Comment on egusphere-2026-5420
Title: Mathematical interpretation of accumulated rainfall, temporal dependence, and validation of the proposed radon anomaly identification framework

The manuscript addresses an important problem in geochemical monitoring: separating meteorologically induced variability in soil-gas radon concentrations from anomalies potentially associated with seismic processes. The proposed combination of first-order differencing, event classification, and LOWESS regression provides an interesting framework for exploring nonlinear relationships between radon and environmental variables.
Nevertheless, several aspects of the mathematical formulation, statistical validation, and physical interpretation require further clarification before the proposed framework can be considered a reliable procedure for identifying earthquake-related anomalies. The following observations complement the issues already raised during the public review.

1. Mathematical interpretation of accumulated rainfall differences
A particularly important issue concerns the multi-window rainfall analysis presented in Section 4.5 (lines 317–349).
The authors define accumulated rainfall over a moving window of length w as:

$$R_w(t) = \sum_{i=0}^{w-1} r\,(t-i),$$

where $r(t)$ denotes hourly precipitation. The subsequent application of first-order differencing necessarily gives:

$$\Delta R_w(t) = R_w(t) - R_w(t-1) = r(t) - r(t-w).$$

This identity has important implications for the physical interpretation of the results.
The differenced accumulated rainfall depends exclusively on precipitation at the current time and precipitation exactly $w$hours earlier. Consequently, negative values do not necessarily indicate hydrological recession, soil drainage, or decreasing rainfall intensity. They can arise simply because an earlier precipitation event leaves the moving accumulation window.
Similarly, differences between LOWESS curves obtained with 24-, 72-, and 240-hour windows cannot, by themselves, demonstrate different subsurface response times or hydrological memory. Such differences may partly reflect the mathematical properties of the transformed precipitation series.
I therefore suggest reconsidering the interpretation of negative rainfall differences and distinguishing the effects of the moving-window transformation from independently demonstrated hydrological processes.
Explicit lagged-response analyses, for example through cross-correlation, distributed-lag models, or event-aligned response functions, would be necessary to substantiate the proposed delayed infiltration and drainage mechanisms.

2. Temporal dependence, seasonality, and statistical validation
The statistical properties of radon time series deserve greater attention in the proposed methodology.
Previous investigations have demonstrated that soil-radon observations may exhibit multiple seasonal components, nonstationarity, site-dependent relationships with environmental parameters, and long-range temporal dependence (Siino et al., 2019; D'Alessandro et al., 2020; Siino et al., 2020).
These characteristics are directly relevant to the present study, particularly because the analysis uses consecutive hourly observations.
First-order differencing does not necessarily eliminate temporal dependence or seasonal structure. Furthermore, the leave-one-out cross-validation procedure used to select the LOWESS smoothing parameter (line 300) may underestimate prediction errors when training and validation observations remain temporally correlated.
It would therefore be useful to examine the autocorrelation structure of the transformed observations and model residuals, and to assess the stability of the results using temporally blocked cross-validation or another dependence-aware validation strategy (Roberts et al., 2017).

The analysis would also benefit from uncertainty estimates for the fitted nonlinear relationships, preferably obtained through procedures that preserve the relevant temporal dependence.
These checks are particularly important for establishing whether the observed nonlinear patterns represent stable environmental responses rather than structures influenced by serial dependence, uneven sampling density, or smoothing choices.

3. Interpretation of the ablation analysis and methodological novelty
The conclusions drawn from the ablation analysis in Section 5.5 appear stronger than the evidence presented in Figure 12.
In particular, Figure 12c (*LOWESS Without Event Classification*) already displays a pronounced nonlinear, approximately U-shaped rainfall–radon relationship.
This suggests that event classification is not strictly necessary for the emergence of the nonlinear pattern, although it may improve its interpretation or distinguish different meteorological regimes.
Consequently, the statement that the nonlinear relationship emerges only from the combined application of differencing, LOWESS smoothing, and event classification requires qualification.
A quantitative comparison of the individual processing configurations would be more informative than visual comparisons alone. Such an assessment could consider predictive errors, confidence intervals, temporal stability, and sensitivity to classification thresholds and LOWESS bandwidth.
The methodological contribution should also be distinguished more explicitly from previous applications of locally estimated scatterplot smoothing to radon time series. For example, İçhedef et al. (2025) already employed LOESS to investigate temporal variations and changes in radon concentration trends.
The potentially original contribution of the present manuscript therefore appears to lie principally in the event-space transformation and conditioning strategy, rather than in the application of local smoothing itself. A direct comparison with existing approaches would help establish this distinction.

4. Physical interpretation of meteorological influences
The manuscript attributes the observed rainfall-dependent patterns to several processes, including rainfall capping, piston effects, infiltration, and post-rainfall drainage.
These mechanisms are physically plausible and consistent with previous experimental evidence. In particular, Tommasone Pascale et al. (2015) documented pronounced rainfall-induced soil-radon responses associated with capping, infiltration, and wash-out processes.
However, the simultaneous occurrence of different physical mechanisms complicates their identification from the observed statistical relationships.
An approximately U-shaped relationship between radon differences and rainfall differences does not uniquely identify the governing transport mechanisms. In particular, contemporaneous regression curves do not directly establish the temporal ordering of precipitation, pore-pressure variations, gas transport, and radon response.
I suggest that the authors distinguish more explicitly between observed statistical associations and physical mechanisms proposed to explain them.
The interpretation could be strengthened by considering rainfall intensity, antecedent precipitation, event duration, and lagged radon responses separately, rather than interpreting the sign of differenced accumulated rainfall as a direct indicator of a particular hydrological regime.

5. Validation of earthquake-related anomaly identification
The manuscript repeatedly suggests that the proposed framework improves discrimination between meteorologically induced fluctuations and potentially earthquake-related radon anomalies.
However, the reported results primarily establish nonlinear relationships between radon and meteorological variables. They do not yet provide a quantitative demonstration of improved earthquake-related anomaly identification.
This distinction is important because identifying meteorological effects and demonstrating predictive discrimination of seismic anomalies are separate statistical problems.
A more convincing validation would require an explicitly defined anomaly-detection procedure, together with quantitative performance measures such as false-alarm rates, detection sensitivity, and comparisons with an appropriate null model.
The evaluation should ideally include temporally independent observations and, where possible, independent monitoring stations.

Recent work by Salh et al. (2026), which combines meteorological analysis with alternative machine-learning approaches for soil-radon anomaly identification, provides a relevant methodological comparison.
If a quantitative anomaly-detection validation cannot be included, I would recommend restricting the conclusions to the demonstrated characterization of nonlinear meteorological influences, without claiming an established improvement in earthquake precursor discrimination.

6. Reproducibility and measurement characterization
Finally, some additional clarification would strengthen the reproducibility of the analysis.
In particular, the manuscript should specify the treatment of missing observations, the exact LOWESS implementation and parameter-selection procedure, and the construction of any meteorology-reduced radon residuals.
It would also be useful to clarify whether the reported temperature and humidity measurements represent actual soil conditions or measurements associated with the RAD7 sampling system, and how instrumental humidity effects were controlled.
Public availability of the processed observations and analysis code would considerably facilitate independent verification of the proposed methodology.

Concluding remarks
The manuscript presents an interesting application of nonlinear statistical methods to an important problem in geochemical monitoring. The identification of a nonlinear relationship between precipitation changes and soil-radon variability is potentially valuable.
However, the mathematical properties of differenced accumulated rainfall, the temporal dependence of the observations, and the distinction between nonlinear characterization and validated anomaly detection should be addressed more rigorously.
In particular, reconsidering the interpretation of the multi-window rainfall analysis and quantifying the incremental contribution of event classification would substantially strengthen the methodological foundation of the study.
The references listed below provide relevant experimental evidence, statistical methodology, and previous applications that may help place the proposed framework in a more comprehensive and scientifically robust context.

References
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