Reviewer 1 asked about the reasoning for the analysis and asked for an explanation of the econometric methods used, a physical explanation for the conclusions, and a perfect model test.

1. **Reasoning:** The analysis I performed was designed to determine if the observed GSAT-ERF relationship is non-linear over the last five decades, and if a simple econometric model could be constructed from observational data that would accurately and reliably reproduce observed global warming on a decadal time scale. The hypothesis was that the GSAT response to rapidly increasing ERF may differ from the GSAT response to ERF during periods when ERF is relatively stable or rising very slowly (see Figure 1 in the manuscript). From 1871-1970 for non-volcanic years (ERF > 0), ERF varied from 0.02-0.68 Wm^{-2} and GSAT increased by about 0.3°C (Figure 1). During this relatively stable climatic period, the relationship between ERF > 0 and GSAT was linear ($\nu = 1.47$, 95 % CI 0.12–2.82; BIC prefers the linear fit). In the 1970's ERF began to rise rapidly as did GSAT. From 1977-2025, ERF increased by about 2.6 Wm^{-2} and GSAT increased by about 1.0°C, reaching 1.5°C in 2024. These increases are largely due to increased anthropogenic greenhouse gas emissions and decreased sulfur dioxide emissions, especially after 1980. The hypothesis was based on accumulating evidence, as summarized in the manuscript, that as GSAT increases, feedbacks in the climate system may become non-linear. As reviewer 1 says: "The author is right that at high temperatures we expect feedbacks to become non-linear." As reported by McKay et al (10.1126/science.abn7950) current warming already lies within the lower end of some tipping-point uncertainty ranges, and six tipping points become likely between 1.5 and 2°C.
2. **Econometric methods:** I felt modeling the relationship between GSAT and ERF would best be addressed by examining GSAT and ERF data from 1977-2025 obtained from a well-documented and curated database (the IGCC 2025 update using AR6 methods as reported by Forster et al. (2026), 10.5194/essd-18-3889-2026). I wanted to use analytical methods that could provide reliable estimates for trending time series and which did not require mechanistic assumptions that could introduce bias. I found that Pretis (2020) had used econometric regression methods to model climate data. He showed that climate physics can be represented in a discrete-time econometric framework with only a handful of physically interpretable parameters. The total-ERF ARDL specification is the discrete-time counterpart of the surface equation of a two-layer energy-balance model with the deep-ocean temperature held constant. Beyond this theoretical reason, econometric methods are well suited for non-stationary data (meaning the data exhibit long-term stochastic trends over time), such as GSAT and ERF, because they use cointegration techniques to estimate the long-run relationship. GSAT in any given year is heavily dependent on past temperatures. Econometric methods specifically account for this autocorrelation by using lagged dependent variables (such as $GSAT_{t-1}$). Further, econometric analyses can account for natural climate cycles (such as ENSO) that introduce transient variability into the system. Finally, econometric methods are agnostic to the underlying mechanisms. They do not require specifying the physics of energy transfer within the system or allocating variance to specific components of the model.

3. **Physical explanation:** The conclusions of the analysis are summarized in the first paragraph of the discussion: 1) The long-run GSAT ERF relationship is most likely of the form GSAT = $\mu + \theta \cdot ERF^{\nu}$, 2) the ARDL model predicts GSAT from 1977-2025 with high fidelity and 3) that if ERF continues to increase by 0.7 $Wm^{-2}decade^{-1}$, then GSAT in mid-century will likely exceed AR6 estimates and reach levels not experienced in the last 3 million years. No other significant findings are implied.

   The reviewer finds the non-linear GSAT ERF relationship "surprising". To address this assessment, I evaluated the GSAT-ERF relationship using the Finite-amplitude Impulse Response (FaIR) simple climate model. When the FaIR model is driven by the IGCC 2025 ERF history and its GSAT output is regressed on its ERF input over 1977–2025, the relationship is not a straight line (Fig. R1). The straight line is rejected in all 841 members for total ERF. In FaIR, this non-linear relationship is produced by the thermal inertia of the ocean acting on the abrupt changes in volcanic forcing. FaIR's feedback parameter is constant and its forcing-to-temperature equations are linear. GSAT depends on the accumulated forcing history through heat exchange between the surface layer and the deeper ocean on three timescales, rather than on the current year's ERF alone. With volcanic eruptions removed from FaIR's forcing, the relationship is close to a straight line ($\nu = 0.97$).

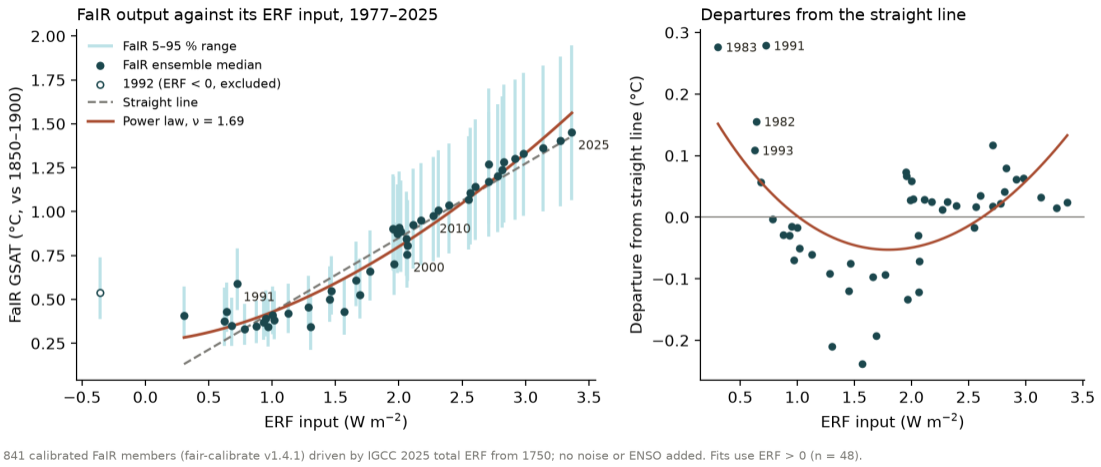


Figure R1. FaIR GSAT plotted against its ERF input, 1977–2025. Left: ensemble median of 841 calibrated FaIR members (fair-calibrate v1.4.1) driven by IGCC 2025 total ERF from 1750, with the 5–95 % range, a straight line and a power law (v = 1.69) fitted to years with ERF > 0 (n = 48). Right: departures of the ensemble median from the straight line.

   In FaIR, the curvature arises mainly from the ocean's thermal inertia acting on the volcanic forcing. FaIR shows that this memory alone produces curvature under the historical forcing. The ARDL model is agnostic to mechanism, including changes in cloud and sea-surface temperature patterns and uncertainty in aerosol forcing. To address whether or not the same effect found in the FaIR analysis explained the non-linear GSAT-ERF relationship in the ARDL model, ERF was partitioned into natural components (volcanic

and solar ERF, NAT) and anthropogenic ERF (AN). The sum of AN + NAT exactly reproduced total ERF. At the risk of over-parameterizing a model with 49 observations with four independent variables, AN and NAT were subjected to exactly the same analysis as total ERF. GSAT was a non-linear, power-law function of AN: $GSAT = 0.32 + 0.21 \times AN_{t-1}^{1.49}$ ($R^2 = 0.898$); a linear relationship was rejected ($p = 0.017$). A partitioned ARDL model (P-ARDL) yields $GSAT_t = 0.26 + 0.20\ GSAT_{t-1} + 0.16\ AN_{t-1}^{1.49} + 0.15\ NAT_t + 0.11\ ONI_t$. The P-ARDL model explained 97 % of GSAT variability from 1977 to 2025 ($R^2 = 0.968$) and was preferred over the total-ERF ARDL model ($\Delta BIC = -6.5$). Residual diagnostics indicate no evidence of residual autocorrelation or heteroskedasticity and bounds testing ($F = 53.5$) supports cointegration. The P-ARDL model was validated by expanding-window recursive out-of-sample prediction from 1994 to 2025 (Theil's U = 0.45). The P-ARDL model as a free-running simulation, tracks observed GSAT more closely than the calibrated FaIR ensemble (RMSE = 0.055 against 0.094 °C). The estimated power law exponent for AN was 1.49 (bootstrap 95 % CI 1.04–2.02) compared with 1.89 for total ERF. The reduction in the power law exponent is attributable to the volcanic forcing that is in NAT. Nevertheless, after controlling for volcanic effects, the AN exponent remained >1 ($p = 0.017$). With the eruption years (1982–1984 and 1991–1994) dropped or interpolated instead, the exponent on total ERF is 1.46–1.53.

4. **Perfect model test:** In the climate literature a perfect-model test treats one model's simulation as the "true" climate: a new model is trained on the "true" model's historical output, used to project the "true" model's future, and the new model is scored against the "true" model's future, which is known (Abramowitz et al., 2019, https://esd.copernicus.org/articles/10/91/2019/). AOGCM output is beyond the resources of this study, so the test was carried out with the calibrated FaIR ensemble (841 members, fair-calibrate v1.4.1) as the "truth".

   Each FaIR member was driven by the IGCC 2025 ERF history to 2025. From 2026 to 2050, natural ERF was held at its 2015–2024 mean and anthropogenic ERF was increased linearly so that total ERF reaches 5.15 W m^{-2} in 2050, the pathway used for the projections in the manuscript. Pseudo-observations for 1977–2025 were formed from each member's GSAT plus the observed ENSO effect (0.112 °C per unit ONI) and white noise with the standard deviation of the observed P-ARDL residuals (0.057 °C). The procedure used for the observations was then applied without change: ν was estimated from the static power law on AN_{t-1}, the P-ARDL was fitted with ν imposed, and GSAT was projected from 2025 to 2050 with ONI set to zero. Applied to the observations, the same code reproduces the published coefficients and gives 2.48 °C in 2050 (66 % interval 2.25–2.74 °C).

   The exponent estimated from FaIR's historical output, 1.47 (5–95 %: 1.28–1.65), is almost the same as the power-law exponent estimated from observations, 1.49. The 2050 P-ARDL projections, however, exceed FaIR's 2050 GSAT in every member, by a median of 0.43 °C (5–95 %: 0.18–0.72 °C). FaIR's 2050 value lies within the P-ARDL 66 % interval in 22 % of members (Fig. R2, Table R1). With the AN_{t-1} exponent set to one, the P-ARDL reproduces FaIR's 2050 GSAT to within −0.06 °C (5–95 %: −0.19 to 0.03 °C). In

FaIR the curvature of the historical GSAT–ERF relationship arises from the ocean's accumulated response to past forcing, mainly the volcanic forcing, while its response to future forcing is close to a straight line. A power law, such as the P-ARDL model, fitted to that history therefore overstates FaIR's "true" future warming. The historical record alone cannot distinguish curvature produced by ocean memory from curvature produced by a response that depends on the level of anthropogenic forcing.

The perfect model test is conditional on FaIR being the truth, and that is not known for a climate in which ERF is rising rapidly, mostly from anthropogenic forcing, and in which climate feedbacks may become non-linear. FaIR is a constant-feedback model: its forcing-to-temperature equations are linear and its calibration is constrained by the historical record. A perfect-model test shows whether a new model would work if the "perfect" model were the true climate. It cannot show whether the "perfect" model is the true climate, and it is optimistic when the true model and the new model share the same structural assumptions (Abramowitz et al., 2019). The referee notes that feedbacks are expected to become non-linear at high temperatures, and current warming already lies within the lower end of some tipping-point uncertainty ranges (McKay et al., 2022, https://doi.org/10.1126/science.abn7950). If the GSAT response depends on the level of anthropogenic forcing, the FaIR result does not apply. The observed record also differs from a typical FaIR member: the estimated persistence coefficient is 0.20 against a FaIR median of 0.31, GSAT rose 0.94 °C over 1977–2025 against 1.12 °C in the FaIR median, and the gap between the P-ARDL and FaIR projections from the observations (0.06 °C) is smaller than in more than 99 % of FaIR members. The test therefore defines what the P-ARDL would project if FaIR were correct, about 0.4 °C too high, and observations over the coming decade will show which model performs best.

Table R1. Perfect-model test with FaIR as the truth: 2050 GSAT at 5.15 W m^{-2}, 841 members. Values are medians with 5–95 % ranges across members.

Quantity	Value
FaIR 2050 GSAT (truth)	2.40 [1.75, 3.26] °C
ν estimated from FaIR 1977–2025 output	1.47 [1.28, 1.65]
P-ARDL projection of 2050 GSAT	2.85 [2.01, 3.86] °C
P-ARDL error (projection minus truth)	+0.43 [+0.18, +0.72] °C
Truth within the P-ARDL 66 % interval	22 % of members
P-ARDL error with ν = 1	−0.06 [−0.19, +0.03] °C

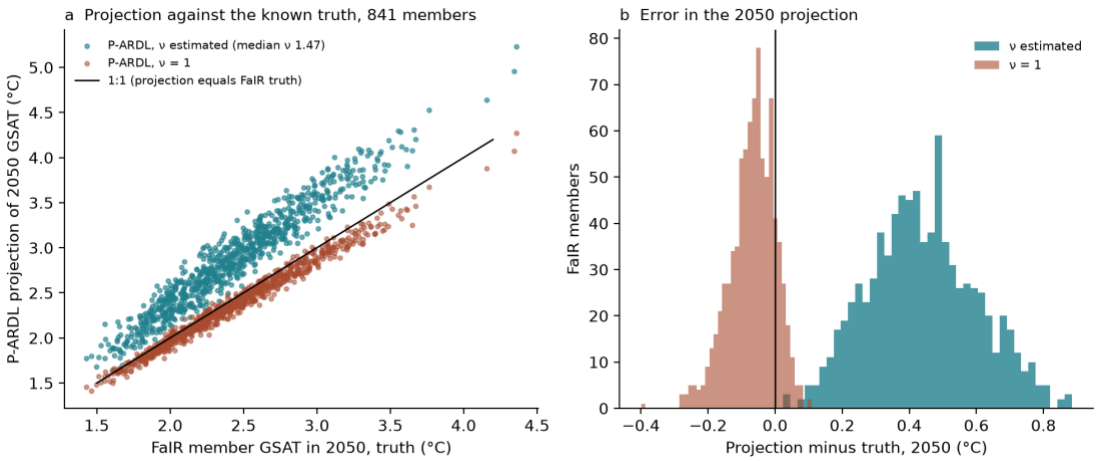


Figure R2. Perfect-model test with FaIR as the truth. (a) P-ARDL projection of 2050 GSAT against each FaIR member's own 2050 GSAT, with ν estimated from the member's 1977–2025 output (teal) and with ν = 1 (rust); the black line is 1:1. (b) Distribution of the projection error across the 841 members. Total ERF reaches 5.15 W m^{-2} in 2050; ONI is set to zero in the projection.

5. **Other comments:**

1) The manuscript makes an empirical claim about the GSAT-ERF relationship. It does not claim to have identified a new climate regime or characterize climate feedback. Language that may imply either will be removed.

2) The constant in the power law analysis reflects that the climate system was not in equilibrium in 1977 and that GSAT (relative to 1850–1900) and ERF (relative to 1750) have different baselines. The same power-law fit to FaIR output also requires a constant (0.26 °C).

3) The reviewer suggests that the curvature may come from other degrees of freedom acting through unforced variability. Unforced climate variability was included in the ARDL model through the inclusion of the ONI variable, which is derived from Niño 3.4 sea-surface temperature data.

4) I do not have access to AOGCM models. An AOGCM test would be of interest but is beyond the resources of this study.

A few additional comments may be helpful to climatologists unfamiliar with the methods used. First, the model captures the net effect of climate feedback jointly in the coefficients and exponents estimated by the regression. Second, the model holds the effect of the deep ocean constant so it cannot be used to estimate equilibrium climate sensitivity. Third, the model is based on 49 years of data and can be used for predictions for about 25 years to mid-century, assuming the climate dynamics that have been operative over the last 49 years continue to be operative in the future and there are no new perturbations that alter underlying climate dynamics. Fourth, the predicted change in GSAT can take up to three years to fully

develop; about 80% occurs in the first year, about 96% by the second year, and >99% by the third year. Fifth, the ARDL model includes the ENSO effect, which results in better performance for the ARDL model compared to FaIR over the last 49 years.