



Identifiability of hydrodynamic forces inferred from instrumented-particle measurements at the threshold of motion

Mohammadreza Zayeri¹

¹Department of Water Sciences and Engineering, Shahid Chamran University of Ahvaz, Ahvaz, Iran

Abstract. Instrumented particles carrying inertial measurement units are increasingly used to infer the hydrodynamic drag and lift acting on a coarse grain at the threshold of motion. Because the grain is still in contact with its supporting grains at that instant, the measured response is produced jointly by the flow and by the bed–particle contact reactions, so the recovery of the hydrodynamic part is an inverse problem whose identifiability has not been examined. This study examines it, using an openly released dataset of 30 threshold conditions and two independent published analyses of a common subset of those runs. Three results follow. First, a recently proposed bounded inversion is degenerate in lift: for every contact angle and every friction coefficient the admissible interval collapses to $[0, W']$, and the inferred lift coefficient reduces to the exact identity $\overline{C}_{L,\text{eff}} = \frac{2}{3} Fr_d^{-2}$, so that its reported constancy is a restatement of a constant critical movability number and carries no information from the sensor. Second, because the two published analyses normalise the same rectified impulse by different durations, their ratio returns the fraction of the onset window during which the acceleration is positive; that fraction is 0.14–0.17, or two to three samples at 100 Hz. Third, taking moments about the contact point rather than the centre of mass annihilates both contact reactions exactly for a spherical grain and yields an identity linking the accelerometer and gyroscope channels; the identity requires a non-negative tangential reaction and is violated in all 30 conditions, the measured angular acceleration being 27–66% of the value implied by the measured linear accelerations. Closing the inversion with the gyroscope channel alone returns $C_D = 0.74\text{--}1.55$, which brackets independent fixed-particle measurements, from the same published data. Instrumented particles measure the net response of a grain, and hydrodynamic coefficients are recoverable from them only when the rotational channel is referred to the contact point or when contact reactions are measured independently.

1 Introduction

1.1 The grain-scale threshold problem

The flow condition at which an individual coarse grain leaves its resting position sets the lower boundary of bedload transport and, through it, the stability of gravel-bed channels, the pace of channel adjustment and the design of restoration and protection works (Buffington et al., 2004; Rickenmann and Recking, 2011; Yager et al., 2007; Wohl, 2015). Since Shields (1936) the condition has been expressed as a dimensionless bed shear stress, $\theta_c = u_*^2 \rho_w / [(\rho_s - \rho_w)gD]$, whose apparent universality has proved difficult to sustain in coarse-grained beds. Measured critical values scatter over an order of magnitude for nominally identical grains, and the scatter is systematic rather than random: it tracks grain protrusion and exposure (Fenton and Abbott,



1977; Kirchner et al., 1990; Hodge et al., 2020; Masteller and Finnegan, 2017), the geometry of the pocket in which the grain rests (Naden, 1987; Komar and Li, 1986; Voepel et al., 2019), grain shape (Maniatis et al., 2020), and the intermittency of the near-bed turbulence (Nelson et al., 1995; Diplas et al., 2008; Celik et al., 2013). A bed-averaged stress is a poor proxy for a force that acts on one grain, over a fraction of a second, in a geometry that differs from pocket to pocket. This is particularly acute in low-slope reaches carrying isolated coarse clasts, where the grains that matter most for bed structure are also the most exposed and the least well described by a bed-averaged criterion.

The alternative is to work at the scale of the grain itself and to write a force or moment balance between the destabilising hydrodynamic drag and lift and the stabilising submerged weight and contact resistance (Wiberg and Smith, 1987; James, 1990; Wu and Chou, 2003; Dey and Ali, 2018; Ali et al., 2025). That balance is only as good as the force estimates entering it, and obtaining those estimates has been the central experimental difficulty of the field for half a century.

1.2 Measuring the forces: three lineages and their limits

1.2.1 Fixed-particle force measurement.

Beginning with Bagnold (1974) and Coleman (1972), and continuing through Schmeeckle et al. (2007), Dwivedi et al. (2011), Celik et al. (2014) and Lamb et al. (2017), drag and lift have been measured directly on a target grain held on a force balance or instrumented with pressure transducers. These experiments are the reference against which everything else is calibrated: for a fully submerged sphere they give drag coefficients of roughly 0.4–0.8 (Schmeeckle et al., 2007; Lamb et al., 2017), consistent with the resolved simulations of Lee and Balachandar (2012). Their limitation is structural rather than technical: a grain that cannot move cannot be observed at the moment it moves. The measured forces therefore correspond to sub-threshold conditions, and above the threshold they become unrepresentative, because in nature the grain would already have departed.

1.2.2 Impulse and energy criteria.

Recognising that near-bed forcing is intermittent, Diplas et al. (2008) showed that entrainment depends on the duration as well as the magnitude of a turbulent excursion, and the idea was developed into impulse and energy criteria by Celik et al. (2010), Valyrakis et al. (2010, 2011, 2013) and Maldonado and de Almeida (2019). This lineage reframed the threshold as an event rather than a state, but it still relies on externally measured flow or force records and does not observe the grain's own response.

1.2.3 Instrumented particles.

The third lineage embeds the sensor in the grain. Since the first smart pebbles (Kularatna et al., 2005; Akeila et al., 2010), inertial measurement units have been used to record the acceleration and rotation of a moving grain in flumes, streams and coastal settings (Frank et al., 2015; Olinde and Johnson, 2015; Gronz et al., 2016; Dost et al., 2020; Pretzlav et al., 2020; Al-Obaidi et al., 2020; Al-Obaidi and Valyrakis, 2021; Alhusban and Valyrakis, 2021). The approach is Lagrangian, non-intrusive, and — uniquely among the three — able to observe the instant of entrainment. Maniatis et al. (2020) developed the first explicit



framework for converting IMU records into drag and lift components, and the Auckland group extended it to systematically varied protrusion (Xie et al., 2022, 2023a, b) and, most recently, to pocket geometry and particle density (Lu et al., 2025, 2026).

1.3 An inverse problem that has not been posed

At the threshold of motion the instrumented grain is still in contact with its supporting grains. The gravity-compensated acceleration it reports is therefore the response to the sum of the hydrodynamic forcing and the bed–particle contact reactions, and recovering the former requires either measuring or eliminating the latter. Neither has been done. The recovery of hydrodynamic coefficients from IMU records is an inverse problem, and to our knowledge its identifiability has never been examined.

That this matters is visible in the published numbers themselves. Applying the same class of method to the same kind of experiment, one laboratory lineage has reported inertial drag coefficients of 0.04–0.07 (Xie et al., 2023a), 2.88 (Lu et al., 2025) and 2.28 (Lu et al., 2026), spanning nearly two orders of magnitude, against the 0.4–0.8 of the fixed-particle reference (Fig. 1). The literature also contains two unremarked signals that something is unresolved. First, Xie et al. (2023a) state explicitly that their inertial coefficients “are not the classical hydraulic coefficients” — a caveat that is absent from the later papers, in which the same quantities are compared directly with Coleman (1972) and Lamb et al. (2017). Second, Maniatis et al. (2020) evaluated the rotational channel about the grain’s centre of mass, found it small relative to the linear channel, and concluded that rotation could be neglected; the possibility that the imbalance reflects an inconsistency rather than a negligible term was not pursued.

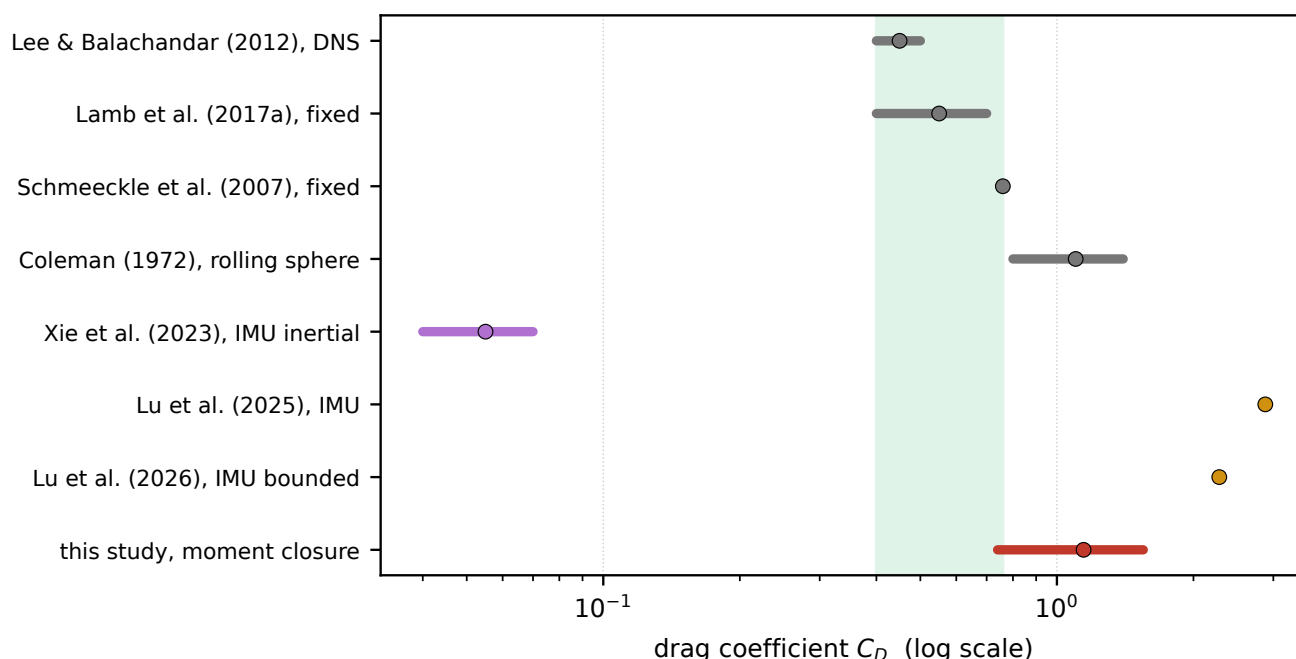


Figure 1. Published drag coefficients for coarse grains near threshold, by measurement approach.



1.4 Contribution of this study

This paper treats the recovery of hydrodynamic forces from instrumented-particle measurements as an inverse problem and asks what is identifiable, what is not, and what additional information closes the gap. The openly released dataset of Lu et al. (2026), together with the independent treatment of a subset of the same experiments by Lu et al. (2025), provides an unusually complete test case: two published analyses of identical runs, a full statement of the inversion, and a public data archive. No new experiments are reported; every result below is obtained from published material and is reproducible from a single archived notebook.

Four contributions are made.

1. **An identifiability theorem.** The bounded inversion introduced by Lu et al. (2026), in which contact reactions are constrained by non-tension and Coulomb conditions and the hydrodynamic components are bracketed, is shown to be degenerate in lift: for every contact angle and every friction coefficient the admissible interval collapses to $[0, W']$, and the resulting midpoint lift coefficient reduces to the exact identity $\overline{C}_{L,\text{eff}} = \frac{2}{3}Fr_d^{-2}$. The reported constancy of that coefficient is therefore equivalent to a constant critical movability number (Novak and Nalluri, 1975; Hager and Oliveto, 2002; Bhat and Ahanger, 2023) and carries no sensor information.
2. **A duration diagnostic.** The two source papers apply the same rectifying gate to identical runs but normalise the positive impulse by different durations. Their ratio returns the fraction of the onset window during which the acceleration is positive — a quantity neither paper reports and which cannot be obtained from either alone.
3. **A moment identity and a consistency test.** Taking moments about the contact point rather than the centre of mass annihilates both contact reactions exactly for a spherical grain, yielding an identity that links the accelerometer and gyroscope channels of the same instrument and can be tested with published data. The test explains, rather than contradicts, the observation of Maniatis et al. (2020).
4. **A closure.** Because the moment equation contains no contact reaction, the gyroscope channel alone constrains the hydrodynamic pair. The resulting coefficients are compared with the fixed-particle reference values.

Section 2 states the data, the geometry, the published inversion and the four analysis methods; Sect. 3 reports the results; Sect. 4 discusses what instrumented particles can and cannot be asked to deliver, and what paired measurement would remove the remaining indeterminacy.

2 Data and methods

2.1 Rationale and scope of the analysis

An instrumented particle measures the gravity-compensated linear acceleration and the angular velocity of its own centre of mass. At the threshold of motion the particle remains in contact with its supporting grains, so the measured response is



produced jointly by the hydrodynamic forcing and by the bed–particle contact reactions. Recovering the hydrodynamic part is therefore an inverse problem, and the analysis below asks whether that inverse problem is solvable, in what sense, and what additional measurement closes it. The analysis is organised as four independent methods: an identifiability analysis of the published bounded inversion (Sect. 2.5), a duration diagnostic constructed from two published treatments of identical experiments (Sect. 2.6), a moment identity that provides a cross-channel consistency test (Sect. 2.7), and a closure of the inverse problem that uses the gyroscope channel alone (Sect. 2.8). All four are applied to published data; no new experiments are reported.

2.2 Source data

The analysis uses the openly released dataset of Lu et al. (2026), comprising 30 threshold conditions obtained with a 60 mm Smart Sediment Particle (SSP) on a fixed bed of 40 mm spheres: six particle densities $\rho_s = 1.19\text{--}2.095 \text{ g cm}^{-3}$ and flow depths $h = 120\text{--}300 \text{ mm}$ ($h/D = 2\text{--}5$). For each condition the dataset provides the onset-window effective linear accelerations $\bar{a}_{x,\text{eff}}$ and $\bar{a}_{z,\text{eff}}$, the characteristic velocity $\langle \bar{u}_{0.15D} \rangle$ measured $0.15D$ above the particle crest, the discharge, the shear velocity obtained from a logarithmic fit, and the window-averaged rotational kinetic energy. Calibration records (shaking-table and pendulum acceleration series, ten ramp-rolling angular velocity runs) are also included. Raw event time series are not part of the release.

A second, independent treatment of a subset of the same experiments is reported by Lu et al. (2025). Table 1 shows that the discharges, cross-sectional mean velocities and characteristic velocities listed there for the saddle configuration coincide, digit for digit, with those listed by Lu et al. (2026) for $\rho_s = 1.395 \text{ g cm}^{-3}$. The two papers therefore analyse identical runs with different processing, which is what makes the diagnostic of Sect. 2.6 possible.

Table 1. Hydraulic conditions reported for the saddle configuration by Lu et al. (2025) (L25) and for $\rho_s = 1.395 \text{ g cm}^{-3}$ by Lu et al. (2026) (L26). The two sets are identical at every common depth, so the papers analyse the same experimental runs.

h (mm)	$Q \text{ (L s}^{-1}\text{)}$		$U_{\text{avg}} \text{ (m s}^{-1}\text{)}$		$\langle \bar{u}_{0.15D} \rangle \text{ (m s}^{-1}\text{)}$	
	L25	L26	L25	L26	L25	L26
120	20.95	20.95	0.323	0.323	0.365	0.365
150	29.52	29.52	0.364	0.364	0.368	0.368
180	37.39	37.39	0.385	0.385	0.365	0.365
210	44.54	44.54	0.393	0.393	0.355	0.355
240	52.64	52.64	0.406	0.406	0.356	0.356



2.3 Pocket geometry and the contact frame

The target sphere of radius $R = D/2$ rests in the saddle formed by three mutually touching bed spheres of radius $r = d/2$. For a close-packed triad the circumradius of the bed-sphere centres is

$$r_c = \frac{d}{\sqrt{3}}, \quad (1)$$

125 and the target centre lies at a height

$$z_c = \sqrt{(R+r)^2 - r_c^2} \quad (2)$$

above the plane of those centres, giving a protrusion above the roughness crest of $P = z_c + R - r$ and a relative protrusion

$$\frac{P}{D} = \frac{z_c + R - r}{D} = 0.906, \quad (3)$$

which reproduces the value $P/D = 0.91$ reported for the experiments and confirms the reconstruction.

130 Two contact configurations are admissible during downstream pocket exit. Rolling over a single downstream sphere defines a contact normal inclined at $\varphi_1 = \arctan(r_c/z_c) = 27.51^\circ$ to the vertical; rolling about the axis through the two downstream contacts, the configuration described for three-dimensional beds by Voepel et al. (2019), gives $\varphi_2 = \arctan(r_c/2z_c) = 14.59^\circ$. Both are carried through the analysis. The contact angle is not reported in the source papers, and Sect. 2.12 quantifies its influence.

135 Because the contact between two spheres is a point on the target surface, the lever arms of the hydrodynamic resultant about that point are

$$a = R \sin \varphi, \quad b = R \cos \varphi, \quad (4)$$

so that $\sqrt{a^2 + b^2} = R$ exactly. The moment of inertia about the contact point follows from the parallel-axis theorem,

$$I_P = I_{cm} + mR^2 = 0.4mR^2 + mR^2 = 1.4mR^2, \quad (5)$$

140 i.e. $I_P/I_{cm} = 3.5$. Equations (4)–(5) are exact for a sphere and are the geometric basis of Sects. 2.7 and 2.8. It is noted that the lever arm $L = \sqrt{0.25D^2 + 0.5Dd} = 45.83$ mm adopted in both source papers after Valyrakis et al. (2010) exceeds $R = 30$ mm and is therefore not the distance from the centroid to a sphere–sphere contact point; the consequences are traced in Sect. 2.9.

2.4 The published inversion, restated

Both source papers gate the acceleration record before averaging. Let $a(t)$ denote a component of the gravity-compensated linear acceleration within the onset window $[t_0, t_0 + T_{\text{win}}]$, $T_{\text{win}} = 0.15$ s, with the sign convention that the onset-driving contribution is positive, and let τ be a noise floor estimated from the pre-event stationary segment. The gated signal and the positive impulse are

$$a_{\text{eff}}(t) = a(t) H(a(t) - \tau), \quad J^+ = \int_{t_0}^{t_0 + T_{\text{win}}} a_{\text{eff}}(t) dt, \quad (6)$$



where $H(\cdot)$ is the unit step function and the integral is evaluated by the trapezoidal rule with interpolation at the zero crossings.

150 Lu et al. (2026) normalise the impulse by the full window,

$$\bar{a}_{\text{eff}} = \frac{J^+}{T_{\text{win}}}, \quad (7)$$

whereas Lu et al. (2025) normalise it by the time during which the gated signal is non-zero,

$$A^+ = \frac{J^+}{T^+}, \quad T^+ = \int_{t_0}^{t_0+T_{\text{win}}} H(a(t)) dt. \quad (8)$$

Equations (7) and (8) share the numerator; they differ only in the normalising duration.

155 The onset-window translational balances used by Lu et al. (2026) decompose the unknown contact interaction into a compressive normal reaction F_{nor} and a tangential reaction F_{tan} ,

$$m\bar{a}_{x,\text{eff}} = F_D - F_{\text{tan}} \cos \varphi - F_{\text{nor}} \sin \varphi, \quad (9)$$

$$m\bar{a}_{z,\text{eff}} = F_L - W' - F_{\text{tan}} \sin \varphi + F_{\text{nor}} \cos \varphi, \quad (10)$$

with $W' = W - F_B = (\rho_s - \rho_w)Vg$ the submerged weight. The contact reactions are restricted to the admissible set

$$160 \quad F_{\text{nor}} \geq 0, \quad 0 \leq F_{\text{tan}} \leq \mu_f F_{\text{nor}}, \quad \mu_f \in [\mu_f^{\min}, \mu_f^{\max}] = [0.2, 0.6], \quad (11)$$

supplemented by a cap on the effective lift, $0 \leq F_L \leq W'$. Optimising Eqs. (9)–(10) over the set (11) yields the published bounds

$$F_{\text{nor}}^{\max} = \frac{m\bar{a}_{z,\text{eff}} + W'}{\cos \varphi - \mu_f^{\max} \sin \varphi}, \quad F_{\text{nor}}^{\min} = \frac{m\bar{a}_{z,\text{eff}}}{\cos \varphi}, \quad (12)$$

$$F_D^{\min} = m\bar{a}_{x,\text{eff}} + m\bar{a}_{z,\text{eff}} \tan \varphi, \quad F_D^{\max} = m\bar{a}_{x,\text{eff}} + F_{\text{nor}}^{\max} (\sin \varphi + \mu_f^{\max} \cos \varphi), \quad (13)$$

$$165 \quad F_L^{\min} = \max(0, m\bar{a}_{z,\text{eff}} + W' - F_{\text{nor}}^{\max} \cos \varphi), \quad F_L^{\max} = \min(W', F_L^{\text{cand}}), \quad (14)$$

with $F_L^{\text{cand}} = m\bar{a}_{z,\text{eff}} + W' + F_{\text{nor}}^{\min} (\mu_f^{\max} \sin \varphi - \cos \varphi)$, and the corresponding coefficients

$$C_D^{\min/\max} = \frac{2F_D^{\min/\max}}{\rho_w \langle \bar{u}_{0.15D} \rangle^2 A_D}, \quad C_L^{\min/\max} = \frac{2F_L^{\min/\max}}{\rho_w \langle \bar{u}_{0.15D} \rangle^2 A_L}, \quad \bar{C} = \frac{1}{2}(C^{\min} + C^{\max}). \quad (15)$$

The corresponding treatment of Lu et al. (2025) omits the contact decomposition and sets $F_D = mA_x^+$ and $F_L = mA_z^+ - F_B$.

2.5 Method 1: identifiability of the bounded inversion

170 The first method establishes analytically what the bounds of Eq. (14) can and cannot resolve. All manipulations were carried out symbolically with SymPy 1.14 and verified numerically on the 30 published conditions.

Proposition 1 (Collapse of the lift interval). *For every $\varphi \in (0, \arctan(1/\mu_f^{\max}))$, every $\mu_f^{\max} > 0$ and every $\bar{a}_{z,\text{eff}} \geq 0$, the bounds of Eq. (14) reduce to $[F_L^{\min}, F_L^{\max}] \equiv [0, W']$.*



Proof. Substituting $F_{\text{nor}}^{\text{max}}$ from Eq. (12) into the argument of the maximum in Eq. (14) gives

$$175 \quad m\bar{a}_{z,\text{eff}} + W' - F_{\text{nor}}^{\text{max}} \cos \varphi = (m\bar{a}_{z,\text{eff}} + W') \left[1 - \frac{\cos \varphi}{\cos \varphi - \mu_f^{\text{max}} \sin \varphi} \right] < 0, \quad (16)$$

so the non-negativity clause returns zero. Substituting $F_{\text{nor}}^{\text{min}}$ into F_L^{cand} gives

$$F_L^{\text{cand}} - W' = \mu_f^{\text{max}} m\bar{a}_{z,\text{eff}} \tan \varphi > 0, \quad (17)$$

so the cap returns W' . Neither result depends on the measured acceleration. $\square$

Corollary 1 (Algebraic identity of the midpoint lift coefficient). *With $A_L = \pi D^2/4$ and $W' = (\rho_s - \rho_w)g\pi D^3/6$, Proposition 1 and Eq. (15) give*

$$180 \quad \bar{C}_{L,\text{eff}} \equiv \frac{W'}{\rho_w \langle \bar{u}_{0.15D} \rangle^2 A_L} = \frac{2}{3} \frac{(s-1)gD}{\langle \bar{u}_{0.15D} \rangle^2} = \frac{2}{3} Fr_d^{-2}, \quad (18)$$

where $s = \rho_s/\rho_w$ and $Fr_d = \langle \bar{u}_{0.15D} \rangle / \sqrt{(s-1)gD}$ is the densimetric particle Froude number, equivalently the movability number of Novak and Nalluri (1975).

Corollary 1 states that the inferred lift coefficient is a deterministic function of the submerged weight and the characteristic velocity and carries no information from the sensor. The drag midpoint is not degenerate; combining Eqs. (13) and (15),

$$\bar{F}_{D,\text{eff}} = m\bar{a}_{x,\text{eff}} + \frac{1}{2} m\bar{a}_{z,\text{eff}} \tan \varphi + \frac{1}{2} (m\bar{a}_{z,\text{eff}} + W') \tan(\varphi + \arctan \mu_f^{\text{max}}), \quad (19)$$

which makes explicit that the contact term carries the submerged weight and that the result is a steeply increasing function of the unreported contact angle.

To establish whether any variant of the admissible set reproduces the published coefficients, an exhaustive forensic scan was performed over two lower bounds on F_{nor} , three upper bounds, the presence or absence of the lift cap, $\varphi \in [6^\circ, 40^\circ]$ in 0.5° steps and $\mu_f^{\text{max}} \in [0.1, 1.0]$ in steps of 0.05, giving 15 732 combinations. Because the feasible region in $(F_{\text{nor}}, F_{\text{tan}})$ is a polytope and the objectives are linear, the extrema of F_D and F_L were evaluated exactly at its vertices rather than by sampling.

2.6 Method 2: duration of positive acceleration

Because Eqs. (7) and (8) share the numerator J^+ , applying them to identical runs yields the ratio

$$195 \quad \frac{\bar{a}_{\text{eff}}}{A^+} = \frac{T^+}{T_{\text{win}}}, \quad (20)$$

that is, the fraction of the onset window during which the gated acceleration is non-zero. This quantity is not reported in either source but follows from their published numbers alone, provided the runs coincide (Table 1) and $\tau \ll |a|$. Equation (20) is diagnostic: a sustained hydrodynamic forcing that drives the particle out of its pocket implies $T^+/T_{\text{win}} \rightarrow 1$, whereas an oscillatory record with a brief excursion implies $T^+/T_{\text{win}} \ll 1$.



200 A complementary admissibility bound is obtained by asking what vertical acceleration is attainable hydrodynamically. At pocket exit the contact reaction vanishes and Eq. (10) gives $m\bar{a}_z = F_L - W'$, so for any admissible lift coefficient $C_L \leq C_L^*$,

$$\bar{a}_z \leq \frac{\frac{1}{2}C_L^* \rho_w \langle \bar{u}_{0.15D} \rangle^2 A_L - W'}{m}. \quad (21)$$

Equation (21) is evaluated with the generous choice $C_L^* = 1$.

2.7 Method 3: moment balance about the contact point

205 The third method introduces the equation that the source papers write for the threshold criterion but do not use to constrain the inversion. Taking moments about the instantaneous contact point P rather than about the centre of mass removes both unknown reactions, because the normal reaction is collinear with $\mathbf{r}_{G/P}$ and the tangential reaction acts at P . With the hydrodynamic resultant lumped at the centroid, the driving moment in the downstream-rolling sense is

$$M_P = F_D b + (F_L - W') a, \quad (22)$$

210 which is the equality form of the rolling criterion of Lu et al. (2025). Substituting Eqs. (9)–(10) together with the exact lever arms (4) gives

Proposition 2 (Cancellation of the contact reactions). *For a sphere in point contact,*

$$M_P = R(F_{\tan} + m\bar{a}_{x,\text{eff}} \cos \varphi + m\bar{a}_{z,\text{eff}} \sin \varphi), \quad \frac{\partial M_P}{\partial F_{\text{nor}}} \equiv 0. \quad (23)$$

For instantaneous rotation about P the transport term reduces to $m\mathbf{r}_{G/P} \times \mathbf{a}_G = m|\mathbf{r}_{G/P}|^2 \alpha$, because $\mathbf{a}_G = \alpha \times \mathbf{r}_{G/P} - \omega^2 \mathbf{r}_{G/P}$, so $M_P = I_P \alpha$ with I_P from Eq. (5). Combining with Eq. (23) yields the exact identity

$$I_P \alpha = m(\bar{a}_{x,\text{eff}} b + \bar{a}_{z,\text{eff}} a) + F_{\tan} R \quad (24)$$

which determines the tangential reaction uniquely from the measured kinematics and, because a resisting contact requires $F_{\tan} \geq 0$, supplies the necessary condition

$$I_P \alpha \geq m(\bar{a}_{x,\text{eff}} b + \bar{a}_{z,\text{eff}} a). \quad (25)$$

220 Equation (25) links the accelerometer and gyroscope channels of the same instrument and is testable with published data. It also explains an earlier observation: Maniatis et al. (2020) evaluated the rotational channel about the centre of mass, obtained $f_{\text{rot}} = I_{cm} \alpha / R$, found it small relative to the linear forces and concluded that it could be neglected. Referred to the contact point the same quantity is larger by the factor $I_P / I_{cm} = 3.5$, and Eq. (25) shows that the residual is not a negligible term but a measure of internal inconsistency.

2.8 Method 4: gyroscope-only closure

Because Eq. (22) contains no contact reaction, it constrains the hydrodynamic pair without recourse to the gated accelerometer signal at all. It defines a line in the (F_D, F_L) plane,

$$F_D b + (F_L - W') a = I_P \alpha, \quad (26)$$



which is bounded to a segment by the physical requirements $F_D \geq 0$ and $F_L \geq 0$. Introducing the lift-to-drag ratio $\lambda = C_L/C_D$,
230 so that $F_L/F_D = \lambda A_L/A_D$, closes the system:

$$F_D = \frac{I_P \alpha + W' a}{b + \lambda (A_L/A_D) a}, \quad C_D = \frac{2F_D}{\rho_w \langle \bar{u}_{0.15D} \rangle^2 A_D}, \quad C_L = \lambda C_D. \quad (27)$$

The bracket $\lambda \in [0.1, 1.5]$ is the range for large particle Reynolds numbers compiled by Bagnold (1956), Patnaik et al. (1992) and Vollmer and Kleinhans (2007), and cited as such by both source papers, so no new assumption is introduced. Results are reported as the interval spanned by the bracket rather than as a single value.

235 It is emphasised that Eq. (26) resolves one linear combination of F_D and F_L exactly; the residual indeterminacy is one-dimensional, against the two-dimensional indeterminacy of the translational system. The correct representation of the residual set is a segment, not the Cartesian product of two independently optimised intervals, and consequently the midpoint of such a product is not in general an attainable force pair.

2.9 Recovery of the angular acceleration

240 The released dataset reports the window-averaged rotational kinetic energy, $\bar{E}_{\text{rot}} = \frac{1}{2} I_r \bar{\omega}_r^2$, from which the root-mean-square resultant angular velocity is recovered as

$$\omega_{\text{rms}} = \sqrt{2\bar{E}_{\text{rot}}/I_r}, \quad I_r = 0.4mR^2 + mL^2, \quad (28)$$

using the moment of inertia defined in the source. Two independent checks support the recovered values. First, the same procedure applied to the corresponding table of Lu et al. (2025), which uses the centroidal inertia, returns $\omega_{\text{rms}} = 1.03 \text{ rad s}^{-1}$
245 against 1.11 rad s^{-1} here, so the two papers describe the same rotation and differ only in the inertia convention. Second, the recovered magnitude is consistent with the onset threshold of 0.7 rad s^{-1} adopted in the source after Gwiazda et al. (2022).

Starting from rest, the angular acceleration is estimated as

$$\alpha = \frac{k \omega_{\text{rms}}}{T_{\text{win}}}, \quad k \in \{1, \sqrt{3}, 2\}, \quad (29)$$

where $k = \sqrt{3}$ corresponds to a linear ramp $\omega(t) = \alpha t$, for which $\omega_{\text{end}} = \sqrt{3} \omega_{\text{rms}}$, and the remaining values bracket slower
250 and faster growth. Every result is reported for all three.

2.10 Threshold metrics

Two dimensionless descriptors of the same 30 thresholds are compared. The critical Shields number and the densimetric Froude number are

$$\theta_c = \frac{u_*^2 \rho_w}{(\rho_s - \rho_w) g D}, \quad Fr_d = \frac{\langle \bar{u}_{0.15D} \rangle}{\sqrt{(s-1)gD}}, \quad (30)$$

255 with u_* obtained in the source from a logarithmic fit. Repeatability is quantified by the coefficient of variation, both pooled over all conditions and within each density group, the latter isolating the scatter that is not attributable to the density contrast.



2.11 Digitisation of published figures

Coefficients plotted but not tabulated in the source were recovered from the published figures. Pages were rasterised at 400 dpi; axes were calibrated on the tick-label centroids; markers were segmented by colour distance to the legend swatches, filtered by connected-component size and aspect ratio, and converted to data coordinates from the blob centroids. The procedure recovers 15 lift and 23 drag markers, whose means (0.966 and 2.224) agree with the dashed means printed in the same figures (0.976 and 2.282), which validates the calibration. The centroid uncertainty is ± 2 px, corresponding to ± 0.02 in C_L .

2.12 Sensitivity and uncertainty protocol

Every reported quantity is accompanied by the full sensitivity range over the parameters that the source papers leave unspecified: the contact angle (φ_1 or φ_2), the friction bracket $\mu_f^{\max}$, the ramp exponent k of Eq. (29) and the lift-to-drag bracket λ . Conclusions are stated only where they hold across the entire range; where a conclusion is parameter-dependent, the dependence is reported rather than a preferred value. Results that require information not present in the public record are identified as such and are not claimed.

2.13 Software and reproducibility

All derivations, tests and figures are contained in a single executed Jupyter notebook (Python 3.12, SymPy 1.14, NumPy, pandas, SciPy, Matplotlib 3.10), which reads only the public dataset and the published PDFs and regenerates every number and figure in this paper. Figures are produced as vector PDFs with TrueType-embedded fonts. The notebook is archived and cited in the Code and data availability statement.

3 Results

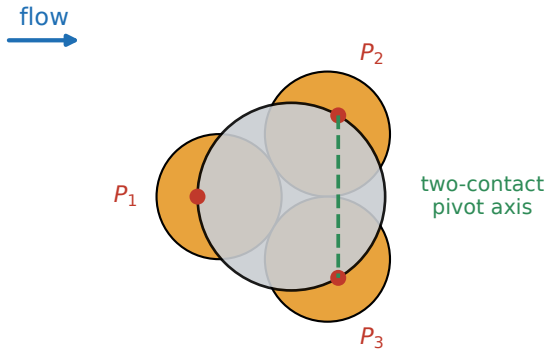
3.1 Reconstruction of the experimental configuration

The close-packed saddle reconstruction of Sect. 2.3 returns $P/D = 0.906$ (Fig. 2b), reproducing the value 0.91 stated in both source papers and confirming that the bed geometry is correctly recovered. The two admissible contact configurations give $\varphi_1 = 27.51^\circ$ for rolling over a single downstream sphere and $\varphi_2 = 14.59^\circ$ for rolling about the axis through the two downstream contacts (Fig. 2a), the latter being the configuration described for three-dimensional beds by Voepel et al. (2019). Neither value is stated in the source papers, although every bound in their inversion depends on it.

Integrity checks on the released table reproduce the particle Reynolds number, the shear Reynolds number and the critical Shields number exactly, and confirm $A_L = \pi D^2/4$. The streamwise projected area implied by the released column is $A_D = 2.7414 \times 10^{-3} \text{ m}^2$, i.e. $A_D/A_L = 0.970$; the reduction is applied but not defined, and no stated geometry reproduces it. The lever arm adopted in both papers, $L = 45.83 \text{ mm}$, exceeds $R = 30 \text{ mm}$ and therefore cannot be the centroid-to-contact distance for a sphere resting on spheres.



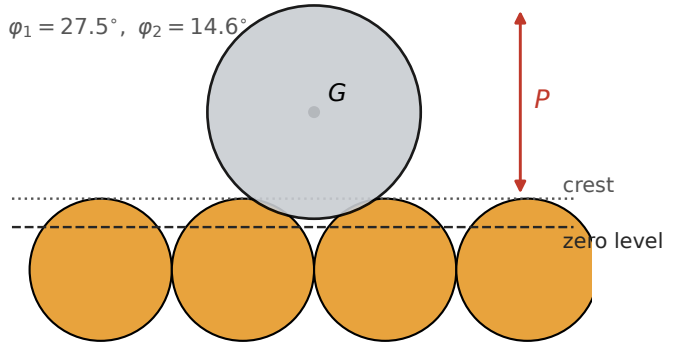
(a) saddle pocket, plan view



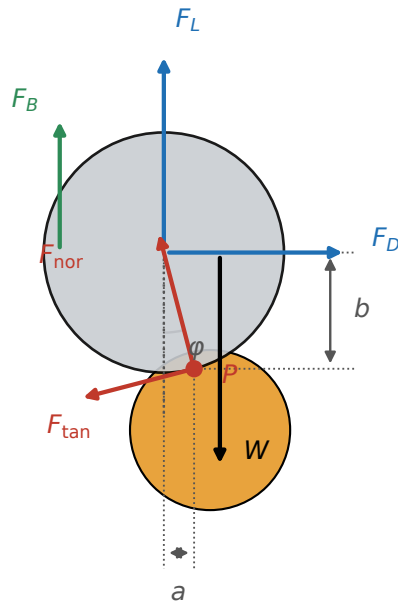
(b) elevation and protrusion

$$P/D = 0.906$$

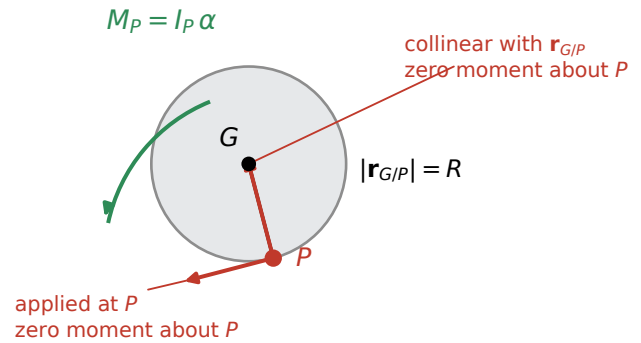
$$\varphi_1 = 27.5^\circ, \varphi_2 = 14.6^\circ$$



(c) forces at the threshold of motion



(d) moment about the contact point



$$I_P \alpha = m (\bar{a}_x b + \bar{a}_z a) + F_{\tan} R$$

Figure 2. Pocket geometry and force system. (a) Plan view of the saddle pocket with the three contact points and the two-contact pivot axis. (b) Elevation, protrusion and the two admissible contact angles. (c) Forces at the threshold of motion with the lever arms a and b . (d) Moment about the contact point: both reactions have zero moment, leaving the identity below.



Table 1 shows that the discharges, cross-sectional mean velocities and characteristic velocities reported by Lu et al. (2025) for the saddle configuration coincide with those reported by Lu et al. (2026) for $\rho_s = 1.395 \text{ g cm}^{-3}$ at all five common depths. The two papers analyse identical runs.

3.2 The lift inversion is degenerate

290 Proposition 1 is confirmed symbolically and numerically. Figure 3a maps the admissible set of Eq. (11) into the (F_D, F_L) plane for a representative condition: the two edges generated by $F_{\text{tan}} = 0$ and $F_{\text{tan}} = \mu_f F_{\text{nor}}$ both leave the strip $0 \leq F_L \leq W'$ immediately, so the non-tension clause and the physical cap return the interval endpoints irrespective of the measured acceleration. Across all 30 conditions the recomputed $F_L^{\min}$ is identically zero and $F_L^{\max}$ is identically W' .

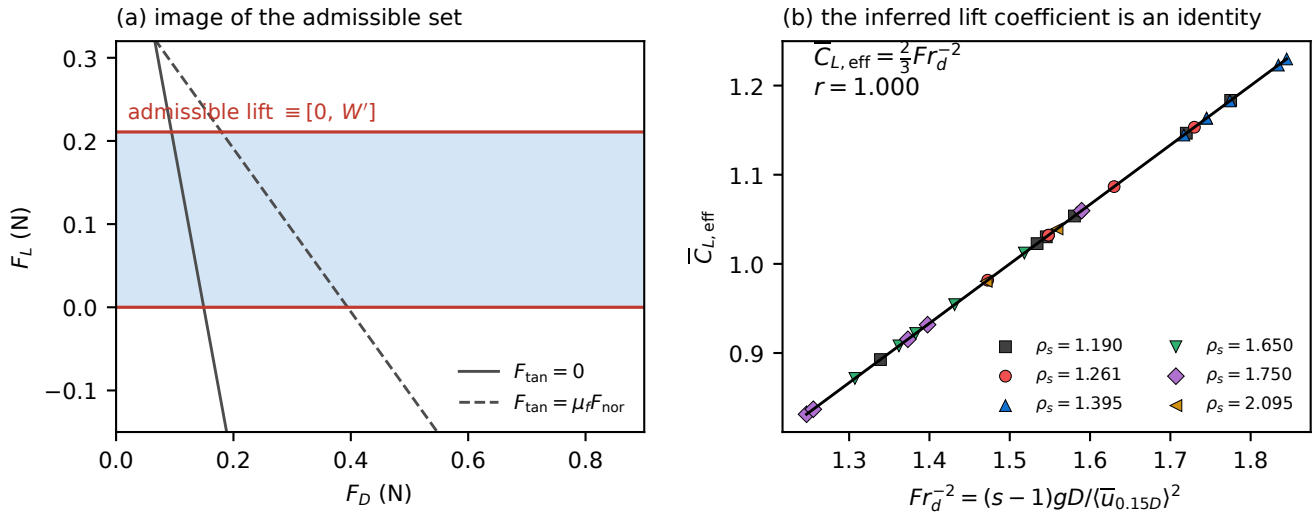


Figure 3. (a) Image of the admissible set in the (F_D, F_L) plane; the lift bracket returns its own endpoints. (b) The inferred lift coefficient against Fr_d^{-2} , $r = 1.000$.

Consequently the midpoint lift coefficient is an algebraic function of the submerged weight and the characteristic velocity. Figure 3b plots the recomputed $\bar{C}_{L,\text{eff}}$ against Fr_d^{-2} : the 30 points fall on the line of slope $2/3$ with a correlation coefficient of 1.000 to twelve decimal places, as required by Corollary 1. The recomputed mean is 1.035 against the 0.976 printed in the source. The statement that $\bar{C}_{L,\text{eff}}$ is nearly constant and only weakly dependent on Re_p , Fr and h/D is therefore equivalent to the statement that entrainment occurs at a nearly constant densimetric Froude number, here $Fr_d = 0.806 \pm 0.046$.

The drag midpoint is not degenerate, but Eq. (19) shows that 76% of it is contributed by the contact term, which carries the submerged weight, and that it depends steeply on the unreported contact angle: the recomputed mean is 2.33 at φ_2 and 3.47 at φ_1 .



3.3 Reproducibility of the published coefficients

Applying the printed equations to the released table does not reproduce the printed figures. Figures 5 and 4 compare the recomputed coefficients with values digitised from the source figures, against relative submergence and particle Reynolds number respectively. The digitisation is validated by its own output: the means of the recovered markers, 0.966 for C_L and 2.224 for C_D , agree with the dashed means printed in the same figures, 0.976 and 2.282.

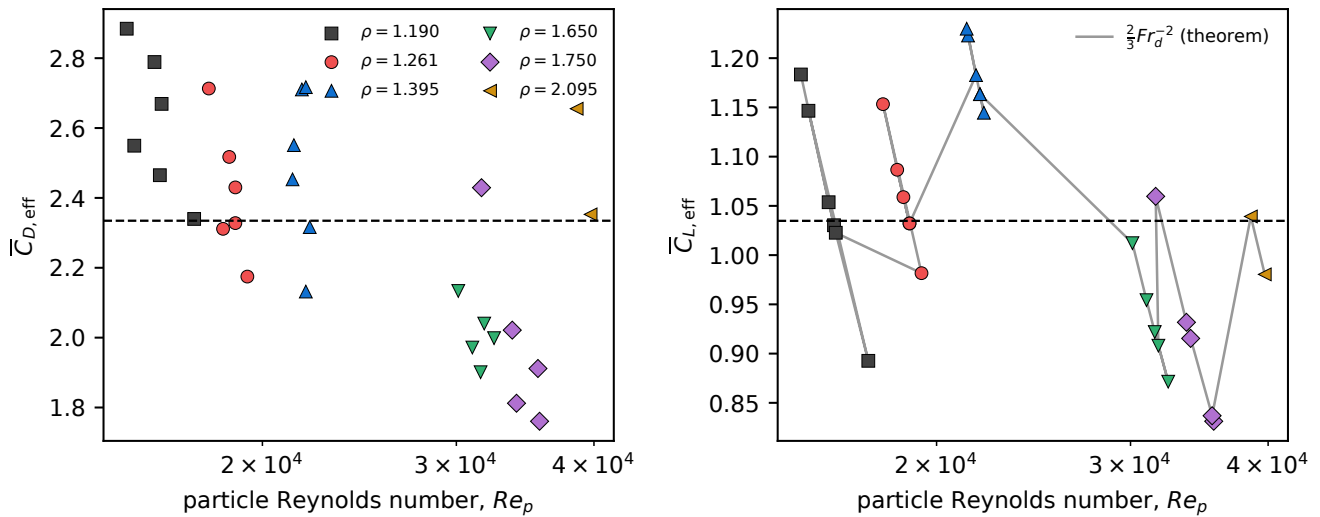


Figure 4. The same against particle Reynolds number, with the theorem line.

Point by point the plotted lift coefficients lie below the value required by Corollary 1 by a factor $k = 0.923$, with a residual scatter of 4.4% after removal of that factor, against a digitisation uncertainty of ± 0.02 . A single change of A_L by +8.3%, of $\langle \bar{u}_{0.15D} \rangle$ by +4.1% or of W' by -7.7% would account for the factor but not for the scatter. The origin cannot be determined from the public record and is not speculated upon here.

The forensic scan over 15 732 combinations of admissible-set variant, contact angle and friction coefficient is summarised in Fig. 6. No combination reproduces the published lift values, for the structural reason established in Sect. 3.2: every member of the family collapses to $[0, W']$ or exceeds it, whereas the published values lie below. The drag residual is minimised near $\varphi \approx 26.5^\circ$, close to the single-contact geometric value, at a root-mean-square error of 0.20 on coefficients of order 2.3.

3.4 Duration of positive acceleration

Figure 7a illustrates the two normalisations of the same gated impulse, and Fig. 7b establishes the property that makes the diagnostic meaningful: applied to a zero-mean signal of standard deviation σ , the gate returns a mean of $\sigma/\sqrt{2\pi} = 0.399\sigma$, so a strictly positive “effective acceleration” is obtained from a record with no net acceleration at all.

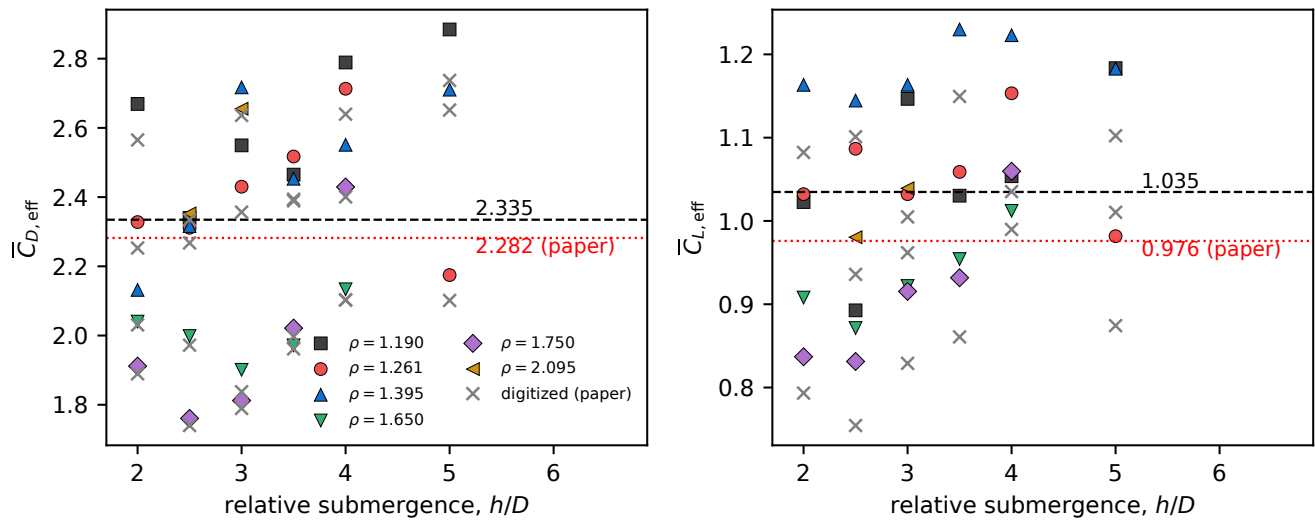


Figure 5. Recomputed and digitised coefficients against relative submergence.

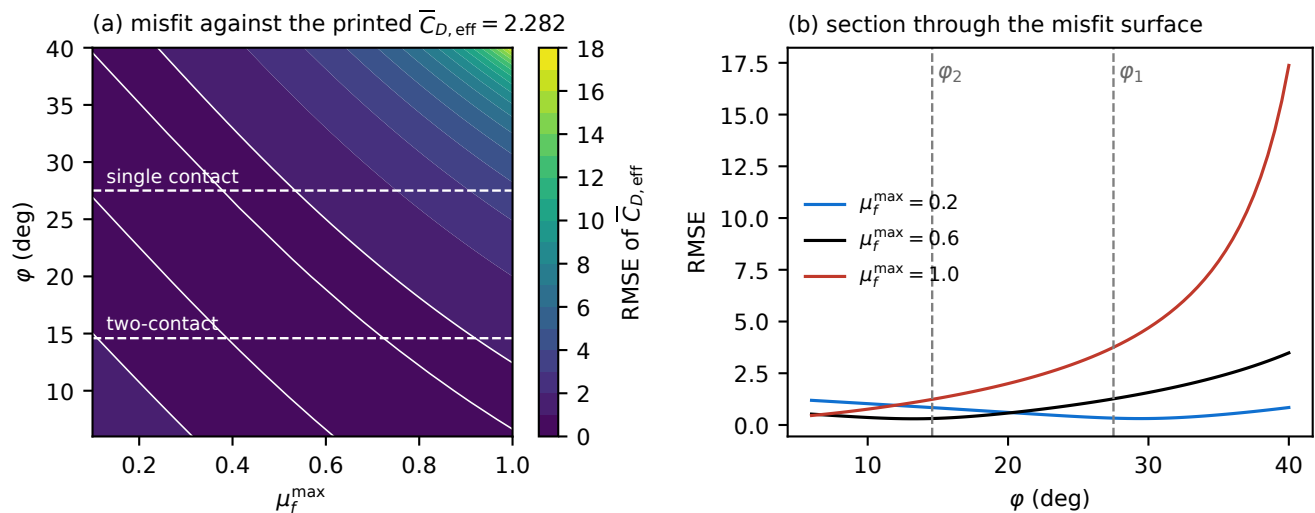


Figure 6. Forensic scan over admissible-set variant, contact angle and friction coefficient.

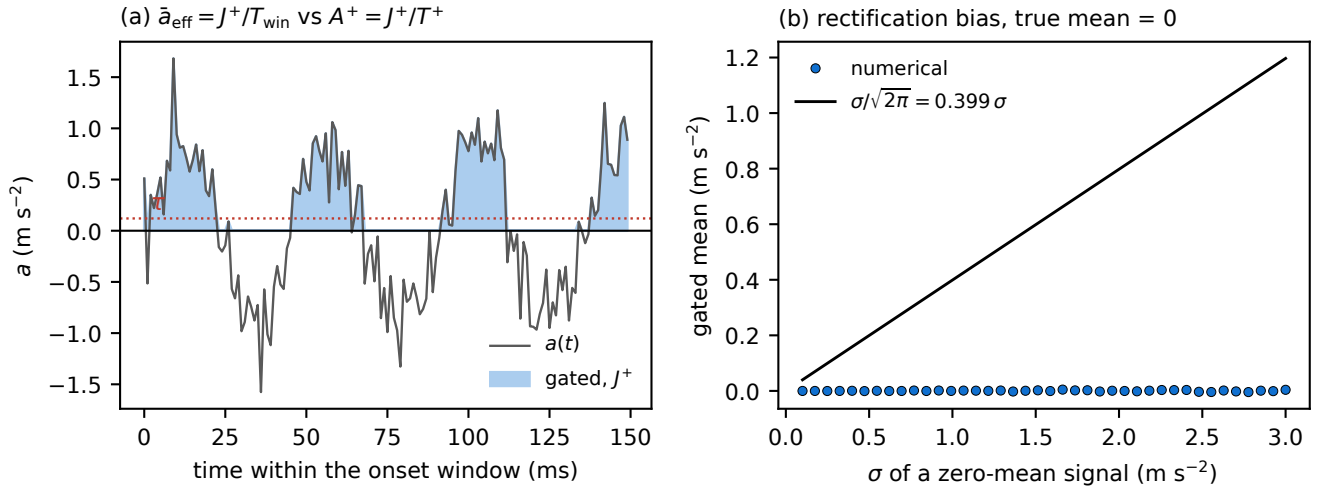


Figure 7. (a) The two normalisations of the same gated impulse. (b) Rectification bias: the gate returns 0.399σ from a signal whose true mean is zero.

Evaluating Eq. (20) on the runs shared by the two source papers gives $T^+/T_{\text{win}} = 0.165$ in the streamwise direction and 0.140 in the vertical (Fig. 8a). The gated acceleration is therefore non-zero for 21–25 ms of the 150 ms onset window, that is, for two to three samples at the 100 Hz acquisition rate. A sustained hydrodynamic forcing carrying the grain out of its pocket would give $T^+/T_{\text{win}} \rightarrow 1$.

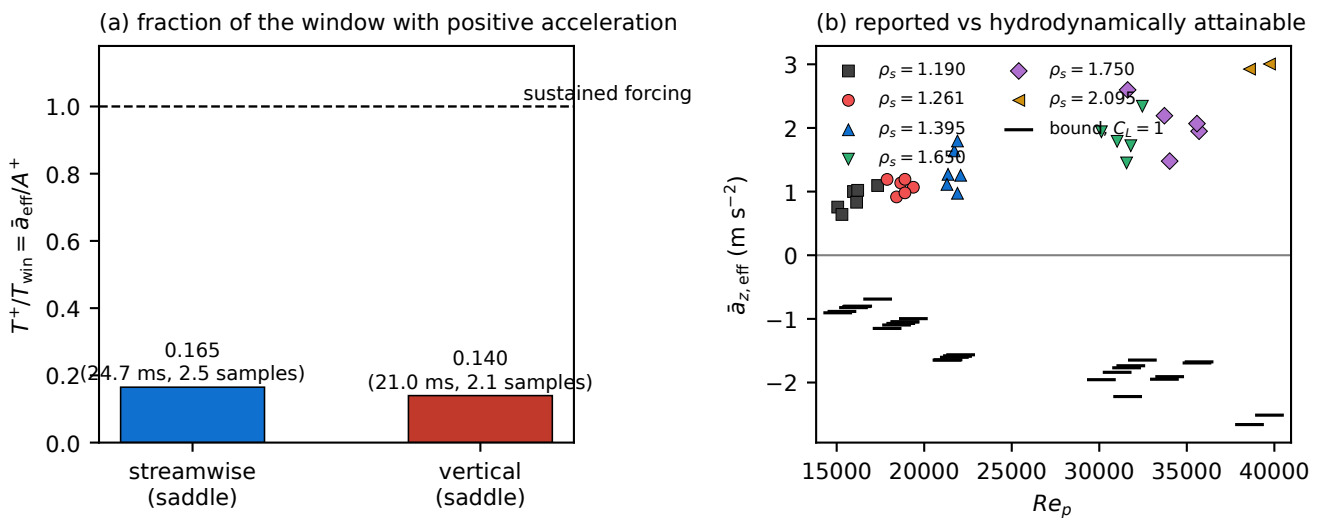


Figure 8. (a) Fraction of the onset window with positive acceleration. (b) Reported vertical acceleration against the hydrodynamically attainable bound.



Figure 8b applies the independent admissibility bound of Eq. (21). Even with the generous choice $C_L^* = 1$, the largest attainable lift is smaller than the submerged weight in every condition, so no admissible lift coefficient can produce an upward
 325 acceleration at pocket exit; the bound is negative throughout, between -0.8 and -2.4 m s^{-2} . The reported vertical accelerations are positive and between 0.6 and 3.0 m s^{-2} . The same calculation applied to the values of Lu et al. (2025) requires $C_L = 10.6$ once the vertical balance is closed with the submerged weight rather than with buoyancy alone.

3.5 Consistency of the accelerometer and gyroscope channels

Proposition 2 is confirmed symbolically: with the exact lever arms of Eq. (4) the derivative of the moment with respect to the
 330 normal reaction vanishes identically, and the moment reduces to $M_P = R(F_{\text{tan}} + m\bar{a}_x \cos \varphi + m\bar{a}_z \sin \varphi)$ (Fig. 2d).

The angular velocity recovered from the released rotational-kinetic-energy table is $\omega_{\text{rms}} = 1.13 \pm 0.04 \text{ rad s}^{-1}$, above the 0.7 rad s^{-1} onset threshold adopted in the source and in agreement with the value obtained by the same procedure from Lu et al. (2025) once the differing inertia conventions are accounted for.

Evaluating the necessary condition (25) gives $F_{\text{tan}} < 0$ in all 30 conditions, for both pivot geometries and all three ramp
 335 models (Fig. 9). The measured angular acceleration is between 27% and 66% of the value implied by the measured linear accelerations, the deficit being largest for the densest particles. A negative tangential reaction would require friction to drive rather than resist the motion; the two channels of the instrument are therefore mutually inconsistent under the rolling mode that both source papers adopt.

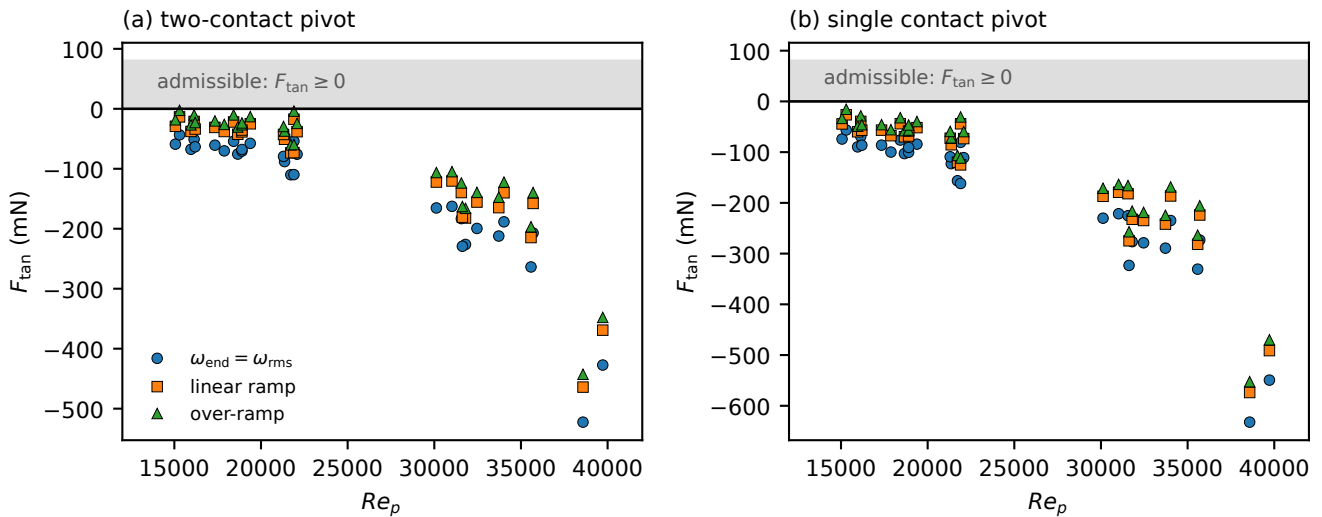


Figure 9. Tangential reaction from the moment identity for both pivot geometries and three ramp models. The admissible region is shaded.

Referred to the centre of mass, as in Maniatis et al. (2020), the same quantity is $f_{\text{rot}} = I_{cm}\alpha/R = 26 \text{ mN}$ against a linear net
 340 force of 301 mN , i.e. 11 %, which invites the interpretation that rotation is negligible. Referred to the contact point it is 92 mN ,



or 37 %, because $I_P/I_{cm} = 3.5$. The apparent smallness of the rotational channel is thus a consequence of the reference point, and the residual is a measure of inconsistency rather than a term that may be dropped.

3.6 Closure and comparison with independent measurements

Because Eq. (26) contains no contact reaction, the gyroscope channel alone constrains the hydrodynamic pair. Figure 10 shows the resulting drag coefficients for both pivot geometries and three values of the lift-to-drag ratio. For the two-contact pivot, $C_D = 1.01 \pm 0.23$ at $\lambda = 0.1$ falling to 0.74 ± 0.16 at $\lambda = 1.5$; for the single-contact pivot the corresponding values are 1.55 ± 0.28 and 0.91 ± 0.16 . The full range across both geometries and the whole lift-to-drag bracket is $C_D = 0.74\text{--}1.55$.

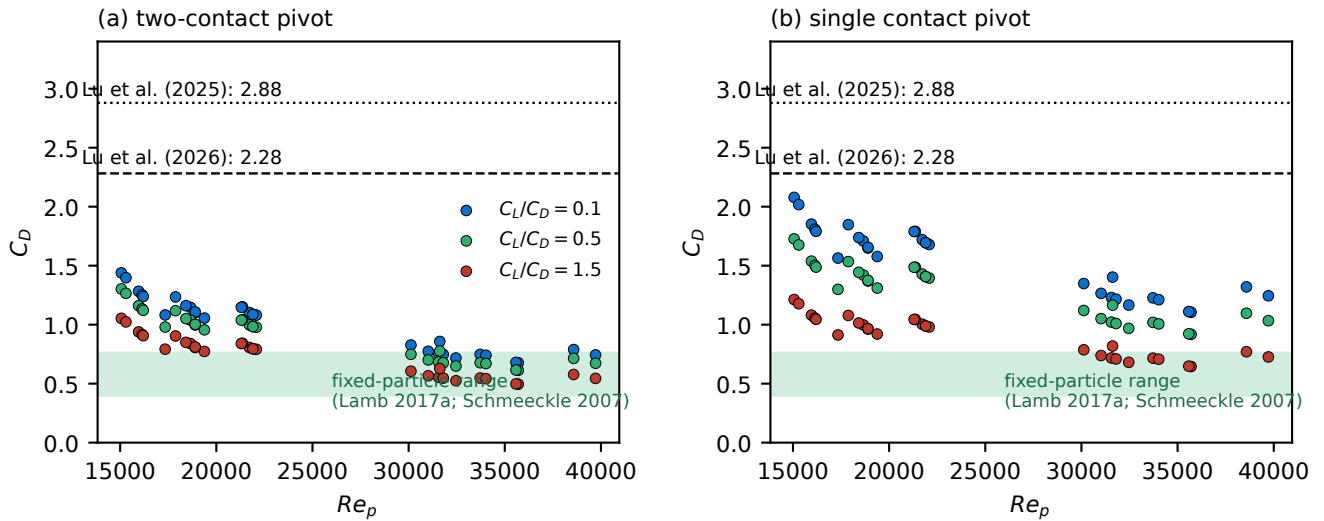


Figure 10. Drag coefficient from the gyroscope-only closure, with the fixed-particle reference band and the published values.

Figure 1 places these values among the published estimates. The closure result brackets the fixed-particle reference range of 0.4–0.8 (Schmeckle et al., 2007; Lamb et al., 2017; Lee and Balachandar, 2012) and overlaps the rolling-sphere values of Coleman (1972), which are expected to be somewhat larger. By contrast, the coefficients obtained from the accelerometer-based inversions applied to the same experiments lie a factor of three to four above that range (Lu et al., 2025, 2026), while the inertial coefficients of Xie et al. (2023a), which the authors explicitly decline to interpret as hydraulic coefficients, lie an order of magnitude below it. That a closure using only the rotational channel recovers the independent reference range, from the same published data, is the principal positive result of this study.

A residual trend of C_D with Re_p , from about 1.4 at $Re_p = 1.5 \times 10^4$ to about 0.7 at 4×10^4 , is visible in Fig. 10. Whether it is physical or an artefact of the angular-acceleration estimate cannot be decided with the available data, and no claim is made.

3.7 Repeatability of the two threshold metrics

The same 30 thresholds expressed in the two dimensionless forms of Eq. (30) differ markedly in repeatability (Fig. 11). Pooled over all conditions, $\theta_c = 0.0071 \pm 0.0016$, a coefficient of variation of 22%, against $Fr_d = 0.806 \pm 0.046$, a coefficient of variation of 5.7%. Within density groups, which removes the density contrast from the comparison, the corresponding values are 17% and 3.2%. The disparity is traceable to the shear velocity, whose within-group coefficient of variation is 8% and which enters θ_c squared; the statistical uncertainty of logarithmic-profile fitting is well documented (Dancey and Diplas, 2008).

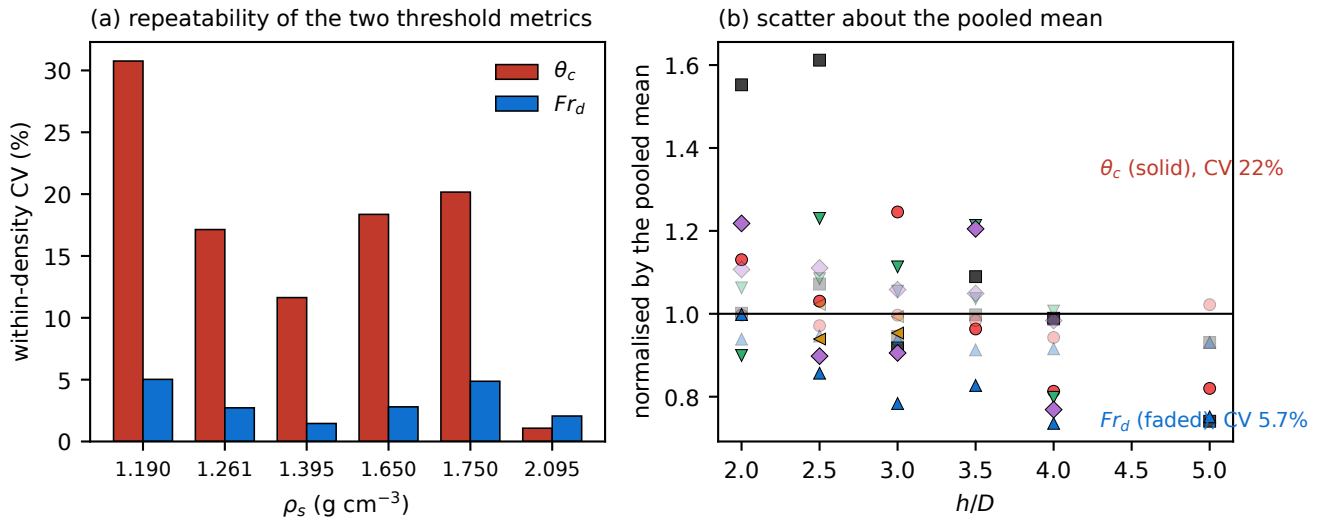


Figure 11. Repeatability of the Shields number and the densimetric Froude number on the same 30 thresholds.

For grains as exposed as these, a velocity-referenced threshold is thus four to five times more repeatable than a stress-referenced one. This is not a new criterion — the quantity is the movability number of Novak and Nalluri (1975), studied under protrusion by Bhat and Ahanger (2022, 2023) — but the quantitative comparison at $P/D \approx 0.9$ appears not to have been made, and it provides the physical reading of the identity established in Sect. 3.2: the degenerate lift coefficient is, up to the factor $2/3$, the inverse squared movability number.

3.8 Sensitivity of the conclusions

Figure 12 summarises which conclusions survive the parameters that the source papers leave unspecified. The collapse of the lift interval, the resulting identity, the cancellation of the contact reactions in the moment equation, and the failure of the consistency test are robust to the contact angle, the friction bracket, the ramp model and the lift-to-drag bracket: they hold everywhere in the parameter space explored. The numerical value of $\overline{C}_{D,\text{eff}}$ and the numerical value returned by the closure are parameter-dependent, and are reported as ranges rather than as point estimates throughout. Table 2 collects every result of this study with its basis and its range of validity.



$F_L^{\min} \equiv 0$	robust	robust
$F_L^{\max} \equiv W'$	robust	robust
$\bar{C}_{L, \text{eff}} = \frac{2}{3} Fr_d^{-2}$	robust	robust
$\partial M_P / \partial F_{\text{nor}} \equiv 0$	robust	robust
$F_{\text{tan}} < 0$ (30/30 runs)	robust	robust
$\bar{C}_{D, \text{eff}}$ value	depends	depends
closure C_D value	depends	robust
	contact angle φ	friction / ramp / λ

Figure 12. Robustness of each result to the parameters left unspecified in the source papers.

Table 2. Summary of the analysis. Robustness refers to the contact angle φ , the friction bracket μ_f , the angular ramp exponent k and the lift-to-drag bracket λ (Sect. 2.12).

Result	Basis	Robustness
$[F_L^{\min}, F_L^{\max}] \equiv [0, W']$	Proposition 1	all parameters
$\bar{C}_{L, \text{eff}} = \frac{2}{3} Fr_d^{-2}, r = 1.000$	Corollary 1	all parameters
$\partial M_P / \partial F_{\text{nor}} \equiv 0$	Proposition 2	all parameters
$F_{\text{tan}} < 0$ in 30/30 conditions	Eq. (25)	all parameters
$T^+ / T_{\text{win}} = 0.14\text{--}0.17$	Eq. (20)	–
$\bar{C}_{D, \text{eff}} = 2.33\text{--}3.47$	Eq. (19)	depends on φ
closure $C_D = 0.74\text{--}1.55$	Eq. (27)	depends on φ, λ
Fr_d CV 5.7% vs θ_c CV 22%	Eq. (30)	–



375 4 Discussion

4.1 What an instrumented particle measures

An inertial measurement unit embedded in a grain reports the motion of that grain, not the forcing applied to it. At the threshold of motion the two differ by the contact reactions, which are of the same order as the hydrodynamic forcing and are not measured. The results above show that this distinction is not a caveat to be noted and set aside: it determines what can be recovered.

380 The degeneracy established in Sect. 3.2 is the clearest expression of the problem. Bracketing an unmeasured quantity is a legitimate response to indeterminacy, and the non-tension and Coulomb conditions used by Lu et al. (2026) are physically sound. What the analysis shows is that those conditions, applied to the vertical balance alone, are not binding: the admissible normal reaction is large enough to absorb any measured vertical acceleration, so the bracket returns its own endpoints. The width of the resulting interval is then a property of the constraint set rather than a measure of the uncertainty in the data, and its
 385 midpoint is not an estimate of anything. This is a general hazard of set-valued inversion and is well recognised in the control and estimation literature (Milanese and Vicino, 1991; Jaulin et al., 2001), where the standard remedy is to check that the feasible set is bounded by the data and not by the constraints. That check does not appear to have been made in the sediment-transport applications of the approach.

The identity $\overline{C}_{L,\text{eff}} = \frac{2}{3}Fr_d^{-2}$ carries a second, more constructive reading. The quantity that the inversion returns is, up
 390 to a constant, the inverse squared movability number of Novak and Nalluri (1975), and its near-constancy across densities, depths and Reynolds numbers is a real and useful experimental result — it is simply a result about the threshold velocity, not about lift. The measurements of Lu et al. (2026) therefore establish, for grains at $P/D \approx 0.9$, that entrainment occurs at $Fr_d = 0.806 \pm 0.046$, and Sect. 3.7 shows that this descriptor is four to five times more repeatable than the critical Shields number computed from the same runs. For highly exposed grains in low-slope beds, where the logarithmic fit for u_* is least
 395 well constrained, a velocity-referenced threshold is the better-conditioned statement of the same physics (Bhat and Ahanger, 2022, 2023).

4.2 Why the rotational channel is the informative one

Sect. 3.5 shows that the two channels of the same instrument cannot both be right under a rolling mode. This is not a new observation so much as a new reading of an old one. Maniatis et al. (2020) formed the rotational balance about the grain's
 400 centre of mass, found the implied tangential force small relative to the linear forces, and concluded that rotation could be neglected. Referred to the contact point the same quantity is larger by $I_P/I_{cm} = 3.5$, and, more importantly, it is the only balance in which the unknown reactions cancel identically (Proposition 2). The apparent smallness of the rotational channel about the centroid is therefore an artefact of the reference point, and the residual is not a negligible term but a measure of how far the linear and angular records are from mutual consistency.

405 The direction of the inconsistency is informative. The measured angular acceleration is too small for the measured linear accelerations, not too large. Under a rolling mode a grain pivoting out of its pocket must rotate at a rate set by its translation; the released records show translation signals that would require rotation two to four times faster than that observed. Two readings



are available. Either the mode is not pure rolling over the onset window — but both source papers document rolling, and the visual record supports it — or the reported linear accelerations are not the net response of the grain. Sect. 3.4 supports the second reading: the gated acceleration is non-zero for only two to three samples of the fifteen in the onset window, and the gate returns 0.399σ when applied to a signal with no net acceleration whatever (Fig. 7b). A brief, rectified excursion of a fluctuating record is not a window-averaged net force, and multiplying it by the particle mass does not make it one.

The independent admissibility bound reinforces this. At the reported threshold velocities no admissible lift coefficient can produce an upward acceleration at all, because the largest lift compatible with $C_L \leq 1$ is smaller than the submerged weight in every condition (Fig. 8b). Vertical accelerations of $1\text{--}3\text{ m s}^{-2}$ upward therefore cannot be a hydrodynamic response; they are consistent with the contact interactions that both source papers describe when they attribute the acceleration spikes to the grain's departure and to collisions with neighbouring grains.

4.3 Consistency with independent force measurements

The closure of Sect. 3.6 uses no accelerometer information and is therefore immune to the gating problem. It returns $C_D = 0.74\text{--}1.55$ across both pivot geometries and the whole literature range of lift-to-drag ratios, which brackets the fixed-particle values of $0.4\text{--}0.8$ reported by Schmeeckle et al. (2007) and Lamb et al. (2017) and the resolved-simulation values of Lee and Balachandar (2012), and sits below the rolling-sphere values of Coleman (1972), as expected for a grain that has not yet accelerated appreciably. That a one-line moment balance applied to the released rotational data reproduces the independent reference range is the strongest available evidence that the rotational channel is the recoverable one.

The comparison with Celik et al. (2014) is instructive on what an independent measurement would add. Those authors instrumented a fully exposed sphere with surface pressure transducers front, back, top and bottom, synchronised with laser Doppler velocimetry, at near-threshold conditions. Because pressure integration isolates the fluid loading from the contact reactions entirely, their record is of a different kind from an IMU record: it measures forcing rather than response. Their finding that the near-bed streamwise velocity governs the entrainment-relevant force events, while lift is intermittent and poorly correlated with the vertical velocity fluctuation, is difficult to reconcile with an analysis in which the vertical channel dominates the force budget. The particle Reynolds numbers differ by two orders of magnitude ($330\text{--}440$ against $1.4\text{--}4.0 \times 10^4$), so the comparison is qualitative; the methodological point is not.

4.4 Limitations

Three limitations bound the results. First, the angular acceleration is recovered from a published table of window-averaged rotational kinetic energy and therefore inherits the moment-of-inertia convention of the source, in which the lever arm exceeds the centroid-to-contact distance for a sphere; every conclusion is reported for three growth models spanning a factor of two in α , and none changes. Second, the closure assumes rolling without slip about a point contact with no rolling-resistance moment. Rolling resistance would add a term to Eq. (24) of the same sign as F_{tan} and would therefore deepen, not relieve, the inconsistency. Third, the residual dependence of the closure coefficient on Re_p (Fig. 10) cannot be attributed to physics or to the angular-acceleration estimate with the available data, and no claim is made about it. More generally, the raw event time



series are not public, so the analysis operates on window-averaged quantities throughout; the diagnostics developed here are constructed precisely so as not to require them.

4.5 Implications for measurement design

The analysis points to a specific and modest addition to the experiment rather than to its replacement. Because the moment
 445 about the contact point is the only balance free of contact reactions, the angular record is the quantity to preserve and report at full resolution: onset-window time series of $\omega(t)$, from which α follows directly, remove the need for the ramp assumption of Eq. (29) and make the closure exact. Reporting the contact angle, the lever arms and the inertia convention would remove the remaining parameter dependence.

Beyond that, the indeterminacy is removed only by measuring the contact reactions. Instrumenting the supporting spheres, or
 450 seating the grain in a load-cell pocket, would make Eqs. (9) and (10) determinate and would convert the bounds into estimates; both source papers propose this, and the present analysis quantifies what is at stake. A pressure-instrumented grain in the manner of Celik et al. (2014), run concurrently with an IMU-equipped grain of the same size and exposure, would provide the paired forcing-and-response record that the field currently lacks.

5 Conclusions

455 The recovery of hydrodynamic drag and lift from instrumented-particle records at the threshold of motion is an inverse problem, and this study has examined what it can identify.

1. The bounded inversion of Lu et al. (2026) is degenerate in lift. For every contact angle in the admissible range and every friction coefficient, the interval collapses to $[0, W']$, so that $\overline{C}_{L,\text{eff}} \equiv \frac{2}{3}(s-1)gD/\langle \bar{u}_{0.15D} \rangle^2 = \frac{2}{3}Fr_d^{-2}$, confirmed on all
 460 30 published conditions with a correlation of 1.000. The reported constancy of the coefficient restates a constant critical movability number and contains no sensor information. The drag midpoint is not degenerate but is dominated by the contact term and varies between 2.33 and 3.47 with the unreported contact angle.
2. Applied to the published record, the same threshold data give $Fr_d = 0.806 \pm 0.046$ (CV 5.7%) against $\theta_c = 0.0071 \pm 0.0016$ (CV 22%). For grains at $P/D \approx 0.9$ the velocity-referenced threshold is four to five times the more repeatable descriptor.
- 465 3. Because two published analyses normalise the same rectified impulse by different durations, their ratio measures the fraction of the onset window during which the acceleration is positive. That fraction is 0.14–0.17: two to three samples of fifteen. The gate returns 0.399σ from a signal with no net acceleration, and at the reported threshold velocities no admissible lift coefficient can produce an upward acceleration at all.
4. For a spherical grain in point contact the moment about the contact point annihilates both reactions exactly, giving $I_P\alpha =$
 470 $m(\bar{a}_x b + \bar{a}_z a) + F_{\tan}R$. Requiring $F_{\tan} \geq 0$ produces a consistency test between the two channels of the instrument,



which fails in all 30 conditions and for every parameter choice. Referred to the centre of mass the same quantity is smaller by a factor 3.5, which explains why the rotational channel has been reported as negligible.

5. Closing the inversion with the gyroscope channel alone returns $C_D = 0.74\text{--}1.55$, bracketing independent fixed-particle and resolved-simulation values, from the same published data.

475 Instrumented particles observe the one thing that fixed-particle experiments cannot — the instant of entrainment — and that capability is worth preserving. What the analysis shows is that the linear channel alone cannot deliver hydrodynamic coefficients, and that the rotational channel, referred to the contact point, can. Reporting onset-window angular velocity at full resolution, together with the contact geometry and the inertia convention, would make the closure exact; measuring the contact reactions would make the full inversion determinate.

480 *Code and data availability.* The source data are those of Lu et al. (2026), archived at <https://doi.org/10.5281/zenodo.18645887> under CC-BY 4.0. All derivations, tests and figures in this paper are reproduced by a single executed notebook, archived at [DOI to be assigned], which reads only that archive and the published articles.

Competing interests. The author declares no competing interests.

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485 dataset, without which this analysis would not have been possible.



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