

Comments from CC2 (Saeed Hariri, 11 Jul 2026):

<https://doi.org/10.5194/egusphere-2026-512-CC2>

Recommendation after author response: Minor revisions

Overall assessment

I thank the authors for their constructive and detailed responses. Taken together with the Supplement and the Zenodo repository, the replies address the substance of my main concerns: the topography development is shown to be present in the code, the numerical configuration has been specified, and the authors confirm that quantitative wave diagnostics (energy propagation, group velocity, wave-wave interactions) already exist for every case. I am satisfied that the underlying model and experiments are sound and that no new simulations are required.

However, most of this information currently lives in the responses, the Supplement, or the repository rather than in the manuscript, and a few claims and inconsistencies still need to be tightened. The gap between “the authors can answer these questions” and “the paper answers these questions” is real for a model-description paper, and closing it is more than a cosmetic pass. I therefore recommend moderate (minor) revisions: the changes are well defined and achievable in one round, but they are substantive enough that I would like to see the revised manuscript before it proceeds.

Author response:

Dear Reviewer,

We appreciate the time and effort you have dedicated to providing feedback on our manuscript, and we are grateful for your insightful comments and valuable suggestions for improving our paper.

The revised manuscript is attached at the end of this file with changes highlighted.

Point-by-point evaluation of the major comments

MC1 — Model-development focus / distinguishing inherited vs. new equations

Substance addressed; the paper must now carry it.

The topography explanation and the concrete code reference (aeolus2main.py, variable Dh10, the $\langle \nabla p_i \rangle$ formulation on lines 125–129) resolve where the new development lives, and the contrast with Rostami et al. (2023) is a clear statement of novelty. In the revision I ask the authors to add a short paragraph or compact table separating (1) inherited mcTRSW equations, (2) established Aeolus 2.0 developments, and (3) additions specific to this paper, and to state briefly how ERA5 fields are projected onto the two-layer (h_i , b_i , v_i) variables.

Because a central result is the inertia-gravity-wave burst, please also state explicitly whether the imposed α -Gaussian anomaly is introduced in balance or as an intentionally unbalanced perturbation, so the burst is correctly understood as the adjustment of that anomaly. I note and credit that all fields are shown relative to a reference run, which cancels the background spin-up shock; please make that point explicit in the text, as it underpins the interpretation of the difference fields.

Author response:

The comments were addressed via corresponding updates in the revised manuscript. See highlighted insertions in the Methodology section as follows:

“Rostami et al. (2023) introduced the Aeolus framework using conceptual experiments in an atmosphere initially at rest, omitting background flow and topography while applying a similar circular buoyancy disturbance. In contrast, the present work includes these essential features to evaluate how bottom topography interacts with background zonal flow and vertical wind shear to distort evolving pressure systems. Bottom topography effects enter the governing **Equations (1a-b, 3a-b)** through the pressure gradient term ($\langle \nabla p_i \rangle$), computed for the two-layer formulation using the topographic height field (h_b). Furthermore, beyond the core dynamical **Equations (1b-h)**, the thermodynamic parameterizations for condensed liquid water ($\mathcal{C}$) and downward convective flux ($\mathcal{D}$) have been updated to account for topographic forcing. It is worth noting that the model uses realistic bottom topography, nonzero over continents and mountainous regions throughout the simulation domain.”

“The inherited mcTRSW equations from previous studies using Aeolus 2.0 (Rostami et al., 2023, 2025a,b) correspond to the versions of **Equations (1a-h, 3a-b)** without the pressure gradient terms ($\langle \nabla p_i \rangle$) accounting for bottom topography height (h_b) or the moisture-related and thermodynamic terms ($\mathcal{C}$, $\mathcal{D}$, γ , γ^* , $\mathbb{F}_i$) in their current implementation; the current forms of **Equations (3c-f, 4a-b, 5)** were kept the same, with changes only in the numerical implementation for the inclusion of the new background flow.”

MC2 — Numerical configuration

Answered in the response; must be moved into the manuscript, with inconsistencies reconciled.

The reply provides grid (384×768 , $\sim 0.5^\circ$), 1-minute timestep, 4 outputs/day, ~ 102 -day runs, per-case cost (~ 3 h, 20 GB), Dedalus v2.2207.3 under Python 3.10.14, and library versions. This answers reproducibility and should become a dedicated numerical-implementation subsection. Please add the dissipation/dealiasing scheme and the stability basis for the 1-minute step, which remain unstated.

Two internal inconsistencies to fix: “102 numerical-days ($4 \times$ per day = 408 time-steps)” conflates output frames (408) with integration steps ($\sim 1.5 \times 10^5$ at $\Delta t = 1$ min); and the run is described as “16 cores / 16 GB RAM” while the machine is given as a 24-core / 32 GB system. Please report one consistent configuration.

Author response:

The comments were addressed via corresponding updates in the revised manuscript. See insertions in **Appendix A**. The following paragraph was added to the Methodology section:

“The numerical implementation of the Aeolus 2.0 model, including the spectral discretisation, dealiasing strategy, time-stepping scheme, and hardware configuration, is detailed in **Appendix A**. A comprehensive list of prognostic variables, diagnostic variables, physical constants, model parameters, and reference fields, along with their dimensional scales and units, is provided in **Appendix B**.”

Clarification required.

Figure 1 states the solver uses the spin-weighted spherical harmonics method, whereas the MC2 response describes a Cartesian domain processed with Fourier series, and Eq. (3)/line ~158 imposes a no-normal-flux boundary condition (implying a bounded domain, not a periodic sphere). Please state unambiguously the coordinate system and spectral basis, the latitude-grid type, and the meridional/pole treatment. This bears directly on the β -gyre, Rossby-wave, and equatorward IG-propagation claims, so it should be settled explicitly rather than left to the figure schematic.

Author response:

The comments were addressed via corresponding updates in the revised manuscript. See insertions in **Appendix A**.

The apparent inconsistency stems from a misleading phrase in our **Supplementary Material** regarding the computational geometry. We apologize for this ambiguity and provide the following clarification.

The Aeolus 2.0 model employs spin-weighted spherical harmonics as its spectral basis on the full sphere, consistent with the schematic in Figure 1 and the description in the methodology section. The spectral transform is implemented within the Dedalus framework (Burns et al., 2020; Lecoanet et al., 2019; Vasil et al., 2019), which uses a tensor product of Fourier bases as a computational convenience for performing spherical harmonic transforms. Specifically, a Fourier basis in longitude (interval 0 to 2π) and a Fourier basis in colatitude (interval 0 to π) together enable efficient spin-weighted spherical harmonic transforms (see `sphere_wrapper.py` and `sphere128.py` in <https://zenodo.org/records/18065000> > Aeolus2.0_model).

The latitude grid employs Gauss-Legendre quadrature points in the meridional direction, providing optimal accuracy for spherical harmonic transforms (see `jacobi128.py` in <https://zenodo.org/records/18065000> > Aeolus2.0_model; Boyd, 2001). The poles are treated naturally as regular points in the spectral domain through the mathematical properties of spin-weighted spherical harmonics (Ripa, 1993), with no special boundary conditions required at the poles.

The no-normal-flux boundary conditions appearing in Equation (3) are expressed in flux-conservative form in the continuous equations and are satisfied naturally by the spectral method on the full sphere. This formulation ensures the conservation of total mechanical energy as described by Equation (4a) in the manuscript (Ripa, 1993; see `equations_aeolus2main.py` in <https://zenodo.org/records/18065000> > Aeolus2.0_model).

We added **Appendix A** accordingly to remove the misleading "Cartesian domain" phrasing and to explicitly state that the model uses a spin-weighted spherical harmonic spectral transform method on the full sphere with a regular latitude-longitude grid using Gauss-Legendre quadrature in the meridional direction.

This clarification is relevant for the interpretation of our dynamical results, as the β -gyre formation, Rossby wave propagation, and equatorward inertia-gravity wave radiation are all phenomena that depend inherently on the spherical geometry and the full Coriolis force treatment (as implemented in `equations_aeolus2main.py` and `sphere128.py` in <https://zenodo.org/records/18065000> > Aeolus2.0_model). The spherical harmonic spectral method accurately captures the latitudinal variation of the Coriolis parameter and the associated wave dynamics without the artifacts that would arise from a Cartesian plane approximation.

MC3 — Validation

Expected addition, not optional.

The authors confirm that quantitative wave diagnostics exist for all cases in the repository.

For a model paper, at least the key ones should appear in the manuscript. Concretely, I ask the revision to include:

- (i) energy conservation for the dry adiabatic runs (a direct verification the authors already have);
- (ii) a numerical value for the external/barotropic L_d at 20°N to support the “~32° aligns with the deformation radius” statement, which is currently asserted; and
- (iii) one or two of the already-computed wave-propagation diagnostics (e.g., group velocity or phase speed) compared against shallow-water expectations.

None of this requires new runs, but it is what moves the validation from qualitative to quantitative, and it is the main reason I am recommending moderate rather than minor revisions.

Author response:

The comments were addressed via corresponding updates in the revised manuscript.

For changes regarding item (i) see insertions in **Appendix A**. The following text was added:

“As a verification of the numerical solver, the total mechanical energy (Hamiltonian, **Equation (4a)**) in dry adiabatic integrations was monitored. The total energy remained conserved to within machine precision (relative fluctuations $< 10^{-6}$) over the simulation period. This confirms that the discretization, spectral transforms, and time-stepping scheme (SBDF2) faithfully preserve the adiabatic invariants of the governing equations. The moist-convective (MC) experiments employ the identical numerical core, with only the addition of physically derived source/sink terms for latent heat release ($\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}_s$, and γ). Therefore, this verification ensures that any energy conversions observed in the moist runs are physically meaningful and not numerical artifacts. This validation thereby underpins the interpretation of energy budgets across all experimental configurations (barotropic/baroclinic, weak/strong, dry/MC) presented in this study.”

For changes regarding item (ii) see insertions in **Appendix B, Table B3**. The term L_{d20N} was inserted.

For changes regarding item (iii) see insertions in Results section. **Figures 11** and **12** were added to the manuscript moved from the Supplementary Material, and the following text was added:

“Tracking of the divergence envelope yields zonal and meridional propagation speeds (c) of approximately 52 m/s (**Fig. 11** and **12**). These values correspond to an equivalent depth of about 275 m ($H_{equiv_depth} = c^2/g$), consistent with the first baroclinic mode of the two-layer system ($H_1 = 6000$ m, $H_2 = 4000$ m) and the expected phase speed of the non-dispersive Kelvin wave. The slower envelope propagation provides Rossby wave group velocities of approximately 7 m/s in both zonal and meridional directions, in agreement with classical beta-plane theory at 20°N. These diagnostics reinforce that the Aeolus 2.0 dynamics are consistent with shallow-water theory.”

MC4 — “Heatwave” terminology and framing

Wording change expected.

Please soften the title and abstract framing toward idealized buoyancy-anomaly dynamics relevant to subtropical blocking and heatwave-like circulation responses, given that land–atmosphere coupling, radiation, and boundary-layer processes lie outside the model’s scope. This does not affect the results.

Author response:

The comments were addressed via corresponding updates in the revised manuscript. The title was adapted to:

“Dynamics of Subtropical Buoyancy Anomalies and Heatwave-Like Circulation Responses in the Intermediate-Complexity Atmospheric Model Aeolus 2.0”

The changes made in the abstract are highlighted in the draft.

MC5 — Qualitative interpretation

Addressed; light tightening expected.

The pointer to divergence and wind diagnostics is noted. Please scale terms implying resolved fine-scale structure (“mesoscale convective systems,” “frontal boundaries,” “rainfall organization”) to what a two-layer $\sim 0.5^\circ$ model actually represents, or tie them explicitly to the supporting diagnostics.

Author response:

The comments were addressed via corresponding updates in the revised manuscript. The changes made are highlighted in the draft.

MC6 — Mathematical notation

Copy-edit; confirm one specific fix.

Please confirm the intended form of Eq. (4b), which as printed lists $\delta\mathcal{H}/\delta v_k$ twice with different right-hand sides.

Author response:

The comments were addressed via corresponding update in the revised manuscript. The intended equation had a typo, the corrected form is as follows:

$$\frac{\delta\mathcal{H}}{\delta v_k} = h_k v_k, \quad \frac{\delta\mathcal{H}}{\delta h_k} = \zeta_k, \quad \frac{\delta\mathcal{H}}{\delta \tilde{v}_k} = h_k \tilde{h}_k$$

MC7 — Figures

Copy-edit.

The caption, colorbar, and vector-scaling improvements described in the response are sufficient; I will confirm in the revision.

Author response:

The comments were addressed via corresponding updates in the revised manuscript. **Figures 2-5** were updated with wind velocity key and time-step numbers. The scales were added to the tables in **Appendix B**.

Two points to clarify on the novelty claims

These are clarifications, not new work.

First, since the experiments are centered over the open North Atlantic (20°N, 40°W), please state whether the bottom-topography height h_b is nonzero anywhere in the shown runs; if it is effectively zero, the topography capability should be presented as a documented model feature rather than a demonstrated result of this paper.

Second, the background-flow novelty is conveyed most directly by indicating where its effect is isolated; if a background-on/background-off contrast already exists among the runs, a brief mention would substantiate the claim.

Author response:

The comments were addressed via corresponding updates in the revised manuscript.

On the first point: the model use realistic bottom topography, and h_b is indeed nonzero over continents and mountainous regions throughout the simulation domain. The buoyancy anomaly itself is centered over the North Atlantic, where h_b is locally small, but the model's topography is active everywhere else in the domain and enters the pressure-gradient terms in Equations. The global circulation that develops in response to the anomaly therefore evolves in the presence of realistic topography, and the topographic forcing terms in the thermodynamic parameterizations are active throughout the integration. We have added a sentence in the Methodology to clarify this point, so the reader understands that topography is not a passive capability but an active component of the shown runs.

On the second point: the background-flow novelty is isolated by direct comparison with Rostami et al. (2023), who performed conceptually similar buoyancy-anomaly experiments in an atmosphere initially at rest, without background wind or potential-temperature gradients. The present runs all use ERA5-derived background fields, so the contrast between this study and that earlier work is in a general perspective the background-on versus background-off comparison.

Specific comments

The remaining specific comments (Abstract and Introduction framing, Section 2.1 tables of prognostic variables/parameters/scales, the symbol-parameter table, Section 2.2 / Eq. 5 typesetting and definition of the α -Gaussian, softening “validates the approach” in the Conclusions, and a language pass) are well defined and can be handled in this revision. The Section 2.1 tables and the symbol table in particular would substantially improve readability.

Author response:

The comments were addressed via corresponding updates in the revised manuscript.

Summary

The science and the model are sound, and the author responses resolve the questions of substance; what remains is to bring that substance into the manuscript and to tighten a few claims. For the revision I ask the authors to:

1. move the numerical-configuration and topography details into the paper, and reconcile the inconsistencies (geometry statement; steps vs. output frames; cores/RAM);
2. add the quantitative validation elements above — dry-case energy conservation, a numerical L_d value, and one or two of the already-computed wave diagnostics;
3. clarify whether topography and background flow are active/isolated in the shown experiments;
4. state whether the imposed anomaly is balanced, and make the reference-run differencing explicit;
5. soften the “heatwave”/“validates the approach” framing;
6. correct Eq. (4b), improve the figure captions, and complete a language pass.

These are achievable in a single round, and I look forward to reviewing the revised manuscript.

Author response:

We are grateful to the reviewer for the constructive feedback. All comments have been carefully considered, and the manuscript has been revised throughout in response. In particular, we have clarified the novelty claims, the physical interpretation, and the energy conservation, and have included information on wave speeds. We believe these revisions have strengthened the paper.

Sincerely,

Sullyandro Oliveira Guimarães
Corresponding Author
Doctoral Researcher
Potsdam Institute for Climate Impact Research (PIK)
Research Domain I - Earth System Analysis
P.O. Box 60 12 03, 14412 Potsdam
E-mail: sullyandro@pik-potsdam.de

Subtropical Extreme Heatwave Dynamics in the Intermediate-Complexity Atmospheric Model Aeolus 2.0

Dynamics of Subtropical Buoyancy Anomalies and Heatwave-Like Circulation Responses in the Intermediate-Complexity Atmospheric Model Aeolus 2.0

Sullyandro O. Guimarães^{1,2}, Masoud Rostami^{1,3}, Stefan Petri¹

¹Potsdam Institute for Climate Impact Research (PIK), Member of the Leibniz Association, Potsdam, Germany

²University of Potsdam, Potsdam, Germany

³Laboratoire de Météorologie Dynamique (LMD), Sorbonne University (SU), Ecole Normale Supérieure (ENS), Paris, France

Correspondence to: Sullyandro O. Guimarães (sullyandro@pik-potsdam.de)

Abstract. Heatwaves Localized buoyancy anomalies in the subtropical and mid-latitude regions, which arise from localized heating can ripple through the atmosphere and persist for weeks, yet the mechanisms behind this chain of events remain difficult to disentangle. Here we use the Aeolus 2.0 model, which is a moist-convective thermal rotating shallow-water (mcTRSW) framework of intermediate complexity, to explore how idealized buoyancy anomalies evolve under both dry and moist conditions, innovatively including background flow effects within an idealized setting to enhance realism. We find that localized buoyancy forcing heating sets off a rapid atmospheric adjustment: air converges near the surface, diverges aloft, and quickly organizes into paired cyclonic (lower layer) and anticyclonic circulations (upper layer). Earth's rotation then distorts these structures, producing asymmetries, spiral moisture bands rainbands, and band-driven feedbacks in the model's bulk moisture fields rainband-driven feedbacks. Moist convection greatly amplifies this response by releasing latent heat, which fuels sustained instability and rainfall precipitation-like organization in the column-integrated moisture diagnostics. Inertia-gravity waves emerge as a key pathway for redistributing heat and momentum, while Rossby waves and beta gyres gradually reshape anomalies, tilting them poleward and breaking them into smaller vortices. The simulations also reproduce well-known structural observational signatures, including comma-shaped water vapor patterns and mesoscale vortices structures. Together, these results show how a simple localized buoyancy heat source can trigger a cascade of atmospheric responses that link convection, wave dynamics, and large-scale circulation. By capturing these processes, Aeolus 2.0 provides a bridge between theoretical frameworks and full climate models, offering new insight into idealized buoyancy-anomaly dynamics relevant to subtropical blocking and heatwave-like circulation responses. the dynamics that sustain extreme heatwaves in a warming world.

Keywords: Moist-convective Thermal rotating shallow water (mcTRSW) model, Heatwaves, Aeolus 2.0, Extreme forcings.

1 Introduction

Heatwaves are occurring more often and with greater intensity around the globe, particularly impacting subtropical and mid-latitude regions. Recent instances of such heatwaves have been recorded in North America, Europe, and Asia (Gao et al., 2023, Wang et al., 2025). Their origins are based in different physical causes, as those formed from the atmosphere, like moisture and radiation, and other from the surface drivers. Sometimes, these causes can mix and create complex effects, a combination of atmospheric dynamics and land-surface interactions, with feedback mechanisms that can amplify their intensity and duration (Petoukhov et al., 2013; Guimarães et al., 2024; Miralles et al., 2019; Zeppetello et al., 2022; Hao et al., 2022, Barriopedro et al., 2023).

Blocking circulation patterns are often mentioned as the phenomena responsible for the association of persistent atmospheric high-pressure anomalies and linked heatwaves (Schaller et al., 2018; Kautz et al., 2022; Nabizadeh et al., 2021; Rouges et al., 2023). In this study, the term heatwaves is used in a connotation to describe the phenomenon involving the gradual formation and sustained presence of large-scale, localized regions of positive buoyancy. These regions develop due to elevated potential temperatures over several days and are analyzed within a framework influenced by the background zonal flow. By using an atmospheric model of intermediate complexity, we are able to isolate and better understand the key physical processes that drive the emergence and evolution of these anomalies, as will be discussed in the model description session. A defining feature of these buoyant structures is their persistence - they are capable of remaining active and hold over multiple consecutive days, highlighting their significance in the broader context of extreme weather patterns.

Limited studies have provided detailed simulations or conceptual models that capture these atmospheric blocking structures effectively, commonly identified by the appearance of ridge-like high-pressure systems within upper-tropospheric geopotential-height contours (Sousa et al., 2021; Schemm and Sprenger, 2019; Kautz et al., 2021). In response to this gap, the present work investigates whether extreme surface heating can contribute to the development of atmospheric ridges in the upper troposphere, building on theoretical understanding of the atmospheric response to thermal forcing and on recent modeling and observational studies that link strong diabatic or surface heating with the amplification of upper-level ridges (Hoskins and Karoly, 1981; Zhang and Wang, 2018; Rostami et al., 2023; Liu et al., 2025).

From our findings, upper-tropospheric ridges can emerge in tandem with localized surface warming events. These simultaneous developments are linked to surface-induced heating, which gives rise to a thermal low-pressure zone and associated cyclonic circulation near the surface, acting in contrast to the anticyclonic conditions in the upper layer. Modelled and observational evidence support the coexistence of these lower and upper-atmospheric features (Pfahl and Wernli, 2012; Rostami et al., 2023). It is worth emphasizing that a comprehensive theoretical framework capable of fully explaining all facets of heatwaves has yet to be established. Rather than replicating real-world heatwave events, this research aims to explore the

fundamental evolution of buoyancy-driven anomalies under the influence of the innovative background wind velocity effects, baroclinicity, moist convection, and the full Coriolis force, in order to capture their essential dynamics.

65 In simplified atmospheric conditions that exclude moisture processes, geostrophic balance tends to form through the generation of horizontal temperature contrasts following localized heating. Such "dry" environments are governed by adiabatic dynamics, where the absence of phase transitions ensures total energy conservation, as described by [Holton and Hakim \(2013\)](#). In contrast, real atmospheric environments are shaped by more complex, non-adiabatic processes that depend heavily on vertical layering, atmospheric stability, wind shear, and, critically, the influence of moist convection. Moist convection significantly alters the adjustment dynamics in subtropical and mid latitude systems, exerting a strong influence on both short-term weather developments and broader climate behavior.

Gaining insight into the behavior of these buoyancy anomalies is essential for improving our ability to anticipate atmospheric responses under a range of weather and climate scenarios. The moist-convective Thermal Rotating Shallow Water (mcTRSW) model serves as a valuable tool in this effort, offering a simplified framework to investigate how temperature variations, moisture content, and large-scale atmospheric circulation interact in shaping these dynamic processes ([Kurganov et al., 2020](#); [Rostami et al., 2022](#); [2023](#)). In this study, we use the stand-alone version of the atmospheric model Aeolus 2.0 ([Rostami et al., 2024a](#); [2025](#)) to investigate the dynamics of localized extreme heatwaves in the subtropical atmosphere under both dry and moist-convective scenarios. See [Guimarães et al. \(2025\)](#) for the Aeolus 2.0 code and data specific to this study.

2 Methodology

80 2.1 Model Description

To better understand how buoyancy anomalies influence atmospheric behavior -particularly during extreme heat events - we rely on a modeling framework that combines conceptual clarity with physical robustness. The historical foundation of our approach lies in the moist-convective Rotating Shallow Water (mcRSW) model, which offers a robust theoretical basis for simulating moist processes in the atmosphere. This model incorporates key features such as phase transitions of water vapor and the associated latent heat release, following the foundational principles established by [Gill \(1982\)](#) and further developed by [Bouchut et al. \(2009\)](#) and [Lambaerts et al. \(2011ab; 2012\)](#). The mcRSW framework represents the evolution of column-integrated humidity and the impact of condensation in a consistent and simplified manner, making it well-suited for idealized studies. Over time, the model has been refined to include additional moist-convective processes such as vaporization, precipitation, and precipitable water ([Rostami and Zeitlin, 2018](#)), and it has been applied successfully in numerous studies of large-scale atmospheric jets and vortices on Earth and other planets. Building on this foundation, we apply the mcTRSW model Aeolus 2.0 ([Rostami et al., 2024a](#); [2025a](#)) in our experiments. This tool developed at the Potsdam Institute for Climate Impact

Research (PIK) allows us to explore the dynamics of subtropical and mid-latitude heatwaves under both dry and moist-convective conditions, offering new insights into how temperature, moisture, and circulation interact to shape extreme events.

95 The dynamical core of Aeolus 2.0 is constructed upon a multi-layer formulation of the mcTRSW model. The underlying principles and mathematical formulation of the multi-layer TRSW system, including its capacity to represent vertical structure and thermodynamic interactions, are thoroughly presented in [Rostami et al. \(2025a; 2025b\)](#). Unlike conventional shallow water models that are typically based on the assumptions of layer homogeneity, incompressibility, and hydrostatic equilibrium, the TRSW framework relaxes these constraints by allowing for horizontal variability in thermodynamic and material properties across stratified layers. This enhanced representation is achieved through a vertical integration of the atmospheric primitive
100 equations, formulated within a pseudo-height isobaric coordinate system. Such a configuration enables the model to more accurately capture the vertical structure and thermodynamic gradients of the atmosphere (see **Fig. 1**), thereby extending its applicability to a broader range of dynamical and thermodynamical regimes.

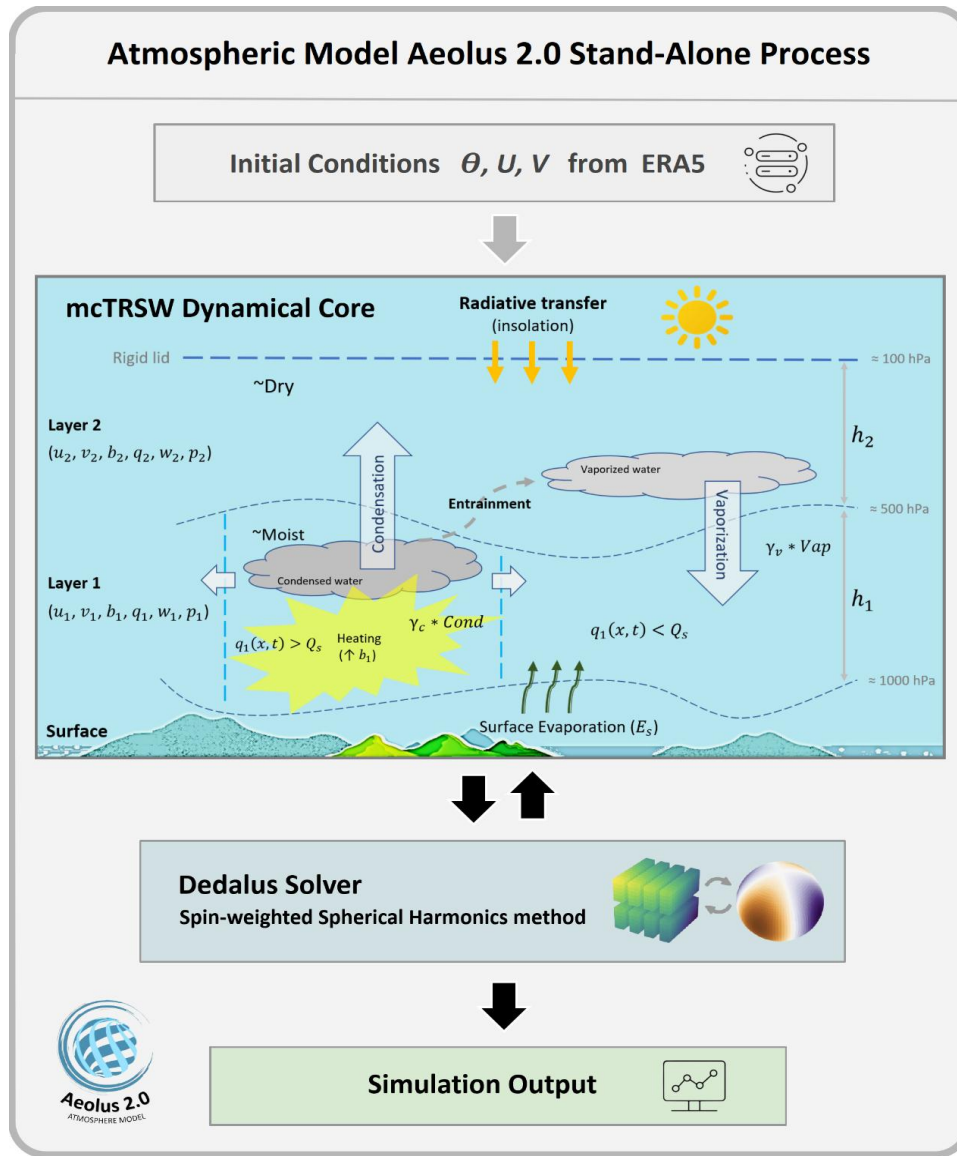


Figure 1: Schematic illustration of the Aeolus 2.0 model stand-alone process. Elements in here were adapted from Rostami et al. (2024a) and Dedalus Project (dedalus-project.org, Burns et al., 2020). Where: u and v (wind velocity); b (buoyancy); q (water vapor); w (precipitable water); p (pressure); and h (layer thickness); Q_s (saturation vapor pressure); Vap (vaporization); $Cond$ (condensation); $\gamma_c * Cond$ and $\gamma_v * Vap$ (Entrainment terms); and θ (potential temperature).

The model offers a robust and coherent representation of horizontal potential temperature gradients and their temporal evolution under the influence of moist convection. This is achieved through a formulation that remains both physically consistent and computationally transparent. A key strength of the mcTRSW framework lies in its ability to fully capture the nonlinear characteristics of moist convection, integrating essential processes such as latent heat release, phase transitions of

water vapor, and the dynamic modulation of buoyancy. These capabilities enable the model to explore the underlying mechanisms of extreme weather phenomena, including surface heat fluxes and the central role of moist convective activity.

The two-layer version of the Aeolus 2.0 mcTRSW model is built upon a set of fundamental equations that describe its behavior and dynamics. These governing equations, thoroughly explained in [Rostami et al. \(2023; 2024a; 2025a; 2025b\)](#) and [Cao et al. \(2025\)](#), form the foundation of the model and are summarized below to provide a clear understanding of the system's core structure and functionality. Additional flux-form equations implemented in Aeolus 2.0, following [Cao et al. \(2025\)](#), are provided in the **Supplementary Material**.

Nondimensionalization is applied to simplify the system dynamics, with the barotropic equatorial Rossby deformation radius (L_d) serving as the characteristic horizontal length scale (L). Here, $L_d = (\sqrt{gH_0}/\beta)^{1/2}$, $H_0 = H_1 + H_2$ represents the total unperturbed layer depth, H_i is the resting-state values for thickness, g is the gravitational acceleration, and β is the meridional gradient of the Coriolis parameter. The scaling relations for velocity components (u, v) and time (t) are given by: $L \sim L_d$, $(u, v) \sim \beta L_d^2$, and $t \sim 1/(\beta L_d)$, as enlightened in [Rostami et al. \(2025a\)](#).

$$(\partial_t + \mathbf{v}_1 \cdot \nabla) \mathbf{v}_1 + f \hat{\mathbf{z}} \times \mathbf{v}_1 = -\langle \nabla p_1 \rangle \quad (1a)$$

$$(\partial_t + \mathbf{v}_2 \cdot \nabla) \mathbf{v}_2 + f \hat{\mathbf{z}} \times \mathbf{v}_2 = -\langle \nabla p_2 \rangle - \frac{(1 - \gamma)}{b_2 h_2} (\mathbf{v}_2 - \mathbf{v}_1)(\mathcal{C} - \mathcal{D}) \quad (1b)$$

$$\partial_t h_1 + \nabla \cdot (h_1 \mathbf{v}_1) = \frac{1}{b_1} [(1 - \gamma)(-\mathcal{C} + \mathcal{D}) - (1 - \gamma^*) \mathbb{F}_1] \quad (1c)$$

$$\partial_t h_2 + \nabla \cdot (h_2 \mathbf{v}_2) = \frac{1}{b_2} [(1 - \gamma)(+\mathcal{C} - \mathcal{D}) - (1 - \gamma^*) \mathbb{F}_2] \quad (1d)$$

$$\partial_t b_1 + \mathbf{v}_1 \cdot \nabla b_1 = \frac{1}{h_1} [(+\mathcal{C} - \mu \mathcal{E}_s) + \mathbb{F}_2] \quad (1e)$$

$$\partial_t b_2 + \mathbf{v}_2 \cdot \nabla b_2 = \frac{1}{h_2} [(-\mathcal{C} + \mathcal{D}) + \mathbb{F}_2] \quad (1f)$$

$$\partial_t q_1 + \nabla \cdot (q_1 \mathbf{v}_1) = -\mathcal{C} + \mathcal{E}_s \quad (1g)$$

$$\partial_t q_2 + \nabla \cdot (q_2 \mathbf{v}_2) = +\mathcal{C} - \mathcal{D} \quad (1h)$$

The model applies a Lagrangian (or material) derivative to describe the time evolution of atmospheric quantities within each layer. This derivative is expressed as $\partial/\partial t + \mathbf{v}_i \cdot \nabla$, where $\mathbf{v}_i = (u_i, v_i)$ denotes the vertically-averaged horizontal velocity vector in each layer i , and ∇ represents the horizontal gradient operator. This formulation allows the model to track changes not only over time but also as air parcels move horizontally, capturing the essential dynamics of advection and temporal evolution in a layered atmospheric system. Each layer's thickness is represented by h_i , with layers numbered sequentially from the surface upward, as presented in **Fig. 1**.

To define the structure of the pressure gradient terms in the governing equations, we specify the relevant velocity and pressure-related quantities for each model layer. In this formulation, $\langle \nabla p_i \rangle = \nabla \tilde{p}_i - \tilde{h}_i \nabla b_i$ represents the vertically-averaged horizontal gradient of the pressure field. For the 2 layers schema, the lower layer gradient is $\langle \nabla p_1 \rangle = (1/2)h_1 \nabla b_1 + b_1 \nabla (h_b + h_1 + h_2)$, and in the upper layer $\langle \nabla p_2 \rangle = \nabla (h_1 b_1) + (1/2)h_2 \nabla b_2 + b_2 \nabla (h_b + h_2)$, where h_b is a bottom topography height, $\tilde{p}_1 = (h_b + h_1 + h_2)b_1$, $\tilde{p}_2 = h_1 b_1 + (h_b + h_2)b_2$, $\tilde{h}_1 = h_b + (1/2)h_1 + h_2$, and $\tilde{h}_2 = h_b + (1/2)h_2$.

To characterize buoyancy within a given layer (b_i), the model uses a vertically averaged formulation $b_i = g \bar{\theta}_i / \theta_s$, where $\bar{\theta}_i$ is the vertically-averaged potential temperature in layer i , and θ_s represents a reference surface-level potential temperature.

The model further includes several moisture-related and thermodynamic parameters. These include $\mathcal{C}$, which accounts for condensed liquid water in the lower layer (also referred as CLWC), playing in upward motion; $\mathcal{D}$, downward convective flux term; $\mathcal{E}_s$, representing surface evaporation through a bulk aerodynamic formula; μ , an amplitude equivalence term between $\mathcal{C}$ and $\mathcal{E}_s$; and γ and γ^* , which quantify the thermodynamic contributions of latent heat from condensation and radiative cooling (Newtonian relaxation), respectively. These parameters can vary with vertical level. In complement, external thermal forcings are incorporated in the term $\mathbf{F}_i$, including insolation and a Newtonian cooling term.

The rotational and directional effects are included through the Coriolis parameter f and a vertical unit vector $\hat{\mathbf{z}}$ to define orientation in the model domain.

It is important to point out that, in **Equations (1a-h)**, we simplify the system by assuming that the buoyancy term (b_i) is much larger than the moisture term (q_i/h_i), where q_i refers to the vertically integrated specific humidity. The model also includes the effects of precipitable water and the vaporization of condensed water. The Condensed Liquid Water Content (CLWC) is parameterized using the Betts–Miller convective adjustment scheme (Betts and Miller, 1986), which provides a simplified yet effective representation of moist processes. The numerical implementation relies on the Dedalus framework (Lecoanet et al., 2019; Vasil et al., 2019), offering a flexible, high-precision platform for solving the partial differential equations governing geophysical fluid dynamics.

In an adiabatic setting, the thermogeostrophic balance can be described using the following **Equations (2a-b)**. A constant geopotential level is assumed at the lower boundary. Under the following scaling, the thickness and buoyancy fields are expressed as: $\mathbf{v}_i \sim U$, $(x, y) \sim L$, $h_i \sim H_0(\delta_i + \lambda \eta_i)$, $b_i \sim B_0(B_i + \lambda b'_i)$, $f \sim f_0$, where U is the characteristic horizontal velocity scale, $\delta_i = H_i/H_0$ is the thickness in nondimensional scale, H_0 is thickness at rest, η_i is the deviation of the δ_i thickness, B_0 is the buoyancy at rest, B_i is the buoyancy resting state, b'_i is the buoyancy perturbation, ϕ is the latitude, λ is a small nondimensional parameter, f_0 denotes the Coriolis parameter at a reference latitude, and $R_0 = \frac{U}{f_0 L} \sim \lambda$ is the Rossby number.

$$R_0 \frac{d\mathbf{v}_1}{dt} + \sin(\phi) \hat{\mathbf{z}} \times \mathbf{v}_1 = -\frac{Bu\lambda}{R_0} \left(\frac{\delta_1}{2} \nabla b'_1 + B_1 \nabla(\eta_1 + \eta_2) \right) \quad (2a)$$

$$R_0 \frac{d\mathbf{v}_2}{dt} + \sin(\phi) \hat{\mathbf{z}} \times \mathbf{v}_2 = -\frac{Bu\lambda}{R_0} \left(\delta_1 \nabla b'_1 + \frac{\delta_2}{2} \nabla b'_2 + B_1 \nabla \eta_1 + B_2 \nabla \eta_2 \right) \quad (2b)$$

The evolution equations of a two-layer TRSW model in a "dry" atmosphere are defined ahead - **Equations (3a-f)**, to examine the dynamics underlying positive buoyancy anomalies and their connection to pressure perturbations. This formulation helps to clarify how gradients in potential temperature are linked to pressure anomalies within a simplified dry environment.

$$(\partial_t + \mathbf{v}_1 \cdot \nabla) \mathbf{v}_1 + f \hat{\mathbf{z}} \times \mathbf{v}_1 = -\langle \nabla p_1 \rangle \quad (3a)$$

$$(\partial_t + \mathbf{v}_2 \cdot \nabla) \mathbf{v}_2 + f \hat{\mathbf{z}} \times \mathbf{v}_2 = -\langle \nabla p_2 \rangle \quad (3b)$$

$$\partial_t h_1 + \nabla \cdot (h_1 \mathbf{v}_1) = 0 \quad (3c)$$

$$\partial_t h_2 + \nabla \cdot (h_2 \mathbf{v}_2) = 0 \quad (3d)$$

$$\partial_t b_1 + \mathbf{v}_1 \cdot \nabla b_1 = 0 \quad (3e)$$

$$\partial_t b_2 + \mathbf{v}_2 \cdot \nabla b_2 = 0 \quad (3f)$$

The **Equations (3a-f)** have to be applied in a defined domain $\mathbb{D}$ (latitudes x longitudes), and the boundary conditions begin with the specification of no normal flux condition $\hat{\mathbf{n}} \cdot h_i \mathbf{v}_i = 0$, with $x \in \partial \mathbb{D}$, and $\hat{\mathbf{n}}$ the outward unit vector.

The total energy of the system described by **Equations (3a-f)**, including both kinetic and gravitational potential components, is represented by the following Hamiltonian in **Equation (4a)**, and its functional derivatives in **(4b)**.

$$\mathcal{H} = \int_{\mathbb{D}} d^2x \left[h_1 \left(\frac{1}{2} \mathbf{v}_1^2 + \tilde{h}_1 b_1 \right) + h_2 \left(\frac{1}{2} \mathbf{v}_2^2 + \tilde{h}_2 b_2 \right) \right] \quad (4a)$$

$$\frac{\delta \mathcal{H}}{\delta \mathbf{v}_k} = h_k \mathbf{v}_k, \quad \frac{\delta \mathcal{H}}{\delta h_k} = \zeta_k, \quad \frac{\delta \mathcal{H}}{\delta \mathbf{b}_k} = h_k \tilde{\mathbf{h}}_k \quad (4b)$$

The concept of Bernoulli energy in each layer is represented by the expression $\zeta_i = \tilde{p}_i + (1/2)\mathbf{v}_i^2$. Here, k represents an elementary column part of the layer i . When calculating the Hamiltonian variation $\delta \mathcal{H} / \delta h_k$, the term $\partial \tilde{h}_k / \partial h_k$ generally does not vanish in the process. The free evolution of the system remains stable due to the conservation of its total mechanical energy, as specified in **Equation (4a)**. Under the Boussinesq approximation, the kinetic energy of an individual fluid column in each layer is given by $d^2 x (1/2) h_i \mathbf{v}_i^2$. The absolute potential energy is then $d^2 x h_i \bar{h}_i b_i$, where $\bar{h}_i$ is the height of each horizontal grid element of the layer (column above $z = 0$). This energy conservation holds for the combined kinetic and potential energy of the 2 layers, assuming a stably stratified configuration where $\theta_2 > \theta_1$ (for more details, see [Ripa, 1993](#)).

2.2 Buoyancy Anomaly Forcing

To investigate the dynamics of localized buoyancy disturbances, this study applies numerical simulations focusing on a large-scale positive buoyancy anomaly situated in the subtropical atmosphere. The analysis is carried out under two contrasting scenarios: one representing a dry (adiabatic environment) and the other incorporating moist-convective (MC) processes.

This experimental setup enables a focused examination of the fundamental mechanisms governing the evolution and dissipation of the buoyancy anomaly. With minimal external forcing, the results provide clearer insight into the core dynamical processes, improving the interpretability and robustness of the findings.

The initial buoyancy anomaly is introduced through a background buoyancy and wind velocity, plus a simplified formulation given in **Equation (5)**, expressed as a circular α -Gaussian distribution. Based on the approach of [Rostami et al. \(2023\)](#), the anomaly is spatially confined in both latitude and longitude, ensuring that its effects can be accurately tracked and analyzed throughout the simulation period. Including background effects in addition to the anomaly forcing is a new development relative to [Rostami et al. \(2023\)](#), enhancing the realism of the experiments.

$$b'_i = \varepsilon H_i \sqrt{2e} \frac{1}{\sigma} \Gamma\left(\frac{1}{\sigma} + \frac{1}{2}\right) G\left(\frac{r^\sigma}{2}, \frac{1}{\sigma} + \frac{1}{2}\right), \quad H_1 = H_2 = 0.5 \quad (5)$$

The initial buoyancy anomaly is defined in non-dimensional form as b'_i (perturbation in layer i). This anomaly is controlled by two main parameters: ε , which sets the maximum amplitude, and σ , which controls its spatial extent or steepness. In our simulations, $\sigma = 4$. The radial distance r from the anomaly center to any given location is calculated using the Haversine formula $\chi(\text{Lat}_0, \text{Lon}_0, \text{Lat}, \text{Lon})$, where the center coordinates are $\text{Lat}_0 = 20^\circ \text{ N}$ and $\text{Lon}_0 = 40^\circ \text{ W}$, over the North Atlantic Ocean basin.

190 To shape the spatial structure of the anomaly pattern, we utilize the function $G(r, \xi) = [1/\Gamma(\xi)] \int_r^\xi e^{-t} t^{\xi-1} dt$, where $\Gamma(x)$ is the Gamma function, and ξ is a dimensionless shape parameter. Together, ε and σ determine both the magnitude and the radial decay of the buoyancy anomaly.

The model distinguishes between barotropic and baroclinic configurations. In the barotropic case, both atmospheric layers share the same buoyancy anomaly, such that $b'_1 = b'_2$, resulting in vertically aligned disturbances with symmetric and horizontally expansive flow patterns. In contrast, the baroclinic configuration assumes a nonzero anomaly only in the lower layer ($b'_1 > 0$, $b'_2 = 0$), leading to vertical asymmetry and the development of cyclonic and anticyclonic structures due to the phase shift between layers. The configuration assumes stable stratification, characterized by $B_1 < B_2$, where B_1 and B_2 correspond to the background buoyancy fields in the state of rest.

To explore the influence of anomaly strength, we introduce two values for ε : 0.1, corresponding to a weak anomaly, and 0.2, representing a strong one. These variations allow us to assess the sensitivity of the atmospheric response under both moderate and intense perturbations.

In simulations involving moist convection, the initial specific humidity is prescribed to ensure strong vertical contrasts in moisture content. The lower layer is initialized close to saturation, while the upper layer remains far from saturation. This setup encourages condensation predominantly in the lower layer. Additionally, the model accounts for vertical entrainment of condensed liquid water content (CLWC) from the lower to the upper layer via convective updrafts (Rostami and Zeitlin, 2022).

To assess the potential dynamical impact of the imposed buoyancy disturbance, it is useful to consider its spatial scale. The anomaly, modeled as a circular α -Gaussian profile, spans about 32° in both the zonal and meridional directions at half its peak intensity. Notably, this scale aligns with the Rossby deformation radius typically observed in barotropic conditions at mid-latitudes. This alignment indicates that the disturbance is sufficiently large to interact with the background stratification and rotation in a dynamically meaningful way. Consequently, such a perturbation is capable of generating large-scale responses, including the excitation of Rossby waves and the onset of related atmospheric adjustments.

The model is initialized using monthly averaged wind and potential temperature fields of June 1980 obtained from ERA5 reanalysis data (Hersbach et al., 2022). These initial conditions serve as the baseline for the simulation, allowing the system to evolve naturally under climatologically grounded dynamics. The localized thermal disturbance, presented here as the previous characterized circular buoyancy anomaly, is introduced after the model has freely evolved for five simulated model's days. This anomaly is imposed instantaneously in a single step and is not maintained throughout the integration, ensuring that the subsequent evolution reflects the system's intrinsic response rather than sustained external forcing. This approach allows for a clear isolation of the dynamical processes triggered by the anomaly within a subtropical latitude setting.

220 The model is initialized using monthly averaged wind and potential temperature fields for June 1980 from the ERA5 reanalysis dataset (Hersbach et al., 2022), providing a climatologically grounded baseline for natural dynamic evolution. Lower-layer initial conditions for buoyancy (b_1) and horizontal velocity (v_1) were derived by taking weighted averages of these fields across the 650, 800, 900, and 1000 hPa pressure levels. Similarly, upper-layer fields (b_2 and v_2) were computed across the 250 and 450 hPa levels. Initial layer thicknesses (h_1 and h_2) were prescribed using resting-state fields incorporating local topography and static reference constants. Nondimensionalization was applied to all required variables prior to model
225 integration. Implementation details are provided in Guimarães et al. (2025).

The localized thermal disturbance, presented here as the previous characterized circular α -Gaussian buoyancy anomaly, is introduced after the model has freely evolved for five simulated model's days. This anomaly is imposed instantaneously as an intentionally unbalanced perturbation in a single step and is not maintained throughout the integration, ensuring that the subsequent evolution reflects the system's intrinsic response rather than sustained external forcing. This approach allows for
230 a clear isolation of the dynamical processes triggered by the anomaly within a subtropical latitude setting.

Rostami et al. (2023) introduced the Aeolus framework using conceptual experiments in an atmosphere initially at rest, omitting background flow and topography while applying a similar circular buoyancy disturbance. In contrast, the present work includes these essential features to evaluate how bottom topography interacts with background zonal flow and vertical
235 wind shear to distort evolving pressure systems. Bottom topography effects enter the governing Equations (1a-b, 3a-b) through the pressure gradient term ((∇p_i)), computed for the two-layer formulation using the topographic height field (h_p). Furthermore, beyond the core dynamical Equations (1b-h), the thermodynamic parameterizations for condensed liquid water ($\mathcal{C}$) and downward convective flux ($\mathcal{D}$) have been updated to account for topographic forcing. It is worth noting that the model uses realistic bottom topography, nonzero over continents and mountainous regions throughout the simulation domain.

The inherited mcTRSW equations from previous studies using Aeolus 2.0 (Rostami et al., 2023, 2025a,b) correspond to the
240 versions of Equations (1a-h, 3a-b) without the pressure gradient terms ((∇p_i)) accounting for bottom topography height (h_p) or the moisture-related and thermodynamic terms ($\mathcal{C}$, $\mathcal{D}$, γ , γ^* , F_i) in their current implementation; the current forms of Equations (3c-f, 4a-b, 5) were kept the same, with changes only in the numerical implementation for the inclusion of the new background flow.

The numerical implementation of the Aeolus 2.0 model, including the spectral discretisation, dealiasing strategy, time-stepping
245 scheme, and hardware configuration, is detailed in Appendix A. A comprehensive list of prognostic variables, diagnostic variables, physical constants, model parameters, and reference fields, along with their dimensional scales and units, is provided in Appendix B.

3 Results

250 At the onset of the adjustment process triggered by the artificial imposed circular positive buoyancy anomaly at the initial steps, the system exhibits an almost uniform air movement flowing directed toward the anomaly's center in layer 1 (lower), and opposite in layer 2 (upper), varying only in intensity among the defined cases – weak and strong, in barotropic and baroclinic situations for a moist-convective environment – see **Fig. 2** and **3**. Similarly, simulations for a dry environment show almost the same behavior (**Supplementary Fig. S1-S2**).

255 This initial convergence pattern in layer 1 is driven by the formation of a localized low-pressure area that draws in surrounding air from all directions. As the system evolves, however, the influence of the Coriolis force (arising from the Earth's rotation) becomes increasingly significant. In the Northern Hemisphere, this force causes air to veer to the right, while in the Southern Hemisphere, it deflects to the left. Consequently, the wind field begins to shift away from purely radial inflow, giving rise to asymmetric convergence patterns. These directional imbalances lead to the formation of non-uniform structures in the
260 divergence fields and the condensed liquid water content (CLWC), as depicted in **Fig. 4** and **5**. Additional zonal and meridional divergence temporal evolutions are presented in **Fig. 11** and **12**, and in **Supplementary Fig. S3-S4 S6**.

The development of convergent zones around the buoyancy anomaly plays a central role in shaping moist-convective structures, particularly influencing the organization of precipitable water. These zones foster the emergence of distinct spiral rainband-like patterns in the CLWC and W2 fields encircling the anomaly, which arise from the dynamic interaction between
265 inflowing air masses and the surrounding atmospheric environment (**Fig. 4** and **5, first row**, Layer 1). A key contributor to this behavior is the variation of the Coriolis parameter with latitude, which intensifies the system's asymmetry in a rotating spherical atmosphere.

Moreover, the asymmetric flow induced by the buoyancy anomaly introduces disturbances into the broader atmospheric circulation, potentially influencing nearby synoptic systems such as cyclones and anticyclones. The evolution of buoyancy
270 anomalies and associated frontal systems in both lower and upper layers is primarily governed by geostrophic motions (where the pressure gradient and Coriolis forces are in balance) with additional modulation from ageostrophic effects, which stem from any imbalance between these forces.

The interaction between these circulations contributes to the formation and intensification of frontal-like boundaries in the buoyancy and moisture fields, delineating air masses with contrasting thermal and density properties. This, in turn, promotes
275 the development of precipitation characteristics and organized circulation structures in the model fields meteorological systems. Our analysis indicates that the presence of a strong buoyancy anomaly can act as a catalyst for triggering convective-like organizations in the bulk moisture fields mesoscale convective systems, particularly in the vicinity west of the initially imposed disturbance for the weak case (**Fig. 4, last row**), and northeast in the strong case (**Fig. 5, last row**). These systems

are capable of producing intense rainfall associated with enhanced precipitation-like tendencies and are shaped by several factors, including the amplitude and scale of the anomaly, the vertical stratification of the atmosphere, wind shear, and the availability of environmental moisture.

Our investigation shows that buoyancy anomalies in the lower troposphere can trigger the development of anticyclonic structures in the upper troposphere. More specifically, we find that such anticyclones may also form simultaneously as a result of positive potential temperature anomalies in lower levels. This supports the concept of simultaneous generation, in which cyclonic circulation appears near the surface while anticyclonic circulation develops aloft, creating more persistent and stable weather conditions that trap warm air masses (Sanders and Gyakum, 1980; Zaitchik et al., 2006; Fischer et al., 2007; White et al., 2023; Li et al., 2024). Consequently, in the later stages of our simulation, the main anticyclonic core shifts northward and gradually produces smaller secondary vortices, clearer in the simulations with initial strong buoyance imposition, in both baroclinic and barotropic cases (Fig. 3).

In similar simulations, although more simplified than those presented here, which did not include the effects of topography, Rostami et al. (2023) found comma-shaped patterns of precipitable water evolution in the upper troposphere, as also reported in other observational studies at mid-latitudes (e.g., Carlson, 1980; Carr and Millard, 1985; Reed and Blier, 1986ab; Businger and Hobbs, 1987). These patterns emerge during our simulations, most clearly around 30 days after initialization (Fig. 3 and 5, timestep 1980-06-29). This is related to the cyclonic flow in the lower troposphere and the anticyclonic flow in the upper troposphere (illustrated also by the zonal and meridional wind means in Fig. 10).

When subtropical latitude disturbances reach high amplitude, our simulations show that the geostrophic adjustment process excites inertia-gravity waves, which subsequently propagate equatorward and dissipate in the tropics, playing an important role in redistributing heat and momentum in the lower troposphere. As the atmosphere begins to adjust in our simulations, it quickly responds by releasing a burst of inertia-gravity waves, as shown in more details in baroclinic arranged results for strong buoyance anomaly case (last row in Fig. 3). These early-stage waves are clearly visible in the wave activity diagrams (Figure 8), where we track them by integrating the divergence of the velocity field. The zonal mean of divergence brings the latitudinal aspect for the inertia-gravity waves, remarking for all simulations the particularity of higher values toward the south of the imposed anomaly (Fig. 9). That is because the added moisture and ascending motion in these regions trigger more intense condensation, related to the model layer thicknesses (h_i) and buoyancy variables (b_i ; Equations (1a-h)), which in turn favors stronger wave generation in those southern areas.

Tracking of the divergence envelope yields zonal and meridional propagation speeds (c) of approximately 52 m/s (Fig. 11 and 12). These values correspond to an equivalent depth of about 275 m ($H_{equiv_depth} = c^2/g$), consistent with the first baroclinic mode of the two-layer system ($H_1 = 6000$ m, $H_2 = 4000$ m) and the expected phase speed of the non-dispersive Kelvin wave. The slower envelope propagation provides Rossby wave group velocities of approximately 7 m/s in both zonal and meridional

310 directions, in agreement with classical beta-plane theory at 20°N. These diagnostics reinforce that the Aeolus 2.0 dynamics
are consistent with shallow-water theory.

The comparison between baroclinic and barotropic configurations in the MC and Dry experiments shows that inertia-gravity
wave activity (**Fig. 8**) initially responds intensely in layers 1 and 2, due to the extreme anomaly imposed. This forcing generates
temperature gradients between the atmospheric layers, resulting in the formation of the cyclones and anticyclones previously
315 discussed. As the simulations progress, however, wave activity tends to decline over the two months analyzed, toward an
equilibrium. In particular, the baroclinic case with the strong initial anomaly in MC environment performed the highest peak
of wave activity than in the other experiments, which means the model's vertical scheme can respond correctly to those
different temperature gradient extreme scenarios. The analogous baroclinic configuration for the dry environment does not
simulate an equal aspect of the peak in lower layer, that can be related to the differences in convective formation, where the
320 MC case is able to release more energy to the upper layer by the entrainment. In the upper layer, the behaviors of the MC and
Dry configuration pairs are very close until the end of the integration, with the greatest differences associated with the intensity
of the initial anomaly: the strong cases show higher wave activity compared to the weak cases.

The evolution of buoyancy anomalies is strongly influenced by their interaction with Rossby waves and beta gyres. At the
initial stage, the slower westward phase speed of Rossby waves at higher latitudes causes perturbations at lower latitudes to
325 advance in phase, producing a northeast-southwest tilt in the buoyancy field (**Fig. 2 and 3**). As the system evolves, the
anomalies develop a comma-shaped structure and are advected northward by beta gyres. The strength and trajectory of these
anomalies depend on factors such as the disturbance amplitude, the presence of moist convection, and whether the vertical
structure is barotropic or baroclinic. The asymmetry generated by the beta gyres enhances system instability, leading to core
fragmentation and the formation of secondary mesovortices vortex-like structures (**Fig. 2 and 3**), a feature also documented in
330 tropical cyclones (e.g., Fang and Zhang, 2012; Schecter, 2018). More intense anomalies tend to exhibit stronger meridional
displacements, first drifting northwestward and later turning eastward, a behavior particularly pronounced in barotropic
configurations.

Overall, the effects of these anomalies manifest in opposite ways when comparing total energy fluxes with tropospheric water
content variables. **Fig. 6 and 7** illustrate that the integrated evolution of CLWC, the Bulk of precipitable water (W2), and the
335 Hamiltonian exhibits a relatively inverse relationship. Specifically, positive buoyancy anomalies can trigger heatwaves and
atmospheric instability, enhancing moist convection across model layers and promoting the development of rainbands-like
moisture structures in certain regions (**Fig. 4 and 5**), while producing opposite effects in others, depending on the experiment
configurations. This process is sustained by a positive feedback cycle, in which latent heat release further intensifies
convection. Thus, the atmospheric adjustment continues until the disturbance dissipates, restoring stability.

340 It should be noted that, in the present configuration, the model is not intended to resolve fine-scale structures such as individual convective cells, fronts, or rainbands. Terms such as “rainband”, “frontal”, and “convective system” are therefore used here in a general sense to denote the model’s bulk signatures, for example, in fields such as CLWC, W2, divergence, and buoyancy, which are consistent with the presence of such phenomena.

Buoyancy Anomaly in Moist-Convective Environment - Weak Perturbation

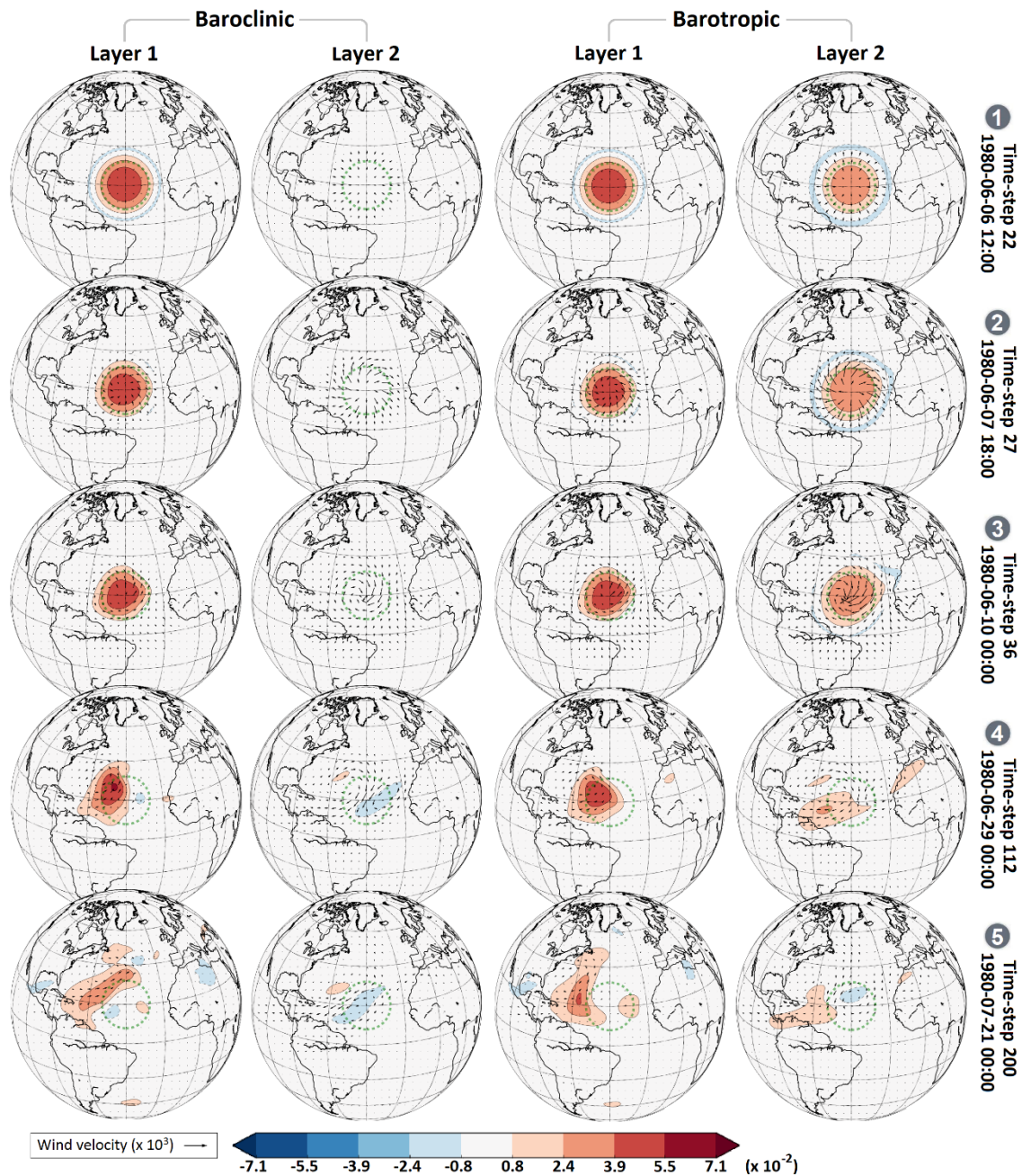


Figure 2: Anomaly evolution of buoyancy by weak perturbation, in the lower (Layer 1) and upper (Layer 2) troposphere, and the corresponding velocity field (represented by arrows) during adjustment in a moist-convective environment, in respect to the reference case. The first and second columns show the results for baroclinic configuration and the third and fourth columns for barotropic. ERA5 monthly-averaged data was used as initial condition to startup the model from 1-June-1980. **The Time-step number is referred to the 6hs output.**

Buoyancy Anomaly in Moist-Convective Environment - Strong Perturbation

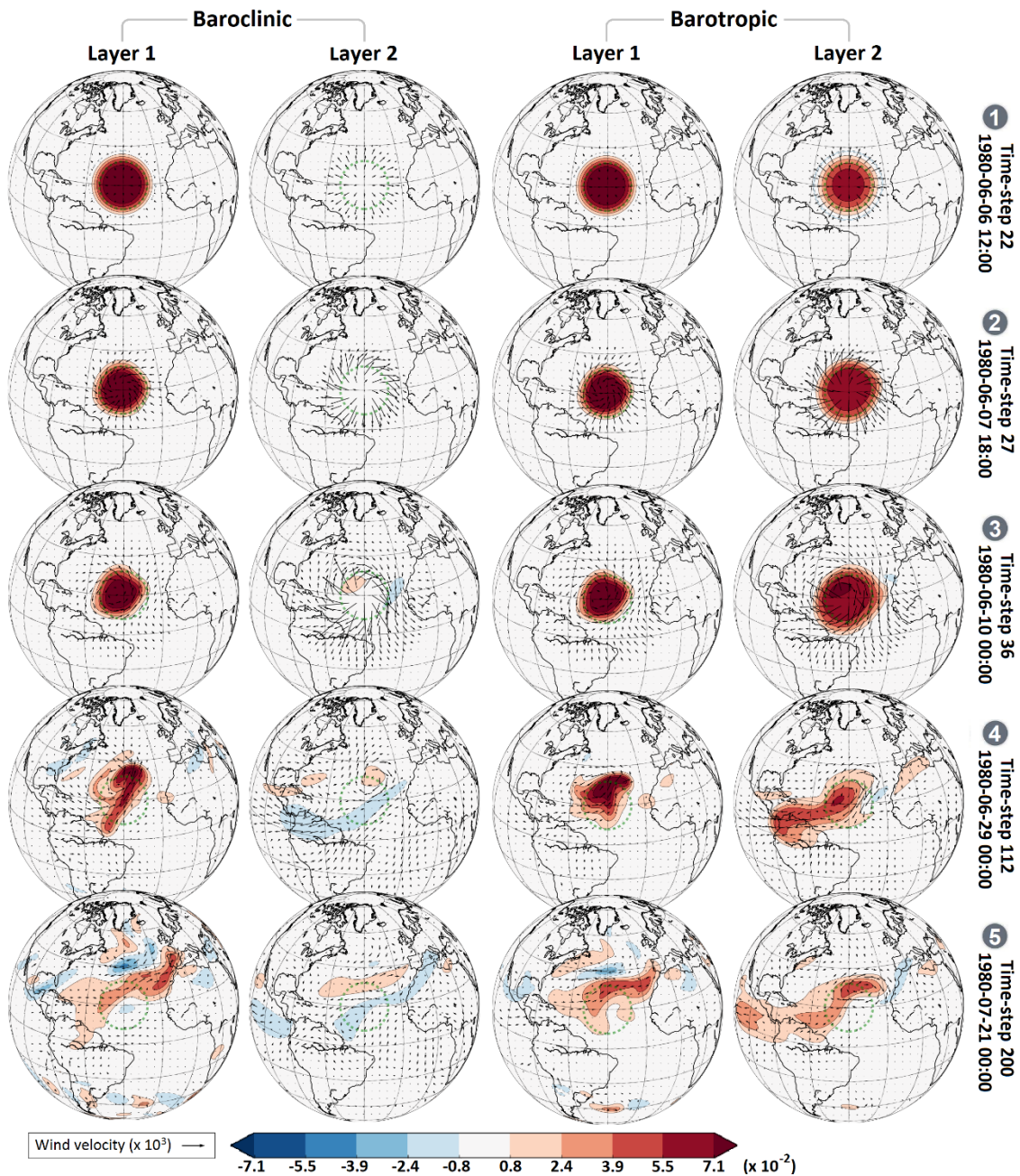
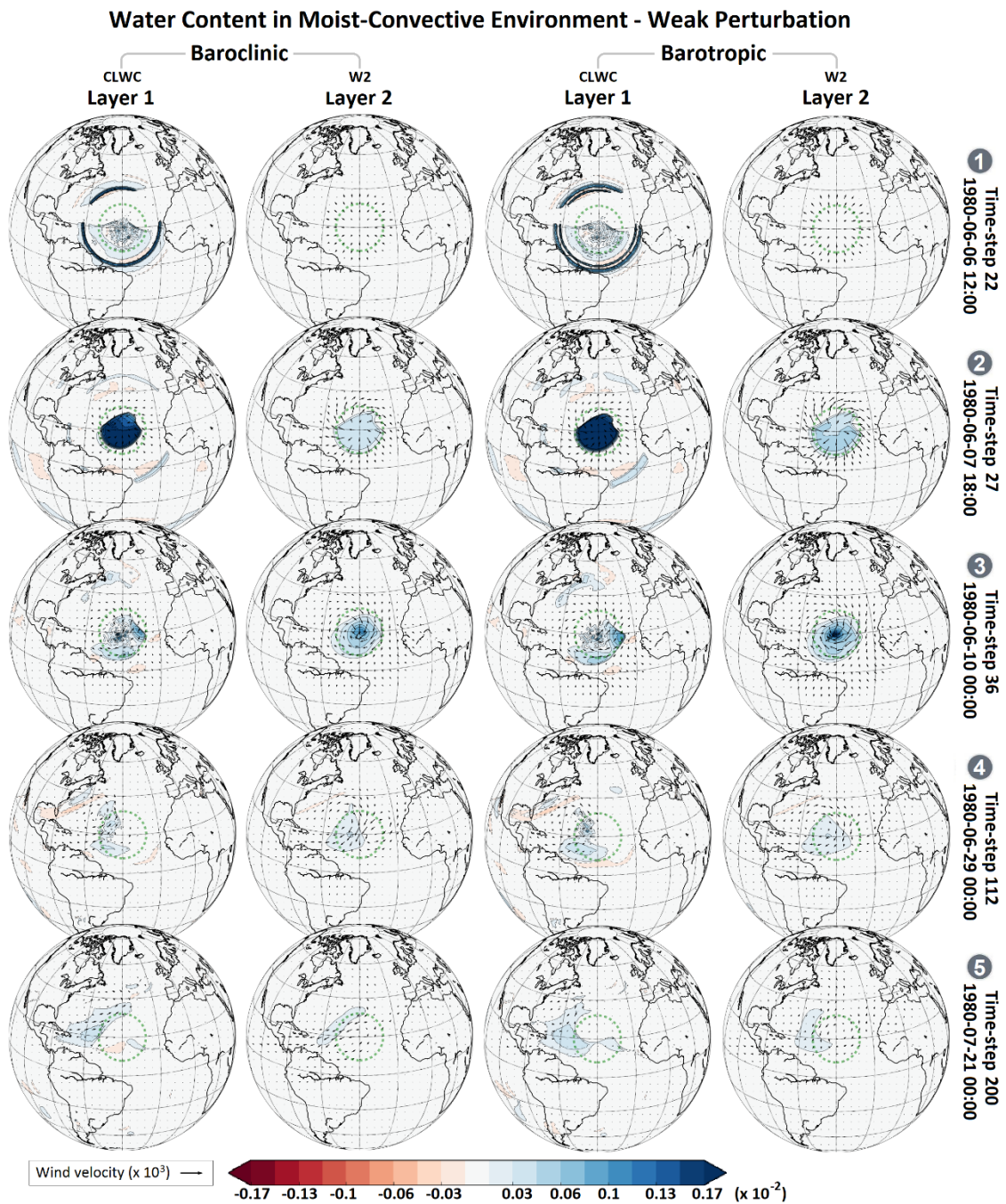
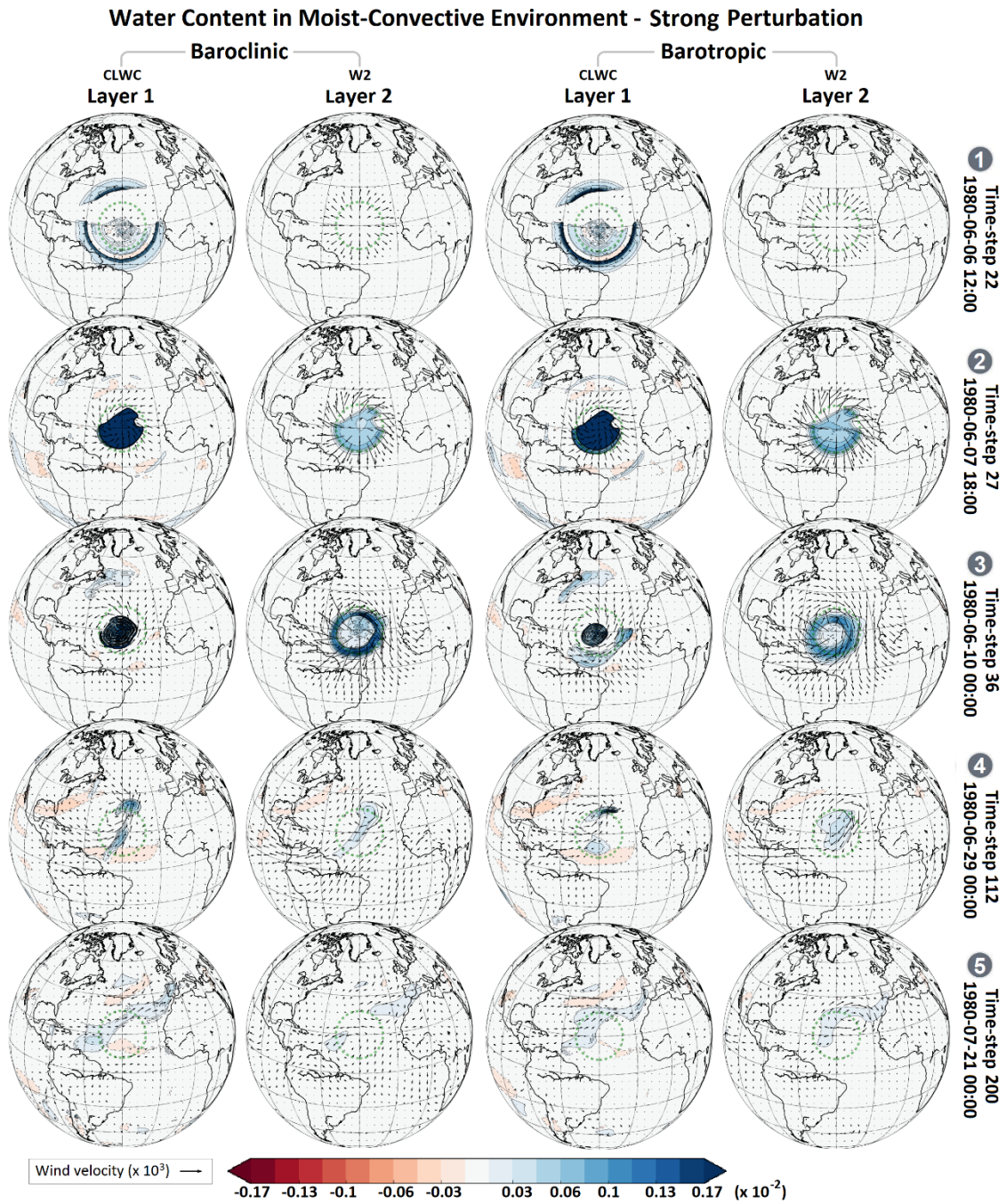


Figure 3: Anomaly evolution of buoyancy by strong perturbation, in the lower (Layer 1) and upper (Layer 2) troposphere, and the corresponding velocity field (represented by arrows) during adjustment in a moist-convective environment, in respect to the reference case. The first and second columns show the results for baroclinic configuration and the third and fourth columns for barotropic. ERA5 monthly-averaged data was used as initial condition to startup the model from 1-June-1980. The Time-step number is referred to the 6hs output.



355

Figure 4: Anomaly evolution of CLWC (Condensed liquid water content) by weak perturbation in the lower troposphere (Layer 1), and W2 (Bulk of precipitable water) in the upper troposphere (Layer 2), and the corresponding velocity field (represented by arrows) during adjustment in a moist-convective environment, in respect to the reference case. The first and second columns show the results for baroclinic configuration and the third and fourth columns for barotropic. The Time-step number is referred to the 6hs output



360

Figure 5: Anomaly evolution of CLWC (Condensed liquid water content) by strong perturbation in the lower troposphere (Layer 1), and W2 (Bulk of precipitable water) in the upper troposphere (Layer 2), and the corresponding velocity field (represented by arrows) during adjustment in a moist-convective environment, in respect to the reference case. The first and second columns show the results for baroclinic configuration and the third and fourth columns for barotropic. The Time-step number is referred to the 6hs output

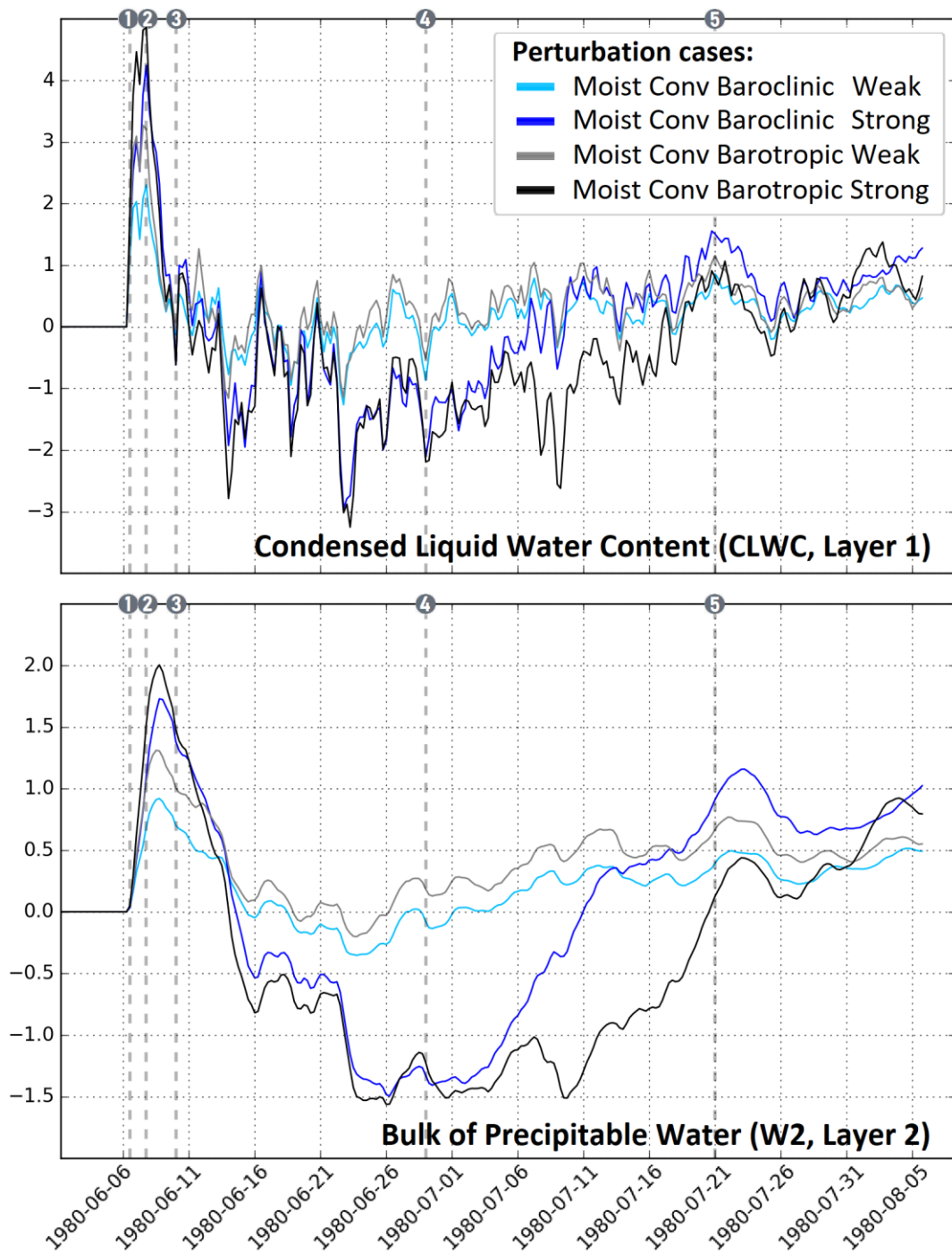


Figure 6: Global anomaly of CLWC and W2 relative to the reference case. Markers on the top border indicate the time-steps of maps displayed in previous figures.

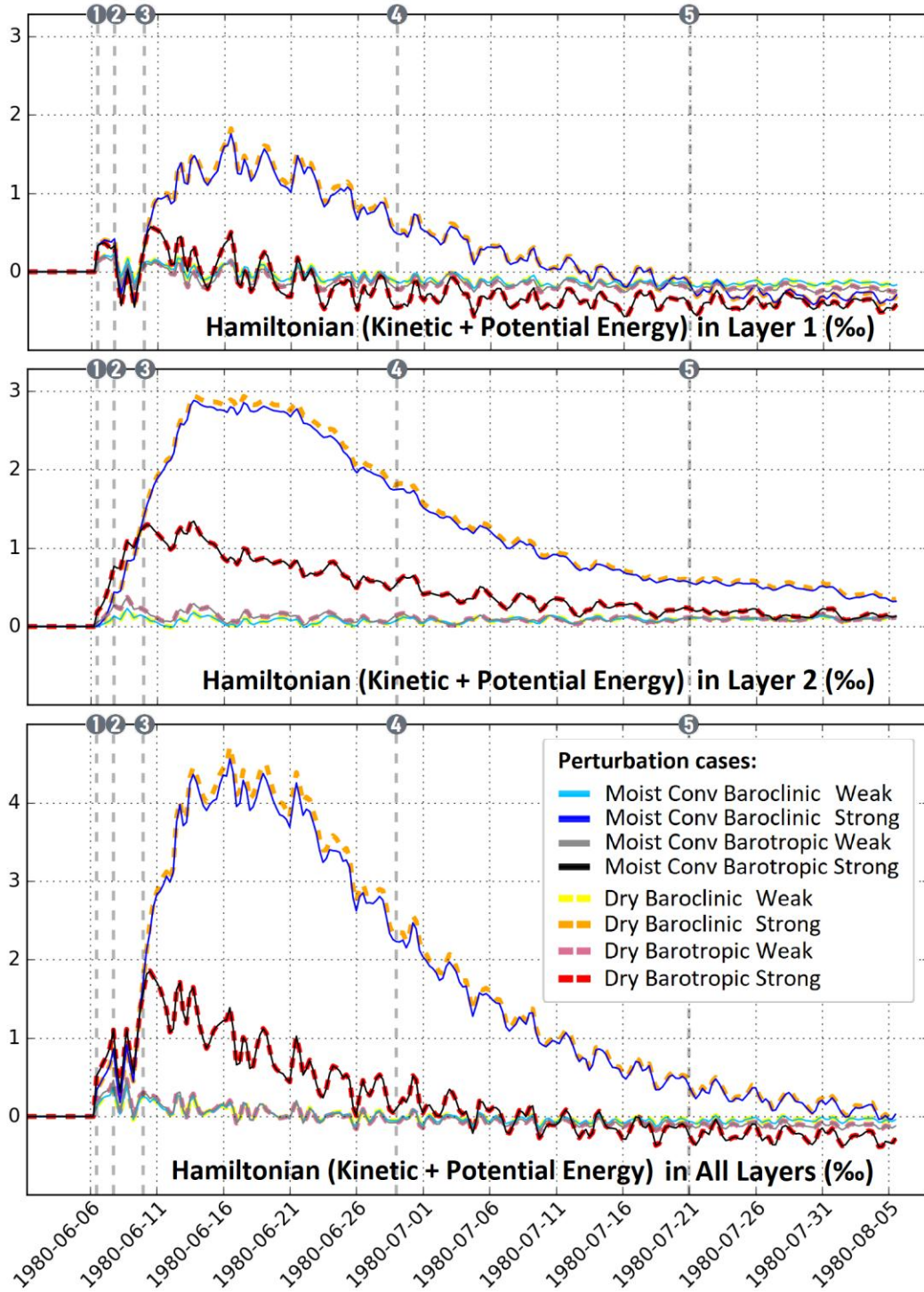


Figure 7: Global mean anomaly of Hamiltonian relative to the reference case, calculated by $(\mathcal{H} - \mathcal{H}_{t=0})1000/\mathcal{H}_{t=0}$. Markers on the top border indicate the time-steps of maps displayed in previous figures.

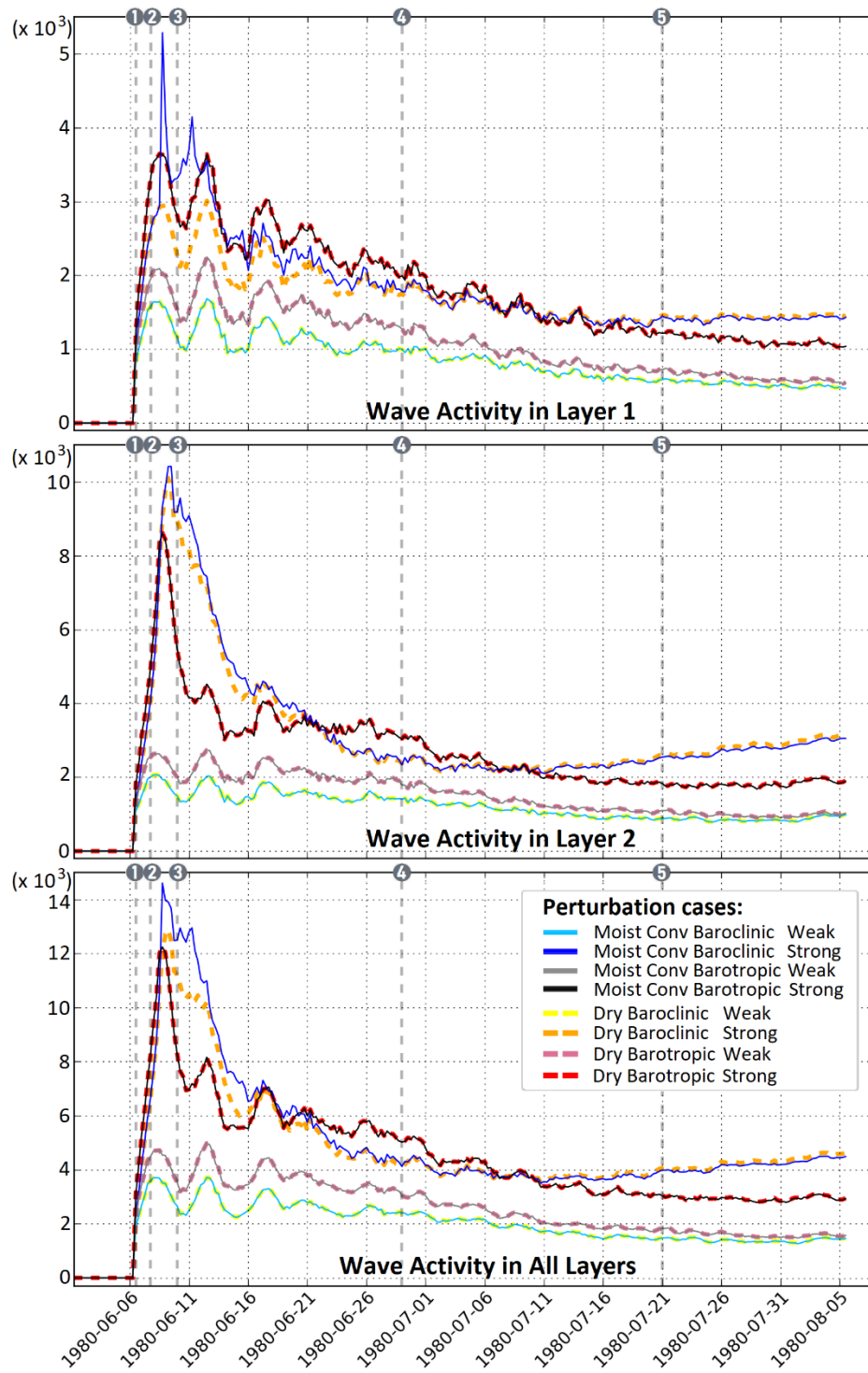
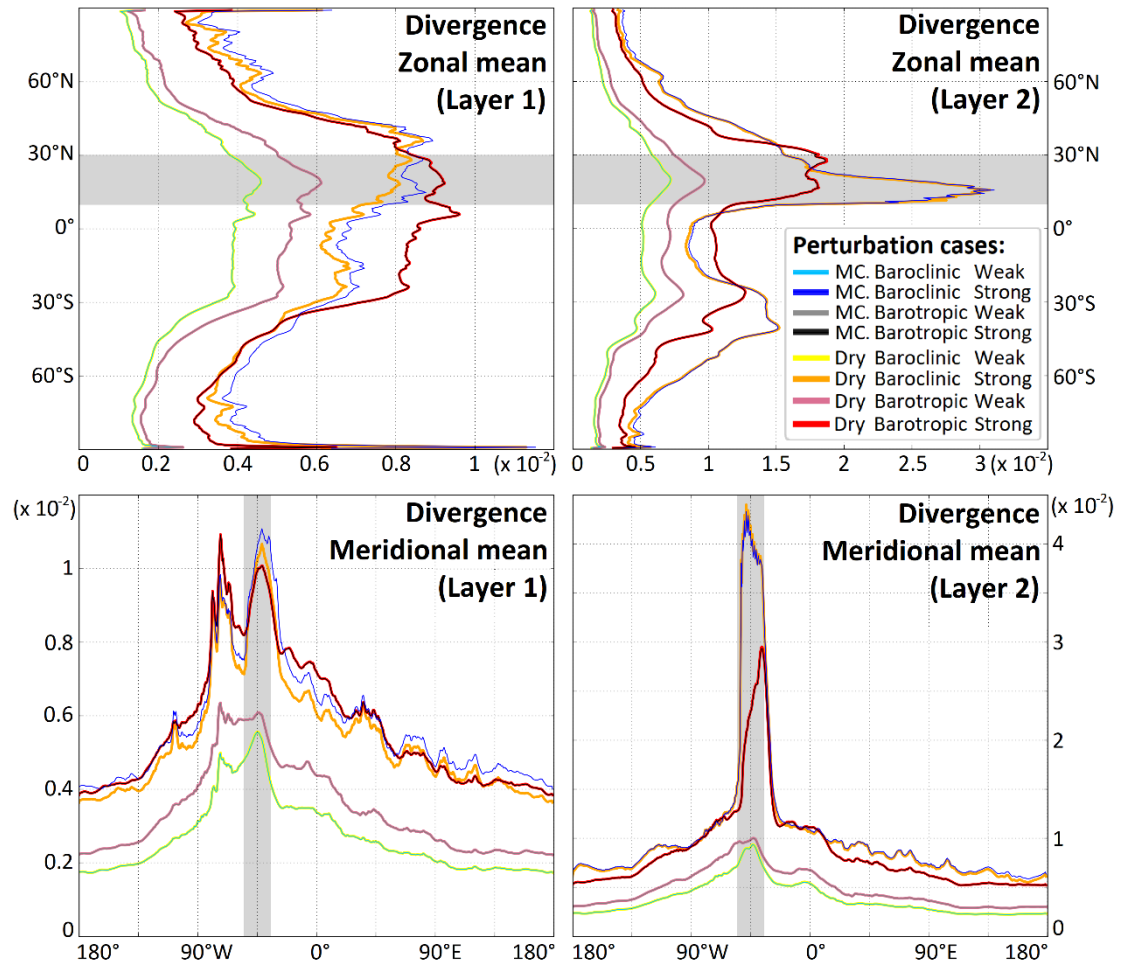


Figure 8: Global anomaly of Wave Activity relative to the reference case. Markers on the top border indicate the time-steps of maps displayed in previous figures.



375 **Figure 9:** Divergence anomaly time-mean (1-June to 5-Aug 1980) relative to the reference case, for moist-convective (MC) and dry situations. The gray shade represents the initial buoyancy perturbation region.

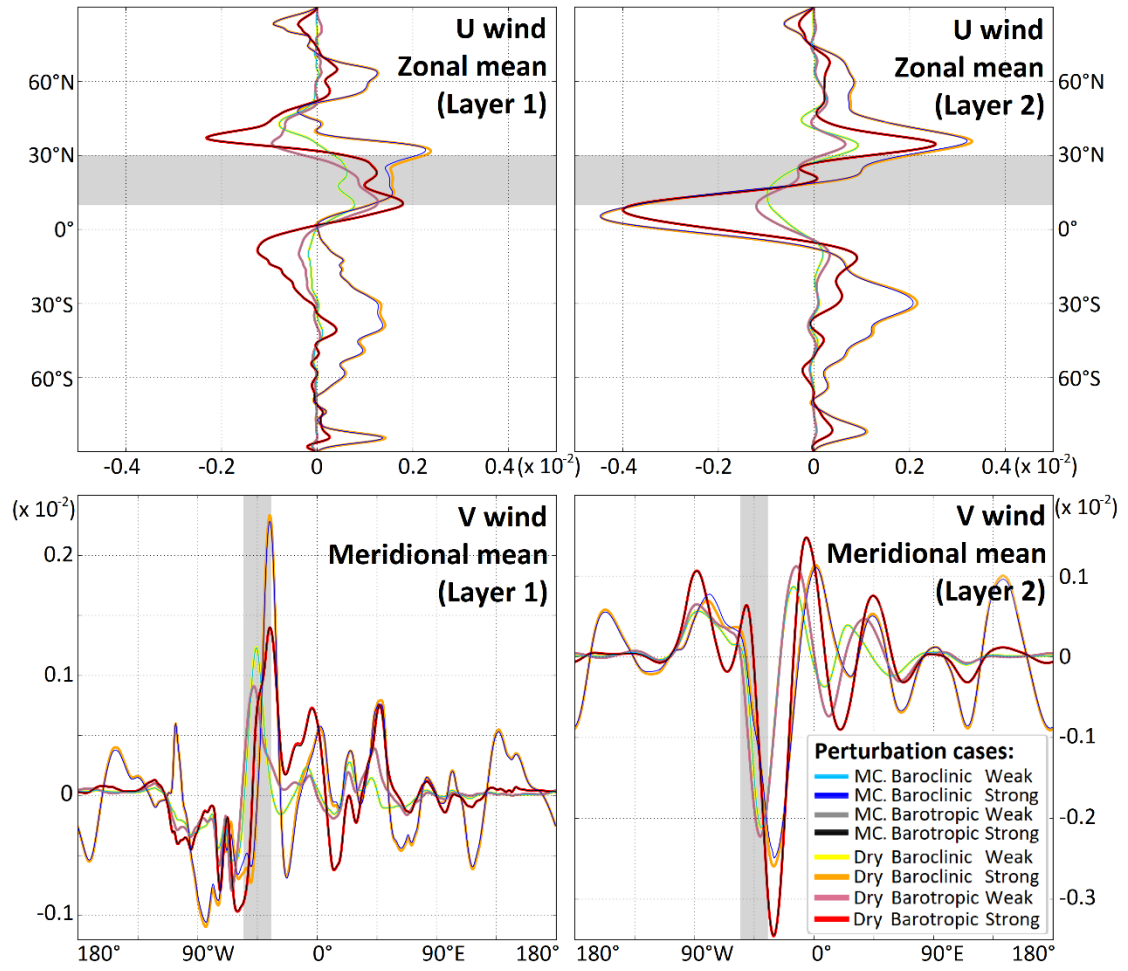
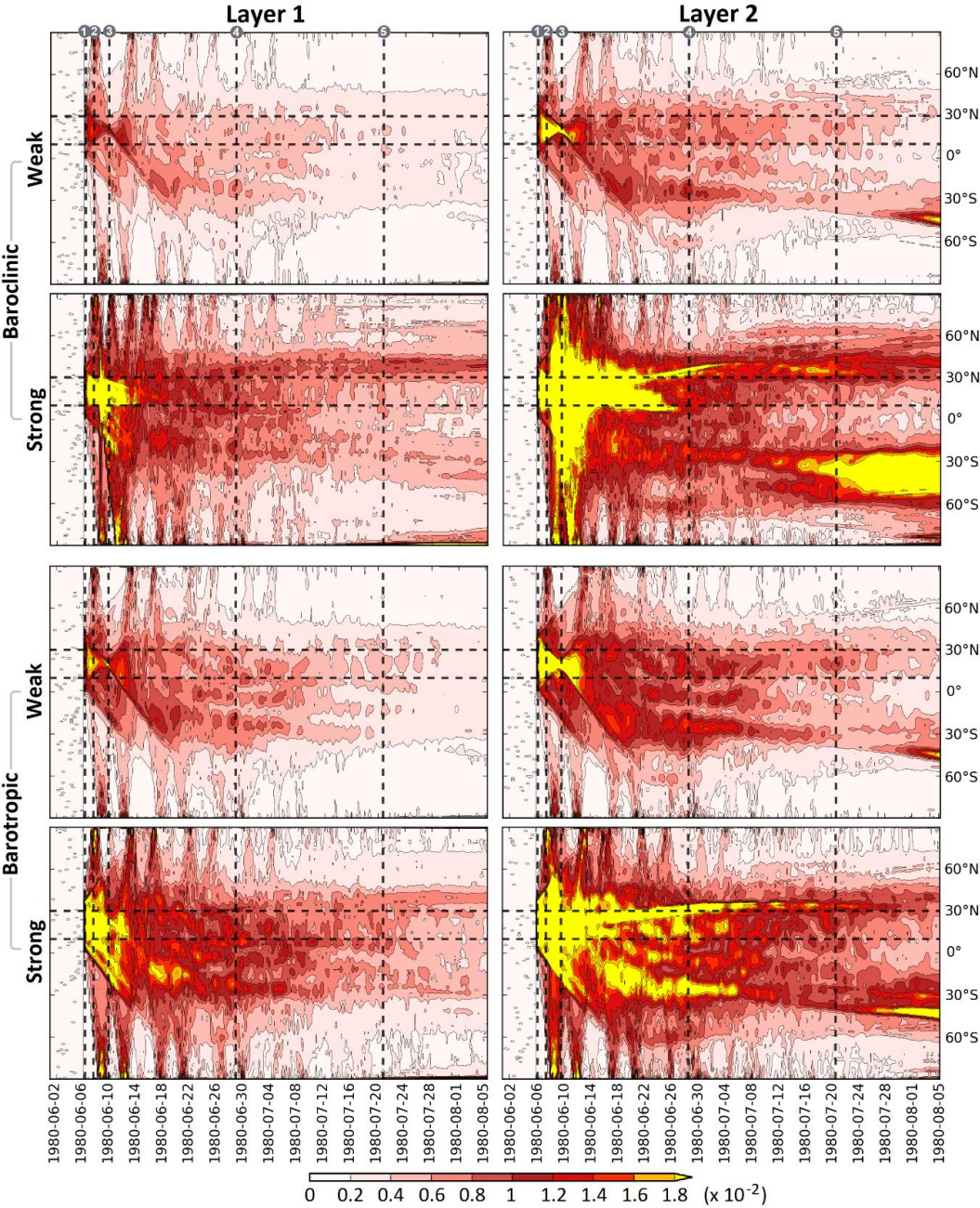


Figure 10: Wind anomaly time-mean (1-June to 5-Aug 1980) relative to the reference case, for moist-convective (MC) and dry situations. The gray shade represents the initial buoyancy perturbation region.

Divergence Zonal mean in MC case



380

Figure 11: Divergence anomaly zonal mean, relative to the reference case, for moist-convective (MC) situations. The horizontal dashed lines represent the initial buoyancy perturbation region. Markers on the top border indicate the time-steps of maps displayed in previous figures.

Divergence Meridional mean in MC case

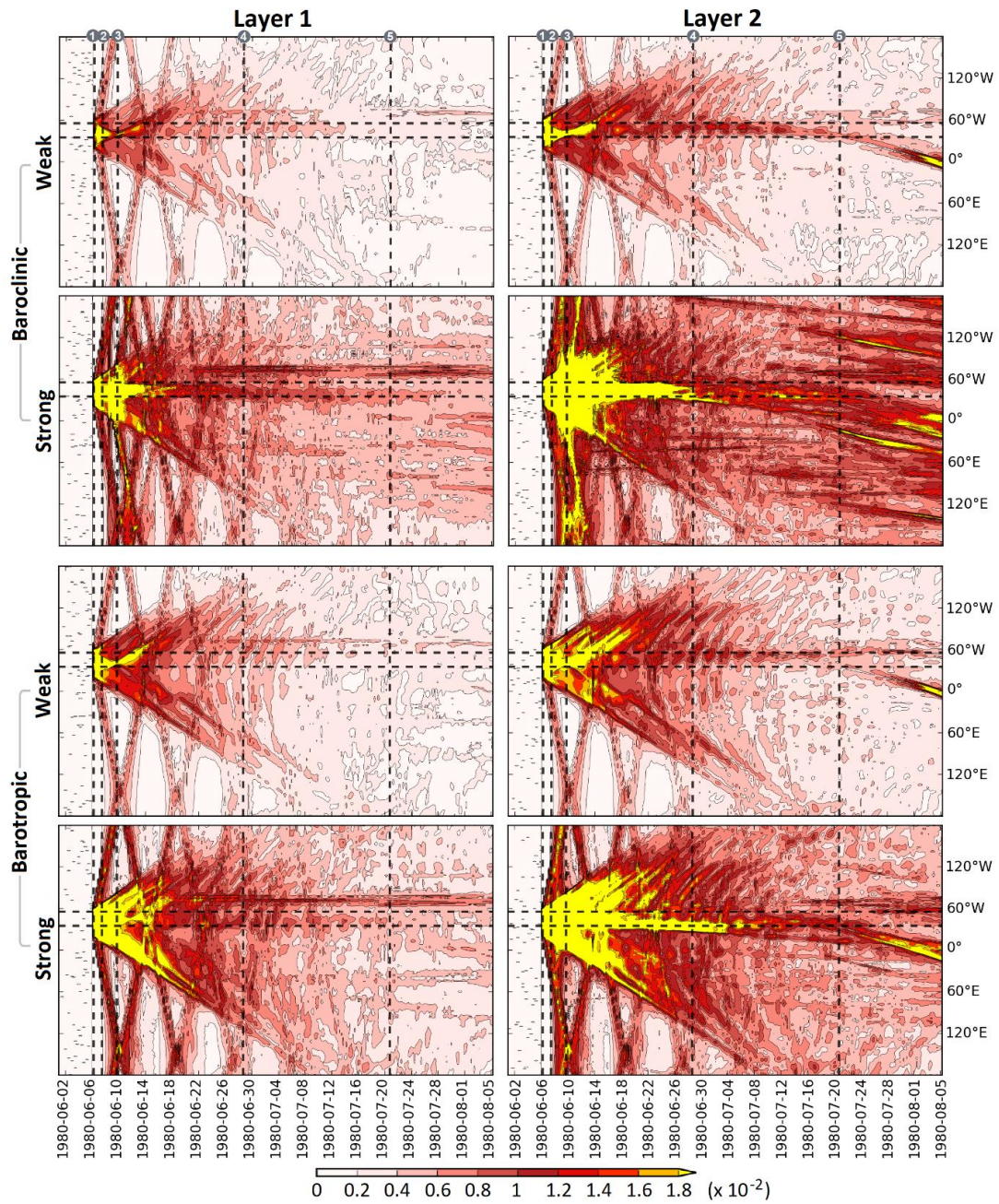


Figure 12: Divergence anomaly meridional mean, relative to the reference case, for moist-convective (MC) situations. The horizontal dashed lines represent the initial buoyancy perturbation region. Markers on the top border indicate the time-steps of maps displayed in previous figures.

This study set out to explore the dynamics of localized extreme heatwaves in the subtropical atmosphere through an intermediate complexity modeling approach using Aeolus 2.0. By imposing buoyancy anomalies under both dry and moist-convective conditions, we were able to disentangle the primary processes that govern the emergence, development, and eventual dissipation of these extreme events. The results provide a coherent picture of how localized heating triggers large-scale atmospheric responses that can persist for extended periods.

A central finding is the role of the imposed buoyancy anomaly in shaping the initial adjustment process simulated by the multi-layer formulation of the moist-convective Thermal Rotating Shallow Water (mcTRSW) model framework.

Early in the simulations, convergence in the lower troposphere and divergence in the upper troposphere set the stage for cyclonic and anticyclonic circulation patterns. These patterns quickly departed from radial symmetry as the Coriolis force came into play, highlighting the sensitivity of heatwave development to planetary rotation and latitudinal gradients.

The simulations further demonstrated the emergence of spiral rainband-like structures encircling the buoyancy anomaly. These mesoscale features, particularly evident in moist-convective scenarios, emphasize the importance of moisture feedbacks in strengthening instability. Latent heat release during condensation amplified convection, generating a positive feedback loop that sustained rainfall bands moisture bands in the column-integrated diagnostics and reinforced the heatwave environment.

Inertia-gravity waves were found to be an integral part of the atmospheric response. Triggered during the early adjustment, these waves propagated equatorward and contributed to the redistribution of heat and momentum. Their presence was strongest in baroclinic configurations with intense initial anomalies, and their decay over time marked the transition toward equilibrium. The sensitivity of wave activity to environmental moisture underlines the value of including moist-convective processes in idealized frameworks.

Another key outcome of this study is the simultaneous generation of cyclonic and anticyclonic structures. Lower-level positive buoyancy anomalies induced cyclonic circulations near the surface, while anticyclones emerged aloft. This vertical coupling, consistent with the simultaneous generation paradigm, promoted stable atmospheric ridges that trapped warm air masses. The northward drift and eventual fragmentation of the anticyclonic core into smaller vortices further reflect the dynamical richness of these systems.

The simulations also reproduced comma-shaped patterns in precipitable water, a well-known observational feature of subtropical cyclones. These structures emerged roughly one month into the integrations, driven by the interplay of lower-tropospheric cyclonic flow and upper-tropospheric anticyclonic circulation. Their occurrence in both moist and dry environments underscores the robustness of this dynamical signature.

Comparisons between baroclinic and barotropic settings revealed systematic differences. Baroclinic experiments, especially
420 under moist-convective forcing, exhibited stronger inertia–gravity wave activity and greater sensitivity to the amplitude of the
initial buoyancy anomaly. In contrast, barotropic environments showed more pronounced meridional shifts of buoyancy
anomalies and a stronger tendency for eastward drift at later stages. These distinctions demonstrate how vertical structure can
influence the path along which heat waves evolve.

Rossby wave dynamics and beta gyres also emerged as fundamental mechanisms. Phase speed differences across latitudes
425 tilted the buoyancy anomalies in a northeast–southwest orientation, later advected poleward by beta gyres. The asymmetries
induced by these gyres amplified system instability, fragmenting the core circulation and giving rise to secondary mesovortices.
Such processes link idealized simulations to real-world observations of instability and breakdown in subtropical and mid-
latitude weather systems.

Energy and moisture diagnostics further highlighted the dual nature of buoyancy anomalies. The integrated evolution of the
430 Hamiltonian energy fluxes and tropospheric water variables revealed an inverse relationship. While positive buoyancy
anomalies strengthened instability and rainfall formation in some regions, they simultaneously suppressed moist processes
elsewhere. This spatial heterogeneity emphasizes the non-linear and regionally dependent character of heatwave evolution.

Taken together, these findings indicate that localized surface heating can initiate complex chains of atmospheric adjustment
that resonate from the mesoscale to the synoptic scale. The interplay of inertia–gravity waves, Rossby wave dynamics,
435 convective feedbacks, and vertical structural differences creates a multifaceted environment where heatwaves can persist,
intensify, and interact with larger-scale circulation patterns.

Compared to studies based on earlier versions of our model, adding certain real-world characteristics, such as topography and
ERA5-derived background wind velocity and potential-temperature fields effects, is a novel development that improves the
fidelity of the experiments, the model simulates the essential dynamics of buoyancy-driven anomalies. The success in
440 reproducing phenomena observed in nature like rainbands, comma cloud patterns such as banded moisture structures, comma-
like patterns, ridged trough coupling validates the is consistent with the overall approach and provides a bridge between theory
and fully comprehensive climate simulations. Building on these results, a comprehensive sensitivity analysis comparing model
runs with and without topography under the present buoyancy anomaly configuration will be pursued in future work.

In conclusion, our results shed new light on the fundamental processes underlying subtropical heatwaves. They underscore the
445 importance of buoyancy anomalies as drivers of multi-scale atmospheric responses, highlight the necessity of accounting for
moist convection, and reaffirm the coupling between cyclonic and anticyclonic circulations. These insights contribute to the
ongoing effort to build a more complete theoretical framework for heatwave dynamics, ultimately improving our ability to
anticipate, model, and respond to extreme events in a changing climate.

Supplementary Materials

450 Supplementary material for this article is available at:

<Link to the Supplementary material document submitted with this article>

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Conflict of Interest

The authors have no conflicts to disclose.

460 Author Contributions

Sullyandro Guimarães: Conceptualization; Methodology; Modelling; Software; Visualization; Formal analysis; Investigation; Validation; Writing – original draft; Writing – review & editing.

Masoud Rostami: Conceptualization; Methodology; Modelling; Software; Visualization; Formal analysis; Investigation; Validation; Writing – review & editing.

465 Stefan Petri: Modelling; Software; Investigation; Validation; Writing – review & editing.

All authors discussed the results, edited and reviewed the manuscript.

Code and data availability

The code and data that support the findings of this study are openly available in Zenodo at [Guimarães et al. \(2025; v2, https://doi.org/10.5281/zenodo.18065000\)](https://doi.org/10.5281/zenodo.18065000), reference number 18065000.

470 The Aeolus 2.0 code included in the above-mentioned repository was adapted with small changes from [Rostami et al. \(2024b; v6, https://doi.org/10.5281/zenodo.13768553\)](https://doi.org/10.5281/zenodo.13768553).

Appendix A - Numerical configuration

The Aeolus 2.0 stand-alone atmospheric model was configured following Rostami et al. (2024a; 2025a). The model solves the governing equations on the full sphere using a spin-weighted spherical harmonic spectral transform method implemented within the Dedalus framework (Burns et al., 2020; Lecoanet et al., 2019; Vasil et al., 2019).

The horizontal discretization employs a regular latitude-longitude grid with 384 latitudes (Gauss-Legendre quadrature points) by 768 longitudes ($\sim 0.5^\circ$ resolution). The spectral representation uses spin-weighted spherical harmonics with maximum degree ($L_{max} = 255$), and the azimuthal order ranges from ($-L_{max}$ to L_{max}) (Rostami et al., 2024a; Guimarães et al., 2025).

The Dedalus implementation represents the sphere through a tensor product of a Fourier basis in longitude (periodic, interval 0 to 2π) and a Fourier basis in colatitude (interval 0 to π), which together enable efficient spin-weighted spherical harmonic transforms (see `sphere_wrapper.py` and `sphere128.py` in Guimarães et al., 2025). The poles are treated as regular points in the spectral domain, consistent with the spin-weighted spherical harmonic basis (Ripa, 1993; Lecoanet et al., 2019).

The numerical time-step for the integration was set to 1 minute, and each case was integrated over a period of 102 numerical days. Output was saved instantaneously every 6 hours (4 times per day), yielding a total of 408 output steps per simulation.

Non-linear terms within the multi-layer mcTRSW equations are dealiased using Dedalus's standard 2/3 rule, implemented via the dealias parameter during spectral basis initialization ($L_{dealias} = 3/2$) to prevent aliasing errors. High-wavenumber energy accumulation and small-scale numerical noise are dissipated through explicit spatial diffusion operators ($\nu \nabla^2$) applied directly to the governing momentum and continuity equations (see `aeolus2main.py` in Guimarães et al., 2025).

To maintain stability at the 1-minute timestep, the Dedalus solver employs an Implicit-Explicit (IMEX) time-stepping scheme (SBDF2; `timesteppers.py` in Guimarães et al., 2025). By implicitly integrating stiff linear terms, such as linear pressure gradients and wave propagation operators, it bypasses the strict speed limitations typically imposed by fast gravity waves. Non-linear advective terms, meanwhile, are integrated explicitly and remain constrained by the standard advective Courant-Friedrichs-Lewy (CFL) condition: $CFL = (U_{max} \Delta t) / \Delta x$. Given the utilized grid spacing ($\Delta x \approx 55 \text{ km}$) and timestep ($\Delta t = 60 \text{ s}$), the advective Courant number safely satisfies this stability criterion, remaining well below the limit ($CFL < 0.2$) for maximum anticipated flow speeds of 50 to 80 m/s.

As a verification of the numerical solver, the total mechanical energy (Hamiltonian, **Equation (4a)**) in dry adiabatic integrations was monitored. The total energy remained conserved to within machine precision (relative fluctuations $< 10^{-6}$) over the simulation period. This confirms that the discretization, spectral transforms, and time-stepping scheme (SBDF2) faithfully preserve the adiabatic invariants of the governing equations. The moist-convective (MC) experiments employ the identical numerical core, with only the addition of physically derived source/sink terms for latent heat release ($\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}_s$, and γ). Therefore, this verification ensures that any energy conversions observed in the moist runs are physically meaningful and

not numerical artifacts. This validation thereby underpins the interpretation of energy budgets across all experimental configurations (barotropic/baroclinic, weak/strong, dry/MC) presented in this study.

505 Further details regarding the Dedalus framework implementation can be found in the code repository ([Guimarães et al., 2025](#)), with additional information in [Rostami et al. \(2024a\)](#) and on the Dedalus Project homepage ([dedalus-project.org](#); [Burns et al., 2020](#)).

For these simulations, we utilized Dedalus version 2.2207.3 running in Python 3.10.14. The software environment relied on several additional libraries: Climlab (0.8.2), Cython (3.0.10), NumPy (1.26.4), SciPy (1.13.1), MPI for Python (mpi4py 3.1.6),
510 and HDF5 for Python (h5py 3.11.0). By enabling high-performance and efficient execution, this framework supplies Dedalus with the essential functionality required for advanced scientific computations.

All simulations were performed on an Alienware M18 R1 machine running Ubuntu 22.04.4 LTS. The system is equipped with an Intel Core i9-13900HX processor (24 cores, 32 threads, 36 MB L3 cache, 5.4 GHz) and 32 GB of DDR5 4800 MHz RAM (2×16 GB). Each studied case utilized 16 cores and 16 GB of RAM, taking approximately 3 hours to complete and generating
515 20 GB of output data.

In **Tables B1–B4**, the units of the model variables are based on the respective dimensional scale. These units reflect the physical dimensions after the model's dimensional scaling has been applied. It is important to note that the model is integrated in a nondimensional form; therefore, the results presented in this work are expressed as nondimensional quantities.

520

Table B1: Prognostic variables.

Variable	Description	Dimensional scale	Units
$\mathbf{v}_1 = (u_1, v_1)$	Horizontal velocity vector, layer 1	βL_d^2	[m s ⁻¹]
$\mathbf{v}_2 = (u_2, v_2)$	Horizontal velocity vector, layer 2	βL_d^2	[m s ⁻¹]
h_1	Layer thickness, layer 1	H_0	[m]
h_2	Layer thickness, layer 2	H_0	[m]
b_1	Buoyancy, layer 1	$g\bar{\theta}_1/\theta_s$	[m s ⁻²]
b_2	Buoyancy, layer 2	$g\bar{\theta}_2/\theta_s$	[m s ⁻²]
q_1	Specific humidity, layer 1	$L_v g/(C_p \theta_s)$	[kg kg ⁻¹]
q_2	Specific humidity, layer 2	$L_v g/(C_p \theta_s)$	[kg kg ⁻¹]
w_1	Precipitable water, layer 1	$L_v g/(C_p \theta_s)$	[kg kg ⁻¹]
w_2	Precipitable water, layer 2	$L_v g/(C_p \theta_s)$	[kg kg ⁻¹]

Table B2: Diagnostic variables and derived quantities.

Variable	Description	Dimensional scale	Units
$\mathcal{C}$	Condensed Liquid Water Content (CLWC), layer 1	Nondimensional moisture tendency	[Q/T]
$\mathcal{D}$	Downward convective flux (downdraft into lower layer)	Nondimensional moisture tendency	[Q/T]
$\mathcal{E}_s$	Surface evaporation flux (bulk aerodynamic formula)	Nondimensional moisture tendency	[Q/T]
W_1	Vaporization rate, layer 1	Nondimensional moisture tendency	[Q/T]
W_2	Vaporization rate, layer 2	Nondimensional moisture tendency	[Q/T]
p_1	Pressure at the lower layer, derived from thickness and buoyancy	$p_1 + gp_1(h_1 + h_2 + h_b)b_1/a$	[Pa]
p_2	Pressure at the upper layer, derived from thickness and buoyancy	$p_2 + gp_2(h_1b_1 + (h_b + h_2)b_2)/a$	[Pa]
$\mathbb{F}_i$	External thermal forcing (insolation + Newtonian cooling)	Buoyancy-height forcing	[HB]
$\mathcal{H}$	Hamiltonian (total mechanical energy of the system)	$\mathcal{H} \sim L_d^2 H_0 (\beta L_d^2)^2$	[m ⁴ s ⁻²]
ζ_i	Bernoulli energy in layer i	$\tilde{p}_i + \frac{1}{2} v_i^2$	[m ² s ⁻²]

525 **Table B3:** Physical constants, fundamental parameters, and reference fields.

Variable	Description	Dimensional scale or value	Units
R_{Earth}	Earth radius	6371000	[m]
g	Gravitational acceleration	9.8	[m s ⁻²]
a	Aspect ratio, $(R_{Earth} + H_0)/L_d$	1.7218	[dimensionless]
Ω	Planetary rotation rate	7.292×10^{-5}	[s ⁻¹]
f	Coriolis parameter	$2\Omega \sin(\phi)$	[s ⁻¹]
f_0	Coriolis parameter at a reference latitude	$2\Omega \sin(\phi_0)$	[s ⁻¹]
β	Meridional gradient of the Coriolis parameter	$2\Omega \cos(\phi) / R_{Earth}$	[m ⁻¹ s ⁻¹]
$\beta_{Equator}$	Meridional gradient of the Coriolis parameter	2.2793×10^{-11}	[m ⁻¹ s ⁻¹]
β_{20N}	Meridional gradient of the Coriolis parameter	2.1480×10^{-11}	[m ⁻¹ s ⁻¹]
L	Characteristic horizontal length scale	–	–
L_d	Equatorial Rossby deformation radius, $(\sqrt{gH_0}/\beta)^{1/2}$	$\sim 3.706 \times 10^6$	[m]
L_{d20N}	20° Rossby deformation radius, $(\sqrt{gH_0}/\beta)^{1/2}$	$\sim 3.818 \times 10^6$	[m]
H_0	Total unperturbed layer depth, $H_1 + H_2$	10000	[m]
H_1	Lower layer resting thickness, $\delta_1 H_0$	6000	[m]
H_2	Upper layer resting thickness, $\delta_2 H_0$	4000	[m]
δ_i	Layer i thickness fraction	$[\delta_1, \delta_2] = [0.6, 0.4]$	[dimensionless]
η_i	Deviation of the δ_i thickness	–	[dimensionless]
λ	Small nondimensional parameter	$\lambda \sim R_0$	[dimensionless]
h_b	Bottom topography height	–	[m]
$\tilde{h}_1$	Height of the center of mass in layer 1	$h_b + (1/2)h_1 + h_2$	[m]
$\tilde{h}_2$	Height of the center of mass in layer 2	$h_b + (1/2)h_2$	[m]
$\tilde{p}_1$	Vertically-averaged pressure in layer 1	$(h_b + h_1 + h_2)b_1$	[Pa]
$\tilde{p}_2$	Vertically-averaged pressure in layer 2	$h_1 b_1 + (h_b + h_2)b_2$	[Pa]
b'_i	Buoyancy perturbation in layer i	$g\bar{\theta}_i/\theta_s$	[m s ⁻²]
B_0	Buoyancy at rest	$g\bar{\theta}/\theta_s$	[m s ⁻²]
B_i	Background buoyancy scaling in layer i	$[B_1, B_2] = [1.1563, 1.2886]$	[dimensionless]
θ_s	Reference surface-level potential temperature	250.0277	[K]
$\bar{\theta}_i$	Vertically-averaged potential temperature in layer i	–	[K]
t	Time	$t \sim 1/(\beta L_d)$	[s]
T_{scale}	Time scale	$a/2 = 0.8609$	[day]

Table B4: Physical constants, fundamental parameters, and reference fields.

Variable	Description	Dimensional scale or value	Units
U	Characteristic horizontal velocity scale	–	–
R_0	Rossby number	$R_0 = \frac{U}{f_0 L} \sim \lambda$	[dimensionless]
Bu	Burger number	$Bu \approx 1$	[dimensionless]
γ	Thermodynamic efficiency for latent heat release	–	[dimensionless]
γ^*	Thermodynamic efficiency for radiative cooling	–	[dimensionless]
μ	Amplitude equivalence factor (condensation / evaporation)	–	[dimensionless]
R_d	Specific gas constant for dry air	287.05	[J kg ⁻¹ K ⁻¹]
R_v	Specific gas constant for water vapour	461.50	[J kg ⁻¹ K ⁻¹]
C_p	Specific heat capacity of dry air at constant pressure	1005	[J kg ⁻¹ K ⁻¹]
L_v	Latent heat of vaporization	2.5×10^6	[J kg ⁻¹]
Q_s	Saturation specific humidity threshold	$L_v g / (C_p \theta_s)$	[kg kg ⁻¹]
ν	Explicit hyperdiffusion coef. (momentum and buoyancy)	1.08×10^{-4}	[dimensionless]
L_{max}	Maximum spherical harmonic degree	255	[dimensionless]
$L_{dealias}$	Dealiasing scale factor for spectral transforms (2/3 rule)	3/2	[dimensionless]
Δt	Time step	60	[s]
Δx	Grid spacing	~55	[km]
U_{max}	Maximum anticipated flow speeds	~50–80	[m s ⁻¹]
CFL	Courant-Friedrichs-Lewy condition	$(U_{max} \Delta t) / \Delta x$	[dimensionless]
$\bar{h}_i$	Height of each horizontal grid element of the layer	–	[m]
k	Elementary column part of the layer i	–	–
ε	Maximum amplitude of the anomaly	0.1 and 0.2	[dimensionless]
σ	Spatial extent or steepness of the anomaly	4	[dimensionless]
r	Radial distance from the anomaly center	–	[m]
ϕ	Latitudes, also referred to as Lat	$[-\pi/2, \pi/2]$	[radians]
ϑ	Longitudes, also referred to as Lon	$[0, 2\pi]$	[radians]
$\mathbb{D}$	Domain, (latitudes x longitudes)	–	–
χ	Haversine formula, $\chi(Lat_0, Lon_0, Lat, Lon)$	–	[radians]
$\Gamma(x)$	Gamma function	–	–
$G(r, \xi)$	Function to shape the spatial structure of the anomaly	–	–
ξ	Shape parameter	–	[dimensionless]

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