Supplementary

1. Correcting trends in indices decadal predictions

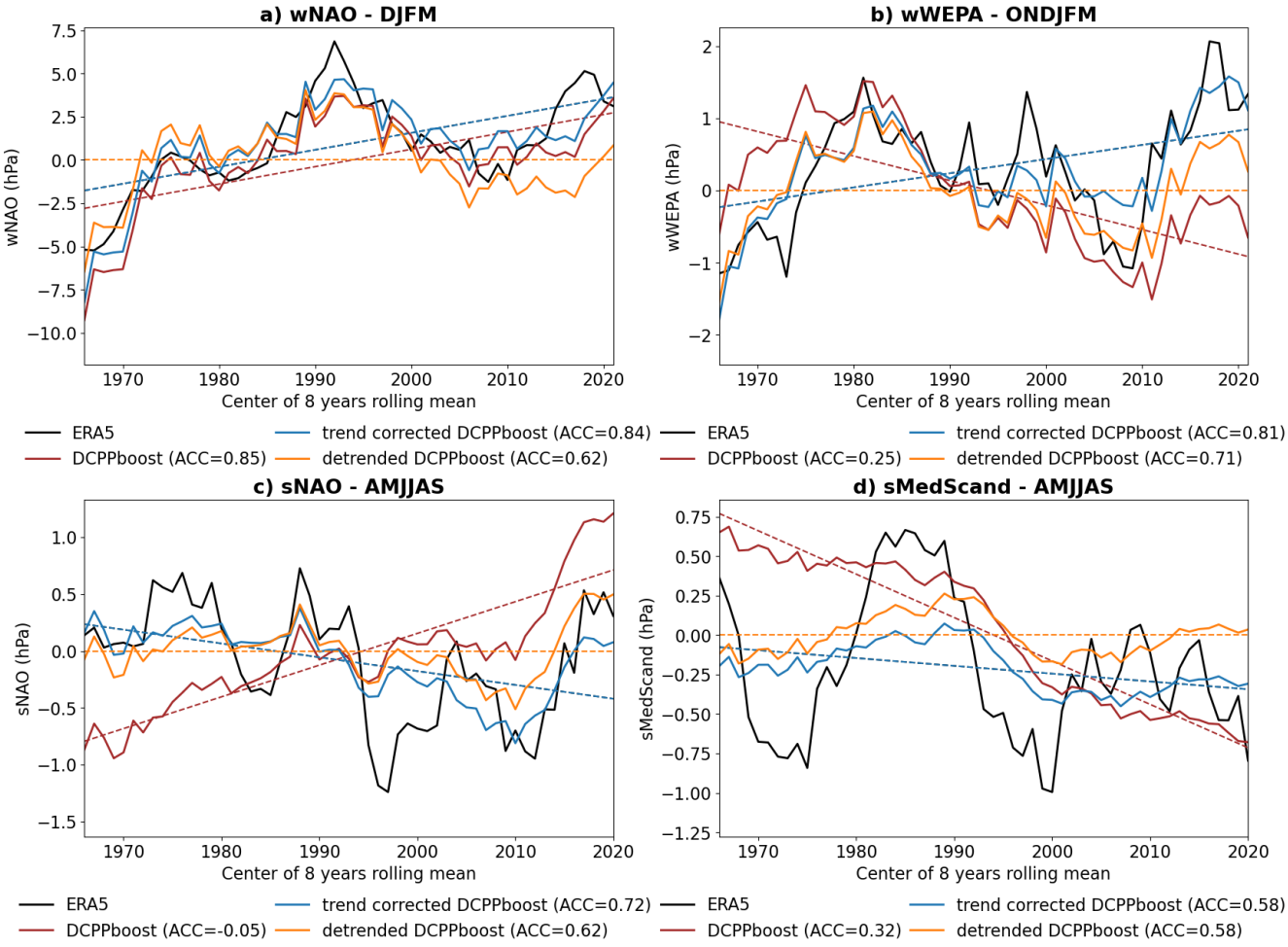

Fig. S1.1: Observed and predicted decadal variability of the wNAO (a), wWEPA (b), sNAO (c), sMedScand (d). The black line represents ERA5 reanalysis, while the burgundy dashed line shows predictions from the boosted DCPP method. The orange and blue lines represent corrected predictions after removing or correcting the trend, respectively. Correlation coefficients between predictions and observations are reported in parenthesis in each panel's legend.

2. Uninitialized IPSL-CM6A-LR Skill scores maps

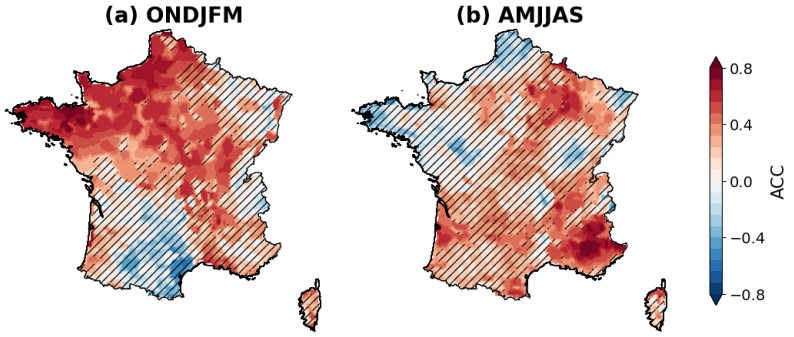

Fig. S2.1: Forecast skill for 8-year mean winter (a) and summer (b) precipitation anomalies over France. Skill maps are based on the uninitialized large ensemble mean IPSL-CM6A-LR. Panels (a-b) show the Anomaly Correlation Coefficient (ACC) computed against SAFRAN precipitation observations over 1966–2019. Hatched areas indicate region where skill scores are not statistically significant at the 95% confidence level, as assessed using a 1000 sample block bootstrap.

3. Subsampling from observed sMedScand

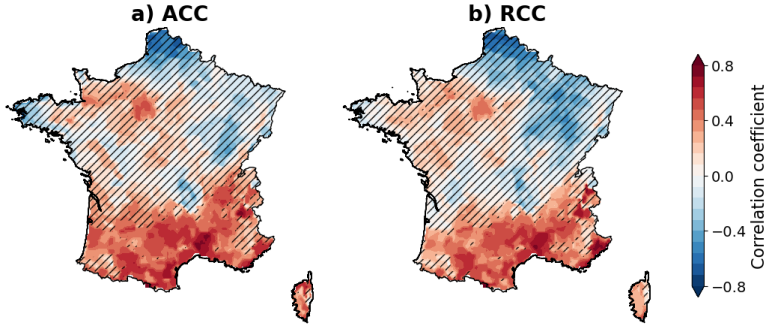

Fig. S3.1: Forecast skill for 8-year mean summer (AMJJAS) precipitation anomalies over France. Skill maps are based on subsampled forecasts using the observed sMedScand index. Panels show (a) Anomaly Correlation Coefficient (ACC) and (b) Residual Correlation Coefficient (RCC)—quantifying skill beyond the forced response—, both computed against SAFRAN precipitation observations over 1966–2019. Hatched areas indicate region where skill scores are not statistically significant at the 95% confidence level, as assessed using a 1000 sample block bootstrap.2

4. Including uAMV in winter subsampling

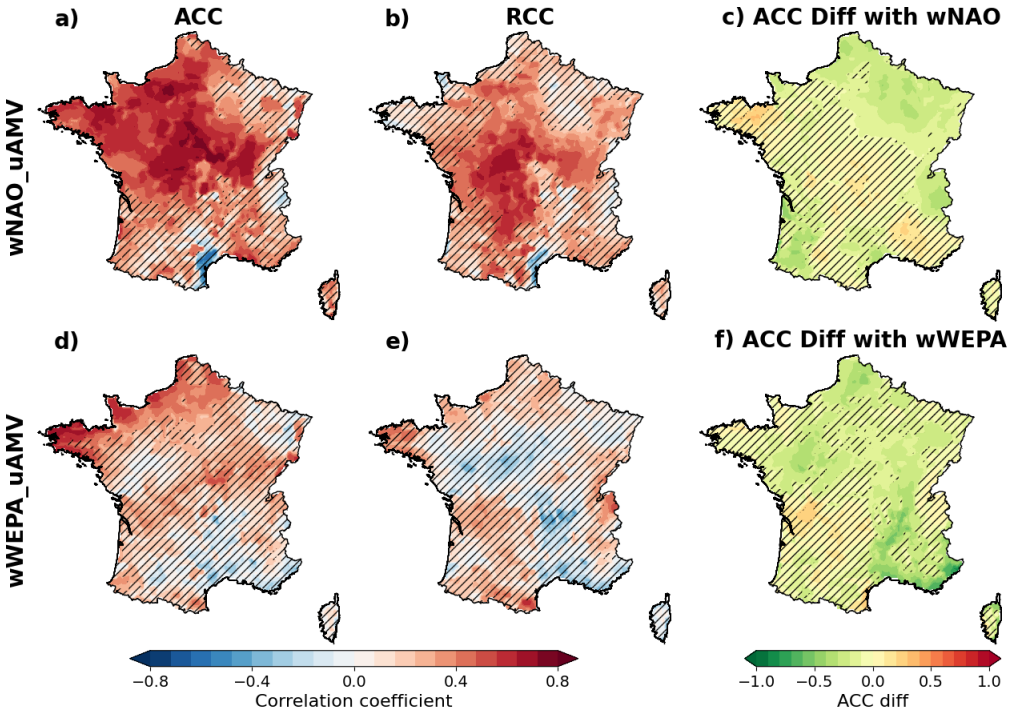

Fig. S4.1: Forecast skill for 8-year mean winter (ONDJFM) precipitation anomalies over France. Skill maps are based on subsampled forecasts using the wNAO+uAMV (a–c) and wWEPA+uAMV (d–f) indices. Panels show (a,d) Anomaly Correlation Coefficient (ACC), (b,e) Residual Correlation Coefficient (RCC)—quantifying skill beyond the forced response—and (c,f) the ACC difference with the reference prediction using wNAO (c) and wWEPA (f) alone. Scores are computed against SAFRAN precipitation observations over 1966–2019. Hatched areas indicate region where skill scores are not statistically significant at the 95% confidence level, as assessed using a 1000 sample block bootstrap (a-b, d-e) and with a Steiger's test for ACC differences (c,f).

5. Exploring other indices linked to winter precipitation second principal component

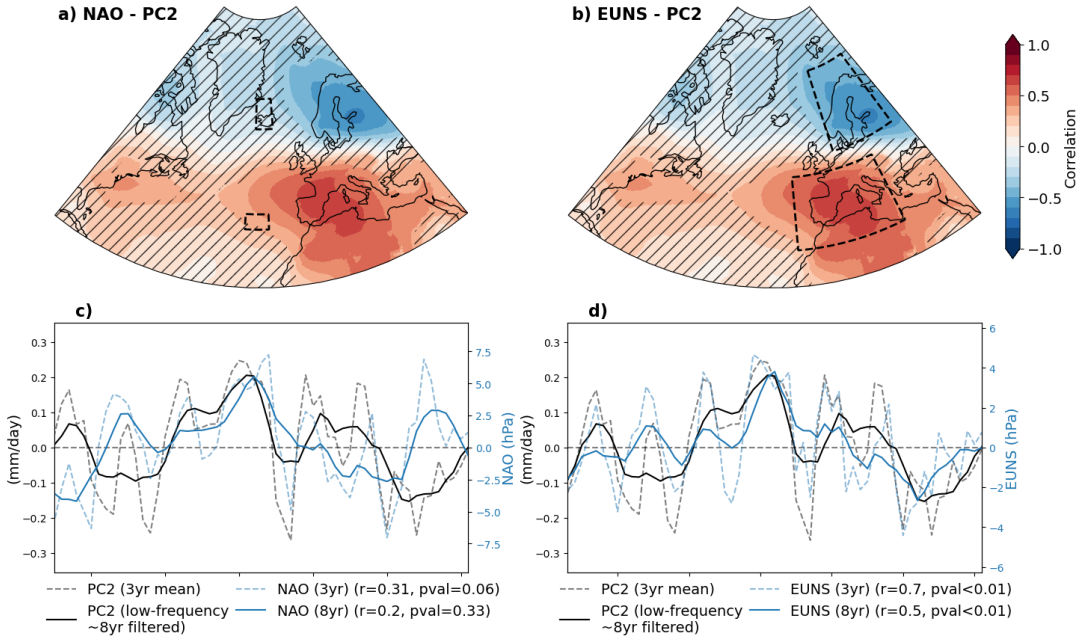

Fig. S5.1: Relationship between winter (ONDJFM) precipitation variability and large scale North Atlantic circulation. Panels (a) and (b) display spatial correlation coefficients between the second precipitation PC (PC2) and SLP anomalies over the North Atlantic. Hatched areas denote statistically non-significant correlations at the 95% confidence level based on a block-bootstrap test. Black dashed boxes outline the spatial domains used to compute the NAO (panel a) and EUNS indices (panel b). Panels (c) and (d) compare the time series of precipitation PC2 (black lines) and SLP indices (blue lines), including 3-year running mean (dashed) and low-frequency (~8-year) filtered (solid). Reported correlation coefficients (r) correspond to both smoothing scales. Units for the precipitation PCs are mm/day, and for the SLP indices are hPa.

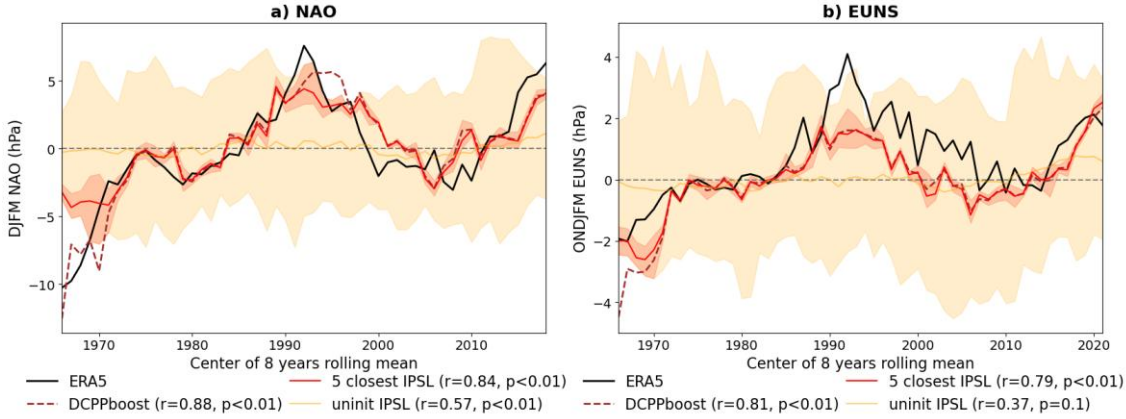

Fig. S5.2: Observed and predicted decadal variability of winter atmospheric indices. Panels (a) and (b) show 8-year running mean of the classic NAO and the Europe North-South (EUNS) indice for extended winter seaons (DJFM and ONDJFM, respectively). The black line represents ERA5 reanalysis, while the burgundy dashed line shows predictions from the boosted DCPP method. The red shading indicates the spread of the five subsampled members, best matching DCPP indices and selected from the uninitialized IPSL-CM6A-LR ensemble in each 8-year time window, with the bold red line marking their ensemble mean. The yellow line and shading

represent the mean and full spread (minimum to maximum) of the entire uninitialized IPSL ensemble, respectively. Correlation coefficients and p-values between predictions and observations are reported in parenthesis in each panel's legend.

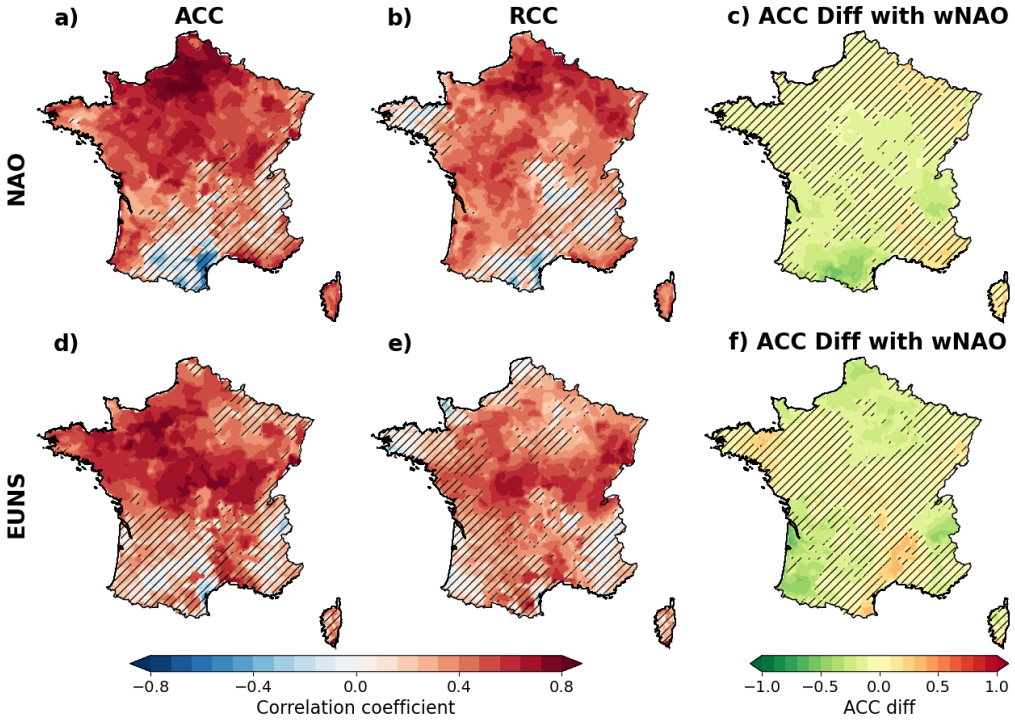

Fig. S5.3: Forecast skill for 8-year mean winter (ONDJFM) precipitation anomalies over France. Skill maps are based on subsampled hindcasts using the NAO (a–c) and Europe North-South (EUNS) (d–f) indices. Panels show (a,d) Anomaly Correlation Coefficient (ACC), (b,e) Residual Correlation Coefficient (RCC)—quantifying skill beyond the forced response—and (c,f) ACC difference with the reference prediction using wNAO. Scores are computed against SAFRAN precipitation observations over 1966–2019. Hatched areas indicate region where skill scores are not statistically significant at the 95% confidence level, as assessed using a 1000 sample block bootstrap (a-b, d-e) and with a Steiger's test for ACC differences (c,f).

6. Results without scaling to the observed variance

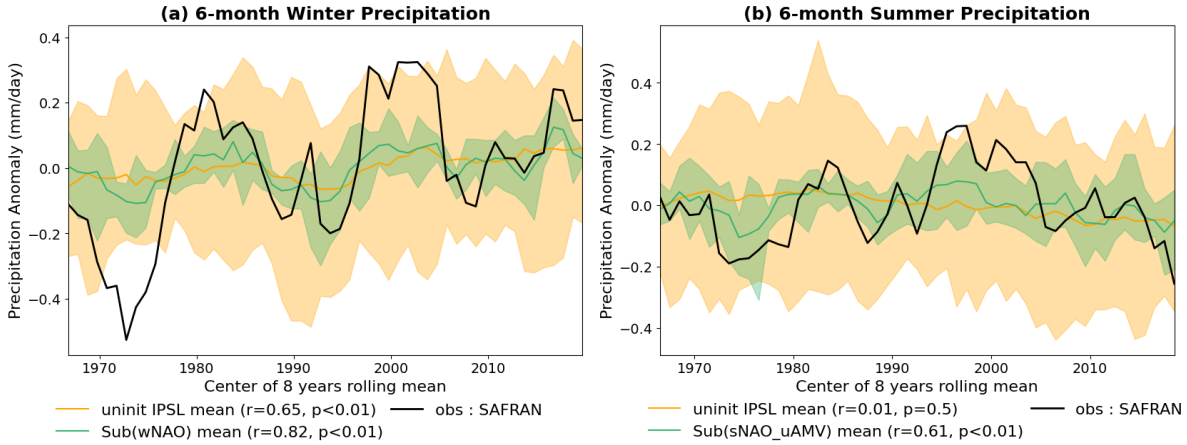

Fig. S6.1: Panel (a) and (b) show 8-year running mean precipitation anomalies for winter (ONDJFM) and summer (AMJJAS), respectively, averaged over the region outlined by dashed boxes in Figs. 8 and 9. The black line shows SAFRAN observations. The green line represents the mean of the five-member sub-ensemble selected based on the wNAO index (panel a) and combined sNAO+AMV indices (panel b), without scaling predictions to the observed variance. The yellow line shows the mean of the full uninitialized IPSL ensemble. Shaded areas indicate the ensemble spread, defined as the 5th to 95th percentile range.

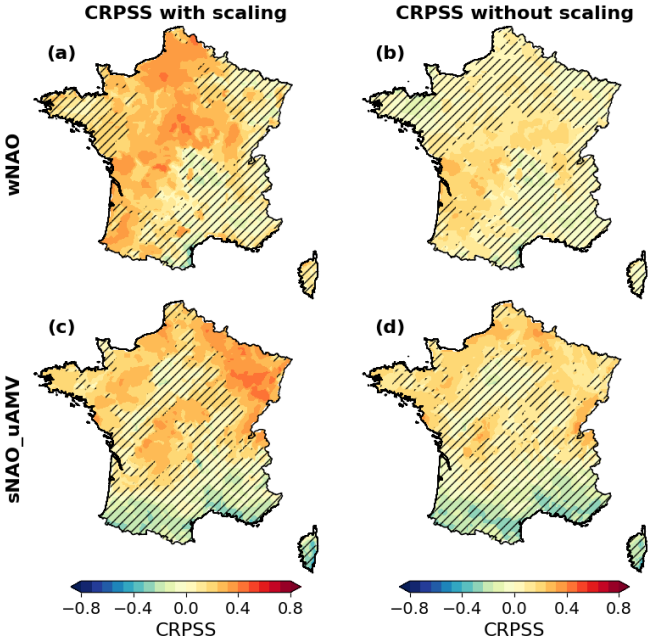

Fig. S6.2: Forecast skill for 8-year mean winter (ONDJFM) and summer (AMJJAS) precipitation anomalies over France. Results are derived from subsampled hindcasts based on the wNAO (a–b) and sNAO+AMV (c-d) indices. Panels show Continuous Ranked Probability Skill Score (CRPSS) computed over 1966-2019, with (a,c) or without (b,d) scaling the predictions to the observed variance. Hatched areas indicate region where skill scores are not statistically significant at the 95% confidence level, as assessed using a 1000 sample block bootstrap.

7. Predicting 5-year mean indices and precipitation

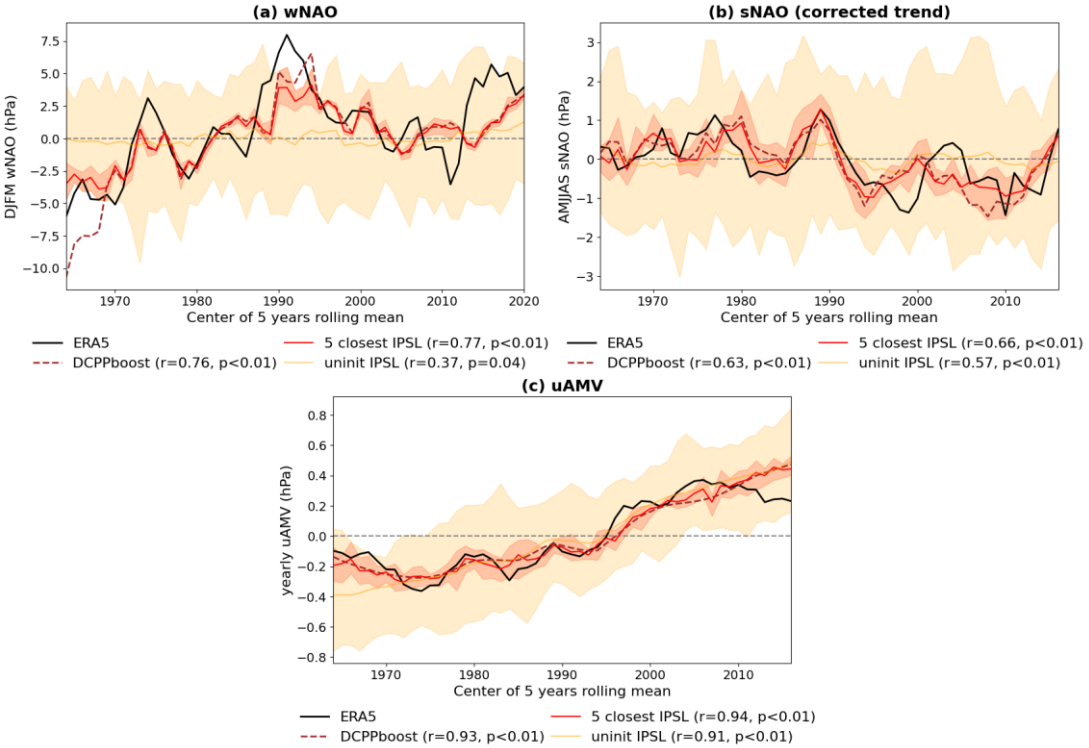

Fig. S5.1: Observed and predicted decadal variability of atmospheric and oceanic indices. Panels show 5-year running mean of the wNAO (a), sNAO (b) and uAMV (c). The black line represents ERA5 reanalysis observations, while the burgundy dashed line shows predictions from the boosted DCPP method. The red shading indicates the spread of the five index-matched members selected from the uninitialized IPSL-CM6A-LR ensemble in each 5-year time window, with the bold red line marking their ensemble mean. The yellow line and shading represent the mean and full spread (minimum to maximum) of the entire uninitialized IPSL ensemble. Correlation coefficients and p-values between predictions and observations are reported in parenthesis in each panel's legend.

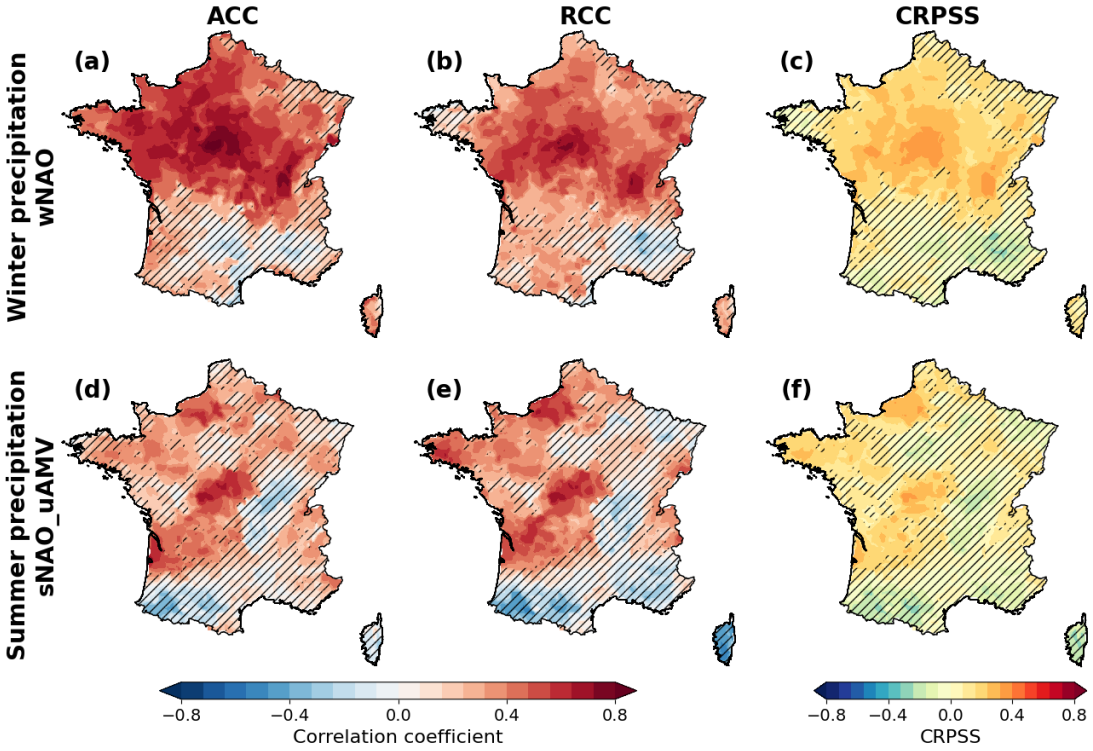

Fig. S5.2: Forecast skill for 5-year mean winter (ONDJFM) and summer (AMJJAS) precipitation anomalies over France. Results are derived from subsampled hindcasts based on the wNAO (a–c) and sNAO+uAMV (d–f) indices. Panels show (a,d) Anomaly Correlation Coefficient (ACC), (b,e) Residual Correlation Coefficient (RCC)—quantifying skill beyond the forced response—and (c,f) Continuous Ranked Probability Skill Score (CRPSS), all computed against SAFRAN precipitation observations over 1966–2019. Hatched areas indicate region where skill scores are not statistically significant at the 95% confidence level, as assessed using a 1000 sample block bootstrap.

8. Sensitivity to sub-ensemble size in the case of sNAO+uAMV subsampling

The forecast skill from the subsampling procedure may depend on the number of members in the selected sub-ensemble. We compared the results in the case of sNAO+uAMV subsampling to predict 8-year mean summer precipitation anomalies, with sub ensemble size from two to eight members.

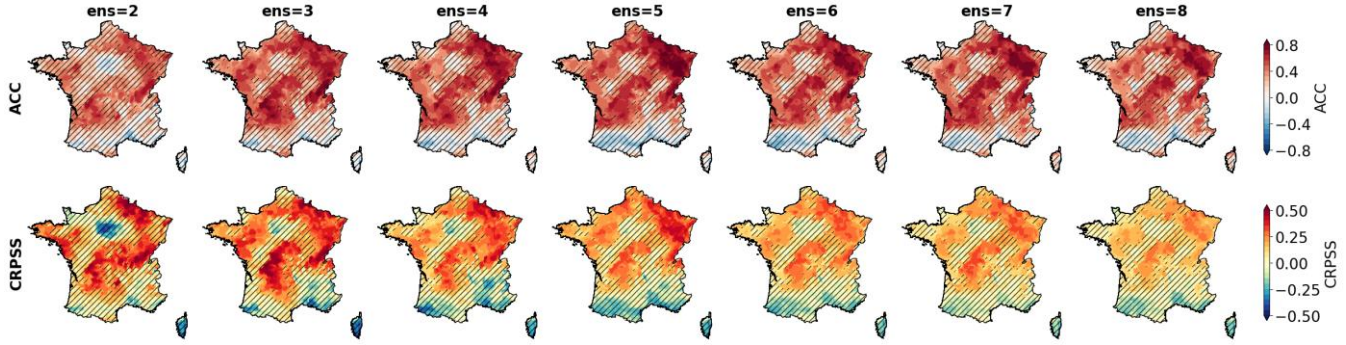

Fig. S8.1: Forecast skill for 8-year mean summer (AMJJAS) precipitation anomalies over France. Results are derived from subsampled hindcasts based on the sNAO+uAMV indices, with sub-ensemble size from 2 to 8. Upper panels show Anomaly Correlation Coefficient (ACC) and lower panels show Continuous Ranked Probability Skill Score (CRPSS), over 1966–2019. Hatched areas indicate region where skill scores are not statistically significant at the 95% confidence level, as assessed using a 1000 sample block bootstrap.

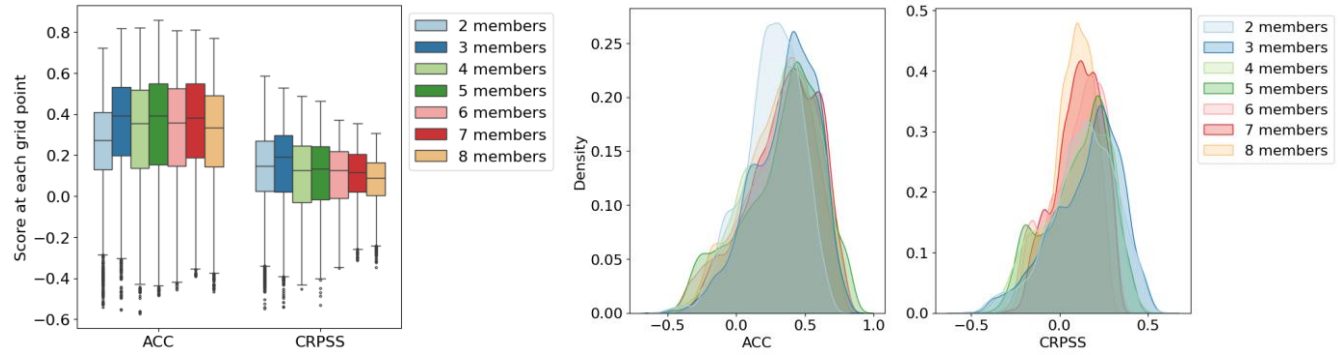

Fig. S8.2: Distribution of forecast skill for 8-year mean summer precipitation anomalies over France derived from subsampled forecasts using sNAO+uAMV indices with sub ensemble size from 2 to 8. Left panel shows the Anomaly Correlation Coefficient (ACC) and Continuous Ranked Probability Skill Score (CRPSS) for each grid point as boxplots. Right panel indicate ACC and CRPSS as continuous probability density curves. Skill scores are computed against SAFRAN precipitation observations over 1966–2019.