(T, 6:10) = \frac{\Sigma_{\text{px}(6 \text{ to } 11)}}{\Sigma_{\text{px}(1 \text{ to } 5)} + \Sigma_{\text{px}(12 \text{ to } 16)}}. \quad (16)$$

This calibration curve is shown, for the T2 symmetric fringe with frequency shift $\Delta f = 0$, in the lower plot of Fig. 18 (a, solid circles). Using the techniques outlined in Sect. A3, a statistical temperature uncertainty $\sigma_{R(T)}$ may be calculated and also the local slope $\partial R / \partial T$ of the calibration curve at any required temperature. The resultant temperature error is then given by $\sigma_T = [\sigma_{R(T)} \times (N_{\text{mol}})^{1/2}] / [dR/dT]$. For the basic calibration curve of Fig. 18 (a), at the selected mid-altitude temperature of 250 K (≈ 5.8 km a.s.l.) and $N_{\text{mol}} = 3.05 \times 10^4$ ph.el., the temperature error is calculated to be $\sigma_T[R(T, 6:10)] \approx 3.5$ K.

This result is somewhat higher than originally estimated from a simple analysis based on line widths. For profiles of near-Gaussian (and also near-Lorentzian) profile it is shown (see e.g. Vaughan, 1989, Appendix 10) that, for a basic "median type" statistical analysis, the standard deviation error σ_w for the line width w (FWHM) is typically a factor 2 times the error of the centre frequency σ_f . From the extensive analysis/discussion of Sect. 5, a best value of $\sigma_f \approx 13$ MHz would be expected (for $T \approx 250$ K and $N_{\text{mol}} = 3.05 \times 10^4$ ph.el.); in consequence σ_w would be ~ 26 MHz. From Fig. 17, the mid-region (~ 250 K) change of FWHM with temperature may be estimated at ~ 12 MHz K $^{-1}$. This would suggest a temperature error of $\sigma_T \approx 26/12 \approx 2.2$ K, i.e. significantly lower than the error for the $R(T, 6:10)$ calculation of ~ 3.5 K above from Fig. 18 (a). These considerations prompted further investigation of the formation of $R(T)$ and the many possible different pixel groupings.

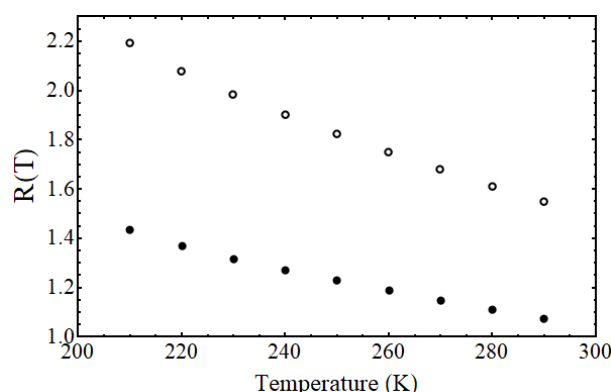


Figure 18. Temperature calibration curves versus temperature for T2 fringes. The lower curve (a, solid circles) is for $R(T, 6 : 10)$, and the upper curve (b, open circles) is for $R(T, 6 : 8)$ which omits pixels 5 and 12 from the denominator.

7.2 Investigation of optimum calibration ratios

Examination of the full set of pixel contents for the symmetric T2 fringe reveals (see Sect. 6.2 and Table 3) that the fractional energy within pixels 5 + 12 is almost invariant, changing by $\lesssim 0.001$ from 210 K to 290 K. In contrast, the other 6 pixel pairs have content differences over the temperature range 80 K amounting to ~ 0.006 to 0.019. These findings suggest that the most efficient calibration ratios would exclude pixels that change little with temperature and that carry little relevant spectral information — most notably px 5 and px 12. Many variants of pixel groupings have been examined, of which the simplest is:

$$R(T, 6:8) = \frac{\Sigma_{\text{px}(6 \text{ to } 11)}}{\Sigma_{\text{px}(1 \text{ to } 4)} + \Sigma_{\text{px}(13 \text{ to } 16)}}. \quad (17)$$

This temperature calibration ratio is shown in the upper curve of Fig. 18 (b, open circles) for the T2 fringe. The resultant standard deviation temperature error σ_T is ~ 3.2 K at 250 K, as shown in Fig. 19. This is a modest improvement on 3.5 K for the calibration ratio $R(T, 6:10)$. It may be noted that a comparable improvement is also found for the ratio $R(T, 4:10)$, i.e. additionally excluding px 6 and px 11 in the numerator.

Of immediate interest in Fig. 19 is the rapid, near-linear reduction of $\sim 19\%$ for σ_T (from 3.2 K to 2.6 K) in the temperature range 200 K to 250 K. This is of course the region over which the molecular scattering profiles remain close to Gaussian, before becoming increasingly flattened and distorted at the higher temperatures and pressures. This 19% linear reduction in error σ_T is quite close to the quasi-linear reduction of line width down to ~ 3.3 GHz FWHM.

Before concluding this analysis, it is worth finally questioning why the error σ_T at 250 K of ~ 3.2 K is still notably greater than the ~ 2.2 K at the end of Sect. 7.1. Inspection of Fig. 17 and the fractional energy content of the 16 detector channels reveals the non-trivial (and changing) energy across the full FSR of 8.8 GHz, which is not recorded within the USR of 6.4 GHz. Simulations show that these are equivalent to ~ 0.051 fractional content at 220 K, increasing to 0.067 at 280 K. Clearly, this

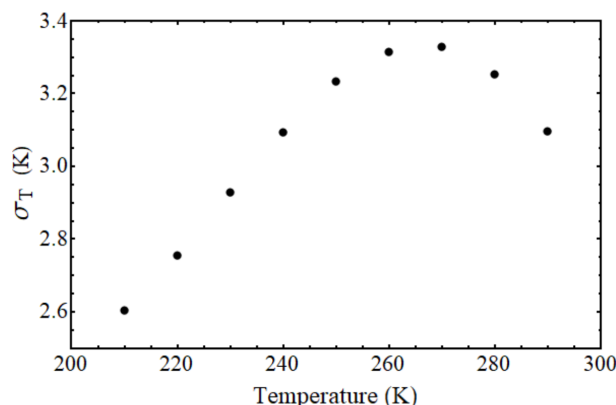


Figure 19. Predicted noise standard deviation σ_T of temperature estimates based on the ratio $R(T, 6 : 8)$, assuming a full N_{mol} of 3.05×10^4 ph.el. T2 fringes.

extra spectral information is not incorporated in Eq. (17). A calculation across the full FSR is readily simulated by forming:

$$R(T, 6:8 + \text{remaining FSR}) = \frac{\Sigma_{\text{px}}(6 \text{ to } 11)}{[\Sigma_{\text{px}}(1 \text{ to } 4) + (\text{signal in } -4.4 \text{ to } -3.2 \text{ GHz})] + [\Sigma_{\text{px}}(13 \text{ to } 16) + (\text{signal in } 3.2 \text{ to } 4.4 \text{ GHz})]} \quad (18)$$

730 It is found that at 250 K, the resultant temperature error σ_T reduces to ~ 2.8 K. Clearly, for estimation of temperature, incorporation of the total signal and its additional spectral information within the full fringe gives a useful 0.4 K improvement in accuracy (i.e. a further 14%).

7.3 Impact of frequency shift and aerosol scattering

7.3.1 Frequency shift

735 The foregoing analyses have concentrated on the symmetrical T2 fringe with Doppler frequency shift $\Delta f = 0$ MHz. Sets of fractional pixel content, for all the molecular profiles, are readily derived for different frequency shifts, leading to the corresponding calibration curves. This is simply illustrated in the data of Table 4, showing the $R(T, 6:8)$ values at temperatures of 220 K, 250 K and 280 K, for the T2 fringe (in the 2nd row), and at the other extreme the T1 fringe (± 200 MHz shift) in the 3rd row. The differences $\Delta R(200 \text{ MHz})$ are in row 4, and the local slope $\partial R / \partial T$ for the T2 calibration curve (as in Fig. 18, 740 b) in row 5.

The last two rows indicate possible systematic/bias errors σ_T in the derived temperatures. Consider the one extreme of an $R(T)$ value for the T1 fringe referenced to the T2 calibration curve; the resultant error ΔR , on division by the local slope, leads to the ΔT errors shown in row 5, ranging between ~ 2.0 K and 3.4 K. In practice, for any such fringe, the Doppler shift should be measured within ~ 15 MHz (see Sect. 5). The experimental measured $R(T)$ values may then be referenced to the

**Table 4.** Simulated data for the temperature calibration curve $R(T, 6:8)$, Eq. (17), for T2 and T1 fringes as illustrated in Fig. 18.

	220 K	250 K	280 K
T2 fringe 0 MHz	2.077	1.821	1.609
T1 fringe ± 200 MHz	2.042	1.798	1.596
$\Delta R(\pm 200 \text{ MHz})$	0.035	0.023	0.013
T2; slope $\partial R / \partial T$	$1.04 \times 10^{-2} / \text{K}$	$0.76 \times 10^{-2} / \text{K}$	$0.64 \times 10^{-2} / \text{K}$
$\sigma_T(\pm 200 \text{ MHz})$	3.4 K	3.0 K	2.0 K
$\sigma_T(\pm 15 \text{ MHz})$	$\sim 0.26 \text{ K}$	$\sim 0.23 \text{ K}$	$\sim 0.15 \text{ K}$

appropriate calibration curve at the closest frequency. In consequence, for quasi-linear spread, the systematic/bias error ΔT should be less than $\sim 3.4 \times (15/200) = 0.26 \text{ K}$ as indicated.

The remaining question is whether the standard deviation σ_T of Fig. 19 changes appreciably between T2 and T1 fringes. Simple calculation indicates a very small increase of $\lesssim 0.12 \text{ K}$ for the T1 fringe across the full atmospheric temperature range. Hence, the pixelation effect only plays a minor role for the temperature retrieval.

7.3.2 Aerosol scattering

Inspection of pixel content for the composite fringe (i.e. incorporating both molecular and aerosol scattering) establishes that a small amount of aerosol scattering (largely concentrated in the 4 central pixels 7 to 10) will greatly distort the delicate balance of the $R(T)$ calibration ratios. Simple calculation for a fringe profile with $\text{ASR} = 1.1$ (and $N_{\text{mol}} = 3.05 \times 10^4$) leads to large systematic bias errors of $\gtrsim 30 \text{ K}$. For measurements in the atmosphere this very high sensitivity has important implications.

In experimental practice it is impossible to maintain that an atmospheric observation is purely molecular with zero aerosol contribution. The previous Sect. 6 has examined measurements of ASR in detail (see e.g. Figure 15). For a relatively strong $N_{\text{mol}} = 3.05 \times 10^4 \text{ ph.el.}$ the statistical standard deviation for vanishingly small N_{aer} is given by $\text{ASR} = 1.0 \pm 0.0069$. At nominal temperature 250 K, this gives an equivalent uncertainty of $\sigma_T = \pm 2.5 \text{ K}$.

The question is therefore whether measurements of ASR, per the calibration techniques of Sect. 6, for a given experimental fringe, can be applied as a correction to the composite fringe data (i.e. the pixel content) to calculate temperature estimates of useful precision? The problem is well illustrated by inserting numerical values, drawn from the VALDI parameters and the 250 K molecular T2 fringe profile, into the composite calibration ratio of Eq. 17:

$$R(T, 6:8) = \frac{N_{\text{mol}} [(0.5185) + (\text{ASR} - 1)(0.9015)]}{N_{\text{mol}} [(0.2847) + (\text{ASR} - 1)(0.0496)]}. \quad (19)$$

For an experimental fringe, one can estimate the value of ASR and use this estimate to remove, to first order, the ASR dependent terms in Eq. (19). The question then is, what are the residual temperature errors after this removal has been applied? For general



discussion, this form of calibration function may be expressed as:

$$R(T) = \frac{N_{\text{mol}}[C_1 + (\text{ASR} - 1)C_2]}{N_{\text{mol}}[C_3 + (\text{ASR} - 1)C_4]}. \quad (20)$$

The calculation of the residual error is given in Appendix A2. The variance for the corrected temperature ratio, here called $R'(T)$, is given by the sum of 3 terms:

$$\sigma_{R'}^2(T) = \frac{K_1 + (\text{ASR} - 1)^2 K_2 + N_{\text{mol}} \sigma_{\text{ASR}}^2 K_3}{N_{\text{mol}}}, \quad (21)$$

where K_1 to K_3 are numerical terms derived from C_1 to C_4 . It is notable that K_1 depends only on the molecular terms C_1 and C_3 (Eq. (20)) and would give the variance in, for example, laboratory conditions of zero-assured aerosol content. In atmospheric conditions the 2nd and 3rd terms (right-hand side of Eq. (21)), contain $(\text{ASR} - 1)^2$ and σ_{ASR}^2 , and establish the strong dependence on any aerosol scattering present. Evaluations based on this analysis are shown in Figure 20 where standard deviation errors σ_T are plotted for ASR = 1.0 to 2.0. These results are for the molecular scattering at intermediate 250 K (standard atmosphere), the T2 fringe and $N_{\text{mol}} = 3.05 \times 10^4$ ph.el. The ASR estimates were produced using the $R(\text{ASR}, 4 : 4)$ ratio, as described in Section 6.2. It is assumed that the slope dR'/dT is, to good accuracy, equal to the slope dR/dT , evaluated at ASR=1. The slope was calculated from the full set of spectra at the different temperatures by fitting a cubic polynomial to the resulting $R(T)$ values and evaluating its derivative at $T = 250$ K.

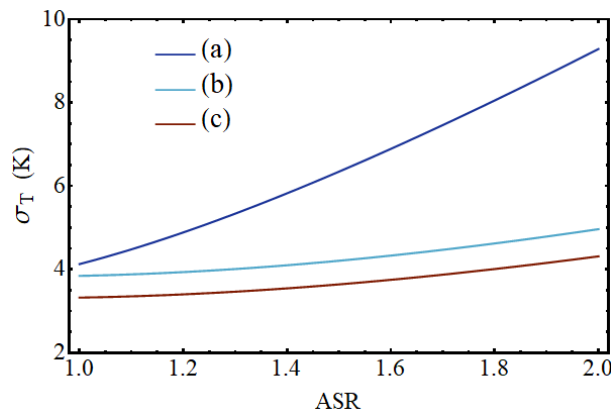


Figure 20. Standard deviation errors σ_T versus ASR for $T = 250$ K and $N_{\text{mol}} = 3.05 \times 10^4$ ph.el. Upper curve (a) $R(T, 6:8)$; middle curve (b) $R(T, 4:8)$, lower curve (c) $R(T, 4:14 + \text{outer regions})$ for the full FSR.

The upper curve (a) shows the results for the calibration ratio $R(T, 6:8)$ per Eq. (19). The temperature error σ_T rises significantly from 4.1 K to 9.5 K at ASR = 2, because of the large value of the 3rd term in Eq. (21): this is due to the very large aerosol term $C_2 = 0.9015$ in Eq. (20) arising from the dominant central pixels 8 and 9.

The mid-curve (b) shows the result of removing these 2 large pixels to form $R(T, 4:8)$, where the numerator = $\Sigma P_x(6, 7, 10, 11)$ and $C_1 = 0.3358$ (molecular) and $C_2 = 0.1946$ (aerosol). The resultant σ_T values rise from 3.7 K to 4.7 K over the range, and are notably smaller than (a), particularly at larger ASR.



As a final note, further simulations with a calibration ratio $R(T, 4:14 + \text{outer regions})$, similar to Eq. (18), may be extended to the full FSR of ± 4.4 GHz, as discussed at the end of Sect. 7.2. Incorporation of this extra spectral information further improves the accuracy σ_T to the range 3.3 K to 4.2 K, as shown in the lower curve (c).

7.4 Summary and Conclusions

790 Measurement of atmospheric temperature with lasers has been discussed since 1981 (Schwiesow and Lading, 1981). It was first demonstrated in 1992 by (She et al., 1992) and (Alvarez et al., 1993), using a barium absorption cell to separate the molecular Raman scattering signal from the aerosol signal. The field was outlined by (Witschas et al., 2014) who presented daytime, ground-based lidar temperature measurements from RB scattering; they employed a Fabry–Pérot interferometer with a frequency-scanned Nd:YAG laser operating at 355 nm. Typical measurement times, for a full atmospheric profile up to
795 ~ 15 km altitude, were 14 or 28 minutes. Comparison with adjacent/contemporary radiosonde data showed good agreement better than ~ 2.5 K. Their statistical error calculations are in close agreement with the analyses presented above, and for the measured signals at different altitudes lay in the range 0.15 K to 1.5 K.

It is particularly notable that in a concluding discussion, (Witschas et al., 2014) pointed to the advantages of simultaneously scanning the entire RB spectrum that would be presented by a Fizeau interferometer. The present analysis for VALDI carries
800 this discussion further and indeed indicates the potential for a temperature profile measurement from space, within a ~ 12 s experimental observation period.

As a very final point, it may be noted that the discussion of Sect. 2.2 suggests that, with improved alignment accuracy etc., a potential signal gain (PSG) of a factor ~ 4 should be available. If, indeed, this were achieved the statistical accuracies above would be improved by the square-root factor ≈ 2 , i.e. approaching standard deviation temperature errors close to 1.5 K.

805 8 VALDI: Optical design and control considerations

The VALDI concept has potential application to atmospheric measurements from ground-based, airborne and spaceborne platforms. Considered separately as a laser Doppler velocimeter, it also has the potential for line-of-sight velocity measurements over a range exceeding ± 1 km, s $^{-1}$. The present paper has largely concentrated on the most demanding atmospheric task of spaceborne operation, drawing from lessons and experience with ALADIN.

810 In Sect. 2 a brief outline of some of the optical problems that became evident during ALADIN operation was presented. These have been examined in detail in Witschas et al. (2022) for both Fabry–Pérot and Fizeau channels, and for the Fizeau channel in particular in Vaughan et al. (2025). In broad summary, the problems may be categorised as: control and monitoring of the outgoing laser beam; optical control of the incoming received signal beam path; and minimisation of parasitic background illumination in the receiver channel. These topics are briefly discussed in the following three sub-sections. It may be noted that
815 the single optical receiver of VALDI should offer in principle the potential for relatively simple and robust solutions.

These topics have also received considerable attention in the design and pre-development activities for the follow-on Aeolus-2 mission (Porciani et al., 2025). In particular, the current Aeolus-2 design concept includes dedicated measures for monitoring



and controlling the outgoing laser beam and for maintaining the alignment of the transmit and receive lines of sight. A co-alignment sensor is foreseen for monitoring the receiver line-of-sight depointing, while a Laser Beam Monitoring Device (LBMD) is planned to monitor the near-field profile of the outgoing laser beam. In addition, laser-frequency monitoring is foreseen through an internal optical path using a fibre. These developments illustrate that several of the control and monitoring issues identified from ALADIN experience are already being addressed in the design of the next-generation Aeolus instrument (Heliere et al., 2023; Berceau et al., 2025; Ciapponi et al., 2026).

8.1 Control and monitoring of the outgoing laser beam

Experience with ALADIN suggests that a monitor of the laser characteristics as the beam first emerges from the laser unit is highly desirable. A multi-channel detector of modest specification, probably no more than 6×6 elements, illuminated with a very tiny fraction of the beam, would provide an immediate measure of pulse energy, together with beam spectral profile and angle. A similar alignment sensor is for instance used by the lidar on-board ESA's EarthCARE satellite (Wehr et al., 2023). This information would give an early diagnostic indication of changes of laser beam power and shape, both within the laser cavity optics and train (due e.g. to induced contamination or damage), as well as changes of cavity structure and alignment. Such changes are a vital component for analysis and understanding of overall system performance. Also for the Aeolus-2 mission, a dedicated laser beam monitoring device is planned to monitor the near-field profile of the outgoing laser beam, while a co-alignment sensor is foreseen to provide information on the relative pointing of the transmit and receive lines of sight.

A further problem is monitoring of laser frequency output. The Fizeau interferometer itself offers an optimum solution and was utilised in ALADIN (Lux et al., 2021). However in this case the beam (i.e. a very small fraction thereof) was introduced directly, with some magnification, into the interferometer. This provided a non-uniform illumination (see e.g. Witschas et al., 2022; Reitebuch et al., 2024). It was indeed possible to construct a rather indirect beam profile by successive tuning of the laser frequency across the full USR range. However assessment of a single set frequency would be subject to systematic bias error due to non-uniform illumination. A beam scrambling optical fibre for provision of more uniform illumination could be a possible solution, or gathering of diffuse light from a suitably positioned scattering surface. Neither solution is trivial and would require detailed analyses and practical investigation for space-qualified equipment. For Aeolus-2, laser-frequency monitoring is instead foreseen through an internal optical path using a fibre, illustrating one possible approach to avoiding the non-uniform illumination encountered with the direct injection used for ALADIN.

8.2 Control of the incoming signal beam

The analysis of Sect. 4.1 establishes that the Fizeau instrumental profile is strongly influenced by the AOI and FOV of the signal beam incident on the interferometer. The experience of ALADIN showed considerable initial and progressive errors that amounted to several tens of microradians (Witschas et al., 2022; Vaughan et al., 2025), and required extensive evaluations. In the laboratory and "hands-on" environment, very precise angular control, typically via successive mirror adjustment, is readily achieved. Remotely guided operation, even for a single optical channel, presents considerable challenges for tunability and



control in a space environment. However it is evident that any large mishap at launch or during operation could, without any remedial/corrective capability, lead to gross loss of signal and optical resolution and amount to a “single point failure” and loss of mission.

8.3 Minimisation of background illumination

855 The illumination entering a VALDI-type instrument would ideally be restricted to an accurately centred square profile of bandwidth no more than the Fizeau FSR. For ALADIN, a workable solution for solar background was achieved with a narrow-band dielectric optical filter combined with a small pinhole to provide a tight spatial filter. Such high optical extinction was in fact a long-standing problem of classical spectroscopy and several solutions were developed over the years (see e.g. Chapter 6 in Vaughan, 1989). These typically involved separate interferometers of carefully selected spacer separation (i.e. spectral range),
860 or successive multi-passing (usually 3 or 5 passes) of a single interferometer. It may be worth revisiting this range of techniques for possible application for the present task.

As a final point, it is worth noting that a uniform parasitic background should not introduce bias errors in a Doppler frequency measurement. However loss of statistical accuracy may be severe and not easily recognised.

9 Summary and Conclusions

865 9.1 Summary

The foregoing Sections 4 to 7 provide extensive analysis and discussion of the principal features and potential performances of the VALDI system, with particular emphasis on spaceborne application. The following Table 5 seeks to provide a simple overall performance guide and summary for this most demanding application.

One of the most significant background findings is the power and flexibility of suitably constructed calibration ratios (and
870 indeed their potential applicability as a wide-ranging analytical technique). It is presently demonstrated for VALDI that suitably constructed ratios can be used to retrieve four atmospheric quantities from a single spectral fringe, with the statistical independence of the two Doppler estimates arising from their use of distinct pixel groups. Moreover these measurement values have a readily calculated statistical accuracy approaching the fundamental photo-detection quantum limits of performance.

Table 5 gives a “broad brush” view of the attainable accuracies with the signal levels described in Sect. 2.2 as estimated
875 from ALADIN operation. However, with the potential signal gains noted at the end of that section, these accuracies would be improved and the errors reduced by a factor in the range 1.4 to 2.4. Individual measurements are of course totally dependent on the individual atmospheric conditions and notably the optical attenuation on the transit path to and from the lidar equipment. For many such scenarios and for large regions of the atmosphere, the present calculations for VALDI establish valuable levels of performance within useful specification.



Table 5. Potential performance of a spaceborne VALDI, drawn from the analyses of Sections 4 to 7. Such calculations are critically dependent on the many atmospheric parameters and assumptions of which actual atmospheric attenuation is highly variable and of prime importance. The performance uncertainties presented above are reasonable estimates including also the experience and guidelines drawn from ALADIN.

	PBL and lower troposphere (1 to 4 km)	Mid and upper troposphere (3 to 14 km)
Aerosol wind measurement (Sect. 4)	Useful measurement requires high spatial resolution. For 12 km path length, $ASR = 2$ and low attenuation, $\sigma_{v_{HLOS}} \approx 1.4 \text{ ms}^{-1}$. This increases to $\sigma_{v_{HLOS}} \approx 3.2 \text{ ms}^{-1}$ for attenuation $F_{att} \sim 5$, but would be reduced for larger ASR.	For 12 km path and low $F_{att} \sim 1$, $\sigma_{v_{HLOS}} \approx 3.6 \text{ ms}^{-1}$ for $ASR = 1.25$. For 87 km path, low attenuation and $ASR \approx 1.25$, $\sigma_{v_{HLOS}} \approx 1.4 \text{ ms}^{-1}$.
Molecular wind measurement (Sect. 5)	Requires molecular signal accumulation over 87 km path. For $ASR \sim 1$ and low attenuation, $\sigma_{v_{HLOS}} \approx 3.4 \text{ ms}^{-1}$. For $ASR \sim 2$ the aerosol scattering in fact improves accuracy so that $\sigma_{v_{HLOS}} \approx 2.4 \text{ ms}^{-1}$. See the discussion of Sect. 5.2.	
Atmospheric scattering ratio (Sect. 6)	Performance varies with the range of ASR under investigation and the optimum choice of calibration ratio. In broad terms: for low attenuation with $ASR \lesssim 1.5$, $\sigma_{ASR} \lesssim 0.015$. Much larger ASR values, of approximately 5 to 50, can be evaluated in favourable conditions, with statistical uncertainties that can be below $\sim 10\%$, depending strongly on attenuation and Doppler-shift uncertainty.	
Atmospheric temperature measurement (Sect. 7)	Has to be derived from the molecular scattering. Highly dependent on low atmospheric attenuation and degraded by aerosol scattering. At best for $ASR \lesssim 2.0$, $\sigma_T \lesssim 4.5 \text{ K}$.	For low attenuation and $ASR \lesssim 1.25$, and the signal gathered across the full FSR, $\sigma_T \approx 2.8 \text{ K}$ at $\sim 5 \text{ km}$, and $\approx 2.3 \text{ K}$ at 12 km .

880 9.2 Conclusions

In conclusion it may be noted that VALDI draws on two important lessons of the ALADIN operation. Firstly the utility of calibration ratios as employed for analysis of molecular data in the Fabry–Pérot channel. Secondly the robust and reliable operation of the ALADIN Fizeau interferometer aerosol channel. The present paper on VALDI combines these aspects in a single interferometric optical channel and a single detector and offers versatile, multi-task, operation from the appropriately
885 selected detector channels of spectral information. The authors are optimistic about experimental verification in a prototype deployment in the reasonably near future.

Code availability. The code that is used for modelling and data analysis (Mathematica, LabVIEW) and for figure plotting (Mathematica, OriginLab) can be provided upon request.

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Data availability. The particular data sets used in this study can be provided upon request.



890 **Appendix A: Error analysis of the $R(A, B)$ and $R(4)$ methods**

A1 Introduction to the ratio measurements

The basic idea of a ratio measurement is to produce a normalised measure of fringe asymmetry relative to some central point, e.g. the middle of the detector array. This asymmetry changes when the fringe moves, to the left or the right, as a result of a change in the frequency. If the relationship between ratio value and frequency is known, through calibration, one can work
895 back from the measured ratio to find the actual change in frequency. It turns out that, for modest frequency changes, the ratio usually varies in a linear fashion, so only the slope is required for calibration purposes.

Consider, first, the $R(A, B)$ ratio. This is given by

$$R(A, B) = \frac{A - B}{A + B} \quad (\text{A1})$$

Here, A and B represent the content of detector pixels; in the simplest case these could be individual pixels but, in general,
900 they are groups of pixels, i.e. the sums of multiple pixel contents. For example, if A and B were simply the central two pixels of the fringe, the difference $A - B$ would be zero for a perfectly symmetric fringe, then positive if the fringe moves in the direction of pixel A (so $A > B$) and negative if the fringe moves in the direction of pixel B . Division by the sum $A + B$ removes dependency on the overall fringe height. Note that this is the same ratio that is used for Rayleigh wind retrieval in Aeolus via the double-edge method.

905 The $R(4)$ ratio uses 4 pixel groupings, and therefore requires at least 4 pixels. If we assign these groupings to consecutive alphabetical letters, following on from A and B , the ratio is defined as

$$R(4) = \frac{(C + D) - (E + F)}{(D + E) - (C + F)} \quad (\text{A2})$$

In the case of 4 pixels, D and E would be the central pixels, with C and F on either side. It can be seen that the numerator of Eq. (A2) is very similar to that of Eq. (A1). Indeed, if A is the group $C + D$ and B is $E + F$ they are identical. The denominator,
910 however, is different. It is the sum of the two largest pixel contents (assuming a monotonic fringe) minus the sum of the two smallest. The idea of this formulation is that it will remove a constant background component, and thus make the calibration easier in the presence of such a background. It should be noted that the background will always add to the statistical uncertainty of all the terms, but for $R(A, B)$ it introduces an additional, and potentially more damaging, bias error.

A2 Errors in the ratio measurements resulting from shot noise

915 We start by analysing a generic ratio Z , before applying the results to $R(A, B)$ and $R(4)$

$$Z = \frac{X}{Y} \quad (\text{A3})$$

Noise can be included by making the terms stationary random variables and writing them as a sum of the mean value and a fluctuating component

$$\bar{Z} + \delta Z = \frac{\bar{X} + \delta X}{\bar{Y} + \delta Y}, \quad (\text{A4})$$



920 where the over-bar indicates a mean value. We will assume that the noise in the denominator is small, i.e. $\bar{Y} \gg \delta Y$. Then, the denominator can be expanded to first order, giving

$$(\bar{Y} + \delta Y)^{-1} \approx \bar{Y}^{-1} \left(1 - \frac{\delta Y}{\bar{Y}}\right), \quad (\text{A5})$$

and

$$\bar{Z} + \delta Z \approx \bar{Z} + \frac{\delta X}{\bar{Y}} - \bar{Z} \frac{\delta Y}{\bar{Y}}. \quad (\text{A6})$$

925 Note that the expectation value of $(\delta Z)^2$ is the variance of Z . Subtracting $\bar{Z}$ from both sides of A6, squaring and taking expectation values gives

$$\sigma_Z^2 \approx \frac{1}{\bar{Y}^2} (\sigma_X^2 + \bar{Z}^2 \sigma_Y^2 - 2\sigma_{XY}). \quad (\text{A7})$$

Here, in addition to the variances, we have the covariance $\sigma_{XY} = \langle XY \rangle - \bar{X}\bar{Y}$, with the angled brackets indicating expectation values.

930 In the case when the fluctuations in Z are sufficiently small that the curve of Z with respect to the quantity of interest (e.g. Doppler shift, ASR, etc.) is, to a good approximation, a straight line, then the variance of that quantity is simply the variance of Z divided by the square of the local slope of the curve.

If the noise is dominated by photoelectron shot noise, we can use the fact that shot noise follows Poisson statistics and the variance is equal to the mean. In the ratio method, X and Y are simply sums and differences of various pixel combinations and
935 their noise will also follow Poisson statistics. Thus, $\sigma_X^2 = \bar{X}$ and $\sigma_Y^2 = \bar{Y}$. In addition, the noise on different pixel outputs is uncorrelated so their covariance is zero.

Applying this to the $R(A, B)$ ratio, and noting that the noise in A is not correlated with the noise in B because they do not contain any pixels in common, use of Eq. (A7) gives

$$\sigma_{R(A,B)}^2 \approx \frac{4\bar{A}\bar{B}}{(\bar{A} + \bar{B})^3}. \quad (\text{A8})$$

940 Note that near the T2 fringe position we have $A \approx B$ and therefore $\sigma_{R(A,B)} \approx (A + B)^{-\frac{1}{2}}$.

The same approach can be used to analyse the noise in the $R(4)$ ratio measurement. Again, the important fact is that the components C, D, E, F use mutually independent pixel groupings so their noise is uncorrelated. The resulting expression is somewhat more complicated than the $R(A, B)$ equivalent,

$$\sigma_{R(4)}^2 \approx \frac{4 [(\bar{C} + \bar{E})(\bar{D} - \bar{F})^2 + (\bar{D} + \bar{F})(\bar{C} - \bar{E})^2]}{(\bar{D} + \bar{E} - \bar{C} - \bar{F})^4}. \quad (\text{A9})$$

945 Note that, although the denominator contains the difference of two terms, it cannot be zero for a monotonic fringe because D and E are chosen to be the central pixels and these always have a greater average content than C and F respectively.

For the T2 position of a symmetrical fringe, one can set $D = E$ and $C = F$, giving the considerably simplified result

$$\sigma_{R(4)}^2 \approx \frac{(\bar{D} + \bar{C})}{2(\bar{D} - \bar{C})^2}. \quad (\text{A10})$$



This can be compared with the $R(A, B)$ result if it is assumed that the same pixel contents are used for both, i.e. $A = C + D$ and $B = E + F$. To compare the noise in the frequency estimates, at the T2 position, one needs to include the slopes of the calibration curves in each case. For the $R(A, B)$ ratio, the slope is

$$\frac{dR(A, B)}{df} = \frac{\gamma}{2(\bar{D} + \bar{C})}, \quad (\text{A11})$$

and for the $R(4)$ ratio it is

$$\frac{dR(4)}{df} = \frac{\gamma}{2(\bar{D} - \bar{C})}, \quad (\text{A12})$$

where

$$\gamma = \frac{d\bar{C}}{df} + \frac{d\bar{D}}{df} - \frac{d\bar{E}}{df} - \frac{d\bar{F}}{df}. \quad (\text{A13})$$

Combining Eq. (A11) and Eq. (A12) with Eq. (A8) and Eq. (A10), respectively, reveals that the frequency errors are the same for $R(A, B)$ and $R(4)$ and are given by

$$\sigma_f \approx \frac{\sqrt{2(\bar{C} + \bar{D})}}{\gamma}. \quad (\text{A14})$$

However, this is only for the special case of the symmetric T2 fringe. For a general fringe position, the frequency errors differ, as shown in Fig. A1 for a Lorentzian fringe with a pixel width on the same order as the fringe width. It can be seen that the predicted frequency errors are similar near zero frequency (which corresponds to the symmetrical T2 position) but that the $R(A, B)$ method has significantly greater errors near the edges of the frequency range. The dashed line is the Cramér–Rao lower bound (see next section). Both methods are close to the CRLB at the T2 position, and the $R(4)$ method remains reasonably close to it over the entire frequency range.

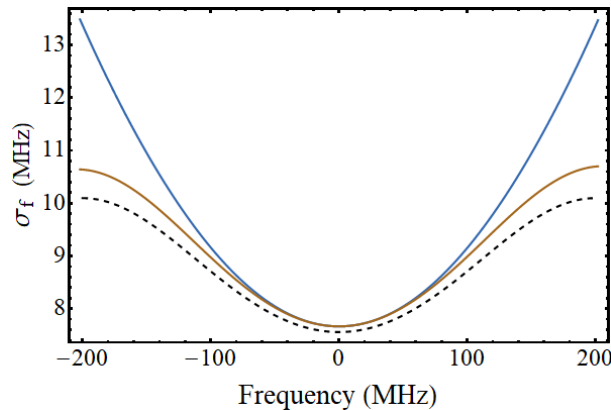


Figure A1. Frequency errors for the $R(A, B)$, blue, and $R(4)$, brown, ratio methods. This is for a Lorentzian fringe of 400 MHz FWHM containing 1600 photoelectrons. The pixel width is 400 MHz and the central 4 pixels are used for both ratios. The dashed line is the CRLB.



A3 Ratio method for ASR and temperature estimation

In Sect. 6, ratio measurements are expressed as a function of the true ASR, including separate contributions from aerosol and molecular scattering, see Eq. (11), for example. These expressions contain four parameters, written as C_1 to C_4 in Eq. (20), which represent four distinct photoelectron contributions to the pixel grouping used in the ratio measurements. In order to
 970 calculate errors for this ASR ratio measurement, one can treat the products $C_i N_{mol}$ as Poisson random variables and use Eq. (A7) to determine the standard deviation of the ratio. This result needs to be divided by the slope with respect to ASR, which can be readily determined by differentiating Eq. (20). The final result for noise in the ASR measurement, used for the noise curve in Fig. 15, is

$$\sigma_{ASR} = \frac{\sqrt{(C_3 + (ASR - 1)C_4)(C_1 + (ASR - 1)C_2)(C_1 + C_3 + (ASR - 1)(C_2 + C_4))}{N_{mol}^{\frac{1}{2}}(C_2 C_3 - C_1 C_4)} \quad (A15)$$

Further use of this ASR ratio occurs in Sect. 7.3. The idea explored in this section is reducing the ASR dependence of the temperature ratio by using the estimate of ASR obtained from the fringe data. This involves subtracting the following quantities from the numerator and denominator of Eq. (20), respectively: $(ASR + \delta ASR - 1)\bar{C}_2$ and $(ASR + \delta ASR - 1)\bar{C}_4$. Here, the estimated ASR has been written as a sum of the true ASR and the error in the estimate, δASR . It is important to note that it is assumed that the fractional pixel content of the aerosol scattering component is known to sufficient accuracy that its error
 980 is negligible compared to the error in the ASR estimate. Thus the terms in $\bar{C}_2$ and $\bar{C}_4$ are taken to be the true average values, without any associated errors. Substituting these into Eq. (20) leads to the following expression for the modified temperature ratio

$$R'(T) = \frac{N_{mol} [\bar{C}_1 + \delta C_1 + (ASR - 1)\delta C_2 - \bar{C}_2 \delta ASR]}{N_{mol} [\bar{C}_3 + \delta C_3 + (ASR - 1)\delta C_4 - \bar{C}_4 \delta ASR]} \quad (A16)$$

Here, noise in the coefficients C_i has been included explicitly. Noticing that this equation has the same functional form as
 985 Eq. (A3), and making the usual assumption that the fluctuating quantities are small, Eq. (A7) can be used to calculate the variance of the modified ratio measurement. Assuming that the noise in the C_i coefficients is mutually uncorrelated and uncorrelated with the noise in the ASR estimate, one obtains Eq. (21) in the main text, where $K_1 = \frac{\bar{C}_1}{\bar{C}_3^2} (1 + \frac{\bar{C}_1}{\bar{C}_3})$, $K_2 = (\frac{\bar{C}_2}{\bar{C}_3^2} + \frac{\bar{C}_1^2 \bar{C}_4}{\bar{C}_3^4})$ and $K_3 = (\frac{\bar{C}_2}{\bar{C}_3} - \frac{\bar{C}_1 \bar{C}_4}{\bar{C}_3^2})^2$.

Appendix B: The Cramér–Rao lower bound (CRLB) and the impact of detector pixellation and RB background

990 B1 The basic CRLB

The CRLB is a value of the standard deviation of the measured frequency which is a minimum in the sense that no unbiased frequency estimator can give a lower standard deviation (Casella and Berger, 2024). This does not, of course, guarantee the existence of an estimator that attains this lower bound, but if one finds an estimator that performs close to the bound then one can be assured that it is, in some sense, a good estimator. In addition, the way in which the CRLB varies as a function



995 of the various parameters involved can give useful insights regarding estimator performance. A result for the CRLB of a one-dimensional spectral line-shape (fringe) in the presence of Poisson-distributed photoelectron noise has been given by Winick (1986). In terms of the square of the CRLB, i.e. the variance, this is

$$\sigma_f^2 = \left(\sum_i \frac{[g'_i(f)]^2}{N_S g_i(f) + \lambda_N} \right)^{-1}, \quad (\text{B1})$$

where

$$1000 \quad g_i(f) = \int_{P_W(i-\frac{1}{2})}^{P_W(i+\frac{1}{2})} S(x) dx$$

and

$$g'_i(f) = \frac{dg_i(f)}{df}.$$

Here, $S(x)$ is the fringe profile, centred at frequency f and normalised to have unit area. N_S is the average photoelectron number across the fringe. λ_N is the average number of background photoelectrons per pixel, and i refers to the i th pixel in the
1005 array. P_W is the pixel width. Thus, the integral represents the output of a single pixel: the i th pixel.

It is convenient to take a factor of N_S outside of the summation so that the summation becomes a function of the ratio $\frac{\lambda_N}{N_S}$. A further useful normalisation is to divide the variable of integration x and the pixel width P_W by the FWHM of the line-shape, Γ_{FWHM} , which results in the term in g_i becoming independent of Γ_{FWHM} . Note that this independence is a consequence of $S(x)$ being inversely proportional to Γ_{FWHM} because $S(x)$ is normalised to have unit area. When the same normalisation by
1010 Γ_{FWHM} is applied to g'_i , the result is a multiplying factor of $1/\Gamma_{\text{FWHM}}$ owing to the differentiation with respect to f . Taking the square of this factor outside of the summation, one can write the CRLB, being the square root of Eq. (B1), in the general form

$$\sigma_f = \frac{C_{\text{CR}} \Gamma_{\text{FWHM}}}{\sqrt{N_S}}. \quad (\text{B2})$$

Here, the summation is now contained within the coefficient C_{CR} . Eq. (B1) can be evaluated numerically for any fringe profile
1015 and set of parameters.

In the case of negligible pixel width a version can be derived in which the summation is replaced by an integral:

$$\sigma_f^2 = \left(\int \frac{[S'(x)]^2}{S(x) + \frac{\lambda_N}{N_S}} dx \right)^{-1}, \quad (\text{B3})$$

where the integral is over the extent of the detector array. This integral can be evaluated analytically for the case of a Lorentzian fringe profile by applying a partial fraction decomposition to the kernel. If the limits are taken to be $-\infty$ to $+\infty$, i.e. assuming
1020 the detector array is large enough to encompass the entire fringe, one obtains the following expression,

$$C_{\text{CR}} = \frac{1}{\sqrt{2}} \frac{(\sqrt{1+c_1} + \sqrt{c_1})^{\frac{3}{2}}}{(\sqrt{1+c_1} + 3\sqrt{c_1})^{\frac{1}{2}}}, \quad (\text{B4})$$



where $c_1 = \frac{\pi}{2} \frac{N_B}{N_S}$, N_B being the average number of background photoelectrons within the fringe FWHM.

In the case of a Gaussian fringe profile, no solution for the integral in Eq. (B3) could be found in terms of known functions. However, results can be easily obtained via numerical integration. Fig. B1 gives results for both the Lorentzian (Eq. (B4)) and Gaussian fringes as a function of the ratio $\frac{N_B}{N_S}$, where N_B is defined with respect to the Gaussian or Lorentzian fringe FWHM. Note that, at $N_B = 0$, the curves give the known values of 0.425 for the Gaussian and 0.707 for the Lorentzian (Rye, 1998; Vaughan et al., 2025).

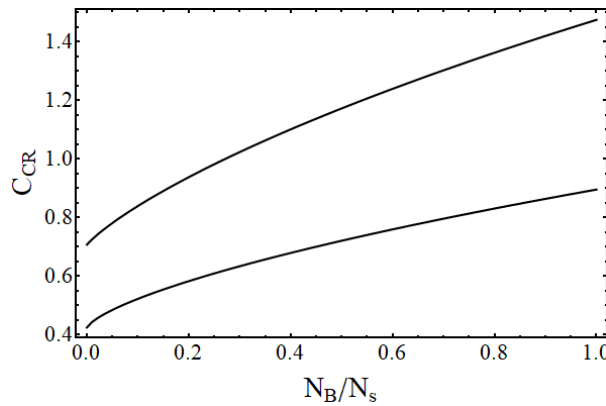
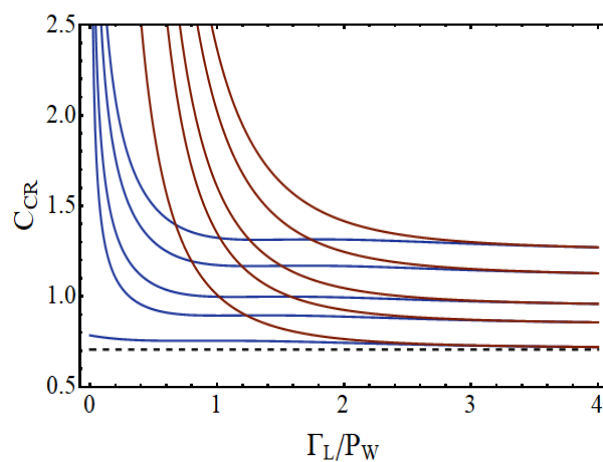


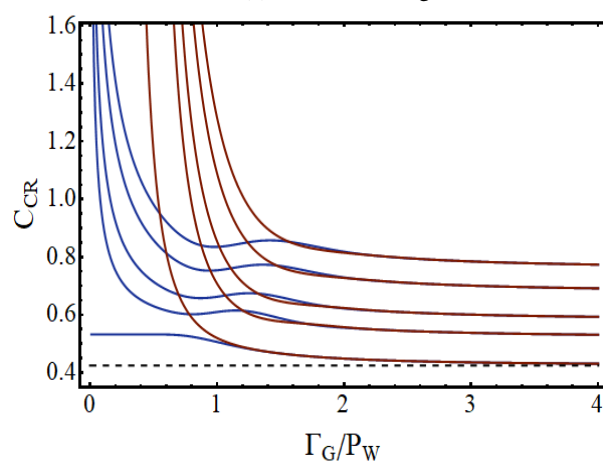
Figure B1. Cramér-Rao coefficient as a function of the background level for a Gaussian (lower curve) and Lorentzian (upper curve) fringe.

Fig. B2 gives results for the effect of changes in the ratio of fringe width to pixel width, obtained via numerical integration of Eq. (B1). Fig. B2a is for a Lorentzian fringe and Fig. B2b for a Gaussian. In these calculations, the pixel width was kept constant and the fringe width varied. The length of the detector array was sufficient that there was always negligible clipping of the fringe at the edges of the array (the fringe centre also being located near the centre of the detector array); this was achieved by having 32 pixels in the array. Calculations were carried out for a number of different background levels and for both T1 and T2 fringe positions.

It can be seen that when the pixel size is sufficiently small relative to the fringe FWHM the CRLB is essentially identical for the T1 and T2 fringes. The CRLB starts to differ when pixel sizes are large, and the point at which the separation occurs is a function of both the background level and the fringe shape. For the Lorentzian fringe, and the range of background levels covered in the figures, the separation occurs for a fringe width between 2 and 3 times the pixel width. For the Gaussian fringe this occurs between 1 and 2 times the pixel width. In this respect, the VALDI pixel size is somewhat non-optimum because the ratio between the Mie fringe FWHM and the pixel size is close to unity. However, based on the predicted background levels, the increase in the CRLB for the T2 fringe is small. However, this is not the case for the T1 fringe, and this shows the importance of operating under conditions close to the T2 optimum (see, also, Fig. A1). The value of the Cramér–Rao coefficient at the point of separation is generally greater than its value at negligible pixel width (see Fig. B1) but not by a large amount, thus there would be little benefit in making pixel sizes smaller beyond this point.



(a) Lorentzian fringe



(b) Gaussian fringe

Figure B2. Cramér–Rao coefficient as a function of the ratio of fringe FWHM to pixel width. The blue lines are for T2 fringes and the brown for T1 fringes. The different curves are for different values of the ratio $\frac{N_B}{N_S}$; the values, in order from bottom to top are: 0, 0.1, 0.2, 0.4, and 0.6. The dashed line is the coefficient for the case of no background and negligibly small pixels.



1045 It is worth noting that, in the special case of zero background, the Cramér–Rao coefficient for the T2 fringe remains finite even when the ratio of fringe width to pixel width goes to zero. This occurs because, under those conditions, the detector pixels act as an ideal edge detector with the fringe centre located exactly at their juncture, and the pixel width is irrelevant. However, as soon as this condition is broken, by moving slightly away from the ideal T2 fringe centring, or adding in some background, the Cramér–Rao coefficient diverges as the fringe width goes to zero, as one might expect.

Appendix C: Nomenclature

v_{LOS}	Doppler velocity along line-of-sight direction
v_{HLOS}	Doppler velocity along horizontal line-of-sight direction
$\sigma_{v_{\text{LOS}}}$	Wind velocity error (standard deviation) in line-of-sight direction
$\sigma_{v_{\text{HLOS}}}$	Wind velocity (standard deviation) error in horizontal line-of-sight direction
σ_f	Frequency shift error (standard deviation)
σ_T	Temperature error (standard deviation)
Γ_{FSR}	Free spectral range
$N_{\text{R}} = R^{1/2}/(1 - R)$	Reflectivity finesse
R	Plate reflectivity
A	Plate absorption
d	Plate spacing
D	Aperture diameter
d_{R}	Reflective coating diameter
α	Fizeau wedge angle
I/I_0	Relative peak intensity
S_{FSR}	Spatial free spectral range
Γ_{FWHM}	Fizeau fringe full width half maximum
Γ_{USR}	Useful spectral range
S_{USR}	Spatial useful spectral range
F_{D}	Frequency dispersion
N_{P}	Number of imaging detector channels
P_{w}	Spectral pixel width
N_{mol}	Signal from molecular backscatter
N_{aer}	Signal from aerosol backscatter
N_{bg}	Background signal



R_b	Ratio of background to signal
ASR	Atmospheric scattering ratio
F_{att}	Attenuation factor
C_{CR}	Cramér-Rao coefficient
T	Temperature
p	Pressure
λ	Wavelength
T1	Fringe with a single dominant pixel with equal energy content on either side
T2	Fringe with two central pixels of equal energy
$R(4)$	4-pixel ratio
$R(A, B)$	2-region ratio
Δf	Relative frequency



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