



Static stability and local stratification constrain where and when oceanic eddies can exist

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Abstract.

Oceanic eddies play a major role in redistributing heat, salt, and tracers. Their three-dimensional structure has historically been studied from a dynamical perspective. Here, we investigate a more fundamental constraint: whether a balanced eddy can remain statically stable when embedded in a given oceanic environment. We prescribe idealized three-dimensional eddies from their vorticity structure and derive the associated density anomaly required for gradient-wind and hydrostatic balance. We then combine these anomalies with background density profiles representative of different regions and seasons to assess the resulting static stability. Our results show that static stability strongly depends on eddy polarity, intensity, vertical extent, and, for subsurface eddies, their position relative to the background pycnocline. Thin and intense cyclonic eddies are particularly prone to gravitational instability, whereas anticyclonic and thicker eddies are generally more stable. The same eddy structure can therefore be statically stable in one region or season but unstable in another, demonstrating that background stratification can constrain where particular eddy structures can exist and propagate. This process may also provide an additional pathway for eddy-induced vertical mixing through convective adjustment. We provide an open notebook that allows users to evaluate the static stability and three-dimensional density signature of prescribed eddies in arbitrary background stratifications.

1 Introduction

- When ocean currents generate mesoscale eddies, they set the initial three-dimensional structure of the eddy vorticity. For example, an unstable deep current can generate a vortex whose vertical extent and vorticity structure reflect the properties of the parent current and the geographical constraints of the generation region (Carton, 2001). Because of the Earth's rotation, this velocity structure is associated with a three-dimensional density anomaly within the eddy, which is required to maintain dynamical balance through gradient-wind and hydrostatic balance.
- At the same time, the background ocean environment is highly variable. Temperature and salinity, and consequently the vertical density variation, can differ substantially between regions and seasons (Talley et al., 2011). When an eddy is embedded in a given environment, the resulting density field is the sum of the local background density and the density anomaly associated with the eddy. Importantly, *there is no guarantee that this resulting density profile remains statically stable*. If the eddy-induced density anomaly is sufficiently strong to reverse the background vertical density gradient, a local density inversion develops,



leading to gravitational instability and subsequent vertical mixing. Thus, whether a balanced eddy with a given shape can exist depends not only on the local dynamics, but also on the stratification of the environment in which it is embedded.

The lifecycle of oceanic eddies has been studied for decades (see the literature of vortex studies since McWilliams and Flierl, 1979). Historically, research has focused predominantly on surface-intensified eddies, whose surface expression has been extensively observed using satellite (from early thermal infrared scanning radiometer-based estimates (Stumpf and Rao, 1975) to high resolution synoptic description (de Marez, 2025)). More recently, subsurface-intensified eddies have received increasing attention, driven in particular by the growing availability of Argo observations (McCoy et al., 2020) and by the development of high-resolution numerical simulations capable of resolving their three-dimensional structure (Gula et al., 2019). Nevertheless, they remain considerably less studied than their surface-intensified counterparts, partly because their signatures are largely hidden from the sea surface and their structure is more closely tied to the surrounding vertical stratification.

Regardless of their vertical structure, most studies of eddy lifecycles have focused on their dynamical stability, examining how vortices evolve and eventually break down through baroclinic, barotropic, symmetric, or other dynamical instabilities (see the literature of vortex stability studies since Gent and McWilliams, 1986). In this context, the vortex community has traditionally used two/three-dimensional numerical simulations in which eddies are prescribed using idealized analytical shapes. A typical approach is to choose an eddy structure, including its vorticity and associated density anomaly, and embed it in a background density profile chosen such that the resulting initial state is statically stable. The eddy can then evolve over a finite time, allowing its subsequent dynamical behavior to be investigated.

This procedure is useful for isolating the mechanisms governing eddy evolution, but it also raises a more fundamental question: *is the prescribed eddy actually compatible with a realistic background environment representative of the region where such an eddy would be observed?* In other words, before considering its dynamical evolution, can the eddy exist in a statically stable state in the first place? Because the oceanic background density profile varies substantially across regions and seasons, a given three-dimensional eddy structure may therefore be statically stable in one environment but unstable in another. Yet, despite its fundamental importance, this aspect of eddy existence is generally overlooked, likely because static stability depends simultaneously on the local background density profile and on the specific three-dimensional structure and intensity of the eddy.

In this study, we do not aim to describe the full dynamical evolution of oceanic eddies. Instead, we consider a more basic constraint: how the background density stratification and the three-dimensional structure of an eddy determine whether a balanced vortex can exist without immediately becoming gravitationally unstable. By considering regional and seasonal variations in temperature and salinity, we explore how changes in the background stratification constrain the range of eddy structures that can remain statically stable. Finally, we provide an easy-to-use notebook that allows users to test whether a prescribed eddy can exist in a given oceanic environment and to calculate its associated three-dimensional density anomaly.

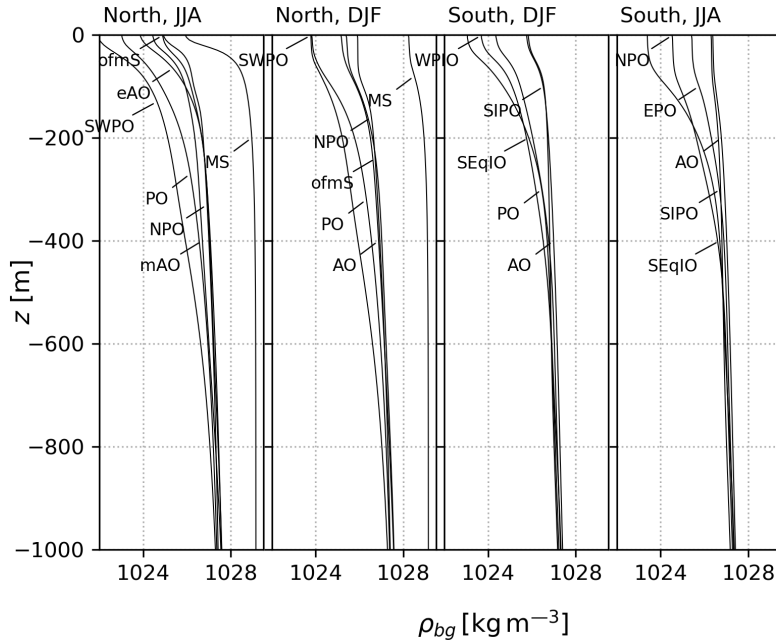


Figure 1. Background density profiles obtained using PCM classification in areas shown in supplementary Fig. B2.

2 On the geographical variation of the density background

In the ocean, the thermohaline properties of the water column can vary substantially from one region to another, from warm and saline Mediterranean waters to the cold, relatively fresh waters of the North Pacific. These regional contrasts are further modulated by strong seasonal variability associated with processes such as summer evaporation, monsoon-driven/river discharge/ice-melt freshwater fluxes, and winter precipitation. As a result, the vertical stratification of the ocean exhibits pronounced spatiotemporal variability, and the background density profile in which an eddy evolves can differ substantially across regions and seasons.

Identifying regions with sufficiently homogeneous thermohaline properties and extracting representative background density profiles could be performed empirically, but such an approach would be both cumbersome and potentially subjective. Moreover, considering classical basin scales average of water-column profiles with substantially different vertical structures could artificially smooth the background stratification and weaken or even remove the vertical density gradients that are central to the present analysis. We therefore adopt an automated approach to characterize the background density profiles. This provides a systematic way of sampling the range of oceanic stratification encountered by eddies, while avoiding an unnecessary regional classification. Since the objective here is not to provide a detailed description of regional variations in ocean thermohaline



properties, but rather to assess their implications for eddy structure and static stability, this approach allows us to retain a more general and theoretical perspective.

We use the Coriolis Ocean Dataset for Reanalysis product (hereafter CORA, Szekely et al., 2019), a global gridded dataset of *in situ* temperature and salinity measurements collected from autonomous platforms (Argo profilers, fixed moorings, gliders, drifters, and instrumented marine mammals) as well as research and ships of opportunity (CTDs, XBTs, and FerryBox systems). The dataset provides monthly temperature (T) and salinity (S) profiles on a $0.5^\circ \times 0.5^\circ$ global grid over the period 1960–2025. We restrict the analysis to 2000–2025 to maximize the representation of autonomous observations, and in particular Argo profiling floats, in the dataset. We further restrict the analysis to the midlatitude ocean, excluding latitudes poleward of 50° and a 20° -wide band centered on the equator. Finally, we consider only open-ocean profiles, defined as locations where the local ocean depth exceeds 2000 m.

To characterize the seasonal and regional variability of the background density profiles, we first average the monthly T/S profiles over three-month periods for the whole period, thereby defining a seasonal climatology. We then apply the PCM classification algorithm (Maze et al., 2017) on the averaged temperature and salinity profiles, independently to each three-month period and in each hemisphere, so as to avoid mixing profiles corresponding to opposite seasons. The number of classes is determined by minimizing the Bayesian Information Criterion (BIC), providing an objective criterion for selecting the number of distinct background water-column types. In the following, we focus on summer (June/July/August, JJA in the Northern Hemisphere and December/January/February, DJF in the Southern Hemisphere) and winter (DJF in the Northern Hemisphere and JJA in the Southern Hemisphere), which encompass the strongest seasonal contrasts in the background stratification.

For each class, we compute the mean temperature and salinity profiles and derive the corresponding mean density profile using the TEOS-10 equation of state for seawater (McDougall and Barker, 2011). The classification yields seven distinct profiles hereafter called ρ_{bg} for summer and six for winter in the Northern Hemisphere, and five profiles for each season in the Southern Hemisphere, see Fig. 1. As expected, the winter profiles generally exhibit a deeper surface mixed layer. Except for the Mediterranean Sea class, all profiles converge toward a similar bottom density of approximately 1027.5 kg m^{-3} . In contrast, surface density varies substantially among the different classes, resulting in marked differences in the depth and structure of the pycnocline.

As an example from the Southern Hemisphere, the Southern equatorial Indian Ocean (SEqIO) exhibits a density difference of 4.2 kg m^{-3} between the base and the top of the pycnocline, compared with only 1.3 kg m^{-3} in the Atlantic Ocean (AO). In both regions, the seasonal cycle primarily affects the depth of the pycnocline, with winter conditions associated with a deeper pycnocline (around 100 m, compared with about 25 m in summer), while the overall density variations across the pycnocline remains relatively similar between seasons.

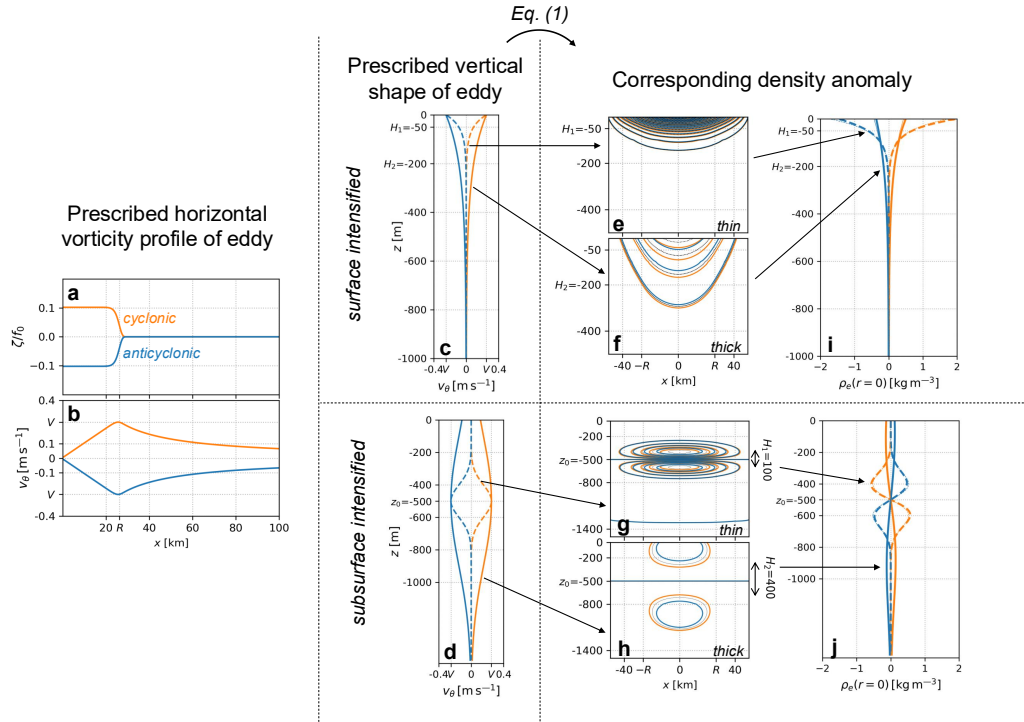


Figure 2. Example of isolated cyclonic (orange) and anticyclonic (blue) eddy prescription. a) Prescribed horizontal profile of vorticity, for $Ro = 0.1$ and b) corresponding azimuthal velocity profile. c) Prescribed vertical velocity profile of surface intensified "thin" ($H = 50$ m, dashed), or "thick" ($H = 200$ m, dashed) eddies. d) Prescribed vertical velocity profile of sub-surface intensified "thin" ($H = 100$ m, dashed), or "thick" ($H = 400$ m, dashed) eddies, at a depth $z_0 = -500$ m. e-h) Density anomaly signature section at the center of eddies prescribed by profile shown in panels a-d; thin grey lines show the result for purely geostrophic equilibrium. i, j) Density anomaly of the same eddies, at their center $r = 0$ where ρ_e is maximum.

100 3 The 3D signature of an isolated oceanic eddy

Oceanic eddies exhibit a wide range of horizontal and vertical structures, from surface-intensified vortices to subsurface-intensified structures, with their characteristic size, intensity, and vertical extent varying substantially across regions and seasons (see *e.g.*, Table 1 in Ciani et al., 2015).

To gain a more general perspective, we can consider a flexible analytical formulation that captures these main structural differences while retaining a limited number of physically interpretable parameters. Also, it is important to note that an isolated eddy can be prescribed from different dynamically related fields, such as pressure, velocity, or density; here, we prescribe the eddy directly from its *relative vorticity profile*, providing the most direct and physically transparent description of its rotational structure.

First, the horizontal profile of surface relative vorticity ζ is defined in radial coordinates (r, θ) referenced at the eddy center



$$\zeta(r, \theta, z = 0) = \pm \omega_0 e^{-(r/R)^\gamma},$$

110 where γ is the steepness parameter ($\gamma = 2$ would prescribe a Gaussian eddy (Ayouche et al., 2021), and $\gamma \gg 2$ a Rankine-like eddy (Barabinot et al., 2025b)), and $\omega_0 = \pm V_0/R$, with V_0 the maximal azimuthal velocity at a distance R (the eddy radius) from the center of the eddy, see example in Fig. 2a. In the following, a cyclonic eddy is defined as an eddy for which $\zeta/f_0 > 0$; with f_0 positive in the Northern Hemisphere and negative in the Southern Hemisphere. By definition the eddy has no radial component of velocity, and its azimuthal velocity can be computed using

$$v_\theta(r, \theta, z = 0) = r^{-1} \int dr \zeta r,$$

115 see Fig. 2b.

Second, on the one hand, *surface intensified* eddies are hypothesized to have velocity following an exponential decrease at depth, such that

$$v_\theta(x, y, z) = v_\theta(r, \theta, z = 0) e^{-z/H},$$

see Fig. 2c. On the other hand, *subsurface intensified* eddies are hypothesized to have velocity depth variations in the form of a Gaussian profile, such that

$$v_\theta(x, y, z) = v_\theta(r, \theta, z = z_0) e^{-(z-z_0)^2/(2H^2)},$$

120 with z_0 the depth of maximum intensity of the eddy, see Fig. 2d. In both cases, H is a proxy describing the thickness of the eddy; it is not strictly speaking the eddy's height, which should be defined in terms of Lagrangian conserved quantity (e.g., the Ertel Potential Vorticity), which was not the point here. In the following, the horizontal velocity field of the eddy used is (u, v) , which is the projection of v_θ into Cartesian coordinates (x, y) .

Therefore, the prescription of an eddy is fully determined by its horizontal parameters, γ , R , and V_0 , together with its vertical
 125 structure, described by H for surface-intensified eddies and by H and z_0 for subsurface-intensified eddies. We set $\gamma = 24$, yielding a profile close to a Rankine vortex, and prescribe the horizontal scale as the local deformation radius, $R = R_D$, while characterizing the eddy intensity through its Rossby number, $Ro = V_0/(fR)$. The parameter space considered here is therefore reduced to Ro , H , and, for subsurface-intensified eddies, z_0 . Note that sensitivity tests show that the present results are largely insensitive to the value of γ . The steepness γ of the eddy profile is, however, important for its finite-time evolution, and hence
 130 for its dynamical stability properties, which are beyond the scope of the present study.

The pressure anomaly field $P'(x, y, z)$ corresponding to the prescribed eddy velocity field is computed *via* the generalized formulation of cyclostrophic equilibrium, called the *Gradient Wind equation*

$$\overbrace{2\mathcal{J}(u, v)}^{\text{ageostrophic component}} + \underbrace{f(\partial_x v - \partial_y u) = \rho_0^{-1} \Delta_h P'}_{\text{geostrophic equilibrium}}, \quad (1)$$



with $\mathcal{J}(u, v) = \partial_x u \partial_y v - \partial_y u \partial_x v$ the Jacobian operator, and $\Delta_h = \partial_{xx} + \partial_{yy}$ the horizontal Laplacian operator. The first term
135 in the left hand side accounts for the ageostrophic part of the flow. Therefore, if the purely geostrophic signature of an eddy
is to be computed, this term is removed from the equation. The corresponding eddy density anomaly ρ_e is then obtained
assuming hydrostatic balance $\partial_z P' = -\rho_e g$. In the following, we consider the eddy induced density anomaly ρ_e at the eddy
centre ($r = 0$) as it is the location where it is maximum.

Surface-intensified eddies exhibit a density anomaly of the same sign as their relative vorticity: cyclonic eddies are asso-
140 ciated with a positive density anomaly, whereas anticyclonic eddies exhibit a negative one. For a given eddy intensity, the
amplitude of this density anomaly increases as the eddy becomes vertically thinner. This results from the requirement of dy-
namical equilibrium in a rotating frame of reference, which constrains the vertically integrated density anomaly to balance
the horizontal pressure gradients associated with the eddy circulation. This relationship between the vertical extent of an eddy
and the amplitude of its density anomaly is central to the analysis developed here. In the examples shown in Fig. 2e,f,i the
145 maximum density anomaly reaches approximately $\rho_e \sim \pm 2 \text{ kg m}^{-3}$ for the thinnest eddies, compared with $\rho_e \sim \pm 0.5 \text{ kg m}^{-3}$
for thicker structures.

Subsurface-intensified eddies exhibit a different vertical density signature, with an anomaly that changes sign across the
depth of the eddy. For anticyclonic eddies, the density anomaly is positive above the eddy core and negative below it, with the
opposite configuration for cyclonic eddies. As for surface-intensified eddies, the amplitude of the density anomaly increases
150 as the vertical scale of the eddy decreases. In the examples shown in Fig. 2g,h,j the maximum density anomaly reaches
approximately $\rho_e \sim \pm 0.5 \text{ kg m}^{-3}$ for the thinnest eddies, compared with $\rho_e \sim \pm 0.1 \text{ kg m}^{-3}$ for thicker structures.

For weak eddies, characterized by small Rossby numbers (*e.g.*, $Ro=0.1$), the density structure obtained under geostrophic
balance is nearly indistinguishable from that obtained from the full balanced solution. As the eddy intensity increases, however,
ageostrophic contributions become increasingly important (Fig. B1). In particular, these contributions enhance the maximum
155 density anomaly associated with cyclonic eddies and can lead to density anomaly inversions within anticyclonic eddies.



4 On the existence of statically stable eddies in a stratified ocean

In the ocean, an eddy can emerge within a given background density field, either through local generation or through the propagation of an eddy generated remotely into a new environment (see discussion section). Upon its emergence, the local density field is expressed as the sum of the background density and the density anomaly associated with the eddy, such that
 160 $\rho = \rho_{bg} + \rho_e$. The central point of the present discussion is that *there is no guarantee that the resulting density field is statically stable*.

Static stability requires the vertical density gradient to remain of the appropriate sign throughout the water column. With z increasing upward, a stably stratified ocean satisfies $\partial\rho/\partial z < 0$ whereas $\partial\rho/\partial z > 0$ locally corresponds to gravitational instability. Equivalently, this condition can be expressed in terms of the Brunt–Väisälä frequency,

$$N^2 = -\frac{g}{\rho_0} \frac{\partial\rho}{\partial z},$$

165 where g is the gravitational acceleration and ρ_0 is a reference density. Static stability therefore requires $N^2 > 0$, while $N^2 < 0$ indicates a locally unstable density profile.

For a density profile ρ resulting from the superposition of a background density profile and the prescribed density anomaly associated with an eddy of vertical scale H , we define the thickness of the statically unstable layer, $\tilde{H}$, as the vertical extent over which $N^2 < 0$. We then introduce the dimensionless parameter $\alpha = \tilde{H}/H$, which measures the fraction of the eddy's
 170 vertical extent that is statically unstable in the water column. Note that multiple unstable layers may occur, not necessarily within the eddy's core, separated by statically stable layers. In such cases, $\tilde{H}$ is defined as the sum of the thicknesses of all statically unstable layers.

4.1 Canonical examples

As a first example, we consider the summer background stratification in the Southern Hemisphere Atlantic Ocean (AO; black
 175 line in Fig. 3a) and superimpose the density anomalies associated with surface-intensified eddies of different vertical scales. Figure 3a shows the resulting density profiles at the eddy center for thin ($H = 50$ m) and thick ($H = 200$ m) eddies, using the density anomalies of eddies prescribed in Fig. 2i. The resulting profiles differ substantially depending on the eddy polarity. For anticyclonic eddies, the negative density anomaly associated with the eddy enhances the background stratification, leading to lower densities near the surface while preserving a statically stable water column. For thick eddies, the density profiles of
 180 cyclones and anticyclones remain broadly similar, although cyclonic eddies are associated with larger densities at the surface. In contrast, the large density anomaly associated with a thin cyclonic eddy reverses the sign of the vertical density gradient near the surface, resulting in $N^2 < 0$ over a thickness of approximately $\tilde{H} \sim 100$ m, corresponding to $\alpha = 2$. Such a density inversion would trigger convective adjustment and vertical mixing. Taking a representative convective vertical velocity of $w_{conv} = \mathcal{O}(1) \text{ cm s}^{-1}$, consistent with observed vertical velocities in oceanic convective plumes (Clément et al., 2024), gives
 185 an adjustment timescale

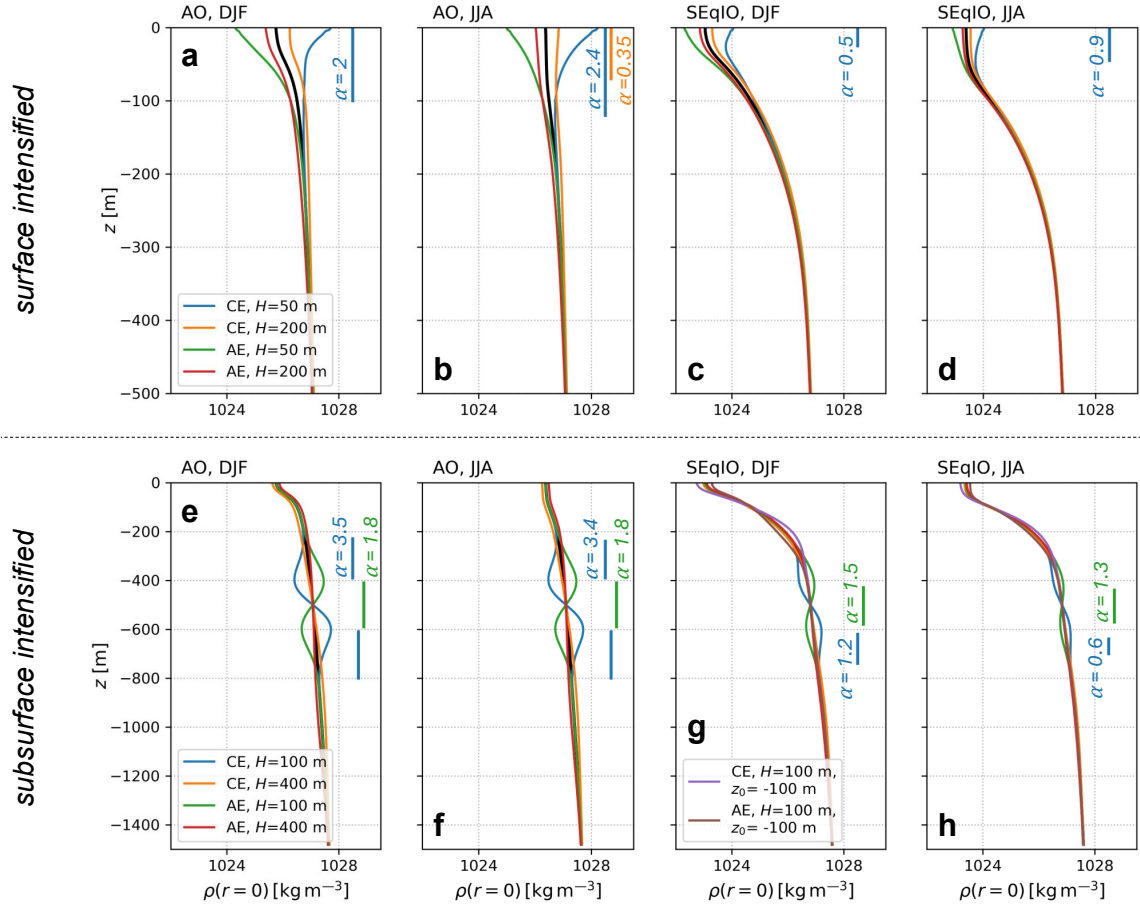


Figure 3. Density profiles obtained by superimposing the density anomalies at the centre of the prescribed eddies shown in Fig. 2 onto the background density profiles for the Atlantic Ocean (AO; a,b,e,f) and Southern Equatorial Indian Ocean (SEqIO; c,d,g,h), for summer and winter conditions.

$$T_{\text{conv}} \sim \tilde{H}/w_{\text{conv}} = \mathcal{O}(1) \text{ h.}$$

This is much shorter than the eddy rotation timescale,

$$\tau = 4\pi/\omega_0 = 4\pi/Rof = \mathcal{O}(10) \text{ days,}$$

for ($Ro = 0.1$) and $f \sim 10^{-4} \text{ s}^{-1}$. By contrast, a purely diffusive adjustment with a typical open-ocean diapycnal diffusivity of $\kappa \sim 10^{-5} \text{ m}^2 \text{ s}^{-1}$ (Bannon and Najjar, 2018) would occur over



$$T_{\kappa} \sim \tilde{H}^2 / \kappa = \mathcal{O}(10) \text{ yr.}$$

Therefore,

$$T_{\text{conv}} \ll \tau \ll T_{\kappa},$$

190 which shows that convective gravitational instability provides a rapid mechanism for modifying the prescribed density structure, on a timescale much shorter than that associated with the rotation and persistence of the balanced eddy. The eddy therefore cannot remain in the prescribed state at this location and season.

We do not attempt to describe the subsequent evolution of the eddy following the onset of gravitational instability, as this evolution depends on additional dynamical and diffusive processes that are beyond the scope of this study. Instead, the presence
 195 of $N^2 < 0$ provides a simple criterion for excluding such an eddy from the set of statically stable balanced states. In this example, surface-intensified cyclonic eddies with $R = R_D$ and $Ro = 0.1$ cannot exist in the summer background stratification of the Southern Hemisphere Atlantic Ocean when $H = 50$ m, whereas thicker cyclonic eddies and anticyclonic eddies remain statically stable. In other words, if the eddy's vertical scale is smaller than the background stratification scale, the eddy cannot remain in static equilibrium. This illustrates the central constraint explored here: *the ability of a balanced eddy to exist depends*
 200 *not only on its dynamical properties, but also on the background stratification in which it is embedded.*

We next consider the same eddies embedded in the winter background density profile of the Southern hemisphere AO (Fig. 3b). During winter, the pycnocline deepens, resulting in weaker stratification near the surface. This reduced surface stratification further increases the susceptibility of thin cyclonic eddies to gravitational instability, with the thickness of the unstable layer reaching $\tilde{H} \sim 120$ m, corresponding to $\alpha = 2.4$. Interestingly, the winter background also leads to an unstable layer for thick
 205 cyclonic eddies. In this case, however, the unstable layer is much thinner than the vertical extent of the eddy, with $\alpha = 0.35$, and the associated density gradient is weak. The resulting instability is therefore substantially weaker than for the thin cyclonic eddy. Although the subsequent adjustment cannot be determined from our analysis, the relatively weak and localized instability suggests that the eddy may undergo only a limited modification of its core during the ensuing convective adjustment, rather than being entirely destroyed. This example further illustrates that *the static stability constraint depends not only on the eddy*
 210 *properties, but also strongly on the seasonal state of the background stratification.*

We finally consider the same eddies embedded in the summer and winter background density profiles of the SEqIO area (Figs. 3c,d). As noted above, the SEqIO is characterized by stronger background stratification across the pycnocline than the AO. This stronger stratification counteracts the density anomalies associated with the eddies and therefore limits the reversal of the vertical density gradient. As a result, thick eddies remain statically stable in both seasons, while thin cyclonic eddies exhibit
 215 a substantially smaller unstable layer than in the AO. Again, the comparison between the Atlantic and SEqIO cases demonstrates that the static stability of an eddy depends critically on the background stratification in which the eddy is embedded, *i.e.*, the location it is studied at.



We next repeat the analysis using the density anomalies associated with subsurface-intensified eddies shown in Fig. 2j. We consider both thin and thick eddies, with an example intensification depth of $z_0 = -500$ m, see Fig. 3e-h. In this configuration, thin subsurface-intensified eddies produce statically unstable density profiles for both polarities, with the instability being particularly pronounced for cyclonic eddies and in the AO. This behavior results from the vertical position of the eddy relative to the background pycnocline. At $z_0 = -500$ m, the eddy core is located well below the pycnocline, where the background stratification is relatively weak. The vertical density anomaly associated with a thin eddy can therefore be sufficiently strong to reverse the sign of the background density gradient and generate $N^2 < 0$.

The importance of the eddy's vertical position is further illustrated by considering the same thin eddies at a shallower intensification depth, $z_0 = -100$ m, in the SEqIO background (Fig. 3g,h). In this case, the eddy is located within a strongly stratified pycnocline, and the background density gradient is sufficiently strong to prevent a reversal of the total density gradient. The resulting density profiles therefore remain statically stable. This result suggests that thin subsurface-intensified eddies are preferentially constrained to strongly stratified regions near the pycnocline, whereas their presence in the weakly stratified ocean interior can lead to gravitational instability. This helps explain the frequent observation of small, coherent subsurface vortices in the ocean interior, hence the name commonly used for such structures: *intrathermocline eddies*.

In contrast, thick subsurface-intensified eddies remain statically stable across the environments considered here. Their larger vertical extent results in a weaker vertical density gradient associated with the eddy anomaly, which is insufficient to overcome the background stratification. Thus, while the vertical position of a thin subsurface eddy can strongly constrain its ability to remain statically stable, thick eddies are much less sensitive to their position within the water column.

4.2 Draw a picture of the global ocean

Building on the examples discussed above, we now extend the analysis to the full set of background density profiles obtained from the classification. For each background environment, we repeat the analysis over a range of prescribed eddy properties and examine the resulting static stability. Because the background profiles are derived from an automated classification, and because the analytical form used to prescribe the eddies is necessarily idealized, we do not aim to provide an exact characterization of the stability limits for each individual oceanic region. Instead, we focus on the main patterns that emerge consistently across the different background environments, and for the different eddy parameters.

We first consider surface-intensified eddies and vary their vertical extent, H , and intensity, Ro (Fig. 4). Several robust patterns emerge. First, anticyclonic eddies remain statically stable over almost the entire parameter space considered. Only the most intense anticyclones ($Ro > 0.5$) exhibit regions of static instability. In contrast, cyclonic eddies are much more susceptible to gravitational instability. The fraction of cyclonic eddies affected by static instability increases systematically with eddy intensity, with stronger eddies producing larger density anomalies and therefore a greater tendency to reverse the background density gradient. The same tendency is observed as the eddy becomes thinner: for a given Ro , decreasing H increases the magnitude of the vertical density anomaly and consequently increases the likelihood and extent of static instability. In some areas and seasons, *e.g.*, the North Pacific Ocean in winter (JJA), no cyclonic case is found to be strictly stable.

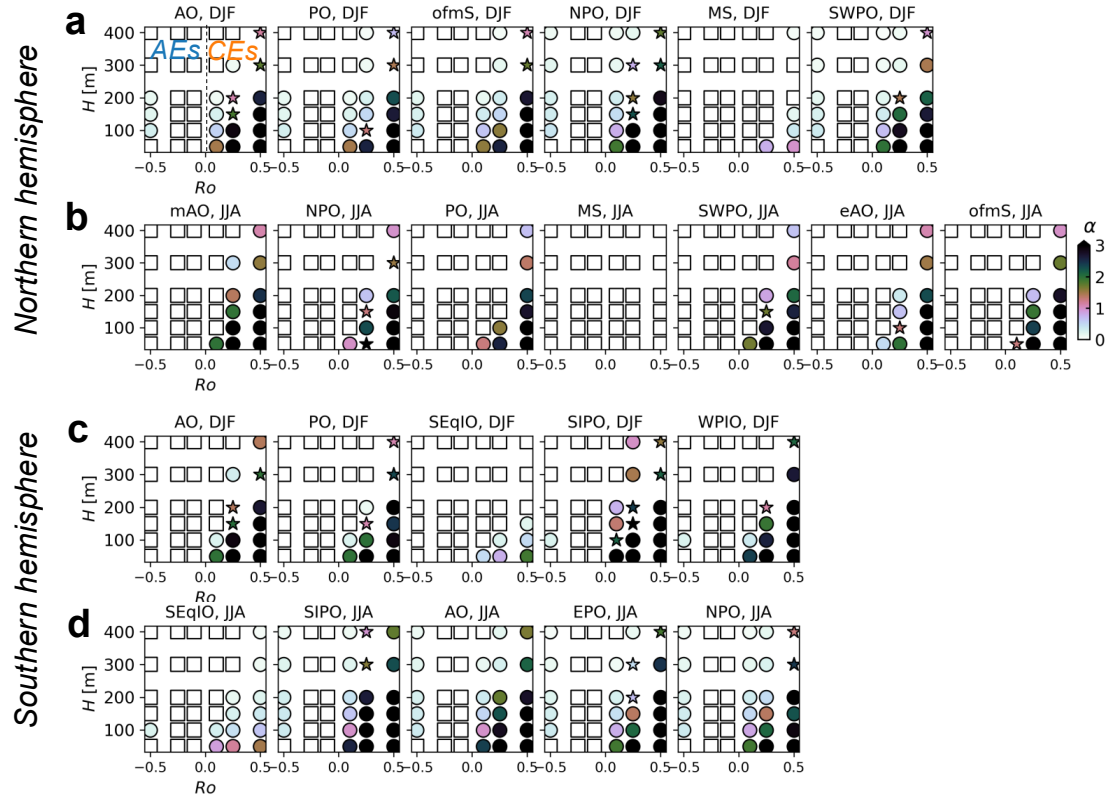


Figure 4. Static stability of prescribed eddies as a function of eddy intensity (Ro , negative values are for anticyclones and positive values for cyclones), vertical extent (H), and background density profile. Square markers indicate statically stable eddies, for which no density-gradient inversion occurs at the eddy centre. Circle markers indicate eddies with one statically unstable layer, while star markers indicate eddies with two or more distinct unstable layers. Marker colour indicates the total thickness of the statically unstable layers normalized by the eddy thickness.

We then repeat the analysis for subsurface-intensified eddies. Because the parameter space is larger, we restrict this analysis to eddies with $Ro = 0.1$, and vary their vertical extent, H , and depth of intensification, z_0 . The contrast between the two polarities is much weaker for subsurface eddies, with broadly similar regions of instability in parameter space, although anticyclones generally exhibit smaller values of α . The tendency for thinner eddies to be more susceptible to instability is again clearly
 255 apparent. In addition, the depth of intensification introduces an important control: thin subsurface eddies are more likely to remain statically stable when their core is located at shallower depths, *i.e.*, closer to the strongly stratified pycnocline. Conversely, when their core lies deeper in the weakly stratified ocean interior, the eddy-induced density anomaly is more likely to reverse the background density gradient and generate static instability.

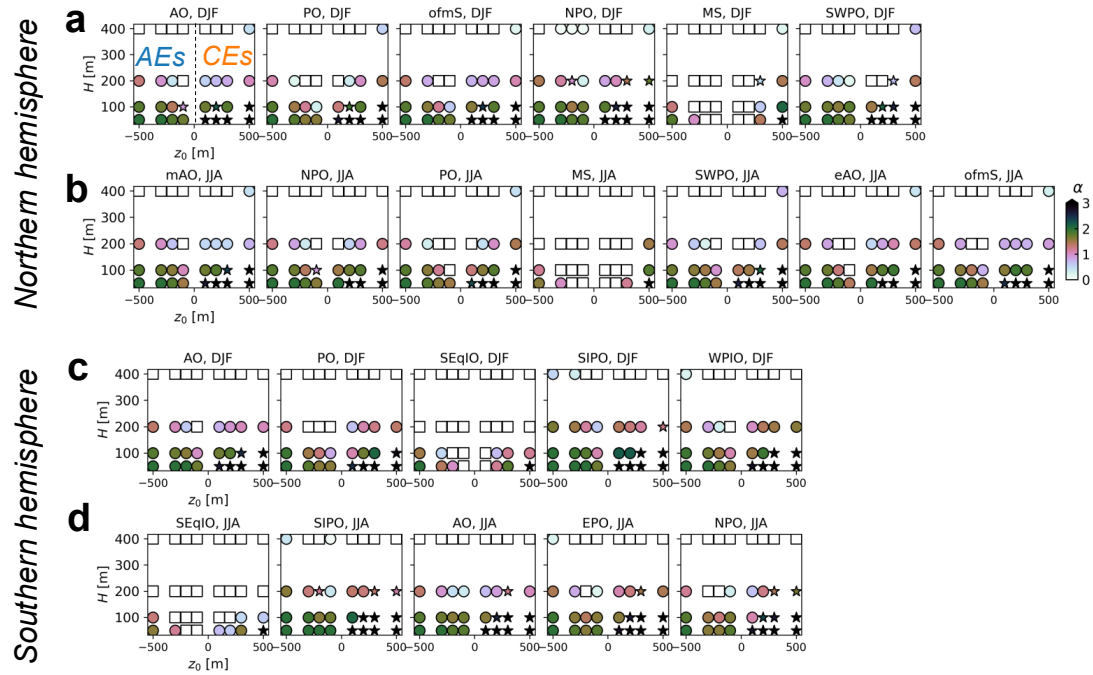


Figure 5. Same as Fig. 4, but for subsurface intensified eddies.

5 Discussion

Our results reveal clear controls on the static stability of oceanic eddies. For surface-intensified eddies, static stability is primarily controlled by eddy polarity and intensity, together with its vertical extent. Cyclonic eddies are systematically more susceptible to gravitational instability, with the strongest and thinnest cyclones being the most affected. Anticyclonic eddies, in contrast, remain largely unconstrained by static stability over most of the parameter space considered here, except at the highest intensities. For subsurface-intensified eddies, the vertical position of the eddy relative to the background pycnocline provides an additional control, with thin eddies being particularly sensitive to the local stratification. These results should nevertheless be interpreted as robust patterns rather than as sharp, universal stability thresholds. The prescribed eddy structure is idealized, and the background density profiles considered here represent a finite sampling of the diversity of oceanic environments. Our objective was therefore not to define a universal boundary between stable and unstable eddies, but rather to identify the physical parameters that control this boundary. These results have important implications for the interpretation of the spatial distribution and evolution of oceanic eddies.

First, the geographical variability of the background density field may constrain the pathways along which eddies can propagate. An eddy with a given structure and intensity may be statically stable in its formation region but become unstable



after entering a region characterized by weaker background stratification. This process may represent an additional source of oceanic mixing. An eddy propagating into an environment whose background density structure is incompatible with its intrinsic density anomaly may develop gravitationally unstable layers within its core, triggering convective adjustment and vertical mixing, therefore "preconditioning" the occurrence of convection (Legg et al., 1998) *via* pure static instability. Such mixing could modify the eddy's water-mass properties and contribute to the redistribution of heat, salt, and other tracers within the ocean. Given the importance of eddy-driven processes to the oceanic budgets of heat, freshwater, and biogeochemical properties, this mechanism may therefore represent an additional pathway through which eddies contribute to vertical exchange. Quantifying its contribution to the total oceanic mixing budget, however, requires explicitly resolving the subsequent evolution of the unstable eddy and is beyond the scope of the present study.

Second, the background stratification may constrain the regions in which particular types of eddies can be generated and persist. An eddy generated by a given instability or current structure is associated with a characteristic three-dimensional vorticity and density anomaly. This structure may be compatible with static stability in some regions but not in others. Static stability could therefore contribute to shaping the geographical distribution of eddies, in addition to the dynamical processes that control their generation, propagation, and decay. In this sense, the spatial distribution of oceanic eddies may reflect not only dynamical properties such as current intensity, available potential energy, instability mechanisms, and external forcing, but also the geographical variability of the background density field; a prescribed jet or boundary current is also associated with a three-dimensional density structure whose compatibility with the background stratification can be assessed using the same static-stability criterion.

The relevance of this constraint can also be considered in the context of observed three-dimensional eddy structures. Observational composites based on *e.g.*, Argo profiles generally reveal density and hydrographic anomalies that depart from the simple vertical structures imposed in idealized numerical experiments (de Marez et al., 2020). In particular, surface-intensified eddies do not generally exhibit a purely exponential vertical decay of their velocity and density signatures. Instead, their vertical structures reflect the interaction between the eddy and the background stratification, as well as the processes involved in their generation and subsequent evolution (Chaigneau et al., 2011). This highlights an important caveat of the present study. In the ocean, eddy density anomaly and background stratification are not independent: eddy formation and evolution involve the continuous deformation and mixing of the surrounding density field, so that the observed density structure results from their dynamical interaction. In particular, the isopycnal surfaces associated with an eddy are not simply superimposed on an otherwise undisturbed background state, but are continuously displaced, stretched, and smoothed by the eddy circulation and by mixing processes. This is also reflected in observational approaches to eddy identification, where eddies are characterized from thermohaline anomalies relative to the surrounding water rather than from a prescribed anomaly embedded in a prescribed background profile (Barabinot et al., 2025a).

Our approach can thus be seen as an *a priori* estimate of the static compatibility of an idealized eddy structure with a given oceanic environment, and our interpretations remain to be tested directly. A systematic comparison between the static-stability criterion developed here and observed three-dimensional eddy structures would be a natural next step. In particular, Argo-derived composites could be used to examine whether the observed vertical density signatures of eddies systematically occupy



the region of parameter space predicted to be statically stable by the present framework. Such an analysis would provide an observational test of whether static stability acts as an effective filter on the three-dimensional structures of oceanic eddies.

310 To facilitate such applications, we provide an accompanying notebook that implements the framework developed here (<https://github.com/demarez/eddy-in-stratif>). The user can prescribe the properties of an eddy, including its polarity, intensity, horizontal and vertical structure, and position within the water column, together with a background density profile representative of the environment of interest. This provides a simple way to assess the static admissibility of an idealized eddy for a particular oceanic environment without requiring a dedicated numerical simulation. The framework can therefore be read-
315 ily applied to specific regions, seasons, or observed eddy properties, and may provide a useful first-order diagnostic before undertaking more computationally expensive dynamical simulations.



Appendix A: A simple analytical example to determine the statically unstable depths

In this appendix, we provide an idealized and generic model of the vertical buoyancy profile of the ocean in the presence of a vortex, for which the depth range of density inversion can be determined analytically. We superimpose the *in situ* buoy-
 320 ancy profile of the ocean at rest onto the buoyancy anomaly associated with the vortex. This simple formulation allows us to determine the depths at which the resulting density field becomes statically unstable.

First, we describe a generic form for the mean vertical profile of buoyancy $b = -g\rho/\rho_0$ in a given region. A typical back-ground buoyancy profile is

$$B(z) = b_1 \exp(z/h)$$

where h is physically related to the depth of the pycnocline. Both b_1 and h are geographically dependent.

325 We then derive the buoyancy anomaly associated with the vortex. We start from the analytical form of relative vorticity for a subsurface-intensified eddy,

$$\zeta(r, z) = \pm \omega_0 \exp(-(r/R)^\gamma) \exp(-(z - z_0)^2/H^2).$$

We define $f(r)$ as the inverse Laplacian of the radial component of the vorticity profile, such that

$$\frac{d^2 f}{dr^2} + \frac{1}{r} \frac{df}{dr} = \exp(-(r/R)^\gamma),$$

with a constant for f , and define the streamfunction as

$$\psi(r, z) = f(r) \exp(-(z - z_0)^2/H^2), \quad V(r, z) = d\psi/dr, \quad \Omega(r, z) = V(r, z)/r.$$

Considering the cyclogeostrophic balance

$$\frac{V^2}{r} + f_0 V = \frac{-1}{\rho_0} \frac{dp}{dr}$$

330 and differentiating it with respect to z , while using the hydrostatic balance $\partial_z p = -g\rho$, gives the buoyancy anomaly induced by the eddy,

$$b'(r, z) = (2\Omega(r, z) + f_0) \partial_z \psi(r, z).$$

The total buoyancy distribution is then given by $b = B + b'$. Static stability requires $\partial_z b < 0$, while the onset of static insta-bility corresponds to the marginal condition $\partial_z b = 0$. We evaluate this condition at the centre of the eddy, where the density anomaly reaches its maximum amplitude.

335 The marginal condition can be written exactly as

$$(b_1/h) \exp(z/h) + 2\partial_z \Omega(0, z) \partial_z \psi(0, z) + (2\Omega(0, z) + f_0) \partial_z \psi(0, z) = 0,$$



and can then be evaluated using the expressions given above. The resulting condition depends explicitly on the vortex parameters γ and ω_0 , as well as on the properties of the background buoyancy profile. The depths at which the marginal condition is satisfied can therefore be determined numerically for a given set of eddy and background parameters. These depths delimit the regions in which the total buoyancy gradient changes sign and the vortex becomes statically unstable.

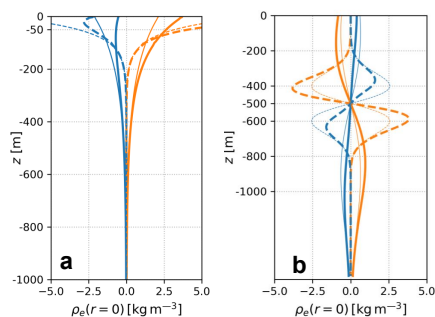


Figure B1. Same as Figs. i,j but for eddies with $Ro = 0.5$.

340 Appendix B: Supplementary figures

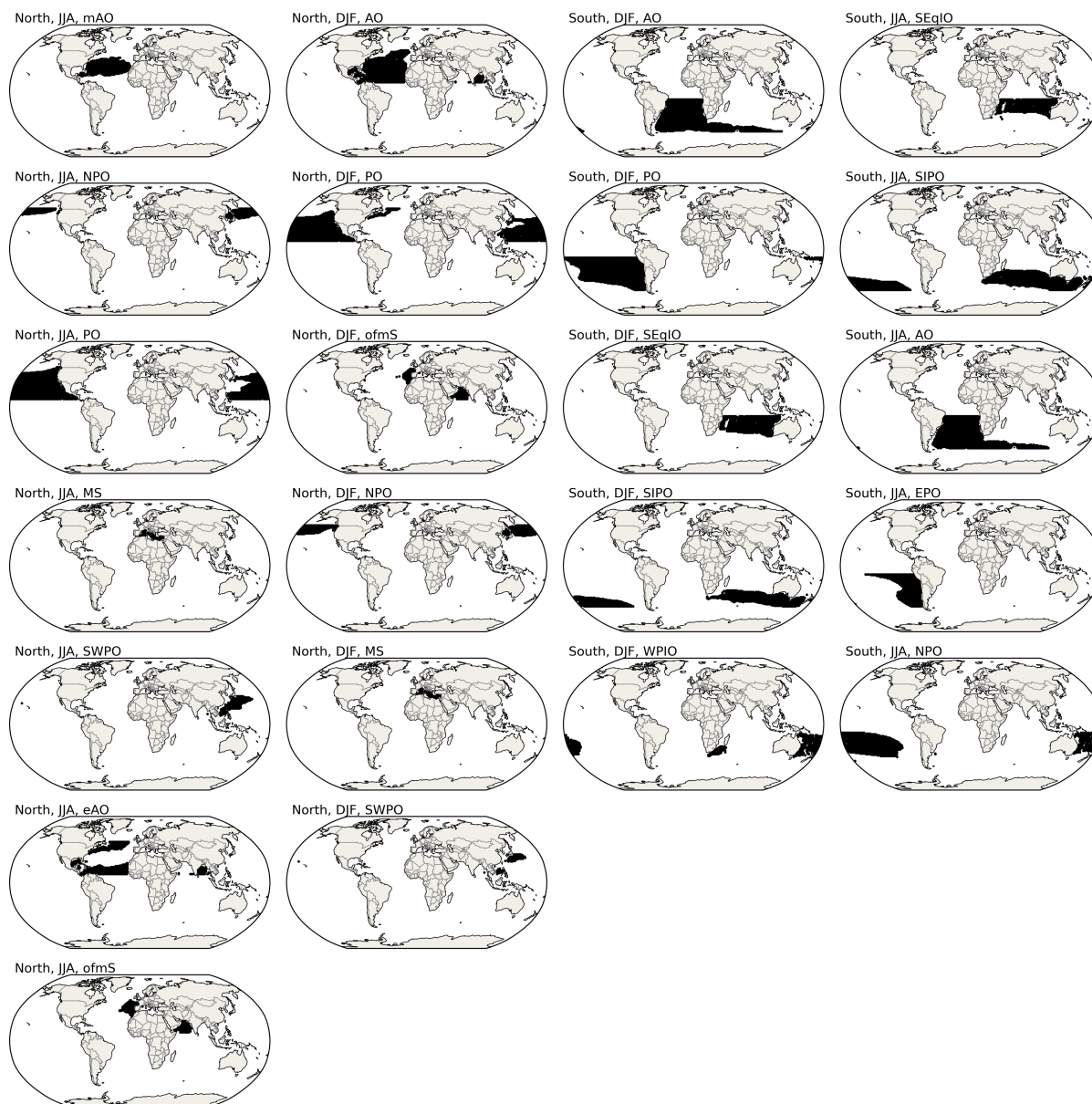


Figure B2. Areas obtained from classification, used to define average profiles in Fig. 1.

Code and data availability. A notebook to reproduce the results presented here is available on the author's github <https://github.com/demarez/eddy-in-stratif>. CORA dataset is available on Copernicus Marine Data Store.



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345 *Competing interests.* The authors declare no competing interests.

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