

Supplementary Materials for

Variable causality in Cenozoic climate-carbon cycle interactions

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Supplementary Text S1
Figs. S1 to S15

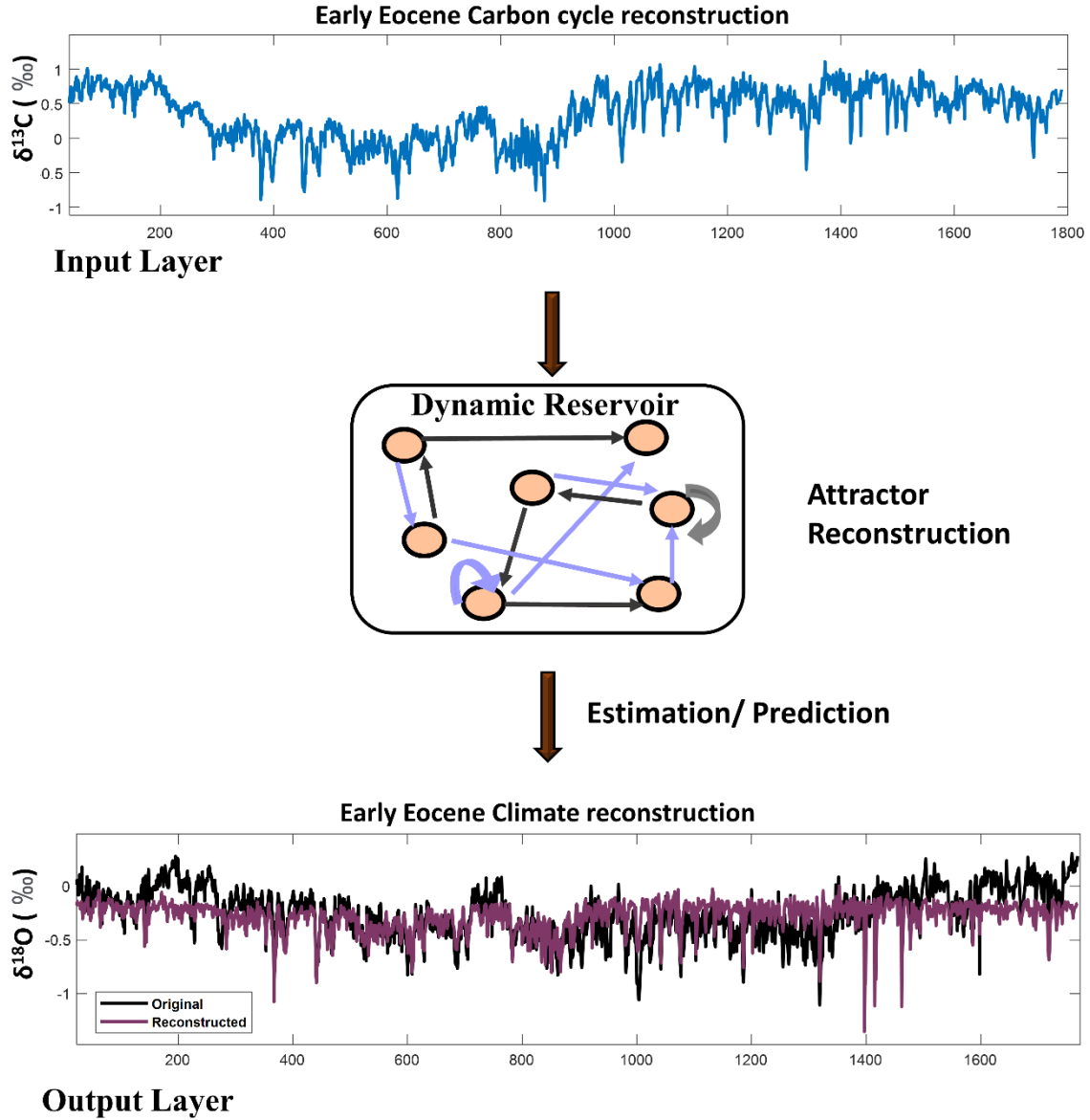


Figure S1: Schematic illustration of RCC framework to determine causality using early Eocene $\delta^{13}\text{C}$ record's ability to predict early Eocene $\delta^{18}\text{O}$ record. The correlation between the original and reconstructed $\delta^{18}\text{O}$ records gives the RCC skill quantifying $\delta^{13}\text{C} \rightarrow \delta^{18}\text{O}$.

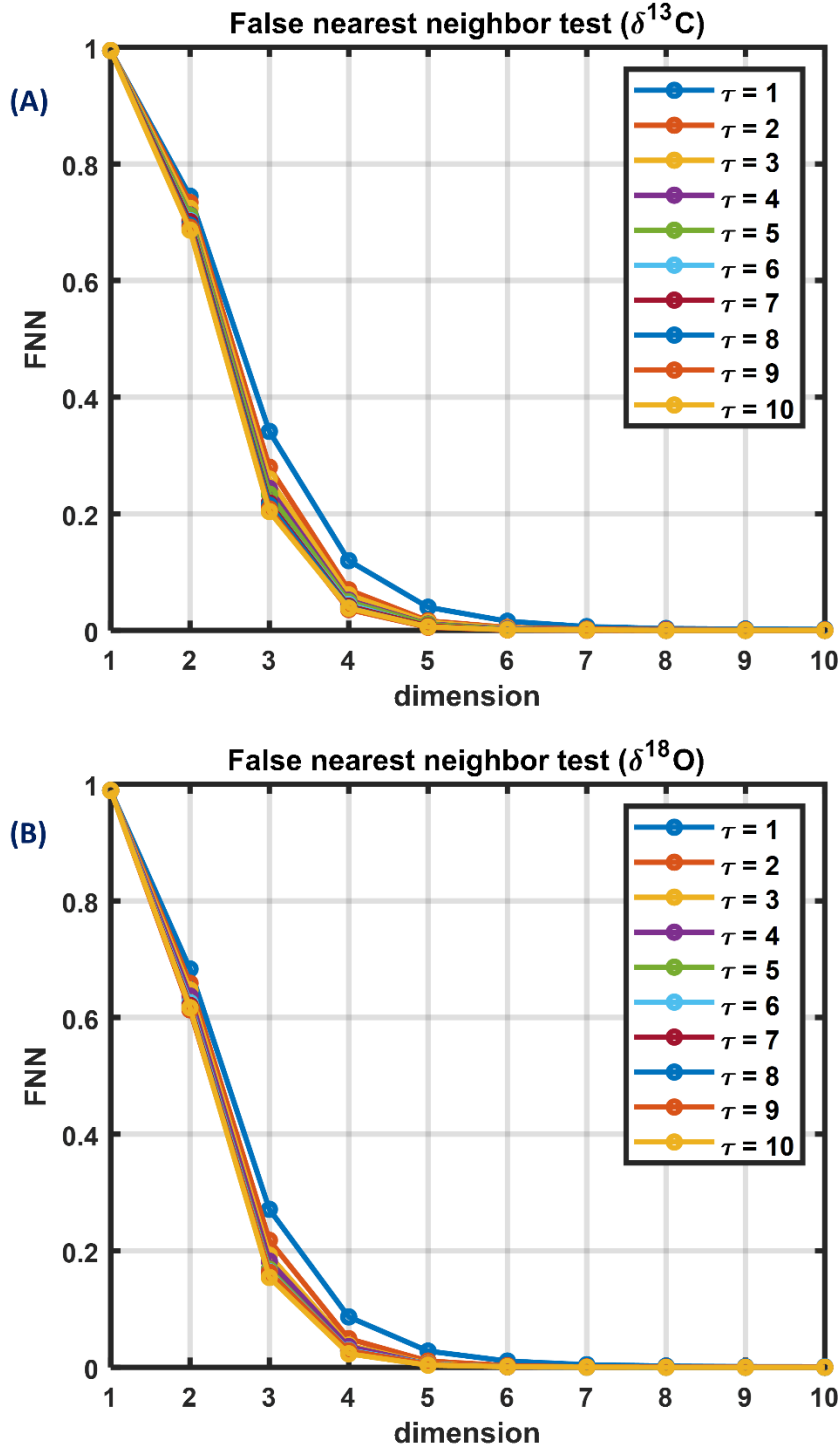


Figure S2: False nearest neighbor for the entire Cenozoic reconstruction using paired benthic foraminiferal of the carbon cycle and the climate, $\delta^{13}\text{C}$ (A) and $\delta^{18}\text{O}$ (B), respectively. The false nearest neighbor was calculated for both time lag- τ and embedding dimension ranging from 1 to 10. For the DCCM analysis, the time lag- τ and embedding dimension was taken as 1 and 6, respectively.

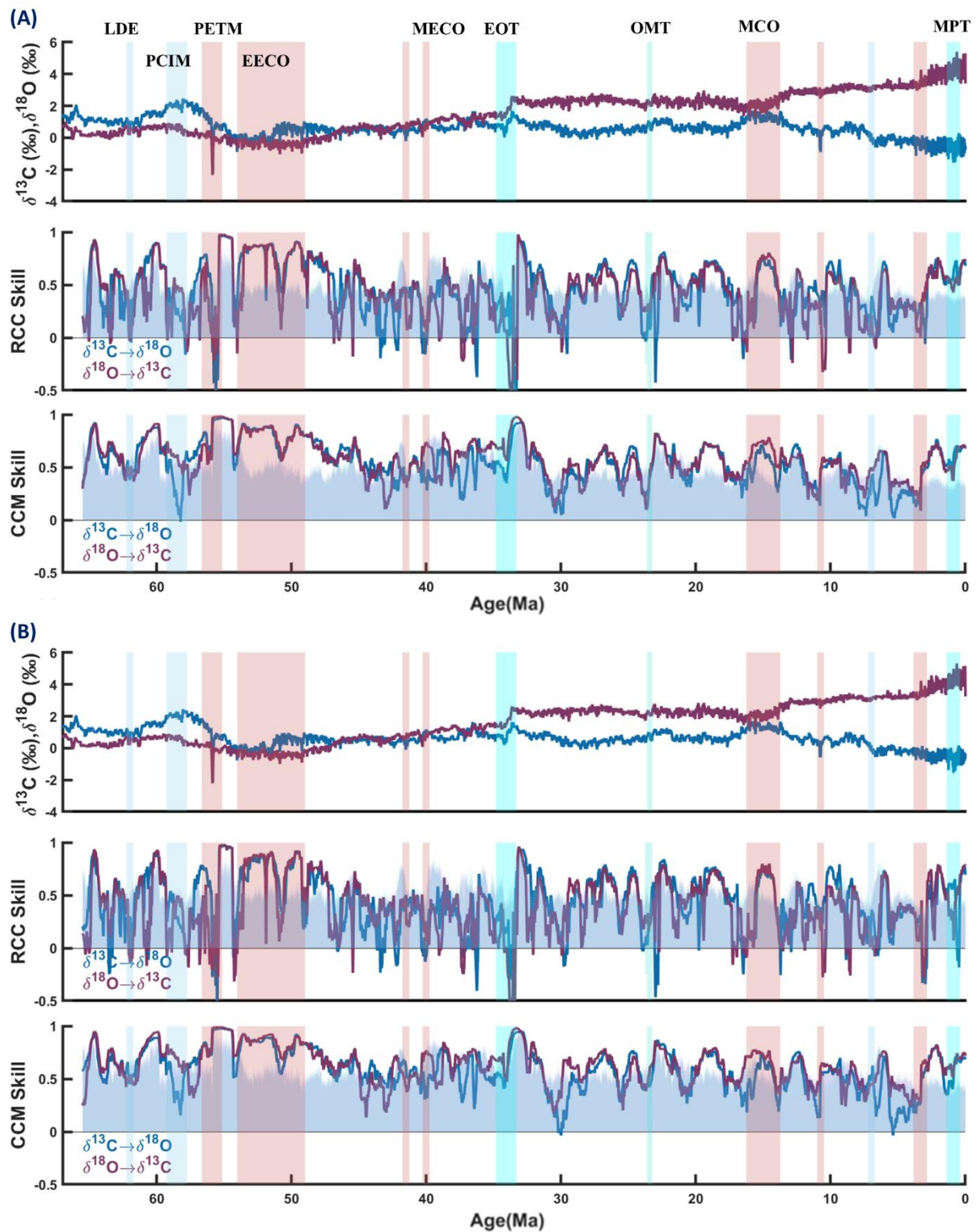


Figure S3: Dynamic causality analysis with a 1 Ma moving window on carbon cycle and climate reconstruction of the Cenozoic era along with 95% confidence level of the skills (shaded region) after reducing variability - A) Quadratic regression (Loess function) over 10 data points for smoothing; B) Quadratic regression (Loess function) over 20 data points for smoothing. The

general patterns in both RCC and CCM causal skills were like those for the original dataset, however, the significance level increased in both A and B, and most of the skills between 40 and 34 Ma become insignificant. This could mean that the data in this region have strong periodicity/seasonality making the null model generation inappropriate.

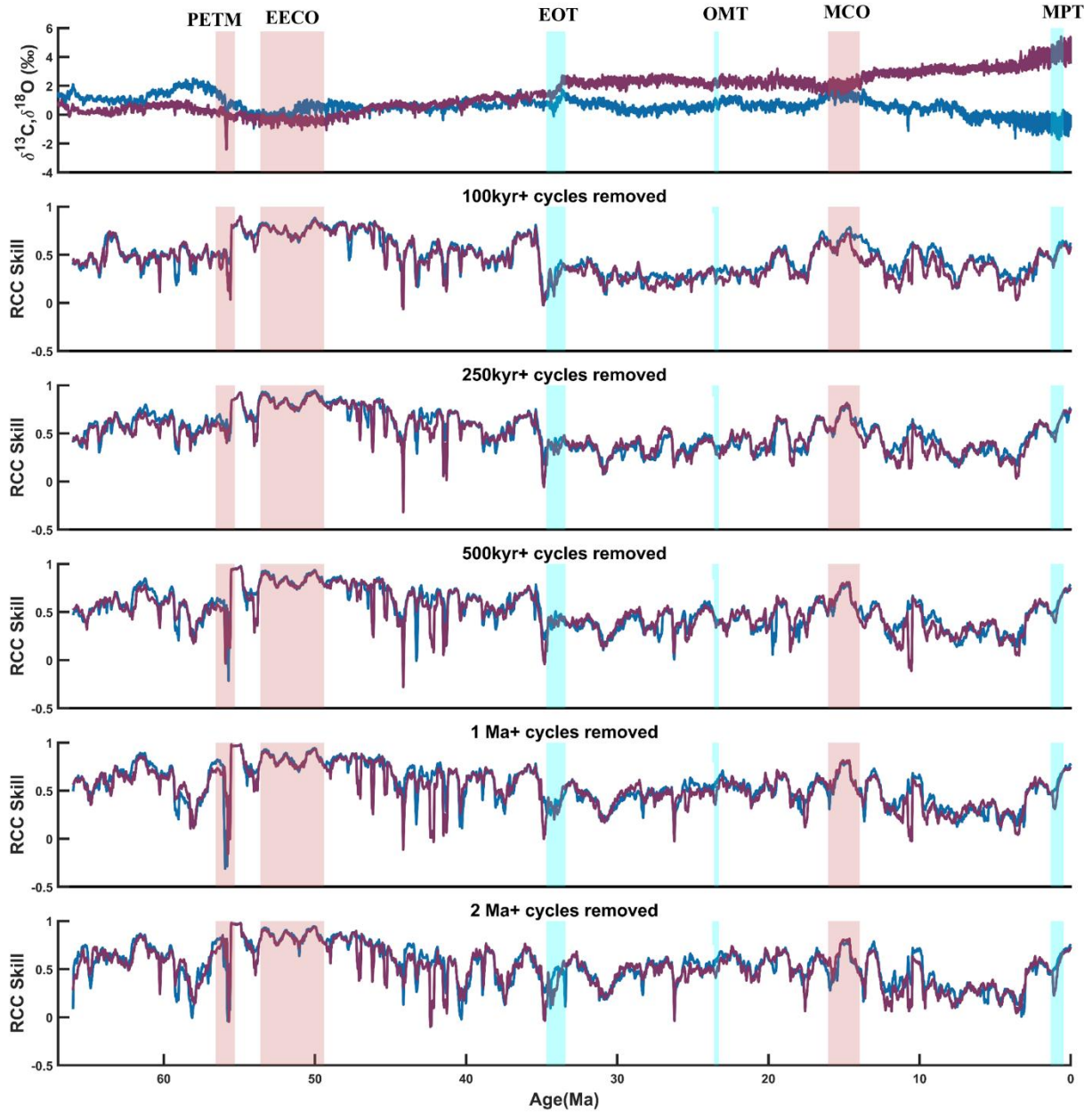


Figure S4: Sensitivity of DRCC analysis to detrending and removing cycles of different timescales, ranging from removing all cycles over 100 kyr to 2 million years. Like window sizes, the general patterns in the dynamics of the RCC skills are similar. When processes over 100 kyr were removed, the cyclicity of RCC skills during Oligocene was reduced, and when processes over 2 Ma were removed, the general patterns were almost like the RCC skills on the original dataset. The major observations discussed in the study are present after removing processes over

different timescales, such as high bidirectional causality during EECO and MCO, increasing causality before PETM and OMT, and negligible causality during PCIM, late Miocene and Pliocene.

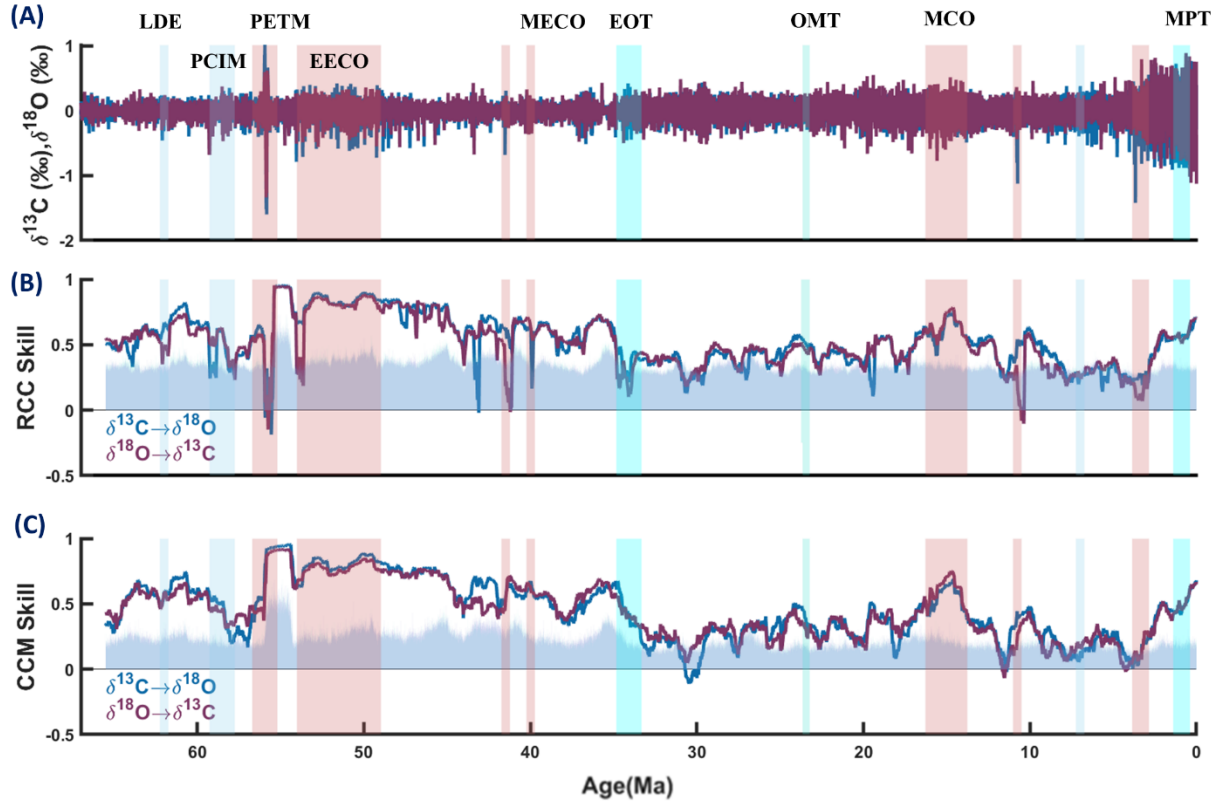


Figure S5: Dynamic causality analysis with a 1 Ma moving window on carbon cycle and climate reconstruction of the Cenozoic era after detrending to remove 500 kyr+ cycles and 95% confidence level of the skills (shaded region). The 95% confidence levels for the entire dataset are around 0.3 with only one region, right after PETM, where the level increases to 0.5. Importantly, after detrending, the causal skills prior to MECO and EOT are significant. However, the causal skills around Pliocene are still insignificant, making them more reliable.

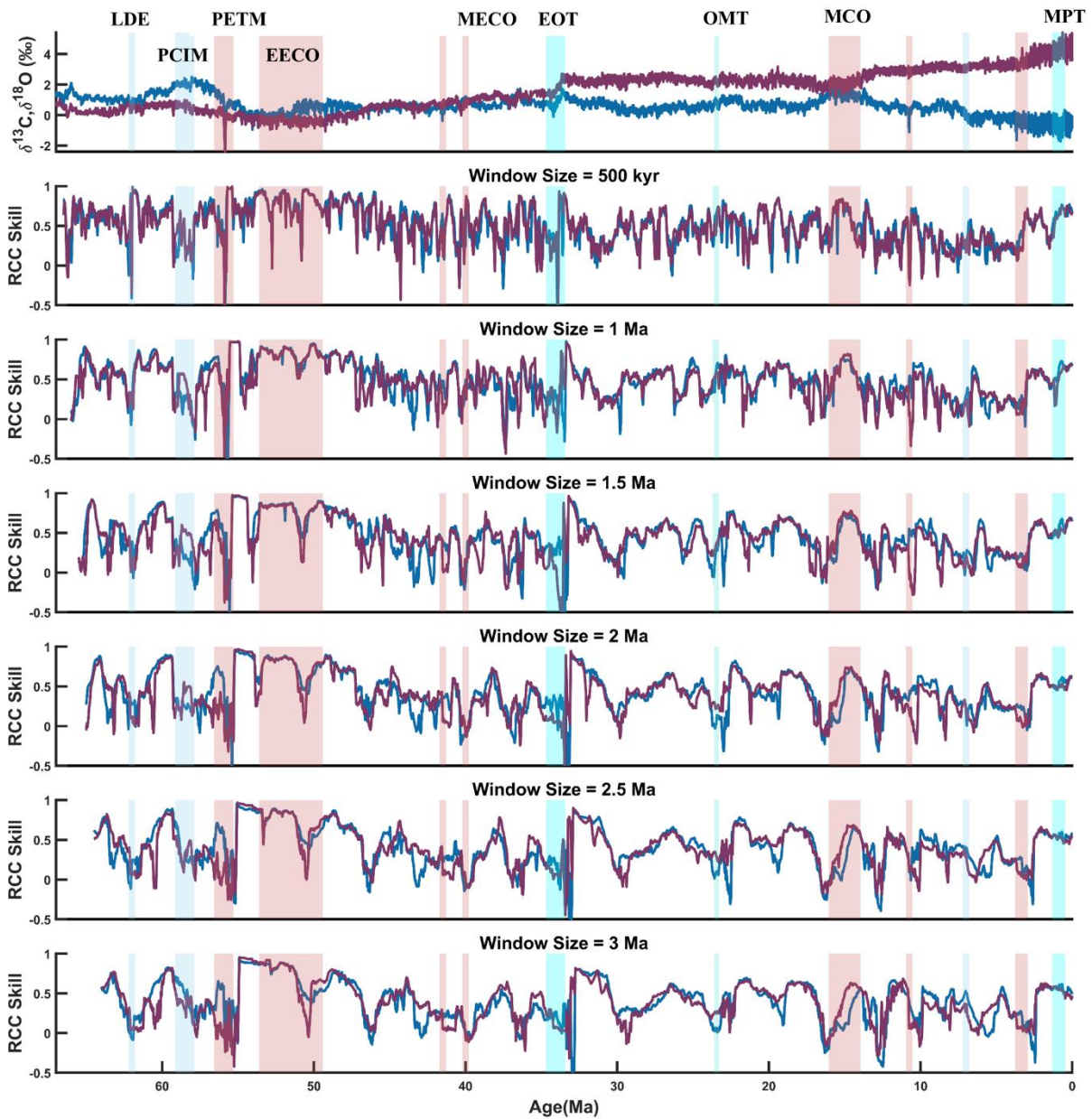


Figure S6: Sensitivity of DRCC analysis to different moving windows, ranging from 500 kyr till 3 Ma. The general patterns in the dynamics of the RCC skills are similar with shorter windows having more variability in the RCC skills, while the RCC skills are more smoothed for longer windows. The major observations discussed in the study are present for all the windows here, such as high bidirectional causality during EECO and MCO, increasing causality before PETM and OMT, and negligible causality during PCIM, late Miocene and Pliocene.

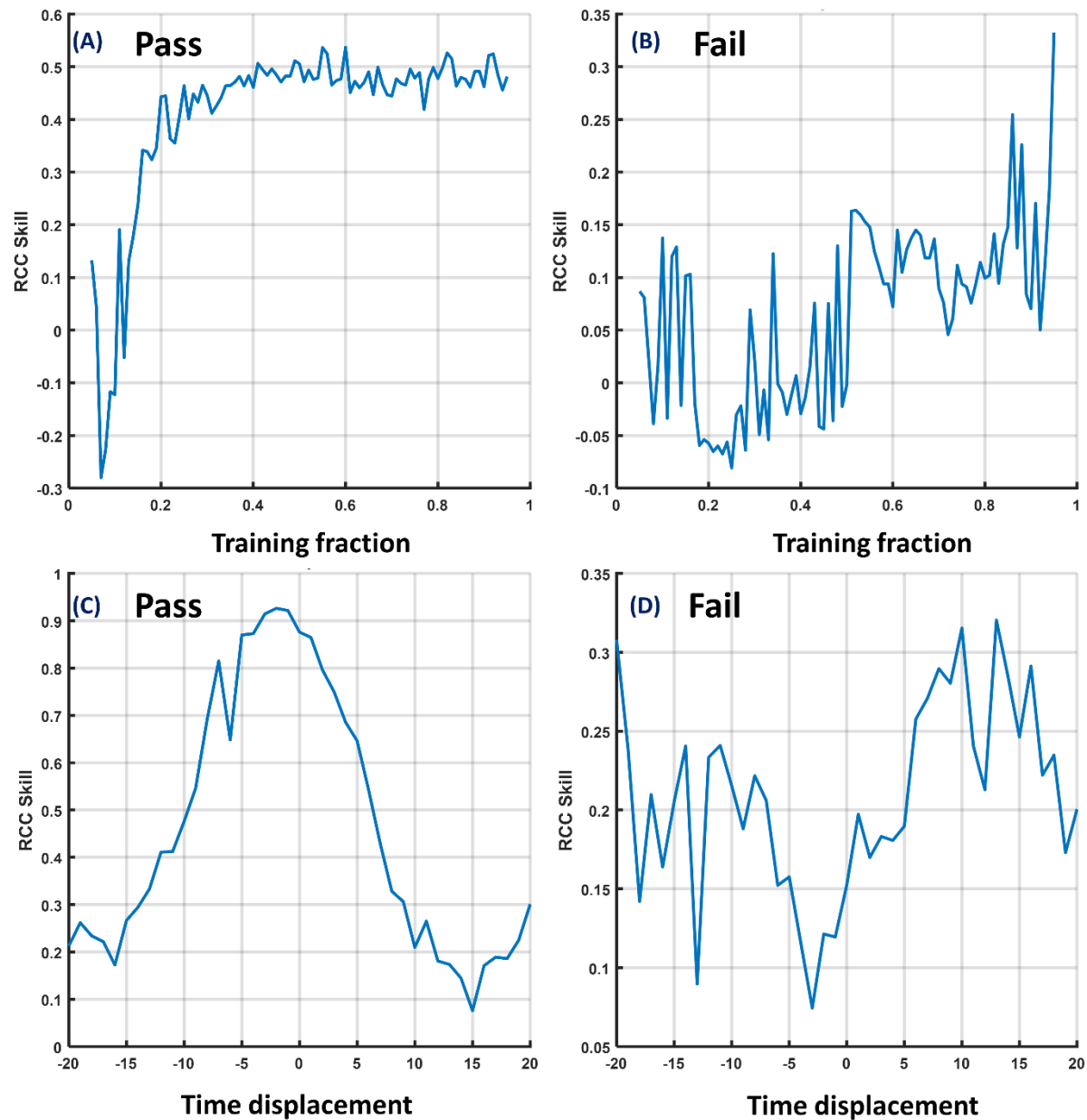


Figure S7: Example graphs to illustrate what is considered as a pass or fail in the convergence and ERCC test depicted in the Figure 2 and 3. A) RCC skills increase with and converge around 0.5 with increasing fraction of training data; B) RCC skills neither increase nor converge to a certain skill with increasing fraction of training data; C) The maximum RCC skill observed for time displacement of -3; D) The maximum RCC skill observed for time displacement of +13.

Supplementary Text

S1 Effect of Signal-to-noise ratio on causal skills

The simple stochastic climate-carbon cycle model developed and employed in Arnscheidt and Rothman 2021 (Arnscheidt and Rothman, 2021) includes the total inorganic carbon in the ocean-atmosphere, the perturbation of the global mean surface temperature from its stable state, and the total amount of inorganic carbon with the heavy ^{13}C . The model includes processes working in the long timescales of millions of years or greater and includes a 400 kyr eccentricity forcing. We used this model to investigate how the causal skills reflecting climate-carbon cycle dynamics change pertaining to the increase of the extent of coupling (μ_X) and the influence of additive noise (μ). For each parameter value of coupling factor and the strength of additive noise, the model was simulated for 100 ensemble trajectories with random Gaussian white noise. The RCC skills of $\delta^{13}\text{C} \rightarrow \delta^{18}\text{O}$ and $\delta^{18}\text{O} \rightarrow \delta^{13}\text{C}$ were calculated for each parameter value and the full spectrum of variability giving a general range where the RCC skills fall. In Figure S1, with increasing coupling factor, the RCC skills increases. As the model is designed to have $\delta^{13}\text{C}$ driving $\delta^{18}\text{O}$ more, than vice-versa, as expected, the RCC skills of $\delta^{13}\text{C} \rightarrow \delta^{18}\text{O}$ are greater than $\delta^{18}\text{O} \rightarrow \delta^{13}\text{C}$ at least for lower coupling factors. When the coupling factor increases, it also becomes more difficult to differentiate the main driver. While when the intensity of additive noise was increased, the RCC skills decreased more for $\delta^{18}\text{O} \rightarrow \delta^{13}\text{C}$, than $\delta^{13}\text{C} \rightarrow \delta^{18}\text{O}$. This suggests that when the signal-to-noise ratio decreases, the RCC skills tend to decrease as it becomes more difficult to differentiate the signal from the high intensity of noise.

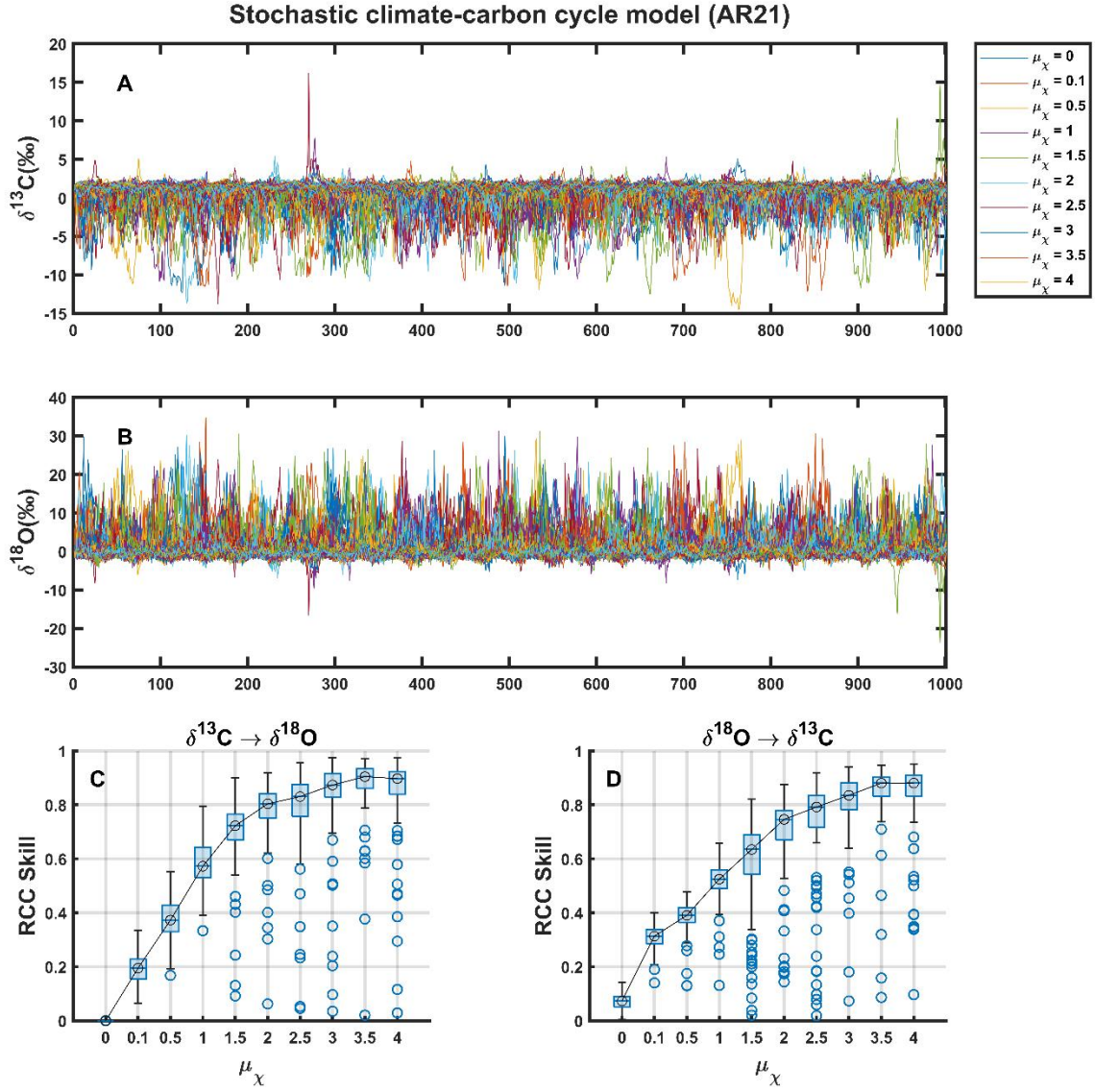


Figure S8: Effect of increasing the coupling factor μ_X on RCC skills of $\delta^{13}C \rightarrow \delta^{18}O$ and $\delta^{18}O \rightarrow \delta^{13}C$. A) $\delta^{13}C$ (‰) model output for 100 ensembles for coupling factor μ_X increasing from 0 to 4. B) $\delta^{18}O$ (‰) model output for 100 ensembles for coupling factor μ_X increasing from 0 to 4. C) Box plot of RCC skills of $\delta^{13}C \rightarrow \delta^{18}O$ for μ_X increasing from 0 to 4. D) Box plot of RCC skills of $\delta^{18}O \rightarrow \delta^{13}C$ for μ_X increasing from 0 to 4.

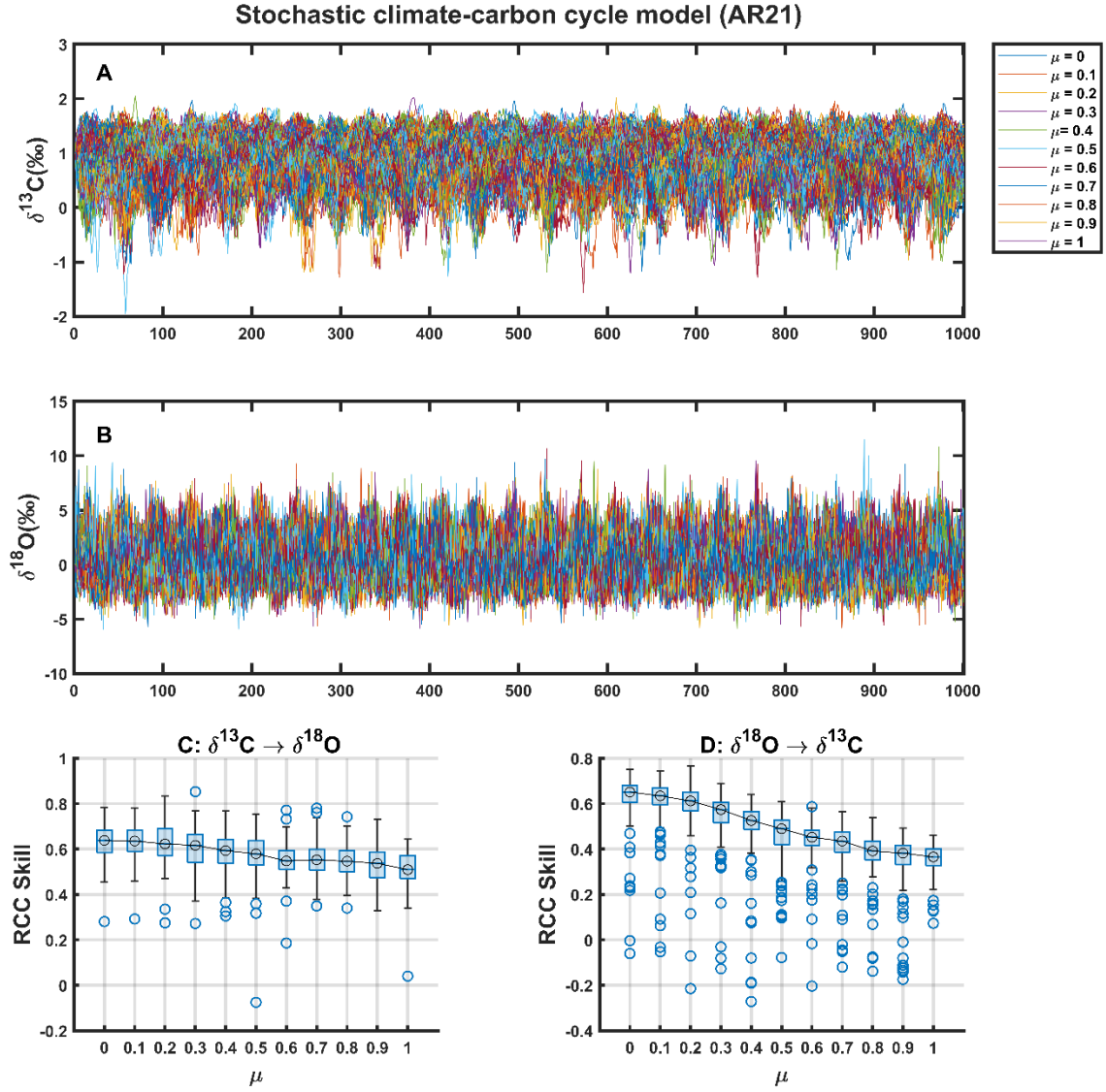


Figure S9: Effect of increasing the coupling factor μ on RCC skills of $\delta^{13}\text{C} \rightarrow \delta^{18}\text{O}$ and $\delta^{18}\text{O} \rightarrow \delta^{13}\text{C}$. A) $\delta^{13}\text{C}$ (‰) model output for 100 ensembles for coupling factor μ increasing from 0 to 1. B) $\delta^{18}\text{O}$ (‰) model output for 100 ensembles for coupling factor μ increasing from 0 to 1. C) Box plot of RCC skills of $\delta^{13}\text{C} \rightarrow \delta^{18}\text{O}$ for μ increasing from 0 to 1. D) Box plot of RCC skills of $\delta^{18}\text{O} \rightarrow \delta^{13}\text{C}$ for μ increasing from 0 to 1.

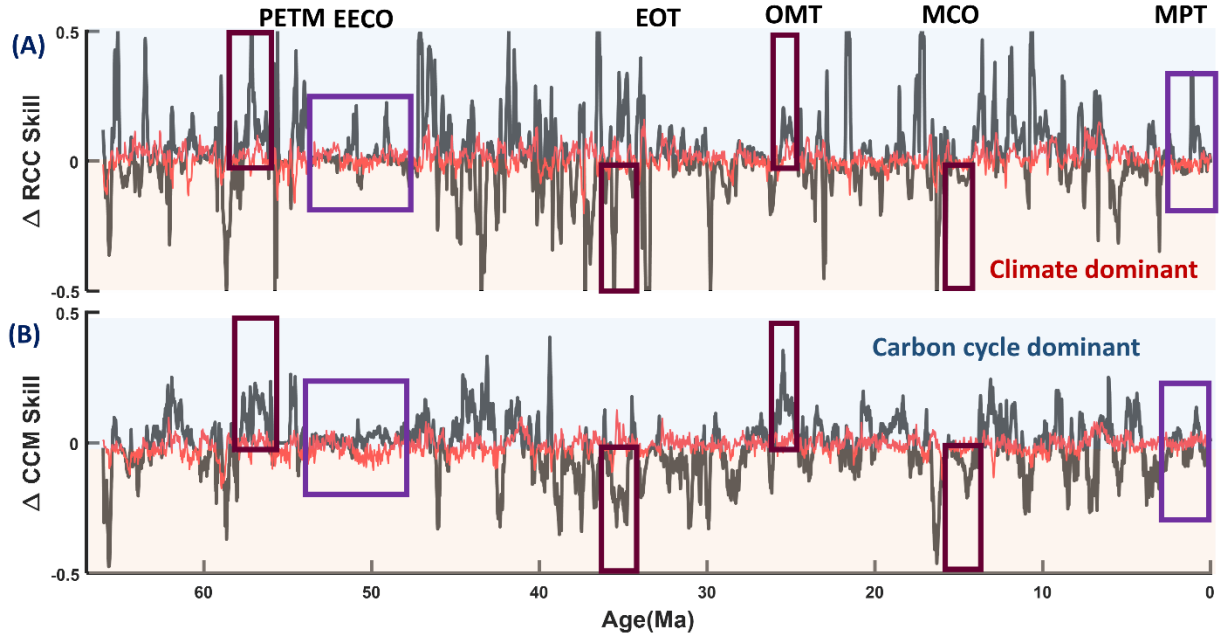


Figure S10: Difference between $\delta^{13}\text{C} \rightarrow \delta^{18}\text{O}$ and $\delta^{18}\text{O} \rightarrow \delta^{13}\text{C}$ causal skills pointing towards the dominant drivers. If the difference is greater than 0, then the carbon cycle is dominant (blue shaded region) and if it's less than 0, then the climate is dominant (red shaded region).

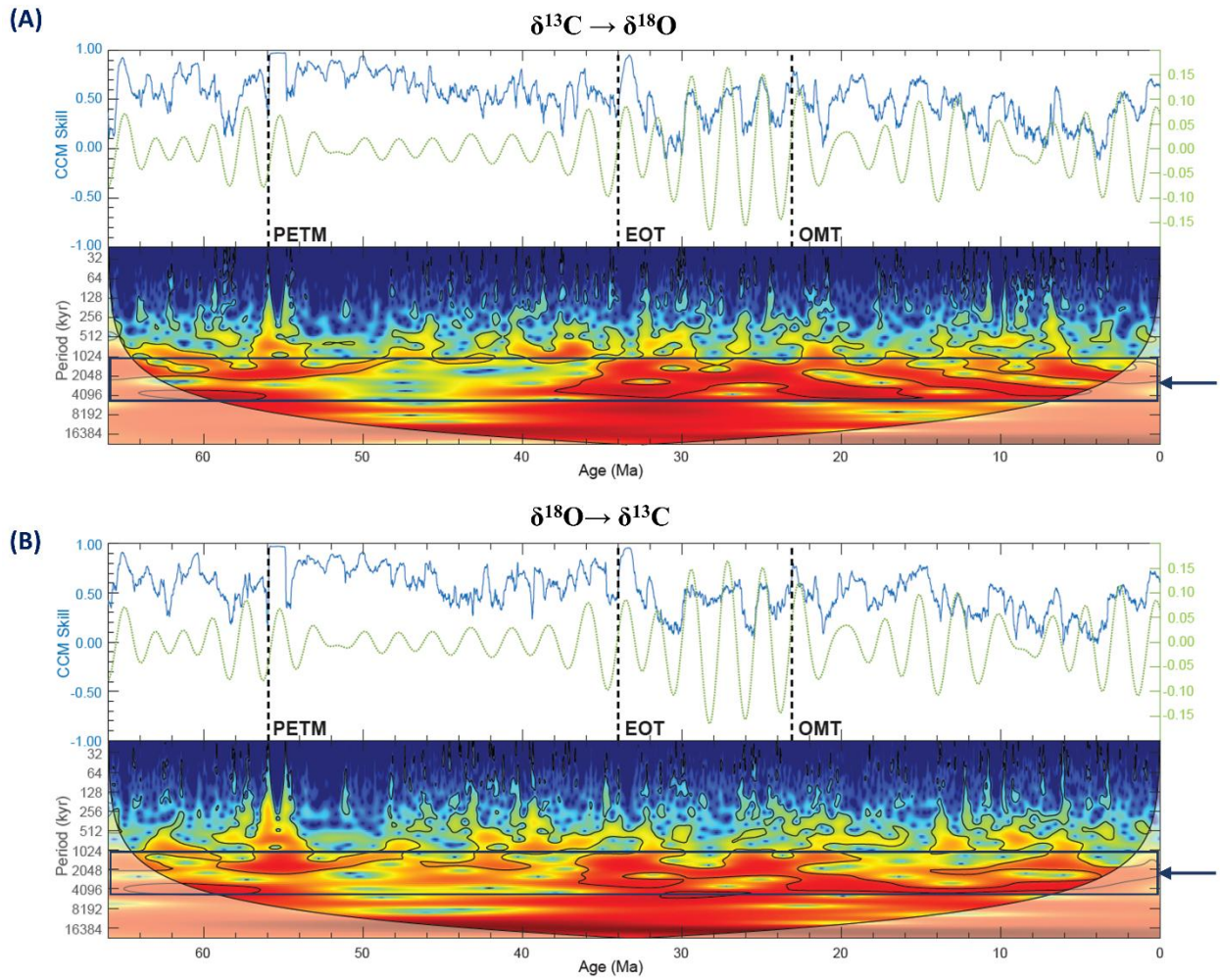


Figure S11: Wavelet analysis and Gaussian filters of the CCM skills. A) Wavelet analysis of CCM Skills of $\delta^{13}\text{C} \rightarrow \delta^{18}\text{O}$ (blue line) and Gaussian filters (green line) B) Wavelet analysis of CCM Skills of $\delta^{18}\text{O} \rightarrow \delta^{13}\text{C}$ (blue line) and Gaussian filters (green line).

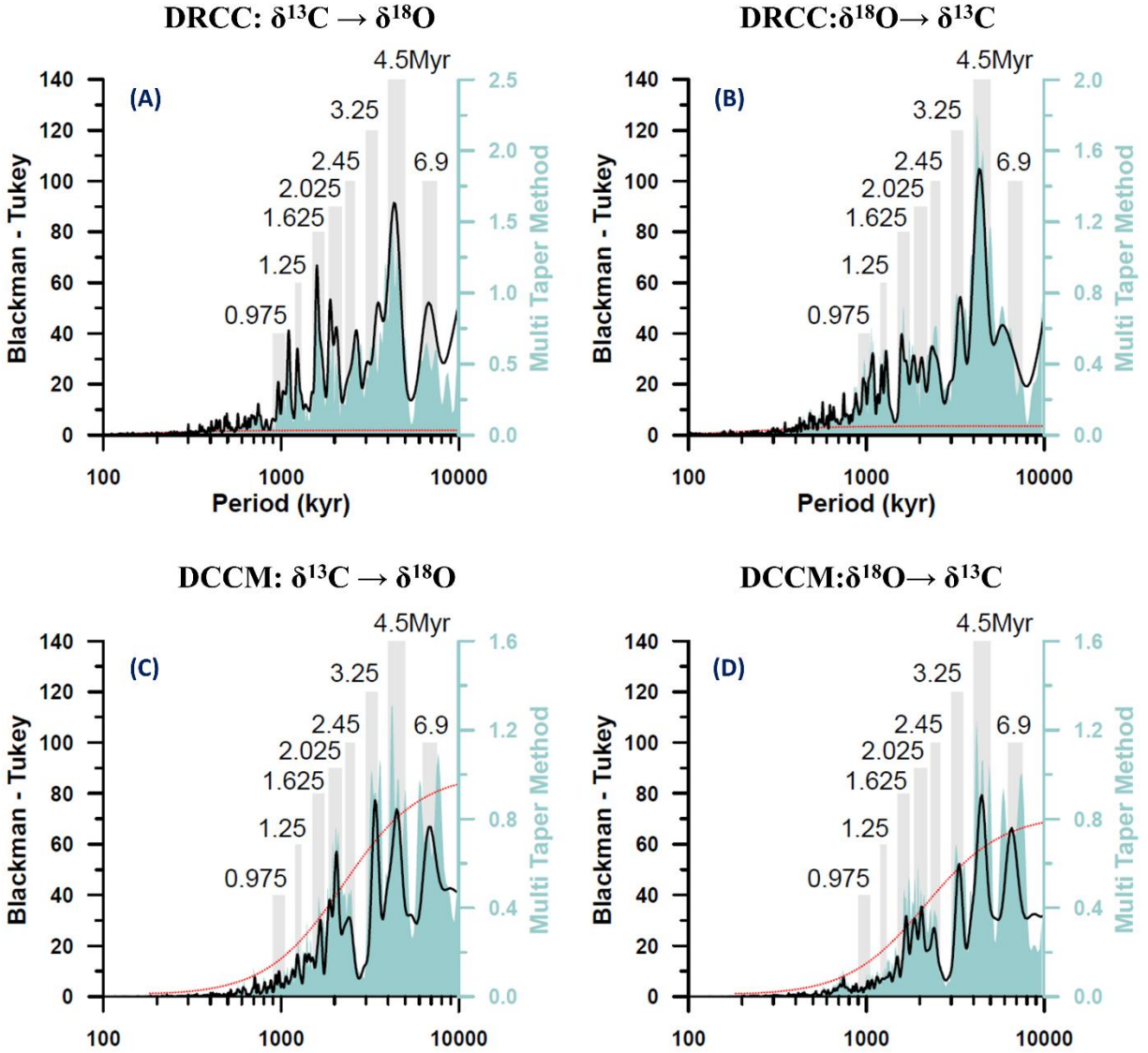


Figure S12: Spectral analysis results of the detrended RCC and CCM skills. For detrending, we applied a third order polynomial trend line. Spectral results of the time series are based on the Multi Taper Method (filled green areas) and the (low confidence/high resolution) Blackman-Tukey (black solid lines) power estimates. The red dotted curves indicate the MTM (LS) (AR1) confidence levels at 90%.

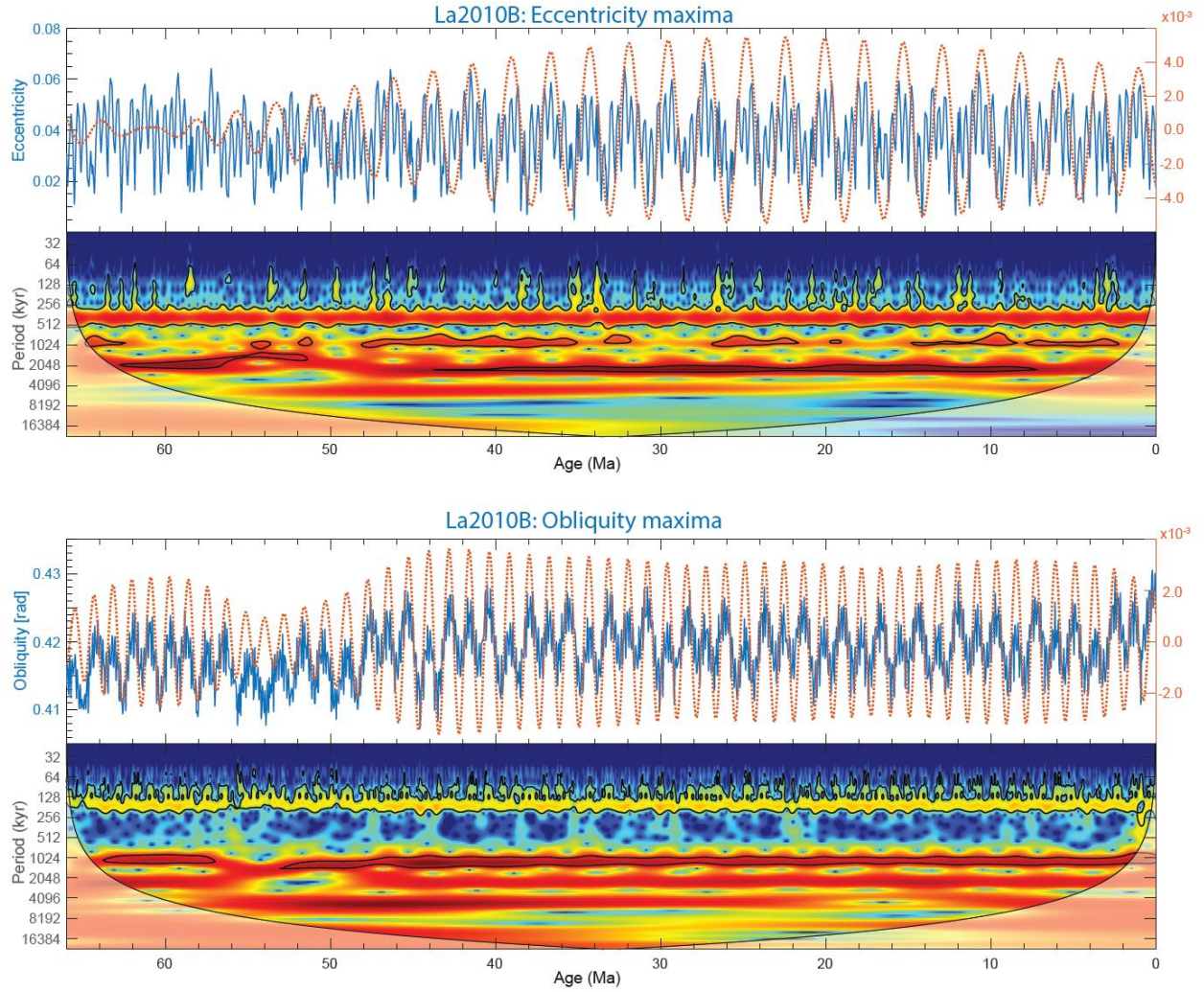


Figure S13: Wavelet analysis and Gaussian filters of the La2010B eccentricity (maxima) and obliquity (maxima) time series. We used the eccentricity (maxima) and obliquity (maxima) time series to better illustrate the long-term modulations of the Earth's orbit and tilt cycles.

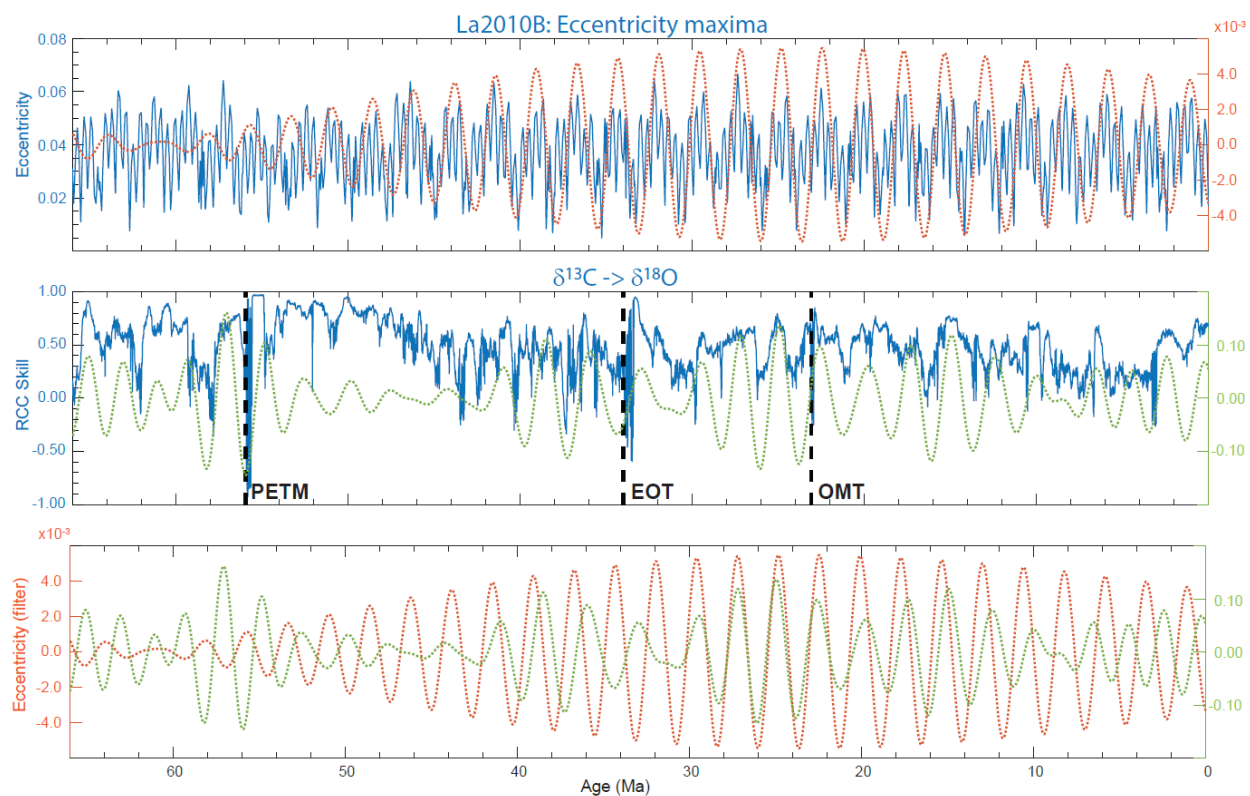


Figure S14 Comparison between the ≈ 2.325 Myr periodicity in the La2010B Eccentricity time series and that of the RCC skills ($\delta^{13}\text{C} \rightarrow \delta^{18}\text{O}$) time series as an example.

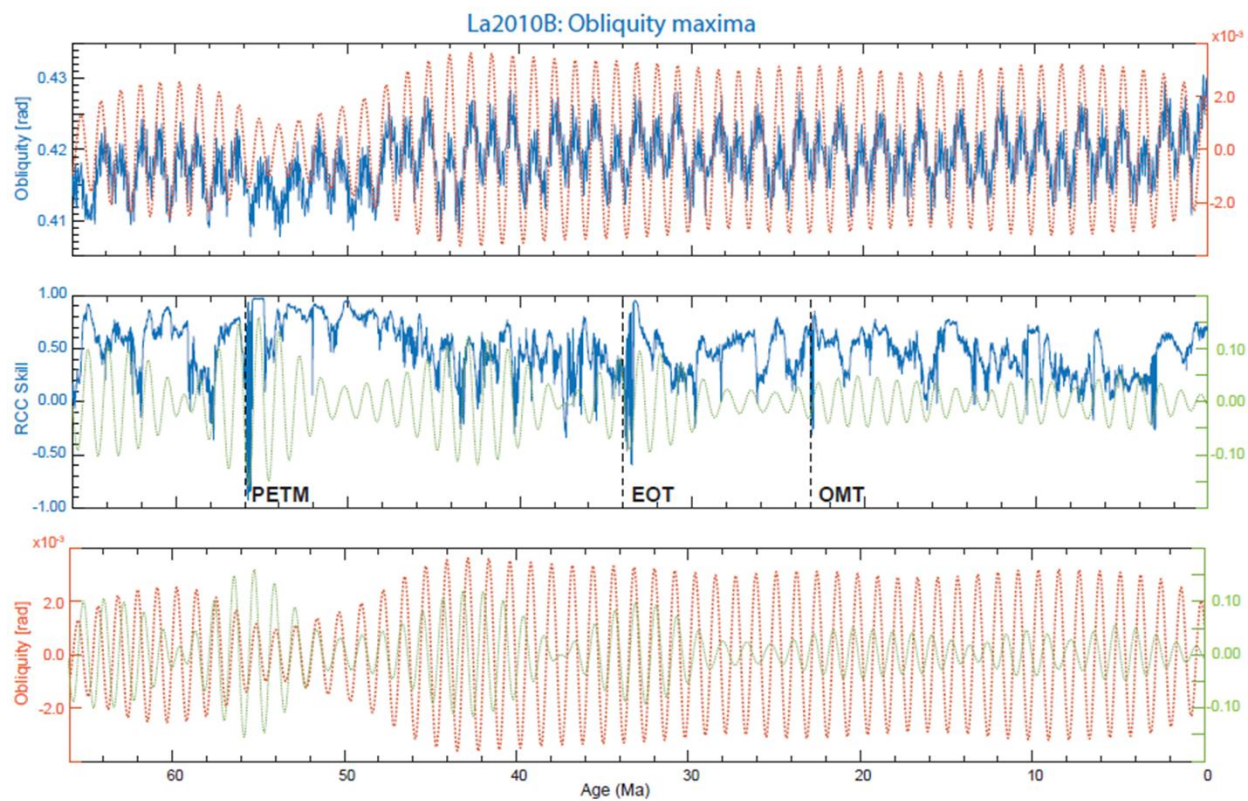


Figure S15: Comparison between the ≈ 1.2 Myr periodicity in the La2010B Obliquity time series and that of the RCC skills ($\delta^{13}\text{C} \rightarrow \delta^{18}\text{O}$) time series as an example.