

Supplementary material for: “Simplifying assumptions made by contrail-cirrus models lead to irreconcilable differences in geometric and optical contrail properties”

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S1 Reformulated CoCiP

This section discusses the premature sublimation of some CoCiP contrails in supersaturated air, and the reformulation to CoCiP implemented by Dr. Tristan Abbott and Dr. Zebediah Engberg from Contrails.org (Breakthrough Energy). Akhtar Martínez et al. (2025) first discovered the premature evaporation of the CoCiP contrail and Abbott diagnosed the root cause of it by analysing the ice growth equation of Schumann (2012). It is important to highlight that the premature sublimation is not a coding bug but rather an emergent property from the ice growth equations in the original CoCiP formulation.

As seen in Sect. 3.2.1 of the main text, CoCiP transports the water vapour with the contrail ice in the Gaussian plume. As the CoCiP contrail descends, CoCiP uses the ambient temperature lapse rate to determine the temperature of the contrail at the new (vertically lower) location. In a stably stratified atmosphere, descending results in the contrail temperature increasing. Despite this, the water vapour in the contrail is isolated from the ambient water vapour and maintains the value at the previous timestep (before air is entrained by the contrail) as if the contrail air was physically separate from the ambient air. When the atmosphere is stably stratified, the temperature increase combined with a constant specific humidity causes a decrease in the relative humidity inside the contrail before the new air is entrained. After new air is entrained into the contrail, CoCiP forces the water vapor in the contrail to be saturated. Hence, if not enough water mass is provided to the contrail through the entrainment of fresh air, the contrail ice is forced to sublimate to satisfy the deficit in relative humidity (even if the ambient air is supersaturated), leading to the (seemingly) premature sublimation of the contrail. This effect is likely to be most evident in cases where the ambient lapse rate is high, the ambient supersaturation is low, or when the contrail sedimentation speed is high.

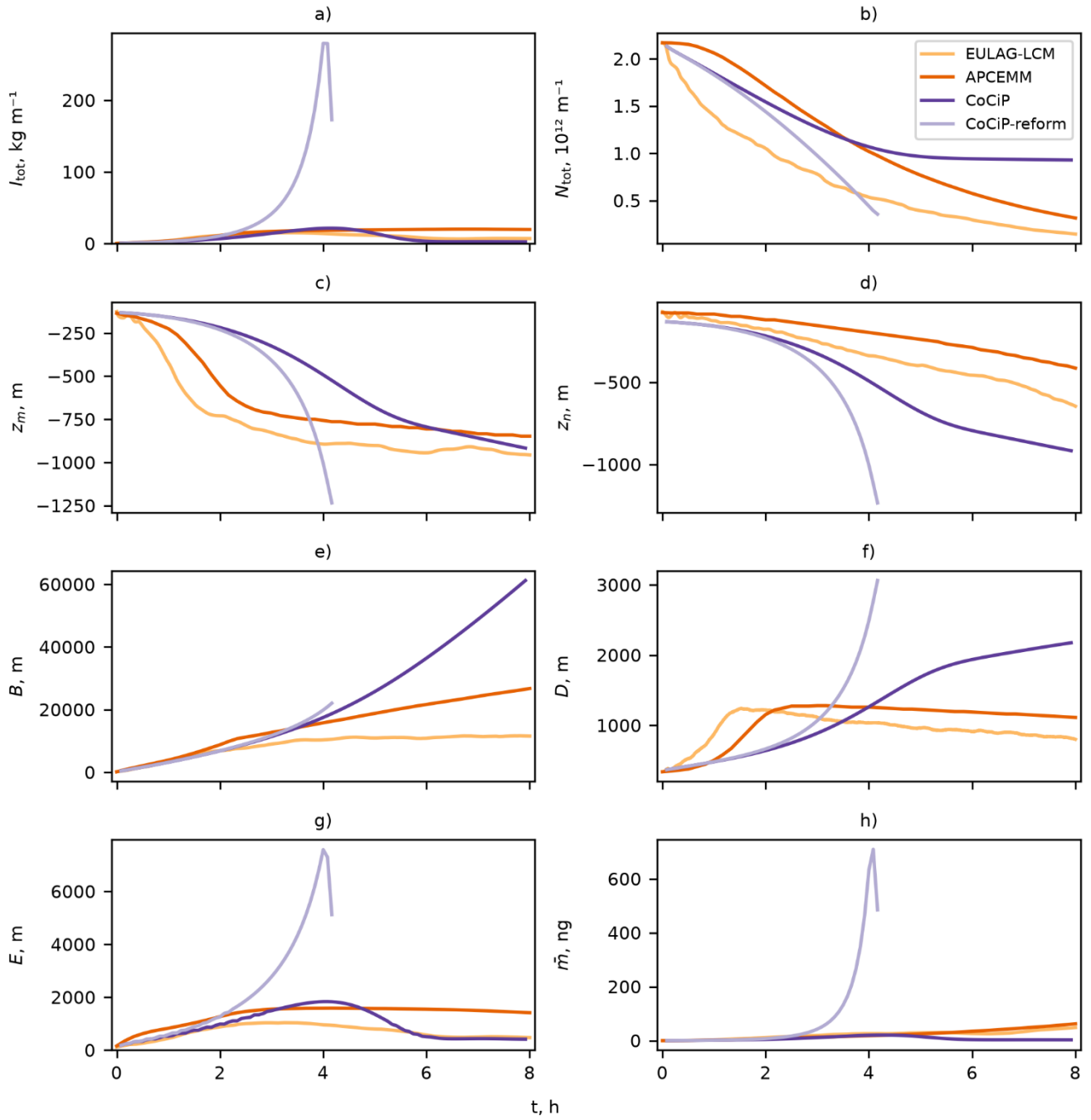
Abbott and Engberg reformulated the CoCiP ice growth equation to prevent the evaporation of ice within a stably stratified ISSR, behaving more like is expected from ice in supersaturated conditions. This reformulated version of CoCiP (hereafter CoCiP-reform) is experimental (although it is publicly available). Abbott et al. provided notes on the development of the

reformulation through a private communication, from which we now provide a brief overview of the reformulation of the ice growth equation in CoCiP.

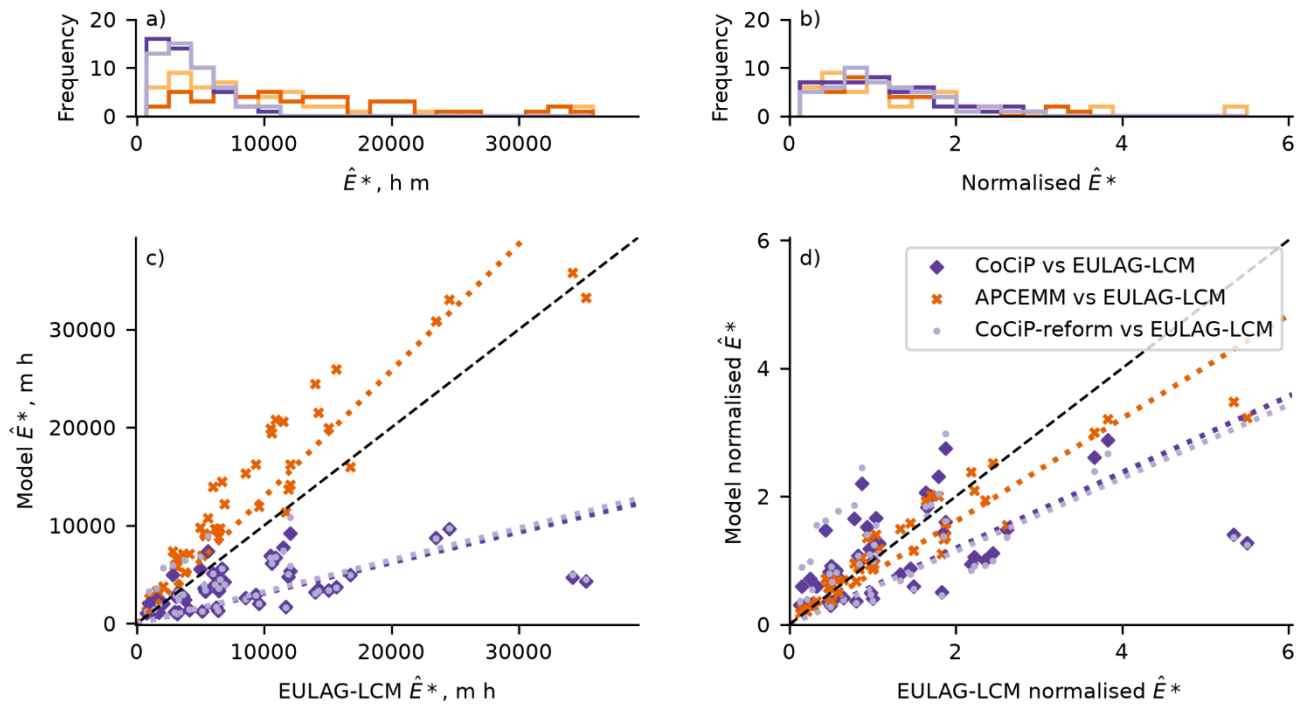
In the CoCiP-reform, the contrail water budget equation is modified to include air mass loss (assumed to be at saturation) at the trailing side of the contrail and air mass intake (assumed to be at the ambient conditions) at the leading side of the contrail – i.e. the contrail is treated as a control volume instead of a “balloon”. The assumption that the air mass leaving the contrail at the trailing edge is saturated assumes that there is enough time for the contrail ice to deplete all supersaturation and hence requires careful consideration when small timesteps are used. Overall, by treating the contrail as a control volume, the previous mismatch between ambient water budget and the water budget in the contrail is eliminated, as can be seen in the results below. Figure S1 shows the time-evolution of the contrail properties for the baseline case. CoCiP-reform simulations do not experience a gradual decrease in total ice mass and total extinction. Instead, they continue to grow in an exponential-like manner until ~ 4 h, at which point the CoCiP contrail enters subsaturated air. The effect of the reformulation on the lifetime total extinction is not straightforward because of the competing effects it causes. When compared to CoCiP, the increased ice mass in CoCiP-reform increases the total extinction locally by up to ~ 4 times, but it also causes an associated decrease in lifetime because of the higher associated settling speed. Therefore, depending on the process that dominates in each simulation, the predicted lifetime total extinction can either increase or decrease (as seen in Figs. S2-S3). In this set of simulations, the CoCiP-reform/EULAG-LCM line of best fit to have a gradient to 0.32 when comparing lifetime total extinction and 0.57 when comparing the median-normalised lifetime total extinction, showing only small changes from the line of best fit from CoCiP (0.31 and 0.59 respectively for the CoCiP/EULAG-LCM relationships).

Figure S3a shows that the cases in which sublimation takes place while the CoCiP contrail lies within the layer with the highest humidity (x-axis less than one) experience the largest increase in lifetime total extinction. Figure S3b shows that on average, CoCiP-reform predicts lifetime total extinctions 1.07 times what CoCiP predicts for the same scenario (given by the gradient of the dotted line of best fit). We note that 30 out of the 48 scenarios (63 %) result in an increase in lifetime total extinction, whereas 18 out of the 48 scenarios (37%) result in a decrease. This highlights that the effects of the reformulation are non-linear and dependent on the region of the parameter space being considered.

Fig. S4 indicates that the sensitivities for CoCiP-reform are like those from CoCiP, except when the updraught velocity changes from 0.00 m s^{-1} to 0.05 m s^{-1} , where the CoCiP-reform sensitivity is negative while the CoCiP sensitivity is positive. This is likely because the CoCiP-reform simulations are more susceptible to increases in the sedimentation velocity than the CoCiP formulation simulations due to the resolution of the premature sublimation.



60 **Figure S1: Time evolution of the total ice mass (I_{tot} ; a), total ice number (N_{tot} ; b), centre of mass vertical position (z_m ; c), centre of number vertical position (z_n ; d), width (B ; e), depth (D ; f), total extinction (E ; g), and the mean ice crystal mass ($\bar{m}$; h) of the contrail simulations using the baseline meteorology produced by EULAG-LCM, APCEMM, CoCiP, and the reformulated CoCiP.**



65 **Figure S2:** Panels a and b contain histograms showing the distribution of the lifetime total extinction in each model. Panels c and d contain parity plots showing the correlation between EULAG-LCM and APCEMM (orange), EULAG-LCM and CoCiP (purple), and EULAG-LCM and CoCiP-reform (light purple) for the lifetime total extinction. The dashed black diagonal line is the line of parity. Panels a and c use the absolute value of the lifetime total extinction, whereas panels b and d use the values normalised by the median value across all simulations by the given model.

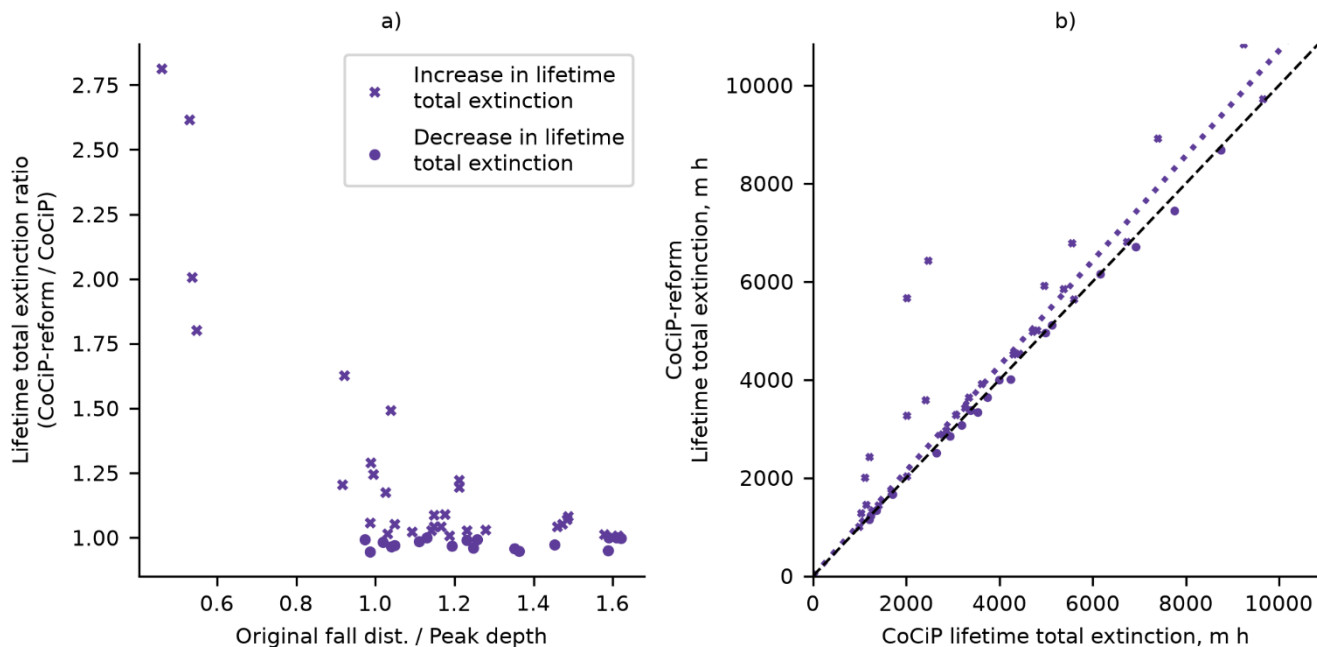
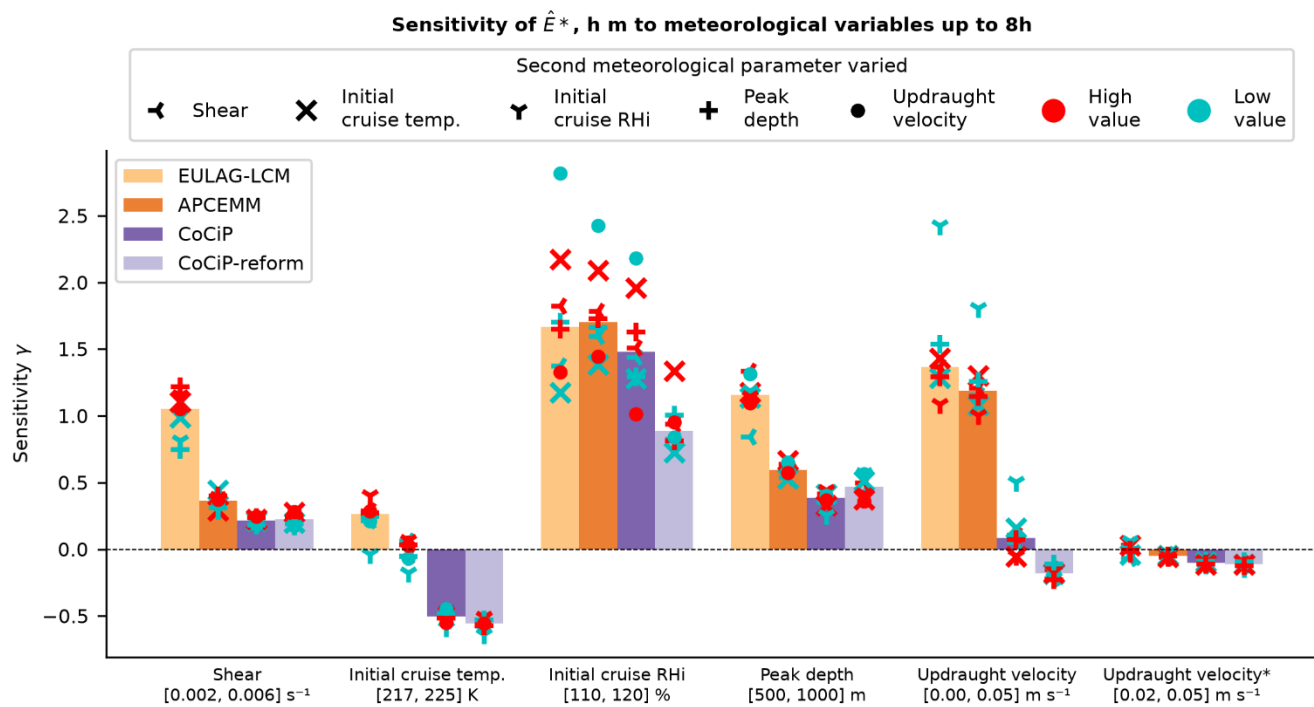


Figure S3: Change in lifetime total extinction between CoCiP and CoCiP-reform as a function of the fall distance before sublimation (panel a) and parity plots showing the correlation between CoCiP and CoCiP-reform (panel b). The dashed black diagonal line is the line of parity.



75 **Figure S4: Primary effect sensitivities (γ_i ; bars) and secondary effect sensitivities ($\gamma_{i|j=state}$; scatter) of the lifetime total extinction**
produced in each model to increases in the corresponding meteorological variable (see Eqs. 9-10). In plain language, the sensitivity
 γ_i is the average effect of increasing a meteorological variable across all runs. Similarly, $\gamma_{i|j=state}$ is the same quantity calculated
for the subset of runs in which a second meteorological parameter is either low (blue; state = lo) or high (red; state = hi) according
80 **to the values in Table 3. Sensitivities cannot be compared across variables, only across models. Runs with zero vertical velocity are**
excluded from all sensitivity calculations.

S2 Visualization of model predictions for each parameter combination

Figures S5-S7 show the runs alongside their corresponding parameter combination. Qualitatively, the runs with the highest normalised lifetime total extinction have high values of RH_i, shear, and layer depth. The runs with high temperature are disfavoured in CoCiP, but not in APCEMM and EULAG-LCM.

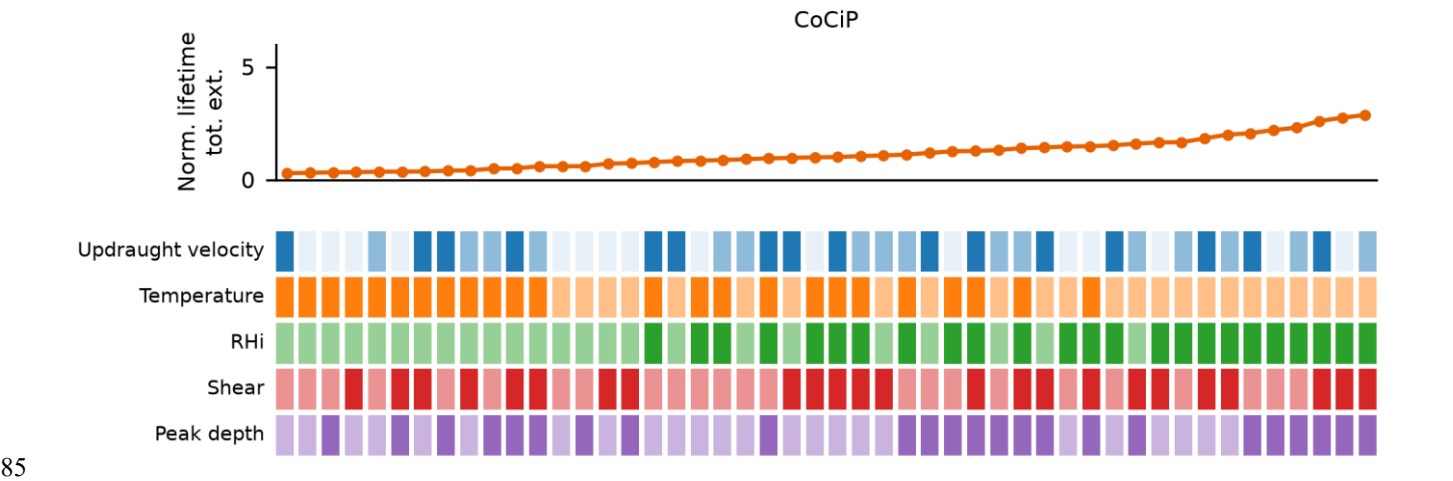


Figure S5: Visualization of the lifetime total extinction (normalised by the median) predicted by CoCiP for each parameter combination. Each column represents one run. The colour in each row represents a parameter value. The dark hue corresponds to the higher parameter value in Table 3, whereas the light hue corresponds to the lower parameter value. For the vertical velocity, the lightest hue corresponds to the $w_{syn} = 0.00 \text{ m s}^{-1}$ case.

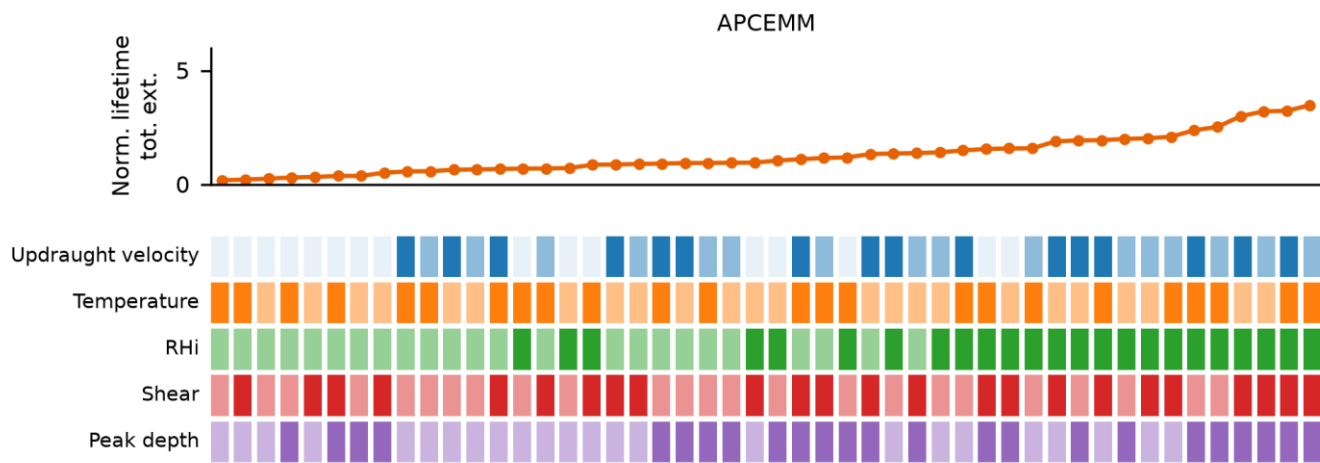


Figure S6: Visualization of the lifetime total extinction (normalised by the median) predicted by APCEMM for each parameter combination. Each column represents one run. The colour in each row represents a parameter value. The dark hue corresponds to the higher parameter value in Table 3, whereas the light hue corresponds to the lower parameter value. For the vertical velocity, the lightest hue corresponds to the $w_{syn} = 0.00 \text{ m s}^{-1}$ case.

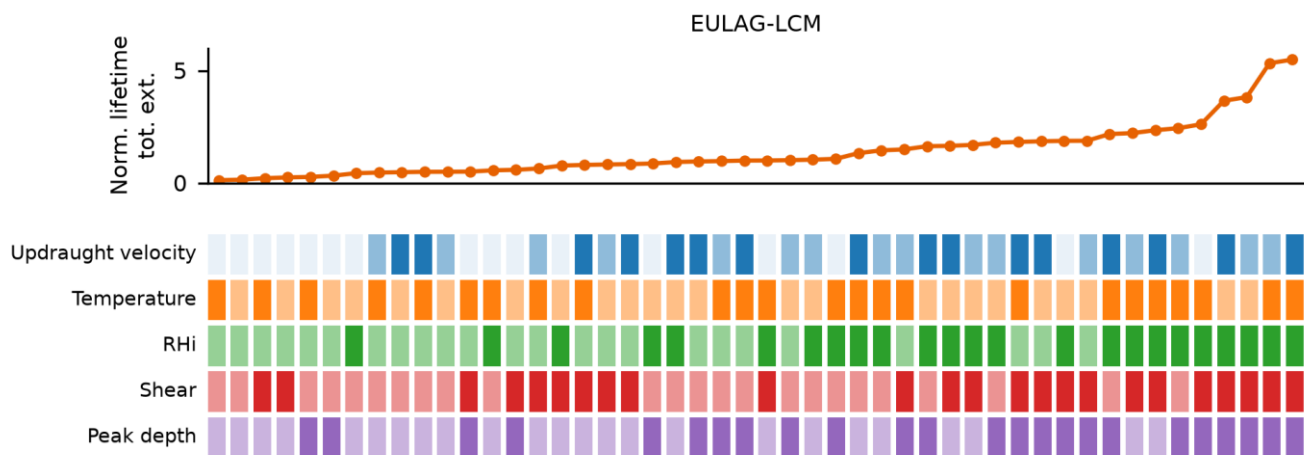


Figure S7: Visualization of the lifetime total extinction (normalised by the median) predicted by EULAG-LCM for each parameter combination. Each column represents one run. The colour in each row represents a parameter value. The dark hue corresponds to the higher parameter value in Table 3, whereas the light hue corresponds to the lower parameter value. For the vertical velocity, the lightest hue corresponds to the w_{syn} from 0.00 m s^{-1} case.

References

Akhtar Martínez, C., Eastham, S. D., and Jarrett, J. P.: Zero-dimensional contrail models could underpredict lifetime optical depth, *Atmos. Chem. Phys.*, 25, 12 875–12 891, <https://doi.org/10.5194/acp-25-12875-2025>, 2025.

Schumann, U.: A contrail cirrus prediction model, *Geosci. Model Dev.*, 5, 543–580, <https://doi.org/10.5194/gmd-5-543-2012>,

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