Comment on the Physical Interpretation of the Refined Runoff–TWS Relationship

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The refined runoff–TWS relationship in Eq. (10) can be written, ignoring the snow-cover term for simplicity, as

$$R_t = \alpha_1(t)S_{t-1} + \alpha_2(t)S_t, \tag{1}$$

where the time-dependent coefficients are related to meltwater intensity.

This equation can also be written as

$$R_t = k(t)\left[\lambda(t)S_{t-1} + (1-\lambda(t))\,S_t\right], \tag{2}$$

with

$$k(t) = \alpha_1(t) + \alpha_2(t), \tag{3}$$

and

$$\lambda(t) = \frac{\alpha_1(t)}{\alpha_1(t) + \alpha_2(t)}. \tag{4}$$

This form may provide another useful physical interpretation of the two coefficients.

The parameter $k(t)$ represents the slope of the runoff–storage relationship. A change in $k(t)$ therefore means that the same amount of storage can produce a different runoff response under different hydrological conditions.

On the other hand, $\lambda(t)$ describes the relative contribution of previous-month storage compared with current-month storage. For example, when $\lambda(t)$ is small, runoff mainly depends on current-month storage. When $\lambda(t)$ becomes larger, the contribution of previous-month storage increases. Therefore, $\lambda(t)$ may be interpreted as an indicator of the effective hydrological memory or lag.

Consequently, changes in $\alpha_1(t)$ and $\alpha_2(t)$ may change both

1. the slope of the runoff–storage relationship, and
2. the temporal memory between storage and runoff.

It may therefore be informative to show the temporal or seasonal variation of $k(t)$ and $\lambda(t)$. This could help indicate whether the improvement of the refined model mainly comes from changes in the runoff–storage slope, changes in the effective response timing, or both.

It may also be useful to examine the joint distribution or correlation between α_1 and α_2 in the MCMC samples. Since S_t and S_{t-1} are expected to be strongly correlated, this could help to assess how independently the two coefficients are constrained.

Finally, the model currently uses storage from the current and previous month. A sensitivity test including an additional lag term, for example S_{t-2},

$$R_t = \alpha_0 S_t + \alpha_1 S_{t-1} + \alpha_2 S_{t-2}, \tag{5}$$

could help to show whether limiting the storage memory to one previous month is sufficient for these large Pan-Arctic basins.