

S1 Doppler-velocity averaging and residual-folding detection

S1.1 Reflectivity-weighted pulse-pair averaging

Doppler velocity was spatially averaged in pulse-pair phase space to account for its periodicity across the Nyquist interval. Let $V_{D,i}^{\text{CLP}}$, $Z_{e,i}$, and $V_{N,i}$ denote the CPR_CLP Doppler velocity, radar reflectivity factor, and Nyquist velocity, respectively, at gate i . The Doppler phase and linear radar reflectivity were defined as

$$\phi_i = \pi \frac{V_{D,i}^{\text{CLP}}}{V_{N,i}},$$

and

$$Z_i = 10^{Z_{e,i}/10}.$$

For an averaging window W_j associated with target gate j , the reflectivity-weighted real and imaginary components were calculated as

$$C_j = \sum_{i \in W_j} Z_i \cos \phi_i, \quad S_j = \sum_{i \in W_j} Z_i \sin \phi_i,$$

and the mean Doppler phase was obtained from

$$\bar{\phi}_j = \text{atan2}(S_j, C_j).$$

Each input gate was converted to phase using the Nyquist velocity of its corresponding CPR ray. The mean phase was converted back to velocity using the Nyquist velocity $V_{N,j}$ of the target ray,

$$\tilde{V}_{D,j} = \frac{V_{N,j}}{\pi} \bar{\phi}_j.$$

Because pulse-pair averaging retains phase but not the Nyquist-branch index, $\tilde{V}_{D,j}$ initially lies on the principal branch. To retain the velocity branch represented in the operationally unfolded CPR_CLP product, the averaged velocity was shifted by an integer multiple of $2V_{N,j}$ to the branch closest to the CPR_CLP velocity at the target gate:

$$V_{D,j} = \tilde{V}_{D,j} + 2V_{N,j} \text{round}\left(\frac{V_{D,j}^{\text{CLP}} - \tilde{V}_{D,j}}{2V_{N,j}}\right).$$

For the climatological statistics, W_j contained three along-track profiles and two vertical range gates ($3 \text{ km} \times 200 \text{ m}$), and the average was retained only when all six gates were valid. The same procedure with a 3×1 window was used to reduce along-track random noise before residual-folding detection.

S1.2 Residual-folding detection

Residual Nyquist folding was identified from vertical discontinuities in the 3-km along-track averaged Doppler velocity. The ray-wise Nyquist velocity V_N was obtained from the corresponding CPR_ECO product. Because the CPR_CLP operational unfolding produces different expected velocity branches below and above the freezing level, separate criteria were applied for $T < 273.15 \text{ K}$ and $T \geq 273.15 \text{ K}$.

For adjacent valid gates below 273.15 K, where the conventional symmetric Nyquist branch is assumed, a pair of velocities v_k and v_{k+1} was identified as fold-like when all of the following conditions were satisfied:

$$\begin{aligned}
|v_{k+1} - v_k| &> 1.5V_N, \\
v_k v_{k+1} &< 0, \\
\max(|v_k|, |v_{k+1}|) &> 0.6V_N.
\end{aligned}$$

These criteria require a large vertical discontinuity, a change in sign, and proximity of at least one velocity to a Nyquist boundary, reducing false detections due to ordinary vertical shear or measurement noise.

For $T \geq 273.15$ K, the operational CPR_CLP unfolding permits an asymmetric velocity branch. With the operational upper-branch limit of 3 m s^{-1} , the expected interval was taken as

$$V_L = 3 - 2V_N, \quad V_U = 3 \text{ m s}^{-1}.$$

A vertically adjacent pair was classified as fold-like when the velocity difference exceeded $1.5V_N$ and the two velocities were located near opposite edges of this branch. Specifically, the edge width was defined as $0.4V_N$, such that

$$v_k \leq V_L + 0.4V_N \text{ and } v_{k+1} \geq V_U - 0.4V_N$$

or

$$v_{k+1} \leq V_L + 0.4V_N \text{ and } v_k \geq V_U - 0.4V_N.$$

Both velocities were additionally required to lie within 0.5 m s^{-1} of the expected branch interval. Adjacent-gate pairs spanning 273.15 K were not evaluated because the assumed velocity branch changes across the freezing level.

A CPR profile was classified as affected by residual folding when at least one fold-like vertical discontinuity satisfying the relevant temperature-dependent criteria was detected. Such profiles were excluded from the target samples used in the climatological statistics.

S2 Supplemental figures

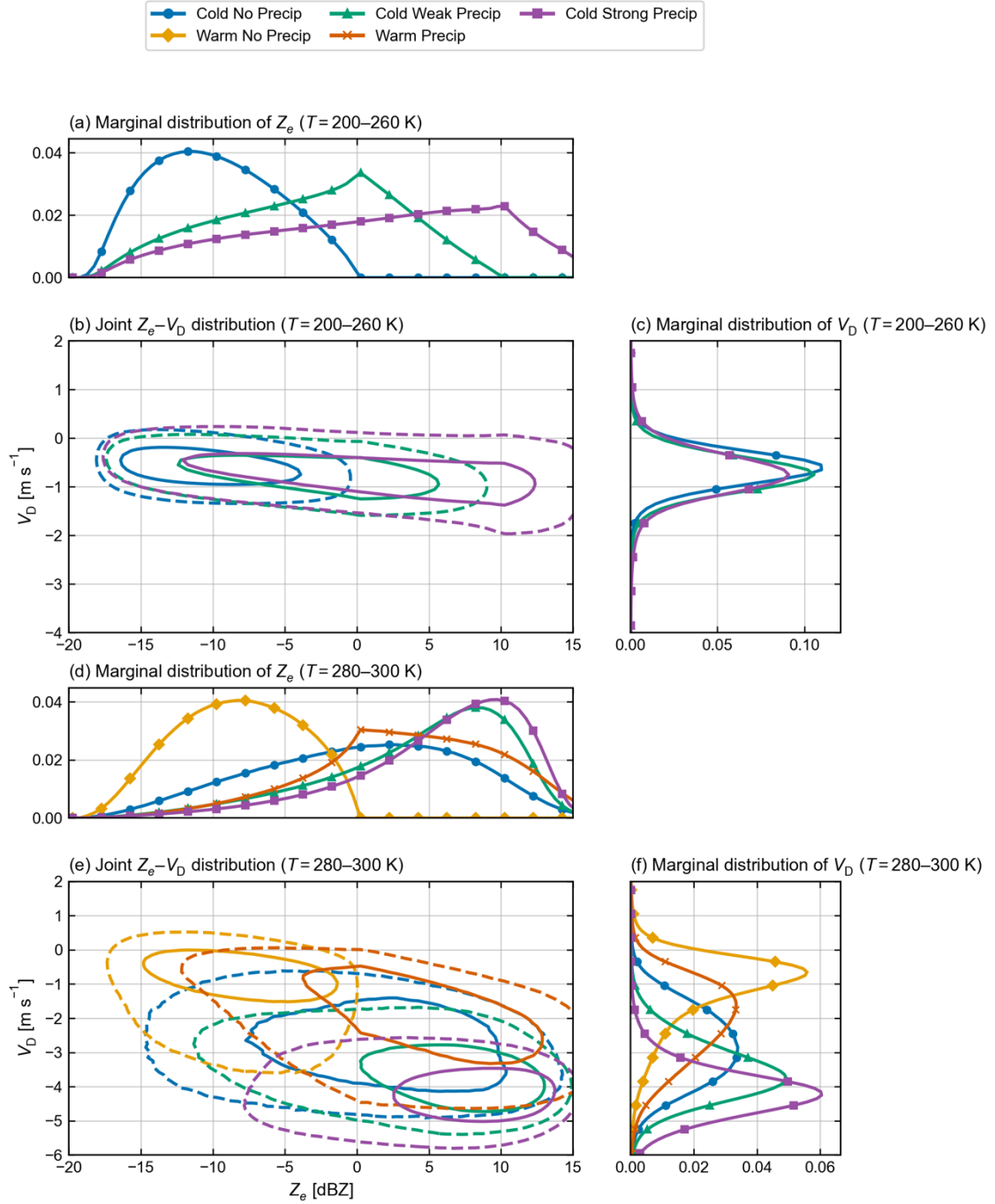


Figure S1. Marginal and joint distributions of Z_e and V_D for each cloud type in the cold (200–260 K; a–c) and warm (280–300 K; d–f) temperature ranges. Panels (a) and (d) show the marginal distributions of Z_e , panels (b) and (e) show the joint Z_e – V_D distributions, and panels (c) and (f) show the marginal distributions of V_D . Solid and dashed contours in the joint-distribution panels enclose 50% and 90% of the probability mass, respectively.

450–550 hPa (middle troposphere)

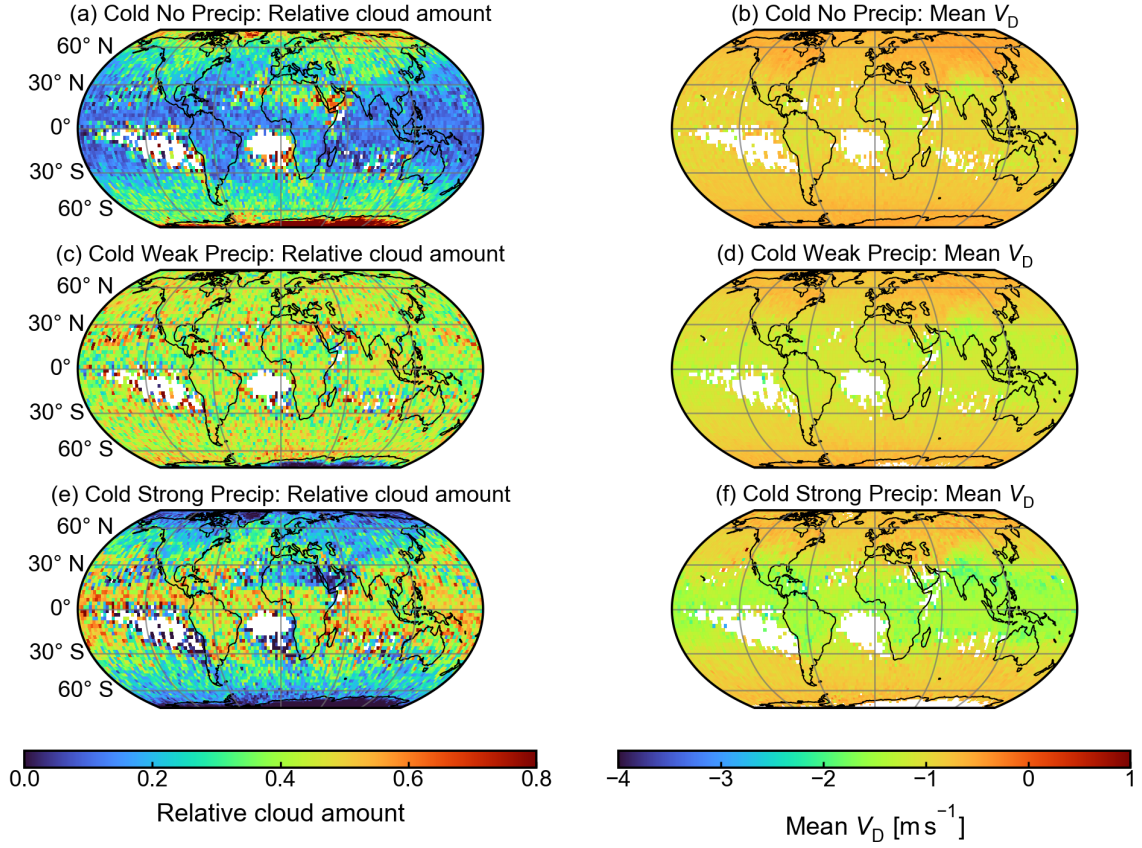


Figure S2. Cloud-type decomposition of the middle-level (450–550 hPa) cloud field. The left column shows the relative cloud amount of each cloud type, defined as its fraction of the total cloudy samples in each grid cell. The right column shows the corresponding mean V_D .

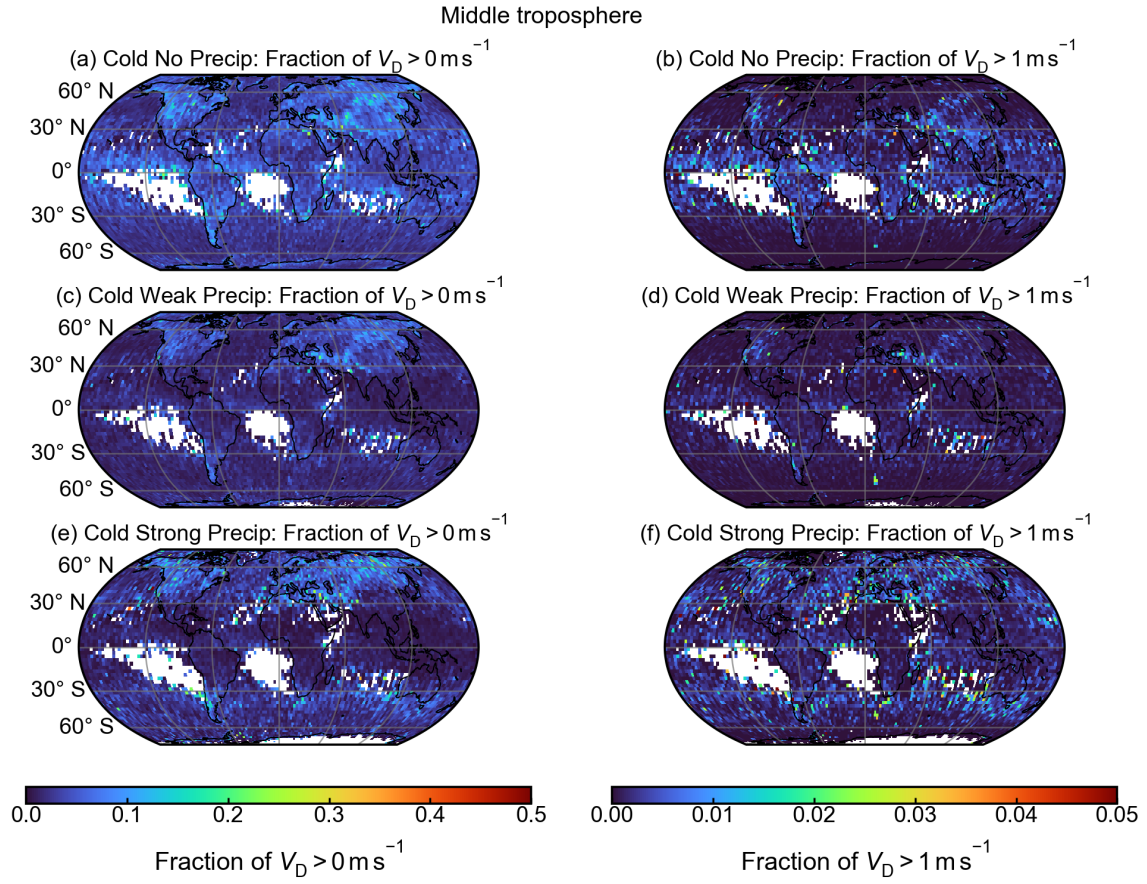


Figure S3. Cloud-type decomposition of the fractions of samples with $V_D > 0 \text{ m s}^{-1}$ (left) and $V_D > 1 \text{ m s}^{-1}$ (right) in the 450–550 hPa layer. Each row corresponds to one cloud type. Same as Fig. 11 but for mid-levels (450–550 hPa).