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Multivariate frequency analysis of threshold-defined streamflow droughts: a mixed-tail framework for duration, severity, and minimum-flow magnitude

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Abstract. Threshold-defined streamflow droughts are commonly described by duration, deficit volume, and minimum-flow magnitude, but these characteristics pose a specific mixed-tail problem for multivariate frequency analysis. Duration and deficit volume increase with drought extremity, whereas minimum-flow magnitude decreases. A consistent trivariate return-period framework for these event characteristics therefore cannot be formulated as a simple upper-tail exceedance problem. Here, we develop an event-based multivariate frequency framework that jointly models drought duration (D), deficit volume or severity (S), and minimum-flow magnitude (M) within Yevjevich's threshold approach. Annual event series are constructed by selecting, in each water year, the drought with the largest deficit volume and extracting D, S, and the minimum discharge M for the same event. Marginal distributions are represented by zero-augmented Weibull models where applicable, and dependence is modelled using a regular vine copula. Analytical AND and OR probabilities are formulated for the resulting mixed upper-/lower-tail setting, representing compound exceedance and at-least-one exceedance risk, respectively. The framework is applied to 46 years of streamflow observations from 235 Austrian catchments, with a focus on the 2003 and 2015 droughts. The analysis provides strengthened trivariate evidence of compound severity in 2003, with 31 % of stations showing $T_{\mathrm{AND}} > 50$ years, whereas 2015 exhibits fewer compound extremes but localized northern hotspots. Non-parametric bootstrap intervals show substantially wider uncertainty for rare AND events than for OR events, reflecting limited information in the joint tail. The multivariate assessment re-ranks events relative to univariate analyses and identifies compound drought hotspots that are not apparent from individual characteristics alone. The framework provides a transferable basis for mixed-tail frequency analysis of streamflow droughts and other threshold-defined environmental extremes.

1 Introduction

Hydrological drought is a sustained period of anomalously low streamflow, commonly defined relative to a threshold (Yevjevich, 1967), and characterised by multiple attributes including duration, deficit volume (severity), and minimum-flow magnitude

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(e.g. Van Loon and Van Lanen, 2012; Van Loon, 2015; Tallaksen and Van Lanen, 2023). These attributes are directly related to impacts: ecological stress is often governed by the persistence and depth of low flows, whereas water-supply deficits depend on cumulative volume shortfalls (Laaha et al., 2017). With increasing drought pressure under climatic and anthropogenic change, operational monitoring and planning frameworks require drought-risk metrics that represent these attributes consistently and jointly (Van Loon et al., 2016).

Frequency analysis provides a formal basis for quantifying the probability and return period of extreme events, but univariate approaches cannot capture dependence among drought attributes. Two complementary bivariate lines of work have emerged. First, copulas have advanced bivariate drought and flood analyses by separating marginal distributions from dependence structure (e.g. Klein et al., 2011; Mirabbasi et al., 2012), thereby accommodating heterogeneous marginals and tail dependence. Second, for threshold-defined hydrological droughts, Jakubowski (2006) introduced an analytical bivariate Generalized Pareto distribution for threshold-defined events, jointly modelling duration and deficit volume. Both approaches consistently reveal strong duration–severity dependence and provide rigorous bases for bivariate frequency analysis.

Streamflow droughts, however, are intrinsically multivariate. Duration, severity, and minimum-flow magnitude are jointly relevant for impact assessment, but they are not equivalent in their direction of extremeness. Duration and deficit volume become more extreme in the upper tail, whereas minimum-flow magnitude becomes more extreme in the lower tail. Consequently, a complete trivariate assessment of streamflow drought events requires a mixed-tail formulation. Bivariate models can miss compound behaviour that matters for decisions, such as events that are simultaneously long, severe, and characterized by very low minimum flow.

Extending drought frequency analysis to higher dimensions requires flexible dependence models. Vine copulas provide a modular framework for representing pairwise and conditional dependence with heterogeneous marginals and potentially asymmetric tail behaviour. For floods, Fischer and Schumann (2021) developed multivariate frequency models using vine copulas, demonstrating their flexibility for tail-dependent hydrological extremes. For droughts, Mirabbasi et al. (2024) applied a vine-copula framework to derive drought type curves using the Joint Deficit Index, highlighting strong severity–duration dependence and moderate links to the time between arrivals.

Within multivariate frequency analysis, AND and OR operators provide complementary return-period concepts. Salvadori and De Michele (2004) introduced these concepts for bivariate hydrological events, and related formulations have since been used in hydrological risk assessment (Klein et al., 2011). The AND operator characterizes the rarity of simultaneous exceedance in all relevant dimensions, whereas the OR operator characterizes the rarity of exceeding at least one critical dimension. More recently, Firoozi (2024) used a copula-based approach to estimate joint probabilities of drought characteristics under AND and OR conditions for duration and severity in non-stationary hydro-meteorological drought events. For streamflow droughts involving minimum-flow magnitude, however, these operators must be adapted to the mixed upper-/lower-tail structure of the drought characteristics.

Despite these developments, a complete trivariate frequency framework for threshold-defined streamflow drought events remains lacking. Existing multivariate drought studies have mainly focused on pairs of characteristics, most often duration and severity, or have treated multivariate drought variables without explicitly addressing the mixed-tail structure of streamflow

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drought events. In the Yevjevich framework, drought duration and deficit volume increase with drought extremity, whereas minimum-flow magnitude decreases. Consequently, a trivariate analysis of duration, severity, and minimum-flow magnitude cannot be formulated as a standard upper-tail exceedance problem. Return-period definitions must consistently combine upper-tail exceedances for D and S with lower-tail exceedances for M.

Here, we develop and apply a mixed-tail trivariate frequency framework for threshold-defined streamflow droughts. We do not introduce vine copulas or AND/OR return periods as new generic statistical concepts; rather, we use them to formulate a hydrologically consistent event-based framework for the three canonical drought characteristics. Specifically, we aim to: (i) construct annual drought-event vectors by selecting physically identical events using deficit volume and extracting duration, severity, and minimum-flow magnitude; (ii) model zero-inflated, nonnegative drought characteristics using suitable marginal distributions; (iii) represent dependence among D, S, and M using a regular vine copula; (iv) formulate analytical AND and OR return periods for the mixed upper-/lower-tail setting; and (v) evaluate the resulting spatial patterns, event rankings, and bootstrap uncertainty for 46 years of observations from 235 Austrian catchments, focusing on the 2003 and 2015 droughts.

2 Methods

2.1 Threshold-defined drought events and event characteristics

A streamflow drought event is defined as a dry spell in the flow record when discharge is below a given threshold (Yevjevich, 1967). We use a constant low-flow threshold corresponding to the flow exceeded on 80 % of days, following conventional practice in low-flow hydrology (Laaha et al., 2017). Events are pooled using the Sequent Peak Algorithm (SPA; Tallaksen et al., 1997; Vogel and Fennessey, 1993) to avoid artificially interrupted events caused by minor rainfall or operational disturbances. The SPA concept is based on depletion and recovery of the storage required to sustain the threshold discharge; two droughts are pooled if the catchment store has not fully recovered when the subsequent drought episode begins.

From the resulting drought-event series, we select in each water year the event with the largest deficit volume (severity), forming an annual series of severity-selected drought events (Laaha et al., 2017). Duration D (days), severity S (m^3), and minimum-flow magnitude M ($\mathrm{m}^3\,\mathrm{s}^{-1}$) are then extracted for this same event. This ensures that the trivariate vector (D, S, M) represents a physically identical drought episode, rather than combining annual extremes of different characteristics that may have occurred during different events. For the selected annual drought event, M is defined as the minimum discharge observed during that event.

For multi-annual events, we assign all characteristics to the last water year of the event and set the corresponding values in the preceding years to `NA`; this avoids double counting and helps preserve the independence of annual events, consistent with the i.i.d. assumption.

All three characteristics are nonnegative with support $[0, \infty)$ and no finite upper bound. In classical frequency analysis, each characteristic is treated individually and modelled as an i.i.d. random variable with distribution function F. We use Weibull-type marginal models because the variables are nonnegative, right-skewed, and bounded below. For M, the choice is also consistent with the Weibull-type extreme-value limit for minima. For D and S, it provides a parsimonious model for

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positive, right-skewed drought-event characteristics. Weibull parameters are estimated by L-moments (Hosking, 1990). For a characteristic $X \in \{D,S,M\}$, the cumulative distribution function (CDF) is

$$G_X(x) = 1 - \exp\left\{ - \left(\frac{x-\mu_X}{\sigma_X} \right)^{k_X} \right\}, \tag{1}$$

where $k_X > 0$ is the shape, $\sigma_X > 0$ is the scale, and $\mu_X \geq 0$ is the location parameter. The distribution G_X has support $[\mu_X, \infty)$, so $G_X(x) = 0$ for $x < \mu_X$.

Annual drought-event series can contain zero values. For D and S, zeros refer to wet years when streamflow remains above the drought threshold, i.e. years without a drought event. For M, zero flows can occur in intermittent or ephemeral rivers. We model this with a mixture comprising a point mass at zero and a continuous two-parameter Weibull component, with location set to $\mu_X = 0$, fitted to the positive observations ($x > 0$):

$$F_X(x) = \begin{cases} p_0, & x = 0, \\ p_0 + (1-p_0)\,G_X(x), & x > 0, \end{cases} \tag{2}$$

where F_X denotes the mixture CDF for characteristic X, G_X denotes the Weibull CDF fitted to the positive portion, and p_0 denotes the empirical relative frequency of zero values in the annual event series. Weibull parameters for the continuous component are estimated by L-moments computed on the positive subsample (Hosking, 1990). Throughout, probability-scale transforms use the full marginals F_X; in particular, for $x = 0$ we have $F_X(0) = p_0$, so the copula-scale variable exhibits a jump of size p_0 at zero.

2.2 Multivariate frequency analysis

2.2.1 Trivariate drought distribution

Let $X = (D,S,M)$ denote drought duration, severity, and minimum-flow magnitude with marginal CDFs F_D, F_S, and F_M as specified above. Define the probability-scale transforms $u = F_D(d)$, $v = F_S(s)$, and $w = F_M(m)$. Following Sklar's theorem, the joint CDF of X can be written as

$$F_{DSM}(d,s,m) = C(u,v,w), \tag{3}$$

where C is a trivariate copula that captures dependence among the margins. Its bivariate margins are obtained by fixing one argument to 1, e.g. $C(u,1,w)$, $C(1,v,w)$, and $C(u,v,1)$.

Throughout, probability statements are expressed on a common drought-extremity scale. For duration and severity we use upper-tail exceedance probabilities $1-u = P(D > d)$ and $1-v = P(S > s)$, whereas for minimum-flow magnitude we use the lower-tail probability $w = P(M \leq m)$. This convention ensures that smaller tail probabilities correspond to more extreme drought conditions for all three characteristics.

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2.2.2 Vine-copula model

To represent dependence among D, S, and M, we specify a regular vine (R-vine) copula on the probability scale under the simplifying-vine assumption. In three dimensions, we use unconditional pairs (D,S) and (S,M) and the conditional pair $(D,M\,|\,S)$.

Estimation proceeds on pseudo-observations (u_i, v_i, w_i) derived from F_X or from empirical ranks, by maximum likelihood under the simplifying-vine assumption. Vine structure and pair-copula families (Independent, Gumbel, Clayton, Student-t, Frank, Joe, Gaussian) are selected by AIC. Simulation and CDF evaluations use the fitted vine copula together with the marginal quantile functions F_X^{-1}. The probabilities in Sect. 2.2.3 are computed from the fitted $C(u,v,w)$ and its bivariate margins.

2.2.3 Mixed-tail AND/OR probabilities and return periods

We consider two drought hazard definitions based on logical operators (Klein et al., 2011), adapted here to the mixed-tail structure of streamflow drought characteristics. Duration and severity are upper-tail variables, whereas minimum-flow magnitude is a lower-tail variable. The AND event occurs when all drought characteristics are beyond their drought-relevant thresholds:

$$\{D > d,\ S > s,\ M \le m\}, \tag{4}$$

and the OR event occurs when at least one characteristic exceeds its drought-relevant threshold:

$$\{D > d \text{ or } S > s \text{ or } M \le m\}. \tag{5}$$

Using the trivariate copula C and its implied bivariate margins, the corresponding probabilities are

$$\begin{aligned} p_{\mathrm{AND}}(d,s,m) &= P(D > d, S > s, M \le m) \\ &= w - C(u,1,w) - C(1,v,w) + C(u,v,w), \end{aligned} \tag{6}$$

and

$$\begin{aligned} p_{\mathrm{OR}}(d,s,m) &= P(D > d \ \vee\ S > s \ \vee\ M \le m) \\ &= 1 - C(u,v,1) + C(u,v,w). \end{aligned} \tag{7}$$

These expressions follow from the inclusion–exclusion principle applied to the events $\{D > d\}$, $\{S > s\}$, and $\{M \le m\}$, together with Sklar's theorem. They are valid for any trivariate copula C and ensure probabilities in $[0,1]$. For annual event series with mean interarrival time of one year, the associated return periods are

$$T_{\mathrm{AND}}(d,s,m) = \frac{1}{p_{\mathrm{AND}}(d,s,m)} \tag{8}$$

and

$$T_{\mathrm{OR}}(d,s,m) = \frac{1}{p_{\mathrm{OR}}(d,s,m)}. \tag{9}$$

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It follows that $T_{\mathrm{AND}} \geq \max(T_D, T_S, T_M)$ and $T_{\mathrm{OR}} \leq \min(T_D, T_S, T_M)$, indicating that the AND event is rarer than any individual event, whereas the OR event is more common. In practice, p_{AND} and p_{OR} are obtained either by direct evaluation of $C(u,v,w)$ and its margins or by Monte Carlo approximation on the copula scale from the fitted vine copula.

145 #### 2.2.4 Empirical probability estimators

For an empirical, model-agnostic assessment, we combine empirical marginals with an empirical copula. Given a sample of size n, let r_D, r_S, and r_M be the ascending ranks of d, s, and m. Define the pseudo-observations as

$$\hat{u} = \frac{r_D}{n+1}, \qquad \hat{v} = \frac{r_S}{n+1}, \qquad \hat{w} = \frac{r_M}{n+1}. \tag{10}$$

Ties, for example at zero, are handled by mid-ranks within the tied block before computing the pseudo-observations. Let C_n
150 denote the empirical copula computed from the pseudo-observations of all years; its lower-dimensional margins are given by $C_n(u,1,w)$, $C_n(1,v,w)$, and $C_n(u,v,1)$. The empirical analogues of the theoretical tail probabilities are

$$\hat{p}_{\mathrm{AND}} = \hat{w} - C_n(\hat{u},1,\hat{w}) - C_n(1,\hat{v},\hat{w}) + C_n(\hat{u},\hat{v},\hat{w}), \tag{11}$$

and

$$\hat{p}_{\mathrm{OR}} = 1 - C_n(\hat{u},\hat{v},1) + C_n(\hat{u},\hat{v},\hat{w}). \tag{12}$$

155 These empirical estimates are primarily used for visualisation and for comparing observed trivariate tail behaviour to the fitted vine-copula model, as they are more susceptible to sampling variability than parametric estimates. Marginal fitting and event characterisation are implemented in the R package `lfstat`, and dependence modelling, simulation, and empirical copula calculations are implemented in `vineCopula`.

2.2.5 Diagnostics relative to univariate return periods

160 Univariate and multivariate frequency analyses answer different questions. Univariate return periods quantify the rarity of a single drought characteristic, whereas T_{AND} and T_{OR} quantify the rarity of compound or at-least-one exceedance events under the fitted trivariate dependence model. We therefore do not interpret differences between univariate and multivariate return periods as validation errors or accuracy gains. Instead, we use them as diagnostics to quantify how strongly the assessment of drought rarity changes when dependence among D, S, and M is taken into account.

165 For the 2003 drought event, we compare univariate return periods for each drought characteristic with the corresponding multivariate return periods. The signed relative deviation is defined as

$$RD = \frac{T_{\mathrm{uni}} - T_{\mathrm{multi}}}{T_{\mathrm{multi}}}, \tag{13}$$

where T_{uni} denotes the univariate return period of a given characteristic and T_{multi} denotes either T_{AND} or T_{OR} for the same event. Positive values indicate that the univariate return period is larger than the multivariate return period, whereas negative

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values indicate the opposite. The relative absolute deviation,

$$RAD = \frac{|T_{\mathrm{uni}} - T_{\mathrm{multi}}|}{T_{\mathrm{multi}}}, \tag{14}$$

summarizes the magnitude of the discrepancy independent of sign.

2.2.6 Uncertainty estimation via non-parametric bootstrap

Sampling and parameter uncertainty were quantified separately at each station using a non-parametric annual bootstrap with $B = 10000$ replicates. For each replicate, the complete annual vectors (D_i, S_i, M_i) were resampled with replacement. Resampling the vector as one unit preserves the observed cross-variable association within each hydrological year.

The marginal distributions, including the zero probability $p_{0,X}$, were re-estimated from every bootstrap sample. The observed target characteristics (d_0, s_0, m_0) for 2003 or 2015 were kept fixed and transformed using the re-fitted marginal distributions. Mid-rank pseudo-observations were then calculated from the resampled annual vectors. The R-vine structure and all pair-copula families selected from the original record were held fixed, while the copula parameters were re-estimated by maximum likelihood in every successful replicate. Consequently, copula parameter estimates vary across bootstrap samples, but the resulting intervals are conditional on the originally selected structure and families and do not include model-selection uncertainty.

For each successful replicate b, $p^{(b)}_{\mathrm{AND}}$, $p^{(b)}_{\mathrm{OR}}$, $T^{(b)}_{\mathrm{AND}}$, and $T^{(b)}_{\mathrm{OR}}$ were recomputed. The 5th and 95th percentiles of the successful bootstrap return periods define the 90 % percentile interval, and the 50th percentile was retained as the bootstrap median. The number of successful fits was recorded for every station. An upper interval endpoint reaching the Monte Carlo return-period bound was flagged as right-censored.

3 Data

We evaluate the proposed multivariate frequency framework using Austrian streamflow records. The dataset comprises daily discharge from 235 near-natural gauging stations across Austria that have been widely used in low-flow and drought studies (Laaha, 2023a, b; Laaha and Blöschl, 2006a; Laimighofer and Laaha, 2022). For the present analysis we consider 46 hydrological years and construct annual series for water years defined from April to March (Laaha et al., 2017). Catchment areas range from 7 to 7,000 km^2, and mean annual low-flow discharge spans 0.03–1,013 $\mathrm{m}^3\,\mathrm{s}^{-1}$. Austria's hydro-climate varies from rain-dominated regimes in the forelands and pre-Alps to snow-dominated regimes in the Alps, providing a broad spectrum of low-flow seasonality.

To illustrate model behaviour, we focus on three exemplar basins that span contrasting regimes and seasonal low-flow mixes (mixed, winter, summer). Two stations were part of Laaha (2023a) in the context of mixed distributions for minimum low flow, and one was used by Laaha et al. (2017) to contrast the 2003 and 2015 droughts. Figure 1 shows the national network and the exemplar stations; Table 1 summarises site characteristics and dependence diagnostics. In line with Sect. 2.1, annual series are formed by selecting in each water year the drought with the largest deficit volume and recording its duration (D), severity (S), and minimum-flow magnitude (M). Years without a drought event are represented by zeros for D and S and by $M = 0$ in

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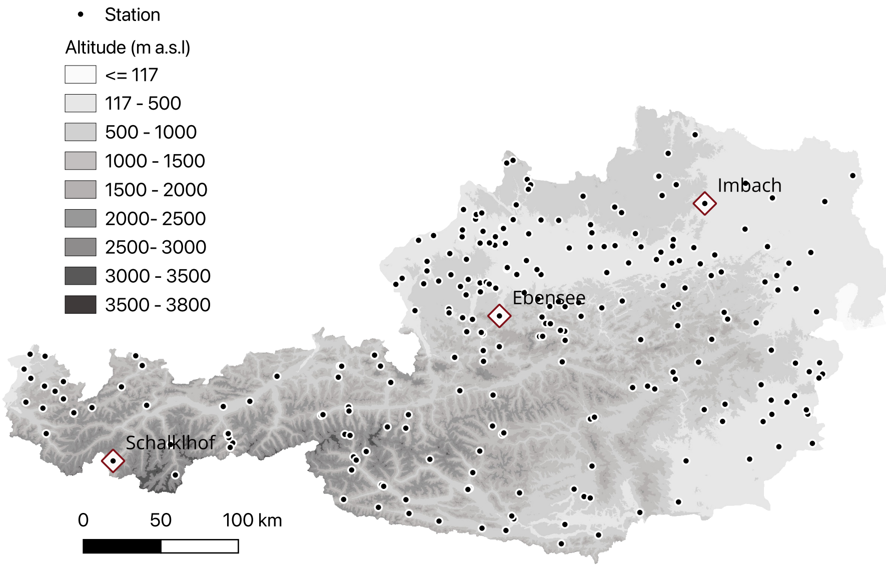

Figure 1. Stream-gauge network in Austria. Points indicate the 235 near-natural stations used for the national analysis; red squares mark the three exemplar stations used for illustration and diagnostics.

Table 1. Exemplar stations used for vine-copula model evaluation. Reported are latitude/longitude, Kendall's τ for the bivariate pairs $\tau(D,S)$, $\tau(S,M)$, $\tau(D,M)$, Kendall's τ for the conditional association $\tau(M,D|S)$, and catchment area.

Catchment	River	Area (km^2)	Altitude (m.a.s.l)	τ (D,S)	τ (S,M)	τ (D,M)	τ $(M,D\,|\,S)$
Ebensee	Langbathbach	40.0	431.17	0.81	-0.72	-0.58	0.17
Schalklhof	Schalklbach	108.0	982.19	0.79	-0.71	-0.57	0.34
Imbach	Krems	305.9	230.59	0.89	-0.80	-0.73	0.18

intermittent rivers, consistent with the mixed marginal modelling. Consistent with the marginal analysis, the exemplar basins differ in the fraction of no-drought years: Ebensee shows a drought almost every year, whereas Schalklhof and Imbach display about 10 % and 25 % no-drought years, respectively, reflecting regime and storage characteristics.

4 Results

4.1 Marginal behaviour and dependence structure at exemplar stations

We first assess marginal behaviour at the three exemplar stations before turning to dependence and joint probabilities. Figure 2 shows empirical cumulative distribution functions (CDFs) for duration (D), severity (S), and minimum-flow magnitude (M) together with fitted Weibull marginals.

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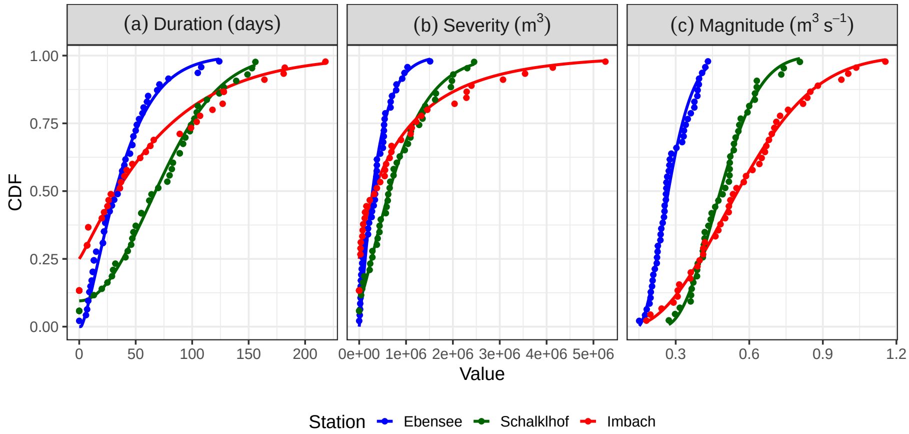

Figure 2. Marginal distributions at the three exemplar stations: (a) duration D (days), (b) severity S (m^3), and (c) minimum-flow magnitude M (m^3 s^{-1}). Lines show fitted Weibull marginals; points show empirical CDFs. Station order in text: Ebensee (blue), Schalklhof (green), Imbach (red).

At Ebensee (Langbathbach; mixed alpine/summer regime, small catchment), the Weibull model provides a good fit for all three characteristics (Fig. 2). A slight discrepancy appears in the upper tail of M, implying a mild overestimation of high minimum flows, i.e. of the least severe low-flow conditions, while probabilities in the range relevant for drought extremes are well captured.

At Schalklhof (Schalklbach; medium-sized, winter-dominated regime), the fitted Weibull distributions closely follow the empirical CDFs for D, S, and M. This supports the suitability of the Weibull family for drought characteristics that behave as upper-tail variables (D, S) and as lower-tail variables (M).

At Imbach (Krems; larger, summer-dominated regime), the Weibull marginals also provide a good overall description. Minor deviations are visible in the upper tails of D and S, but the fits remain adequate for frequency analysis.

A synoptic comparison across regimes reveals systematic differences in spread and behaviour near the origin. Ebensee exhibits the steepest CDFs, Imbach the flattest, and Schalklhof lies in between, reflecting contrasting low-flow regimes. Ebensee is continuous at the origin, indicating a drought event in virtually every water year. In contrast, Schalklhof and Imbach display a point mass at zero, representing years without a drought event, which is captured by the zero-augmented Weibull formulation.

Figure 3 illustrates the evaluation of the vine-copula model based on 1,000 simulated samples generated from the selected pair-copula families at the Ebensee, Schalklhof, and Imbach gauges. For each pair of variables, random realizations were drawn from the fitted copulas and transformed to the original variable scale through the corresponding marginal quantile functions. Grey points represent simulations, whereas black points denote observations.

At Ebensee (Fig. 3a–c), duration and severity show strong positive dependence, while severity and minimum-flow magnitude show negative dependence. These patterns reflect the combined influence of drought persistence and flow minima in a small catchment with mixed seasonal controls. In the selected vine, the main dependencies are represented by Clayton and Student-t

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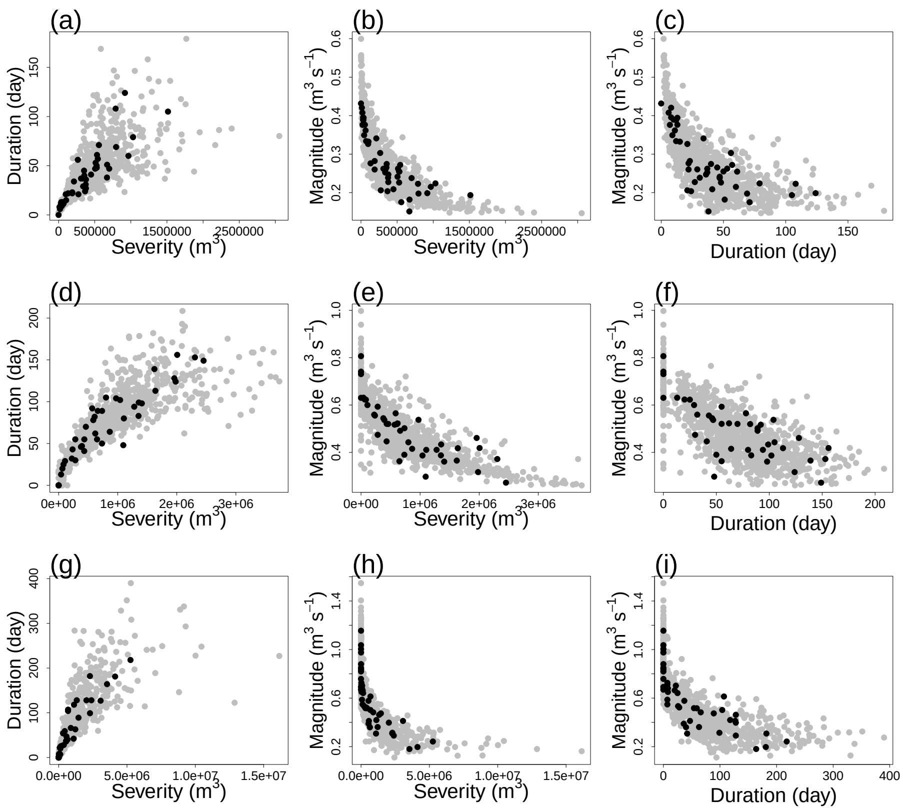

Figure 3. Observed drought-event characteristics and 1,000 samples simulated from the fitted vine copula at the three exemplar stations: Ebensee (a–c), Schalklhof (d–f), and Imbach (g–i). Grey points show simulated values; black points show observations.

copula families, while the conditional dependence between duration and magnitude given severity is captured by a Clayton copula.

At Schalklhof (Fig. 3d–f), the duration–severity and severity–magnitude relationships are also pronounced. The selected copula families indicate strong dependence in the first tree and remaining conditional dependence between duration and minimum-flow magnitude after conditioning on severity. This behaviour is consistent with the winter-dominated regime, where snow storage and melt processes can moderate the relationship between drought duration and flow minima.

At Imbach (Fig. 3g–i), the duration–severity pair exhibits strong dependence, consistent with prolonged summer low-flow periods and cumulative precipitation deficits. The severity–magnitude dependence is represented by a Frank copula, and the conditional dependence between duration and magnitude given severity is weak and approximately symmetric. This suggests that severity acts as a mediating variable, absorbing much of the shared variability between duration and minimum-flow magnitude.

Overall, the exemplar stations show that drought dependence structures differ across seasonal regimes. The fitted vine copulas reproduce the main observed dependence patterns and provide a flexible basis for estimating mixed-tail trivariate drought probabilities.

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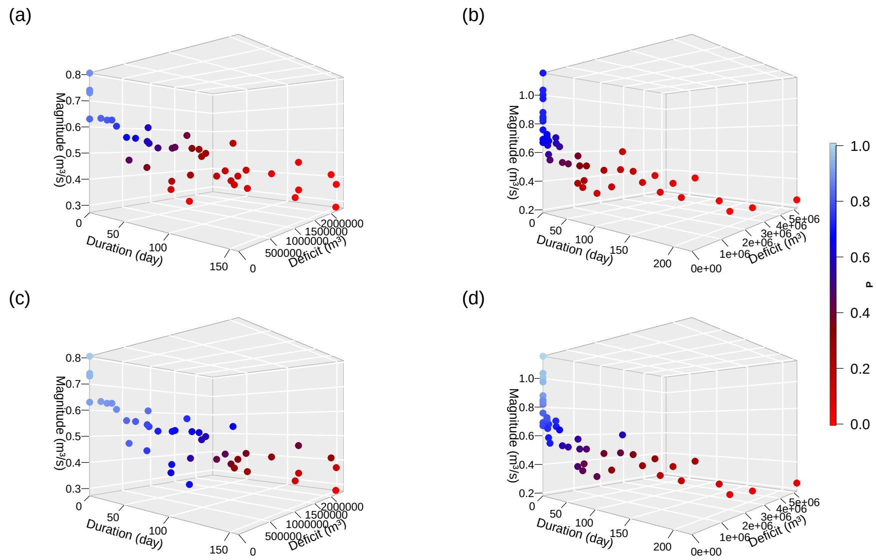

Figure 4. Joint probability plots for drought duration, severity, and minimum-flow magnitude for Schalklhof (a, c) and Imbach (b, d). Points show empirical probabilities; colours show joint probability estimates using AND (a, b) and OR (c, d).

We estimated p_{AND} and p_{OR} at Schalklhof and Imbach using the mixed-tail probability expressions. Figure 4 illustrates joint probabilities for drought duration, severity, and minimum-flow magnitude. Individual drought events are plotted as coloured points representing their empirical joint event probabilities, with the background field indicating fitted joint probability surfaces for the AND and OR operators.

For Schalklhof (Fig. 4a,c), several events with different combinations of duration, severity, and minimum-flow magnitude exhibit similar joint probabilities. For example, the 1977 event lasted 124 days with a severity of 1,620,805 m^3, whereas the 1985 event lasted 139 days with a severity of 2,307,117 m^3, and the 2006 drought event had a shorter duration of 42 days but a severe deficit of 1,094,353 m^3 and the lowest magnitude. These events correspond to similar joint occurrence probabilities, indicating that multivariate drought rarity is governed by the dependence structure and the combined drought characteristics rather than by a single variable alone.

For Imbach (Fig. 4b,d), the 2003 and 2015 events illustrate the role of duration in differentiating events with similar severity and minimum-flow magnitude. Although the 2003 and 2015 events had similar severity and magnitude, the joint probability of the 2003 drought is lower because of its longer duration. The OR probabilities show the expected less restrictive behaviour, while preserving the relative influence of the drought-event characteristics.

4.2 Nationwide dependence structure and multivariate return periods

We analysed data from 235 Austrian catchments to estimate the joint probabilities p_{OR} and p_{AND} for the 2003 and 2015 drought events and derived the corresponding return periods. Figure 5 illustrates the spatial distribution of Kendall's τ between

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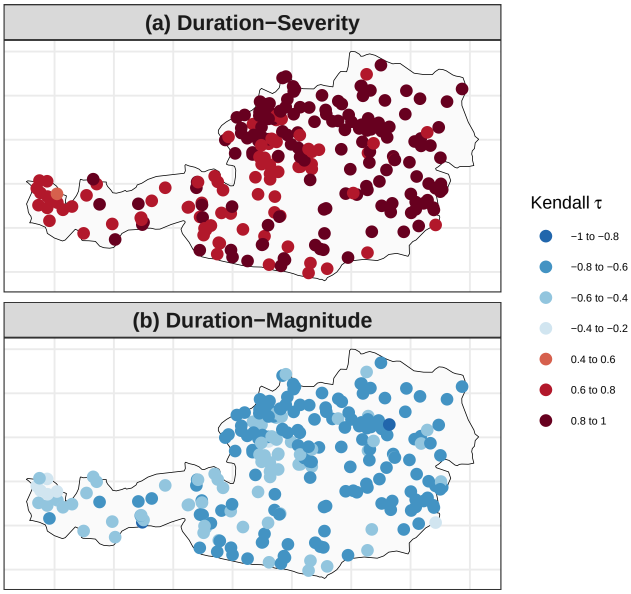

Figure 5. Spatial distribution of Kendall's τ correlation coefficients between drought-event characteristics in Austrian catchments: (a) duration versus severity and (b) duration versus minimum-flow magnitude. Correlations are statistically significant at the 5 % level for all stations.

drought-event characteristics. Duration and severity are positively associated across the network, reflecting the expected link between longer drought episodes and larger cumulative deficits. In contrast, duration and minimum-flow magnitude are generally negatively associated, because longer droughts tend to coincide with lower minimum flows.

The negative association between duration and minimum-flow magnitude is generally weaker in Alpine catchments than in other regions, reflecting the influence of pronounced winter seasonality and snow storage. In south-western Austria, snow-dominated regimes, rapid runoff response, and short but intense low-flow periods can produce similar annual minimum flows across years with markedly different durations and severities. This reduces the dependence between drought duration and minimum-flow magnitude.

The joint probabilities for the 235 Austrian catchments were estimated using the fitted vine-copula framework. Table 2 presents the share of copula families selected across the first and second vine trees based on AIC. In Tree 1, Clayton is prevalent for the duration–severity pair, indicating strong dependence between prolonged low-flow periods and cumulative deficits. For severity–magnitude pairs, Gaussian, Student-t, and Frank copulas are frequently selected, indicating more heterogeneous and often more symmetric dependence structures.

In Tree 2, the selected copula families describe the conditional dependence between duration and minimum-flow magnitude given severity. The frequent selection of the Clayton copula suggests that conditional tail dependence remains important for a large fraction of catchments. The occurrence of Gaussian, Student-t, Frank, and independence copulas in the remaining sites further indicates spatial heterogeneity in the strength and symmetry of conditional dependence across Austrian catchments.

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Table 2. Share of selected copula families in the simplifying vine across 235 Austrian catchments. Entries are proportions (0–1) of stations selecting each family by AIC for the indicated pair; values smaller than 0.01 are shown as "< 0.01".

Pair	Gumbel	Clayton	Indep.	Gauss.	Student-t	Frank	Joe
Tree 1 (D, S)	< 0.01	0.69	0	0.16	0.15	0.02	0
Tree 1 (S, M)	0	0	0	0.43	0.29	0.28	0
Tree 2 $(D, M \mid S)$	0.05	0.48	0.14	0.14	0.03	0.13	< 0.01

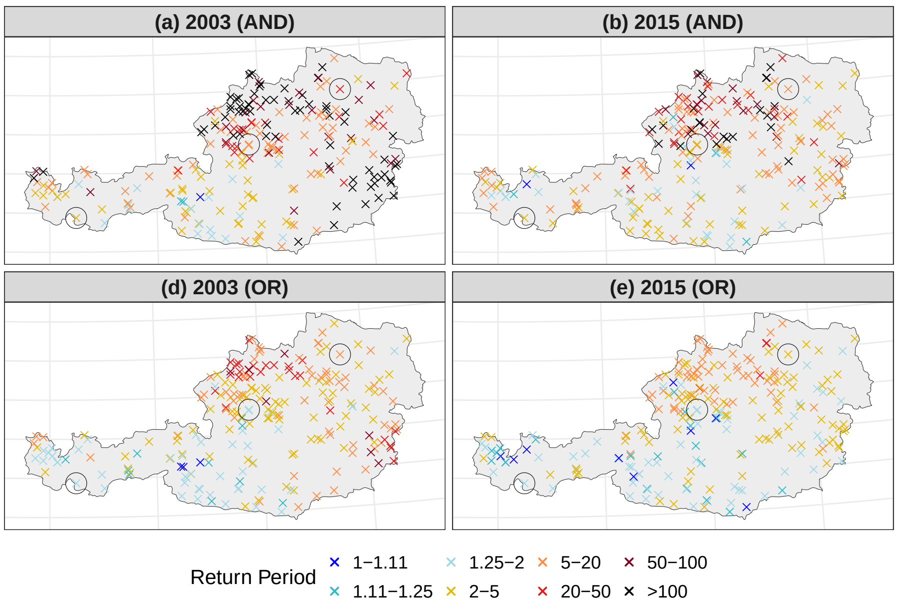

Figure 6. Multivariate return periods for the 2003 and 2015 droughts in Austrian catchments. Left panels show AND return periods (a, c); right panels show OR return periods (b, d).

We estimated return periods for the 2003 and 2015 droughts using the mixed-tail AND and OR probability framework based on drought duration, severity, and minimum-flow magnitude. Figure 6 maps the resulting return periods across Austria, while Fig. A1 presents the corresponding univariate return periods for the two events. Under the AND definition, the 2003 drought shows widespread compound severity, with 31 % of stations exceeding $T_{\mathrm{AND}} > 50$ years. The largest return periods occur mainly in northern, north-western, and eastern Austria, whereas Alpine catchments generally show shorter return periods. The 2015 drought exhibits fewer compound extremes at the national scale but includes localized hotspots, particularly in northern Austria.

Under the OR definition, return periods are shorter, as expected for an at-least-one exceedance event. The OR return period therefore provides a broader screening perspective rather than a measure of simultaneous compound rarity. Together, the AND and OR maps distinguish widespread compound severity in 2003 from the more localized multivariate drought signal in 2015.

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Table 3. Nationwide summary of return-period estimates and their 90 % bootstrap uncertainty for the 2003 and 2015 drought events.

Year	Definition	Stations	Median T	Median 90 % CI width	Q_1 of 90 % CI width	Q_3 of 90 % CI width	Censored upper CI n (%)
2003	AND	231	15.9	15.8	2.5	140.0	49 (21.2 %)
2003	OR	231	3.2	2.0	0.7	12.6	1 (0.4 %)
2015	AND	235	6.8	7.4	1.8	51.8	18 (7 %)
2015	OR	235	2.3	1.2	0.6	3.8	0 (0.0 %)

T denotes the station-specific return-period point estimate. CI width was calculated as the difference between the upper and lower limits of the station-specific 90 % bootstrap interval. Q_1 and Q_3 denote the 25th and 75th percentiles of CI width across stations. Stations with censored upper endpoints were excluded from the CI-width summaries and are reported separately.

Table 4. Relative absolute deviation (RAD, in %) between univariate and multivariate return periods for the 2003 drought event. Reported are summary statistics across all stations (Min, Q1, Median, Q3, Max).

Drought characteristic	Target	Min	Q1	Median	Q3	Max
Magnitude (M)	T_{AND}	0.00	21.10	43.50	75.96	98.99
Magnitude (M)	T_{OR}	0.00	6.20	22.50	126.50	7,599,900.00
Severity (S)	T_{AND}	0.50	23.76	53.86	77.34	97.87
Severity (S)	T_{OR}	0.00	23.49	50.08	90.21	993.04
Duration (D)	T_{AND}	0.30	21.10	49.32	75.48	252.64
Duration (D)	T_{OR}	0.00	9.53	35.13	101.32	3,259.72

Bootstrap uncertainty estimates provide additional information on the robustness of these national-scale return-period patterns. Table 3 summarizes the station-wise return-period estimates and 90 % bootstrap intervals for the 2003 and 2015 events. The number of stations differs from the full network where bootstrap estimation or event evaluation was not successful for all stations. The median return period is larger for AND than for OR in both years, consistent with the greater rarity of simultaneous compound exceedance. Uncertainty is also substantially wider for AND-based return periods, particularly in 2003, reflecting the limited information available in the joint tail of the 46-year annual event series. Censored upper interval endpoints occur mainly for AND-based estimates, whereas OR-based estimates show narrower intervals and almost no censoring.

4.3 Differences between univariate and multivariate return periods

Table 4 summarizes the relative absolute deviation (RAD, %) between univariate return periods and the corresponding multivariate AND and OR return periods for the 2003 drought event. These values quantify the magnitude of the difference between univariate and multivariate drought characterization; they are not interpreted as validation errors because the true return period is unknown. The results show that the discrepancy can be substantial, particularly for OR-based comparisons, where small multivariate return periods can lead to large relative deviations. This indicates that univariate analyses may provide a

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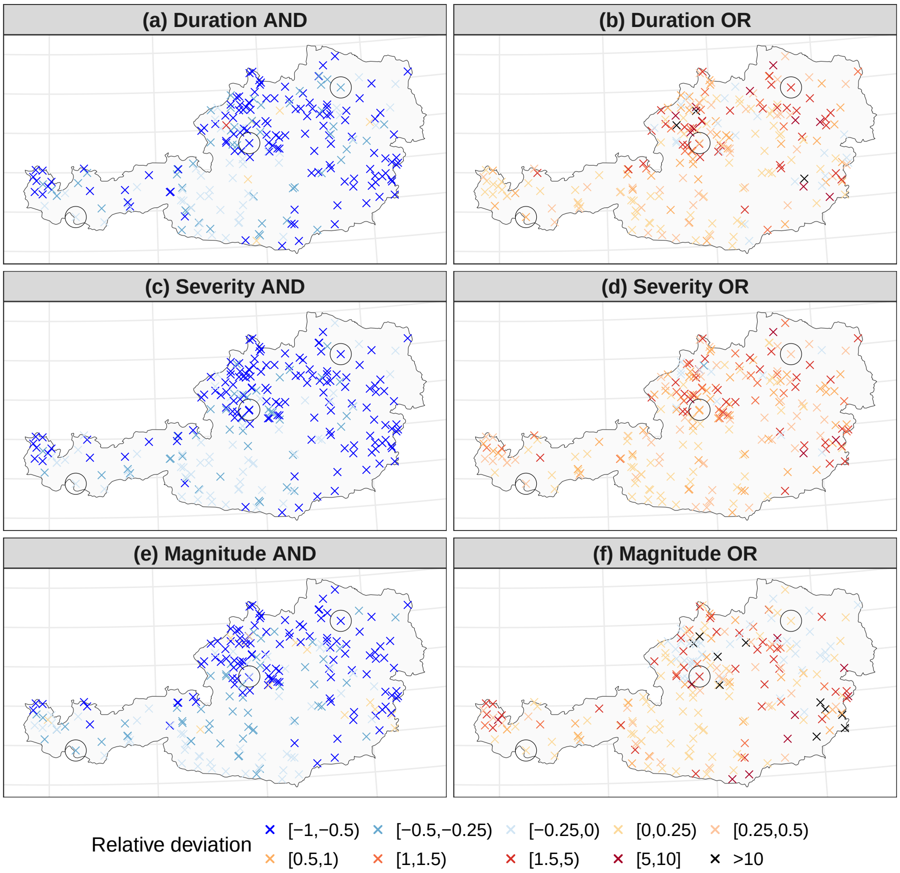

Figure 7. Relative deviation (RD) of 2003 univariate return periods for drought characteristics—duration (a, b), severity (c, d), and minimum-flow magnitude (e, f)—compared with multivariate analysis. Left panels show AND (a, c, e); right panels show OR (b, d, f) return periods.

markedly different assessment of event rarity than a trivariate analysis that accounts for dependence among duration, severity, and minimum-flow magnitude.

Figure 7 shows the signed relative deviation (RD) for the 2003 drought across the 235 catchments. For the AND definition, negative RD values indicate locations where the compound return period exceeds the corresponding univariate return period, consistent with the greater rarity of simultaneous exceedance in all drought dimensions. For the OR definition, positive RD values are expected where the at-least-one exceedance return period is smaller than the univariate return period. The spatial pattern of RD therefore highlights where accounting for the joint dependence structure most strongly changes the interpretation of drought rarity.

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Table 5. Drought characteristics and associated marginal and joint tail probabilities for the 2003 and 2015 events at the three exemplar stations. Duration D, severity S, and minimum-flow magnitude M are shown together with marginal tail probabilities and fitted AND/OR probabilities.

Station	Year	D	S	M	p_{D}	p_{S}	p_{M}	p_{AND}	p_{OR}
Ebensee	2003	24	366919	0.20	0.63	0.40	0.15	0.14	0.64
	2015	56	266265	0.30	0.22	0.52	0.65	0.21	0.72
Schalklhof	2003	78	586103	0.20	0.45	0.58	0.16	0.11	0.65
	2015	41	433292	0.30	0.80	0.70	0.66	0.53	0.89
Imbach	2003	182	2288727	0.30	0.01	0.04	0.07	0.01	0.68
	2015	99	2293765	0.31	0.27	0.04	0.07	0.04	0.78

4.4 Local uncertainty and interpretation of AND and OR return periods

Before interpreting the AND- and OR-based return periods at the local scale, we quantified their parameter-estimation uncertainty for the three exemplar catchments. The observed characteristics of each target event remained fixed and were evaluated using each set of bootstrap parameter estimates. The 5th and 95th percentiles of the resulting return-period distributions were used to construct 90 % uncertainty intervals.

Figures 8 and 9 show the resulting uncertainty intervals for the AND and OR definitions, respectively, at the Ebensee, Schalklhof, and Imbach catchments. The OR-based return periods generally exhibit relatively narrow uncertainty intervals. In contrast, the uncertainty intervals become substantially wider for rare AND events located in the tails of the joint distribution, and some upper interval limits are censored. This contrast is expected because the AND operator evaluates the simultaneous occurrence of all three extreme conditions, whereas the OR operator is governed by a less restrictive union event. It also reflects the limited amount of information available in the tails of the relatively short observational records. Because non-parametric bootstrap samples are restricted to the observed events, they cannot provide additional information beyond the empirical tail, resulting in unstable probability estimates for exceptionally rare event combinations.

To further elucidate the trivariate dependence structure, we projected two-dimensional contour surfaces for drought duration (D) and minimum-flow magnitude (M) using the fitted vine-copula models. Figure 10 illustrates the resulting probability isolines for the Ebensee, Schalklhof, and Imbach catchments. These visualizations clarify how drought extremity is distributed across the joint D–M space under the AND and OR frameworks.

In the Imbach catchment, the 2003 and 2015 drought events exhibit similar magnitudes but distinct durations (Table 5). The contour plots (Fig. 10e–f) indicate different joint probabilities for these events, confirming that duration acts as a key driver of multivariate extremity in this catchment even when magnitude is similar. Conversely, at Ebensee, the 2003 and 2015 events show marked differences in individual duration and magnitude characteristics (Table 5), but the contour surfaces assign approximately comparable joint probabilities. This convergence demonstrates that the multivariate dependence structure balances the contributions of individual characteristics to determine the overall joint hazard.

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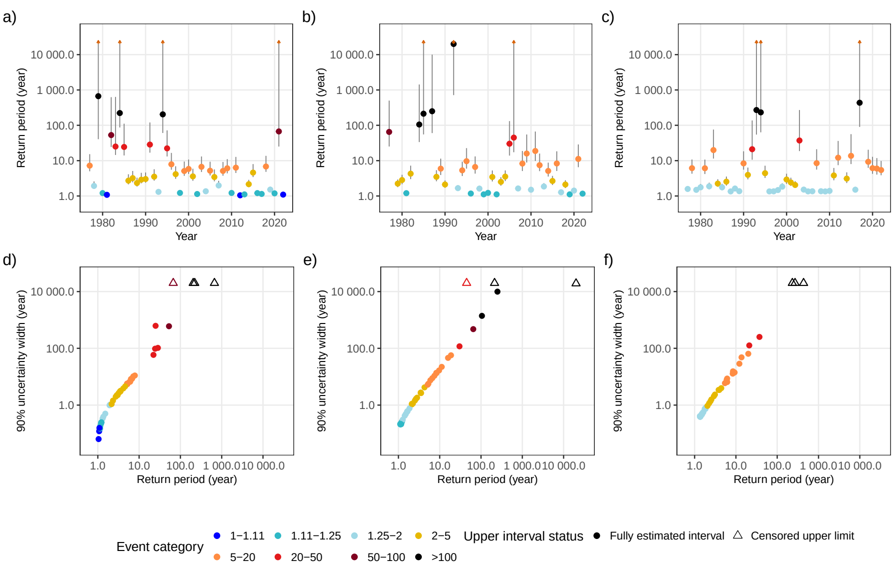

Figure 8. Annual copula-based AND return periods and their 90 % bootstrap uncertainty for the three exemplar catchments. Panels (a–c) show return-period estimates from the original observations together with 90 % bootstrap uncertainty intervals for each water year. Panels (d–f) show the corresponding uncertainty range, calculated as the difference between the upper and lower interval limits, as a function of the estimated return period. Panels (a, d), (b, e), and (c, f) correspond to Ebensee, Schalklhof, and Imbach, respectively. Colours indicate return-period categories. Triangles and arrows identify events for which the upper uncertainty limit exceeded the numerical estimation boundary and was treated as censored.

A similar convergence is observed at Schalklhof, where the 2003 and 2015 events differ in severity but have comparable joint probabilities (Fig. 10c–d). These examples show that multivariate drought rarity depends on the full dependence structure among duration, severity, and minimum-flow magnitude, rather than on any single characteristic alone. Overall, the contour plots illustrate how the trivariate framework modifies the interpretation of drought extremity relative to univariate assessments.

5 Discussion

The main contribution of this study is not the introduction of vine copulas or AND/OR return periods as isolated statistical concepts. Rather, it is their integration into a complete event-based frequency framework for threshold-defined streamflow droughts. This integration is hydrologically non-trivial because the three drought characteristics have different extremal directions: duration and deficit volume increase with drought severity, whereas minimum-flow magnitude decreases. The proposed mixed-tail formulation therefore provides a consistent way to quantify both compound drought rarity and at-least-one exceedance risk for the same physical drought events.

The use of severity-selected annual events is central to the interpretation of the framework. Selecting the event with the largest deficit volume and extracting D, S, and M for that same event avoids combining annual extremes from different drought

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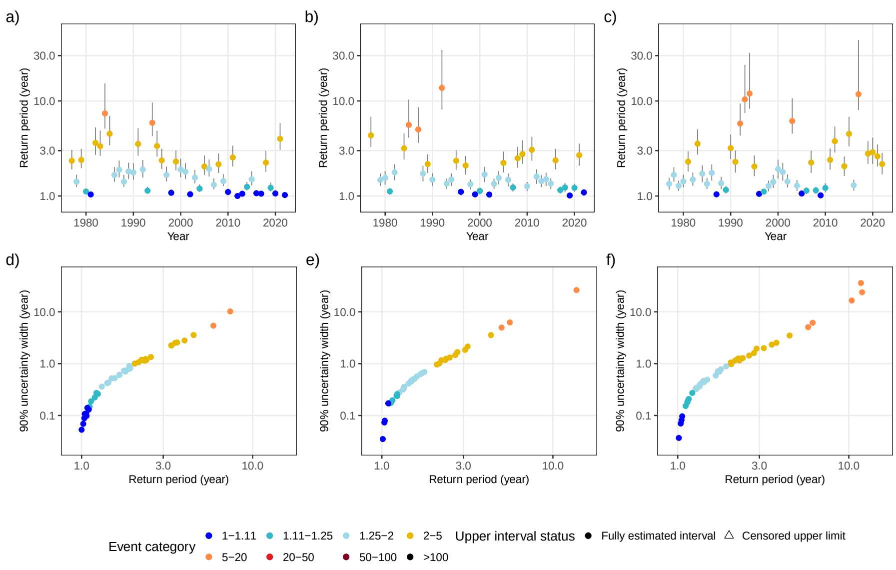

Figure 9. Annual copula-based OR return periods and their 90 % bootstrap uncertainty for the three exemplar catchments. Panels (a–c) show return-period estimates from the original observations together with 90 % bootstrap uncertainty intervals for each water year. Panels (d–f) show the corresponding uncertainty range, calculated as the difference between the upper and lower interval limits, as a function of the estimated return period. Panels (a, d), (b, e), and (c, f) correspond to Ebensee, Schalklhof, and Imbach, respectively. Colours indicate return-period categories.

episodes. This preserves the physical coherence of the trivariate event vector and supports interpretation of the resulting return periods as event-based drought-risk metrics.

The AND and OR return periods provide complementary perspectives. T_{AND} quantifies the rarity of events in which duration, severity, and minimum-flow magnitude are jointly extreme and is therefore suited to identifying compound drought hotspots. T_{OR} quantifies the recurrence of events that are extreme in at least one dimension and is therefore more appropriate for screening or early-warning applications. Other multivariate return-period concepts, such as Kendall return periods, may provide complementary distribution-wide risk measures, but they answer different risk questions.

The bootstrap analysis indicates that uncertainty differs systematically between the two return-period definitions. OR-based return periods are relatively stable because the union event is less restrictive and therefore supported by more observations. AND-based return periods, in contrast, depend on the simultaneous occurrence of extreme values in all three drought dimensions and are therefore more sensitive to sampling variability in the joint tail. This difference is particularly relevant when interpreting long return periods from 46 hydrological years of observations.

Several limitations should be considered when interpreting the results. First, the annual event series span 46 hydrological years, which limits the empirical information available in the three-dimensional joint tail. Consequently, very long AND-based return periods represent model-based extrapolations and may have wide uncertainty intervals. Second, the bootstrap intervals

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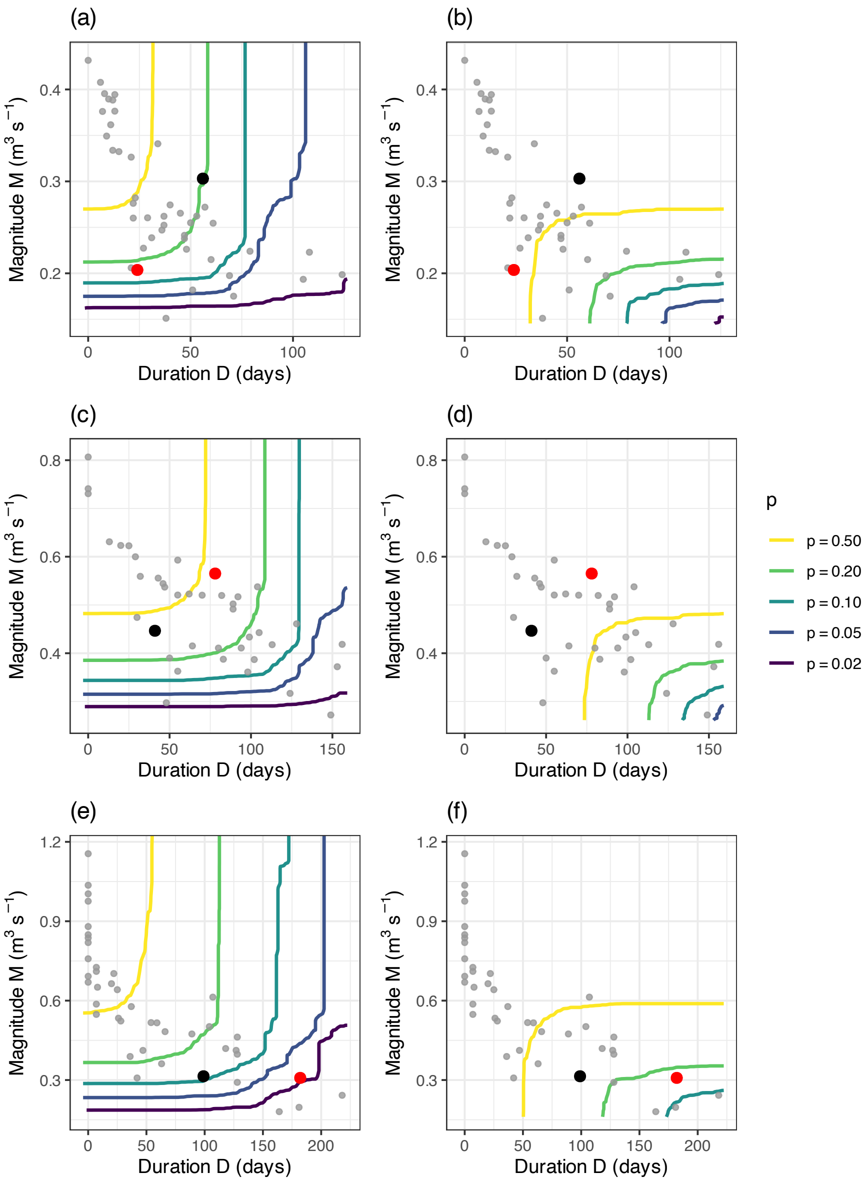

Figure 10. Two-dimensional contour plots of trivariate AND (a, c, e) and OR (b, d, f) probabilities in the duration–magnitude plane for Ebensee (a, b), Schalklhof (c, d), and Imbach (e, f). Grey points show observed drought events; red and black points indicate the 2003 and 2015 drought events, respectively.

quantify sampling and parameter uncertainty conditional on the selected vine structure and pair-copula families; they do not fully account for model-selection uncertainty. Third, the framework is stationary; possible temporal changes in marginal distributions or dependence structure are not addressed here. Fourth, although regular vine copulas provide flexible dependence modelling,

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structural uncertainty associated with alternative dependence models is not exhaustively assessed. Finally, the RD and RAD diagnostics quantify differences between univariate and multivariate return periods, but they do not validate the multivariate model against a known true return period. These limitations motivate future work on non-stationary extensions, model-structural comparisons, and complementary multivariate risk measures.

6 Conclusions

We developed a mixed-tail trivariate frequency framework for threshold-defined streamflow drought events. The framework jointly represents drought duration (D), deficit volume or severity (S), and minimum-flow magnitude (M) for physically identical drought events selected within Yevjevich's threshold approach. Its key feature is the consistent treatment of opposite extremal directions: duration and severity are upper-tail characteristics, whereas minimum-flow magnitude is a lower-tail characteristic. Combining zero-augmented Weibull marginals with a regular vine copula allows the dependence among these characteristics to be represented flexibly, while analytical AND and OR probabilities provide interpretable measures of compound and at-least-one exceedance risk.

Applied to 46 years of observations from 235 Austrian catchments, the framework provides strengthened trivariate evidence of compound severity in the 2003 drought, with 31 % of stations exhibiting $T_{\mathrm{AND}} > 50$ years. The 2015 drought shows fewer compound extremes at the national scale, but localized hotspots, particularly in northern Austria. The exemplar catchments further illustrate that events with similar values in one drought characteristic can have different multivariate return periods depending on the full combination of duration, severity, and minimum-flow magnitude.

Comparison with univariate return periods shows that drought rarity can be substantially re-ranked when dependence among event characteristics is accounted for. These deviations should not be interpreted as validation errors or accuracy gains, but as diagnostics of the additional information provided by trivariate event characterization. Bootstrap uncertainty estimates show that OR-based return periods are generally more stable, whereas rare AND-based return periods can have wide or censored upper intervals because they rely on the joint tail of a relatively short annual event series. The AND return period identifies rare compound droughts in which all characteristics are extreme, whereas the OR return period provides a broader screening metric for events that are extreme in at least one drought dimension.

The framework is relevant for drought-risk assessment because it links standard threshold-based event identification with multivariate return periods that remain interpretable for water-resources applications. It can support the identification of compound drought hotspots and can be transferred to other regions or to other threshold-defined environmental extremes, including groundwater droughts. Future work should address non-stationary extensions, structural uncertainty associated with alternative dependence models, and complementary multivariate risk measures such as Kendall return periods.

Code availability. The analysis code will be made available upon publication at a public repository.

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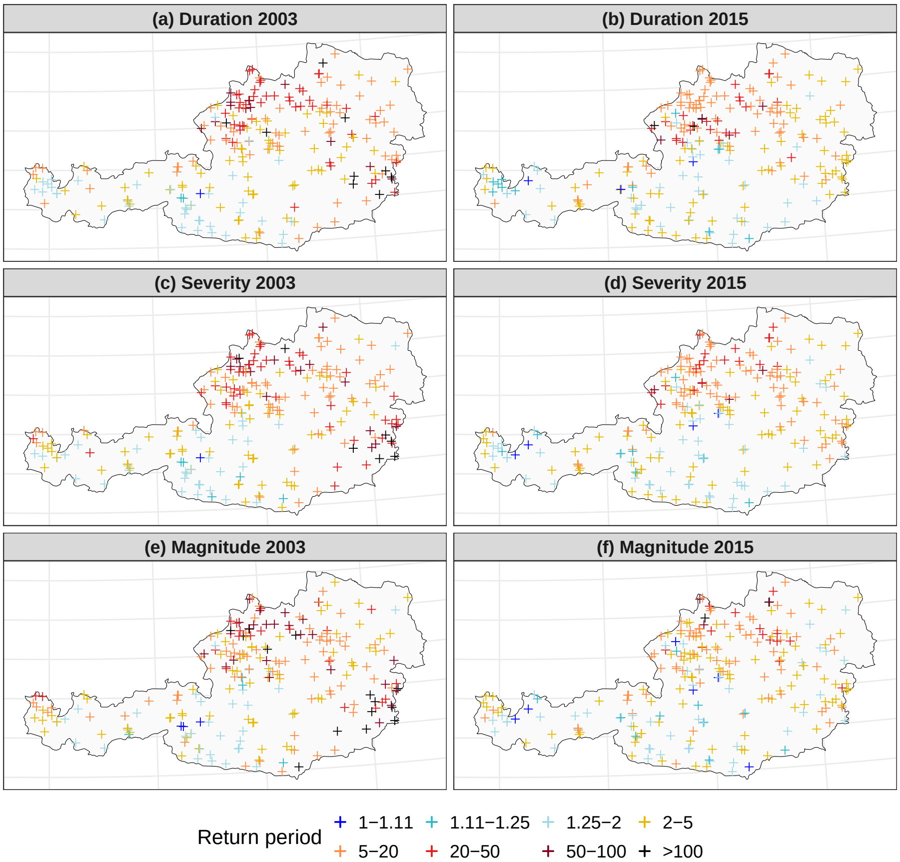

Figure A1. Return periods from univariate frequency analysis for the 2003 (a, c, e) and 2015 (b, d, f) droughts in Austrian catchments: duration (a, b), severity (c, d), and magnitude (e, f).

Data availability. The streamflow data are held by the Austrian Hydrographic Service. Processed data used for the analyses will be made available upon publication where permitted by data-use agreements.

Code and data availability. Data and code will be made available upon publication at a public repository.

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Appendix A: 2003 and 2015 univariate drought events

Appendix B: Computation of 2D contour lines for trivariate AND/OR return periods

We visualize trivariate AND/OR probabilities in the (D,M) plane by reducing the third dimension through a level-specific severity threshold. For a target probability level $p_0 \in (0,1)$ and an event type $\mathrm{EVENT} \in \{\mathrm{AND}, \mathrm{OR}\}$, we define the plotted surface

$$z(d,m) = p_{\mathrm{EVENT}}\big(d,\ s(p_0),\ m\big), \tag{B1}$$

and draw the iso-probability contour

$$z(d,m) = p_0. \tag{B2}$$

There is no analytical solution for $s(p_0)$ in general. We determine $s(p_0)$ numerically by enforcing that the equal-marginal anchor reaches the target level. Let

$$d^\star = F_D^{-1}(1-p_0), \qquad m^\star = F_M^{-1}(p_0), \tag{B3}$$

and find $s(p_0)$ such that

$$p_{\mathrm{EVENT}}\big(d^\star,\ s(p_0),\ m^\star\big) = p_0. \tag{B4}$$

Equivalently, with $v^\star = F_S\big(s(p_0)\big)$ and $s(p_0) = F_S^{-1}(v^\star)$, this reduces to a one-dimensional problem in $v^\star \in (0,1)$:

$$p_{\mathrm{EVENT}}\big(d^\star,\ F_S^{-1}(v^\star),\ m^\star\big) = p_0. \tag{B5}$$

We implement three methods to obtain $s(p_0)$, solving for $v^\star$ and then setting $s(p_0) = F_S^{-1}(v^\star)$:

- `p_quantile`: set $v^\star = 1-p_0$, hence $s(p_0) = F_S^{-1}(1-p_0)$. This is simple and interpretable but does not enforce $p_{\mathrm{EVENT}}(d^\star, s, m^\star) = p_0$.
- `p_global`: solve

  $$p_{\mathrm{EVENT}}\big(d^\star,\ F_S^{-1}(v),\ m^\star\big) - p_0 = 0, \quad v \in (0,1), \tag{B6}$$

  using a full bracket. If no sign change exists, return the closest v by minimizing the absolute mismatch.
- `p_local`: first attempt a local bracket around $v_0 = 1-p_0$; if unsuccessful, fall back to the full bracket; otherwise return the closest v.

Algorithmically, for each event type and probability level p_0, we compute $d^\star = F_D^{-1}(1-p_0)$ and $m^\star = F_M^{-1}(p_0)$, select $s(p_0)$ using one of the methods above, evaluate $z(d,m) = p_{\mathrm{EVENT}}(d, s(p_0), m)$ on a regular (d,m) grid using the fitted marginals and vine-copula model, and plot the contour $z(d,m) = p_0$. Extracted iso-lines can be smoothed for cartographic clarity without altering the target level or axis limits.

Implementation note: all methods are implemented in R and will be made available upon publication.

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Author contributions. F.F. conducted the primary analyses, co-developed the methods, and prepared the initial manuscript draft. J.L. provided and curated the datasets, supported data preprocessing and quality control, contributed to data handling and analysis workflows, and assisted with visualization. G.L. conceived the study, co-developed the methods, provided overall supervision, and led the final manuscript synthesis. All authors contributed to the study design, interpretation of results, and manuscript revision, and approved the final version.

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