



Investigating the radar-rain gauge observation-scale gap through local multifractal singularity

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Abstract.

Rain gauges and weather radars observe rainfall at substantially different spatial supports, from near-point gauge measurements to kilometre-scale radar pixels. Given the strong spatial variability of rainfall, this observational-support mismatch contributes to radar-gauge differences independently of instrumental and retrieval errors. Here, we investigate this scale effect using local rainfall singularity, which describes how rainfall intensity changes with observational scale through a locally estimated power-law exponent. The approach is applied to radar and rain-gauge observations for 10 rainfall events over the United Kingdom. Locally identifiable scaling behaviour is found for a substantial proportion of the observations, with scaling over 1-16 km being more consistent than over larger spatial ranges. The retrieved singularities are then used to extrapolate radar rainfall towards a pragmatic finer support of 50 m. Accounting for local scale dependence substantially modifies radar rainfall estimates but does not systematically improve their agreement with gauges; instead, conventional verification scores generally deteriorate. These results show that observational-support mismatch is only one component of radar-gauge disagreement, alongside radar retrieval errors, gauge uncertainty, and unresolved rainfall variability. Local singularity therefore provides a scale-aware diagnostic for investigating radar-gauge differences, rather than a direct correction towards point-scale rainfall.

1 Introduction

Operational rainfall observations are acquired across a wide range of spatial scales. At the finest scale, rain gauges and disdrometers provide near-point measurements, collecting rainfall over sampling areas typically only a few tens of centimetres in extent. At the mesoscale, weather radar networks operated by national meteorological services commonly produce spatially distributed rainfall estimates with pixel sizes of approximately 1 km. At larger scales, satellite-derived precipitation products generally have pixel sizes of several kilometres. Among these observing systems, rain gauges and weather radars are particularly complementary: gauges provide direct ground-based rainfall observations, whereas radars provide the spatial coverage required to capture the strong heterogeneity of rainfall fields. Gauge observations are therefore widely used as reference information for evaluating radar quantitative precipitation estimates (QPE). However, the observational supports of rain gauges and weather radars differ by many orders of magnitude. Because rainfall exhibits extreme variability across a wide range of



space-time scales (see, e.g., Lovejoy and Schertzer, 1995), measurements obtained by the two instruments do not necessarily represent the same rainfall quantity, even when they are nominally collocated.

This observational-support mismatch has practical consequences because radar-gauge comparison underpins not only radar QPE verification but also radar adjustment and radar-gauge merging. Radar performance is commonly evaluated using comparisons of rain rates or cumulative rainfall depths, scatter plots, and associated scores such as the Nash-Sutcliffe coefficient, root mean square error, normalised bias, and correlation coefficient (Diss et al., 2009; Moreau et al., 2009; Emmanuel et al., 2012; Figueras i Ventura et al., 2012; Tessema et al., 2022). Gauge observations are also routinely used to correct systematic radar errors or are combined with radar estimates to produce merged rainfall fields (Bringi et al., 2011; Ochoa-Rodriguez et al., 2019). Consequently, an observed radar-gauge discrepancy cannot necessarily be interpreted as radar estimation error alone. Conceptually, it may reflect contributions from radar measurement and retrieval uncertainty, gauge measurement uncertainty, and the difference in observational support between the two instruments (Ciach and Krajewski, 1999; Zhang et al., 2007; Gires et al., 2014). The latter contribution is commonly referred to as gauge representativeness or point-to-area error and can remain even in the absence of conventional instrumental error. Neglecting this distinction may therefore affect both the interpretation of radar verification statistics (Ciach and Krajewski, 1999; Zhang et al., 2007; Bringi et al., 2011) and the use of gauges for radar adjustment or merging (Wang et al., 2015; Ochoa-Rodriguez et al., 2019).

One major family of approaches has addressed this problem by treating gauge representativeness as an observational uncertainty that should be statistically distinguished from radar estimation error. The error variance separation framework of Ciach and Krajewski (1999), for example, explicitly recognised that the variance of radar-gauge differences contains contributions from both radar estimation error and the inability of a point gauge to represent rainfall over the radar sampling area. Subsequent studies extended and examined this framework by considering its underlying covariance assumptions, the filtering of gauge representativeness effects from radar verification distributions, and the scale dependence of the resulting error statistics (Habib and Krajewski, 2002; Ciach et al., 2003; Habib et al., 2004; Mandapaka et al., 2009a; Seo and Krajewski, 2011). Of particular relevance, Zhang et al. (2007) demonstrated that sub-pixel rainfall variability, gauge location, and the spatial correlation structure below radar resolution affect the separation of radar and gauge rainfall errors. Related studies have incorporated estimates of representativeness error when evaluating operational radar rainfall products (Emmanuel et al., 2012; Jaffrain and Berne, 2012). Collectively, these studies established that radar-gauge disagreement cannot be interpreted solely as radar estimation error and provided statistical means of quantifying the contribution of gauge representativeness. However, they primarily describe the observation-scale mismatch as an uncertainty component rather than explicitly estimating how rainfall at a particular location and time changes with observational scale.

A complementary family of studies has investigated the same problem by directly characterising rainfall variability at scales finer than the radar pixel. Early work by Kitchen and Blackall (1992) demonstrated that differences in the sampling volumes of radar and gauges can generate substantial representativeness errors in their comparison. More recent studies have utilised high-resolution radar observations or dense networks of gauges and disdrometers to quantify rainfall variability within radar pixels (Moreau et al., 2009; Pedersen et al., 2010; Peleg et al., 2013). For example, Pedersen et al. (2010) used a dense rain-gauge network to quantify spatial rainfall variability within an area of only 500×500 m, demonstrating that substantial variability



can occur even at scales smaller than a typical operational radar pixel. Similarly, Peleg et al. (2013) showed how sub-pixel rainfall variability contributes to radar-gauge uncertainty and how its importance changes with temporal aggregation. These studies provide direct observational evidence that the point-to-area mismatch is physically significant rather than merely a statistical artefact. Their application, however, relies on dense observational networks that are rarely available operationally, and they primarily characterise the magnitude and statistical structure of sub-pixel variability rather than providing a scale transformation for an individual radar observation.

A third approach utilises the scale-dependent structure of rainfall itself. Rainfall variability is known to exhibit scaling behaviour over a broad range of space-time scales, which can be represented within the Universal Multifractals framework (Schertzer and Lovejoy, 1987; Lovejoy and Schertzer, 1995). This provides an alternative to treating unresolved small-scale variability solely as an observational error. Multifractal analyses have demonstrated the relevance of such scaling behaviour for radar rainfall fields and high-resolution rainfall observations (Gires et al., 2011a, 2015a). Of particular relevance to radar-gauge comparison, Gires et al. (2014) used discrete Universal Multifractals (UMs) to stochastically downscale radar rainfall and generate ensembles of realistic virtual rain-gauge-like time series. This enabled the influence of unresolved small-scale rainfall variability on standard radar-gauge comparison scores to be quantified and showed that, once such variability is considered, appropriate target values for these scores are no longer necessarily their theoretical optimum values. This approach therefore explicitly connects rainfall scaling with the observational-support problem. However, stochastic multifractal downscaling represents unresolved variability through statistical scaling properties and ensembles of plausible small-scale rainfall realisations; it does not use the instantaneous multi-scale rainfall structure surrounding a particular location to estimate how rainfall there changes with observational scale.

Beyond rainfall applications, local singularity analysis has also been developed within the broader field of mathematical geosciences. In particular, Cheng (2007a) introduced a spatial local-singularity framework to characterise anomalous enrichment and depletion in geochemical fields through their scale-dependent behaviour. Related developments demonstrated how locally estimated singularity exponents can be used to characterise spatial heterogeneity and identify local anomalies in multifractal geophysical fields (Chen et al., 2007; Cheng, 2007b). Although developed primarily for geological and geochemical applications, these studies demonstrate the broader value of local singularity as a scale-dependent descriptor of spatially heterogeneous fields. To our knowledge, however, this local scale-dependent perspective has not been explicitly exploited to investigate the observational-support mismatch between weather-radar and rain-gauge rainfall measurements.

In this study, we investigate this missing local and instantaneous component of the radar-gauge observational-support problem using the concept of rainfall singularity. Within the multifractal framework, rainfall intensity changes with observational scale according to a power-law relation characterised locally by a scaling exponent (Schertzer and Lovejoy, 1987). Rather than prescribing a statistical distribution of unresolved small-scale variability, we estimate this local singularity from the multi-scale radar rainfall structure surrounding each rain-gauge location and use it to express the corresponding radar rainfall towards finer observational support. An earlier implementation of a singularity-sensitive approach was used by Wang et al. (2015) for radar rainfall adjustment in urban hydrological applications. Here, the purpose is different: local singularity is used as a scale-aware diagnostic framework to investigate how the observation-scale gap itself affects radar-gauge comparison. Importantly, the sin-



95 gularity is estimated from radar-resolved spatial scales; its continuation below radar resolution is therefore an extrapolation
rather than a direct observation of true sub-pixel rainfall. Accordingly, the aim is not to reconstruct the true point-scale rainfall
measured by a gauge, but to investigate how radar rainfall and its apparent agreement with gauge observations change when the
locally observed scaling behaviour is extrapolated towards finer observational support. Specifically, we address three questions:
(1) is there locally identifiable scaling behaviour in radar-observed rainfall that can be used to characterise rainfall variability
across observational scales? (2) how does local scaling alter radar rainfall estimates when they are expressed towards the finer
100 observational support of rain gauges? and (3) to what extent does accounting for observational-scale mismatch change the
apparent agreement between radar and rain-gauge rainfall measurements?

The remainder of this paper is organised as follows. Section 2 introduces the radar and rain-gauge datasets and the selected
rainfall events. Section 3 presents the theoretical framework for deriving the local rainfall singularity and its use for addressing
the observation-scale gap between radar and rain-gauge measurements. Section 4 presents and discusses the results, first ex-
105 amining the validity and characteristics of the retrieved local scaling behaviour and then assessing its impact on radar–gauge
comparison. Finally, Section 5 summarises the main findings, limitations, and perspectives of the study.

2 Data

2.1 Radar data

Nimrod C-band radar rain-rate composite data operated by the UK Met Office are used in this study. Nimrod provides rainfall
110 estimates at 5 min temporal resolution on a 1 km Cartesian grid covering the UK (Golding, 1998; Yu et al., 2020). The
Nimrod system applies algorithmic corrections to radar-estimated precipitation intensities, incorporating real-time rain-gauge
measurements at hourly intervals to improve data accuracy (Harrison et al., 2000; Kidd et al., 2012). For this study, Nimrod data
from 2021 are used for method development and verification. At each time step, the radar image is cropped to a 512×512 km
domain centred on Birmingham (Fig. 1).

115 2.2 Ground rain-gauge records

A set of Environment Agency (EA) rain-gauge records at 5 min resolution over the Severn Trent (ST) region, a sub-area
covering the centre of the study domain, is employed. The EA-ST dataset originally includes 258 stations distributed within
approximately a 250 km radius centred on Birmingham. The observations were originally recorded as times of tips and were
further processed to 5 min resolution to match the temporal resolution of the Nimrod radar dataset. Basic quality control
120 was performed by comparing the EA-ST gauge records against the nearest MIDAS observations (Met Office Integrated Data
Archive System, a publicly available ground-observation dataset operated by the UK Met Office; see Met Office (2019)) at the
1 h scale. Pearson correlation coefficients were computed, and a total of 232 stations were retained for further analysis. These
rain gauges were not used for calibrating Nimrod radar data.

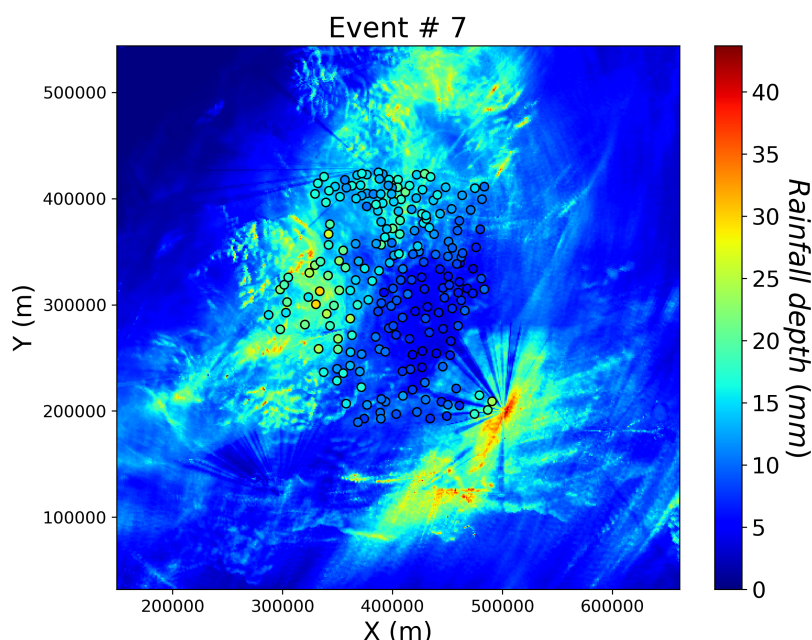


Figure 1. Cumulative rainfall depth during event #7. Background map corresponds to radar product over the 512 x 512 km studied area, while point are available rain gauges.

2.3 Event selection

125 A total of 10 storm events in 2021 were selected for this study based on the following criteria:

1. A given time step is classified as *wet* if more than 10% of the pixels within the study domain register rainfall intensities $\geq 1 \text{ mm h}^{-1}$; otherwise, it is classified as *dry*.
2. A storm event begins with a *wet* time step and continues until a subsequent *dry* period lasting at least 8 h occurs.
3. A storm event ends at the last *wet* time step preceding a *dry* period of at least 8 h.

130 Applying these criteria resulted in an initial pool of 74 candidate storm events. To prioritise events most relevant to hydrological applications, we further filtered the events based on their *wet* coverage over the land area and overall duration. In addition, seasonal diversity was considered to include a representative range of storm types, including convective and stratiform systems. Ultimately, 10 events were selected with an approximately even distribution across seasons.

Summary statistics for the selected events are provided in Table 1. Figure 1 shows the spatial distribution of accumulated
 135 Nimrod rainfall together with the accumulated EA-ST gauge rainfall (mm) for event #7 across the study domain.



Table 1. Summary statistics for the studied rainfall events.

Event #	Start time	End time	Duration	max RG	mean RG	max Rad.	mean Rad
1	2021-01-15 22:00:00	2021-01-16 18:00:00	20	26	6.6	22.9	10.3
2	2021-03-09 22:00:00	2021-03-11 00:00:00	26	81.2	12.8	45.4	12.7
3	2021-05-03 08:00:00	2021-05-04 05:00:00	21	63.6	15.9	33.6	13.5
4	2021-05-08 01:00:00	2021-05-08 21:00:00	20	52.2	16.8	45.9	13.7
5	2021-05-20 09:00:00	2021-05-22 02:00:00	41	107.8	18.4	96.5	18.6
6	2021-05-23 11:00:00	2021-05-24 07:00:00	20	29	9.5	33.5	9
7	2021-07-05 17:00:00	2021-07-06 13:00:00	20	29.8	12.5	32.8	11.8
8	2021-10-02 05:00:00	2021-10-03 02:00:00	21	37.6	14.6	67.8	13.3
9	2021-10-31 03:00:00	2021-11-01 07:00:00	28	53.4	18.1	61	16.3
10	2021-12-07 08:00:00	2021-12-08 05:00:00	21	38.8	9	34.4	9.7

3 Methodology

The proposed framework consists of three main steps. First, the multi-scale radar rainfall structure surrounding each rain-gauge location is used to estimate a local scaling exponent describing how rainfall intensity changes with observational scale. Second, assuming that the identified scaling behaviour persists below the radar resolution, the locally estimated exponent is used to extrapolate the radar rainfall from its original 1 km support towards a finer observational scale. Finally, the resulting scale-adjusted radar rainfall is compared with the rain-gauge observations and with radar rainfall at its original resolution to investigate how accounting for observational-support differences affects the apparent radar-gauge agreement.

3.1 Computing a local singularity

In the framework of multifractals, a non-conservative field at a given resolution λ , defined as the ratio between the outer scale of the phenomenon L and the observation scale l ($\lambda = L/l$), can be written as (Schertzer and Lovejoy, 1987):

$$R_\lambda \approx \langle R \rangle \epsilon_\lambda \lambda^{-H}, \quad (1)$$

where $\approx$ stands for equality in distribution, $\langle R \rangle$ is the average rainfall, H is the non-conservation parameter, and ϵ_λ is a multifractal conservative field. Locally, the latter scales as $\epsilon_\lambda \propto \lambda^\gamma$, where γ denotes the singularity exponent. Combining this relation with Eq. 1 yields:

$$R_\lambda \approx \langle R \rangle \lambda^{\gamma-H}. \quad (2)$$



Equation 2 provides the theoretical basis for characterising the local change in rainfall intensity with resolution or observational scale through the exponent $\gamma - H$. Here, 'local' means that $\gamma - H$ is estimated independently for each rain-gauge location and time step from the multi-scale radar rainfall structure surrounding that location. More precisely, the following procedure is implemented:

- 155 – R_λ is computed over circles centred on each rain-gauge location, with diameters corresponding to powers of two from 1 to 128 km. For each circle, rainfall intensity is calculated as the area-weighted average of all radar pixels having a non-zero intersection with the circle, with the weights determined by the corresponding intersection areas. These circles are illustrated for one rain gauge in Fig. 2; given the zoom level, only the largest circles are clearly visible.
- For two observational scales, the ratio of rainfall intensities can be expressed from Eq. 2 as

$$160 \quad \frac{R_{\lambda_1}}{R_{\lambda_2}} = \left(\frac{\lambda_1}{\lambda_2} \right)^{\gamma-H} = \left(\frac{l_2}{l_1} \right)^{\gamma-H}. \quad (3)$$

The larger scale l_2 is used as the reference, while l_1 varies over the smaller circles. Equation 3 is then expressed in log-log space.

- A linear regression is fitted in log-log space, and its slope provides an estimate of the local exponent $\gamma - H$. The quality of the scaling fit is assessed using the coefficient of determination r^2 , while the standard error of the fitted slope is used to quantify uncertainty in the estimated $\gamma - H$.

170 The nested circles from 1 to 128 km are initially considered to examine the range of scales over which a local power-law relationship is supported by the radar observations. As shown in Sect. 4, this scale-range assessment indicates that restricting the regression to scales between 1 and 16 km provides more consistent local scaling behaviour than including the larger spatial scales. The 1–16 km range is therefore adopted for the subsequent estimation of $\gamma - H$ and the scale transformation described below.

3.2 Use of local singularity to compare radar and rain-gauge data

As discussed in Sect. 1, radar and rain gauges have substantially different spatial observational supports. The locally estimated scaling exponent provides a means for investigating this observation-scale gap. Assuming that the scaling behaviour identified over the radar-resolved scales continues towards finer scales, Eq. 2 can be used to extrapolate rainfall observed by radar at approximately 1 km resolution towards a smaller observational support. The resulting scale-adjusted radar rainfall, denoted by R_{scaled} , is expressed as

$$R_{\text{scaled}} = R_{\text{radar, circle 1km}} \left(\frac{l_1}{l_{\text{target}}} \right)^{\gamma-H}, \quad (4)$$

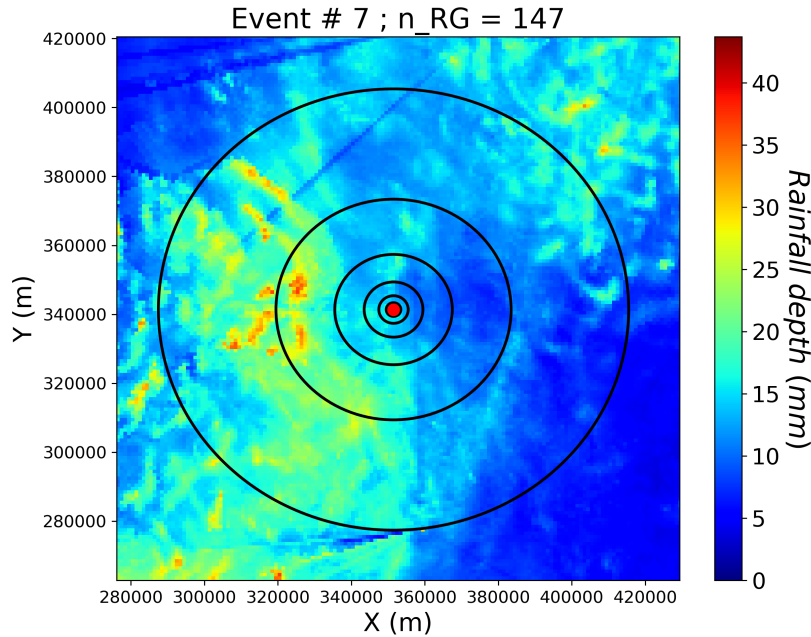


Figure 2. Accumulated radar rainfall during event #7, corresponding to the event shown in Fig. 1. Circles with diameters of 128, 64, 32, 16, 8, 4, 2, and 1 km used to examine the local scaling behaviour at the location of rain gauge #147 are shown. Given the zoom level, only the largest circles are clearly visible.

where $l_1 = 1$ km, $R_{\text{radar, circle 1km}}$ is the radar rainfall averaged over a 1 km-diameter circle centred on the considered rain gauge, and l_{target} is the finer target scale.

180 In this study, l_{target} is pragmatically set to 50 m. This target is substantially finer than the original 1 km radar resolution and therefore enables the effect of reducing the radar-gauge observational-scale gap to be investigated, while avoiding extrapolation all the way to the physical sampling scale of a rain gauge, which is typically of the order of tens of centimetres. The latter would require assuming persistence of the same scaling regime over several additional orders of magnitude. Although scaling behaviour has been reported for rainfall at small observational scales, its persistence and possible scale breaks at such scales
 185 remain open research questions (Lilley et al., 2006; Mandapaka et al., 2009b; Gires et al., 2015b). The 50 m scale should therefore be regarded as a pragmatic finer target support rather than as the physical rain-gauge sampling scale. Furthermore, because $\gamma - H$ is estimated only from radar-resolved scales, the calculation in Eq. 4 represents an extrapolation of the locally
 190 observed scaling behaviour rather than a direct observation of sub-pixel rainfall.

Based on the empirical assessment presented in Sect. 4.1, the scale transformation is applied only when the retrieved singularity satisfies $r^2 > 0.8$ and $-2 < \gamma - H < 0.5$. The r^2 criterion retains cases for which the variation of rainfall across the
 190 radar-resolved scales is sufficiently well represented by the assumed local power-law relationship, while the bounds on $\gamma - H$



avoid extrapolation based on extreme and sparsely represented singularity estimates. When these conditions are not satisfied, no sub-resolution scale transformation is applied and R_{scaled} is set equal to $R_{\text{radar, circle 1km}}$. Thus, observations with poorly constrained local scaling behaviour remain in the radar-gauge comparison, but without introducing an extrapolation towards the 50 m target scale.

The rain-gauge observations R_{RG} are compared against three radar-derived rainfall time series. First, R_{radar} denotes the rainfall intensity of the original 1 km radar pixel in which the rain gauge is located, corresponding to conventional pixel-to-point radar-gauge comparison. Second, $R_{\text{radar, circle 1km}}$ represents radar rainfall at approximately the original 1 km observational scale but spatially centred on the rain-gauge location. Third, R_{scaled} represents the radar rainfall series incorporating the local scale transformation where sufficiently constrained: where the retrieved singularity satisfies the reliability criteria, the radar rainfall is extrapolated towards the 50 m target support using Eq. 4; otherwise, $R_{\text{scaled}} = R_{\text{radar, circle 1km}}$. This three-way comparison enables the effect of centring the radar sampling support on the gauge to be distinguished from the additional effect of the local scale transformation.

The three radar-derived time series are evaluated against R_{RG} using scores commonly employed for radar-gauge comparison (Diss et al., 2009; Moreau et al., 2009; Emmanuel et al., 2012; Figueras i Ventura et al., 2012; Gires et al., 2014; Tessema et al., 2022). In the following equations, Y_t denotes the reference rain-gauge observation, X_t denotes the corresponding radar-derived estimate, and N is the number of paired observations. The following scores are used:

- The normalised bias (*Bias*), whose optimal value is 0:

$$Bias = \frac{\sum_{t=1}^N X_t}{\sum_{t=1}^N Y_t} - 1. \quad (5)$$

Positive values indicate overall radar overestimation relative to the gauges, whereas negative values indicate underestimation.

- The Nash-Sutcliffe model efficiency coefficient (NS), which has an optimal value of 1:

$$NS = 1 - \frac{\sum_{t=1}^N (X_t - Y_t)^2}{\sum_{t=1}^N (Y_t - \bar{Y})^2}, \quad (6)$$

where $\bar{Y}$ denotes the mean of the reference rain-gauge observations. Values below zero indicate that the mean of the gauge observations provides a better prediction than the tested radar-derived time series according to this criterion.



- The root mean square error (RMSE), whose optimal value is 0:

$$RMSE = \sqrt{\frac{1}{N} \sum_{t=1}^N (X_t - Y_t)^2}. \quad (7)$$

- The slope (*Orth. reg.*) of the orthogonal linear regression between X and Y . Orthogonal regression minimises the perpendicular distances between the observations and the fitted line, rather than the vertical distances minimised by ordinary least-squares regression. It is therefore suitable here because neither radar nor rain-gauge observations are assumed to be error-free. A slope of 1 indicates the optimal one-to-one scaling between the two datasets.

4 Results and discussion

4.1 Local scaling behaviour and reliability of singularity retrieval

We first examine whether locally identifiable scaling behaviour can be retrieved from the radar rainfall field and under which conditions the estimated local singularity $\gamma - H$ can be considered sufficiently reliable. Event #7 is used initially for illustration, while the consistency of the findings across the other events is discussed subsequently. Figure 3 shows the temporal evolution of rainfall at rain gauge #147 together with radar rainfall averaged over different observational scales. The corresponding cumulative rainfall depths are shown in the second row. For this particular gauge and event, the radar rainfall estimates are generally lower than the rain-gauge observations. More importantly for the present analysis, the radar rainfall intensity does not vary uniformly with observational scale, that is, both the magnitude and direction of the scale dependence change during the event.

The local singularity is estimated at each time step by expressing Eq. 3 in log-log space and fitting the linear relationship described in Sect. 3. Figure 4 illustrates three representative configurations observed during the event. At time step 66, rainfall intensity changes only weakly across the considered scales, resulting in a poorly identifiable scaling relationship ($r^2 = 0.05$). At time step 101, a clear positive slope is observed, indicating that rainfall intensity increases towards finer observational scales. Conversely, time step 109 exhibits a well-defined negative slope, indicating decreasing rainfall intensity towards finer scales. The latter two examples demonstrate that both positive and negative local singularities can be well represented by the assumed power-law relationship. More generally, the magnitude and sign of $\gamma - H$ describe the local scale dependence of rainfall, while r^2 indicates how well this dependence is represented by a single power law over the considered range of scales.

The range of spatial scales included in the regression has an important influence on the quality of the retrieved scaling relationship. The initial analysis considered nested spatial supports between 1 and 128 km. A sensitivity analysis across all events and rain gauges showed, however, that restricting the regression to the 1–16 km range generally resulted in higher r^2 values than extending the fit towards the larger scales. Figure 4c (i.e. time step # 109) illustrates this behaviour, for which a clear scaling relationship is obtained between 1 and 16 km ($r^2 = 0.96$), whereas the relationship deteriorates when larger scales are included. This behaviour suggests that a single local power-law relationship does not necessarily extend across the complete

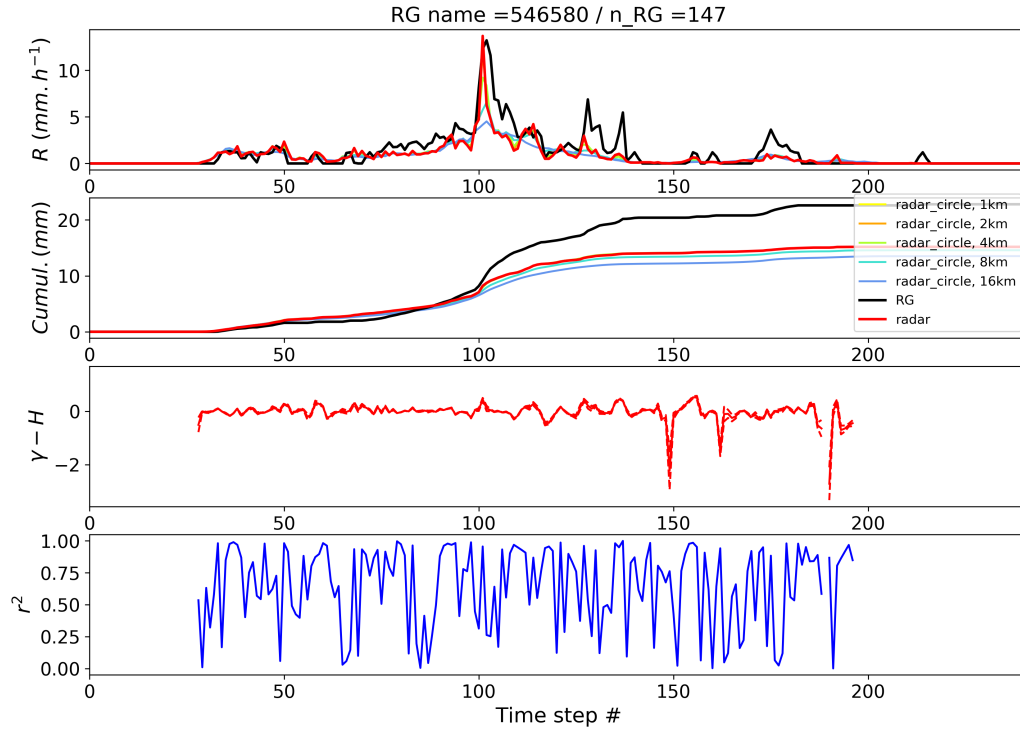


Figure 3. Temporal evolution of rainfall and the retrieved local scaling properties for event #7 at rain gauge #147, whose location is shown in Fig. 2. The first and second rows show, respectively, rainfall intensity and cumulative rainfall depth from the rain gauge and radar observations averaged over different spatial scales. The third row shows the retrieved local singularity $\gamma - H$, and the fourth row shows the corresponding coefficient of determination r^2 of the log–log scaling regression.

1-128 km range. A transition around scales of order 16 km is also broadly consistent with previously reported changes in rainfall scaling regimes (Nykanen and Harris, 2003; Gires et al., 2011b). Based on this analysis, the 1-16 km range is therefore used for the subsequent retrieval of $\gamma - H$.

The occurrence of both positive and negative singularities has a direct physical interpretation in terms of the spatial organisation of rainfall around a gauge. If an intense portion of the rainfall field is located close to the gauge, averaging over progressively larger areas incorporates surrounding regions of lower intensity. Rainfall intensity therefore decreases with increasing observational scale, corresponding to a positive $\gamma - H$ and an increase in rainfall towards finer scales. On the contrary, when relatively weak rainfall occurs near the gauge but more intense rainfall is present several kilometres away, increasing the averaging area can increase the mean rainfall intensity, resulting in a negative $\gamma - H$. The temporal variation of $\gamma - H$ in the

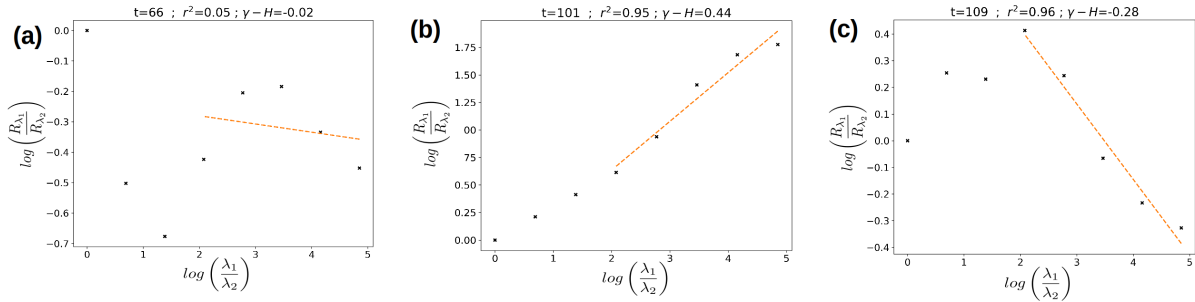


Figure 4. Examples of the retrieval of the local singularity $\gamma - H$ for three representative time steps of event #7 at rain gauge #147. The panels show Eq. 3 in log-log space and the corresponding linear regression used to estimate $\gamma - H$, illustrating different combinations of scaling direction and goodness of fit.

third row of Fig. 3 therefore reflects changes in the local spatial organisation of the rainfall field during the event, expressed through a common scale-invariant descriptor.

The quality of this local scaling description is not uniform in time. As shown in the fourth row of Fig. 3, r^2 varies from values close to zero to values approaching one. To examine the conditions associated with this variability in a more systematic way, Fig. 5 combines $\gamma - H$, r^2 , and $R_{\text{radar, circle 1km}}$ for all rain gauges and time steps of event #7. Figure 5a shows that the lowest r^2 values occur predominantly when $\gamma - H$ is close to zero. This behaviour does not necessarily indicate a failure of the scaling concept. When $\gamma - H \approx 0$, rainfall intensity changes only weakly across the considered scales, leaving little scale-independent signal from which the slope can be estimated. The singularity is consequently weakly identifiable even though the absolute effect of scale on the rainfall estimate is also small.

The relatively few very negative values of $\gamma - H$, particularly those below approximately -1.5 , also tend to be associated with poorer scaling fits. Figure 5b indicates that these cases occur mainly when $R_{\text{radar, circle 1km}}$ is very small. A similar tendency is observed for some of the largest positive singularities. These extreme exponent estimates should therefore be interpreted cautiously, because extrapolation using a large absolute value of $\gamma - H$ can produce substantial changes in rainfall intensity even when the exponent is estimated from weak rainfall signals. By contrast, Fig. 5c shows no simple monotonic relationship between r^2 and the 1 km radar rainfall intensity itself. The reliability of the retrieved singularity therefore depends not simply on rainfall magnitude, but on whether a sufficiently clear scale-independent structure is present in the surrounding radar field.

Similar patterns are observed for the other nine events. Overall, the analysis indicates that local scaling behaviour can be identified for a substantial fraction of the radar observations, while poor fits occur preferentially where the scale dependence is weak or for some extreme singularity estimates associated with very low rainfall intensities. To avoid extrapolating poorly constrained local scaling relationships, the subsequent analysis applies the scale transformation only when $r^2 > 0.8$ and $-2 < \gamma - H < 0.5$. Depending on the event, the r^2 criterion excludes approximately 25-40% of the gauge-time-step pairs, whereas the lower and upper singularity thresholds exclude only approximately 0.1-0.2% and 1%, respectively. For observations that do not satisfy these criteria, no sub-resolution scale transformation is applied, as described in Sect. 3.

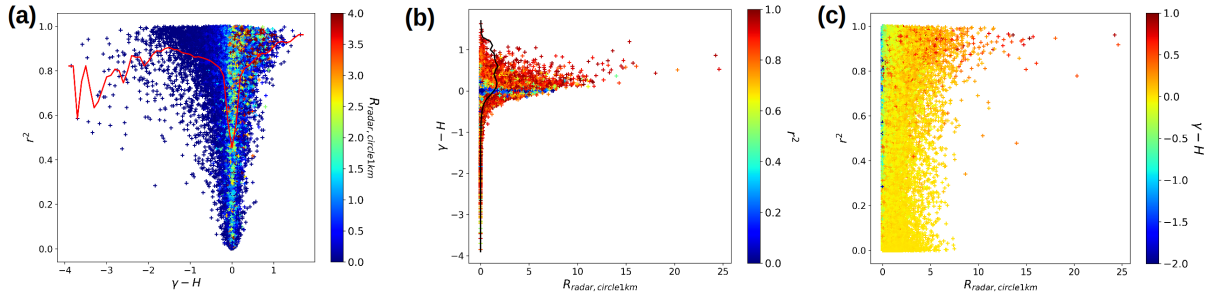


Figure 5. Joint analysis of the retrieved local singularity $\gamma - H$, the goodness of fit r^2 , and the 1 km circular radar rainfall $R_{\text{radar,circle 1km}}$ for all rain gauges and time steps of event #7. (a) r^2 versus $\gamma - H$, with points coloured by $R_{\text{radar,circle 1km}}$; the curve shows the mean r^2 within $\gamma - H$ bins of width 0.1. (b) $\gamma - H$ versus $R_{\text{radar,circle 1km}}$, with points coloured by r^2 . (c) r^2 versus $R_{\text{radar,circle 1km}}$, with points coloured by $\gamma - H$.

Taken together, these results provide a qualified answer to the first research question. The radar rainfall fields exhibit locally identifiable scaling behaviour over the adopted 1-16 km range for a substantial proportion of locations and time steps, and the retrieved $\gamma - H$ captures both increases and decreases in rainfall intensity towards finer observational scales. However, the scaling relationship is not uniformly identifiable throughout an event. In particular, $\gamma - H$ becomes poorly constrained when rainfall varies little across scales, highlighting the need to account explicitly for retrieval quality before extrapolating the local scaling behaviour towards finer observational support.

4.2 Effect of local scaling on radar rainfall estimates

Having established the conditions under which local scaling can be sufficiently identified, we next examine how extrapolating this behaviour towards finer observational support modifies the radar rainfall estimates. Figure 6 compares the rain-gauge observations R_{RG} , the original radar rainfall R_{radar} , the 1 km circular radar rainfall centred on the gauge $R_{\text{radar,circle 1km}}$, and the locally scale-adjusted rainfall R_{scaled} for event #7 at rain gauge #147. The complete event is shown in Fig. 6a, while a selected period is enlarged in Fig. 6b to illustrate the effect of different local scaling configurations.

The scale transformation is inherently bidirectional. From Eq. 4, a positive $\gamma - H$ increases rainfall intensity when extrapolating from 1 km towards the 50 m target support, whereas a negative $\gamma - H$ decreases it. This behaviour can be seen clearly at time steps 101 and 109. At time step 101, the positive local singularity identified in Fig. 4b results in R_{scaled} being greater than $R_{\text{radar,circle 1km}}$. In contrast, the negative singularity at time step 109 (Fig. 4c) reduces the scale-adjusted rainfall relative to the centred 1 km radar estimate. The local scale transformation therefore does not act as a systematic multiplicative enhancement or reduction of radar rainfall; rather, both its direction and magnitude depend on the instantaneous multi-scale organisation of rainfall surrounding the gauge.

At time step 66, by contrast, the local scaling relationship is poorly constrained ($r^2 = 0.05$; Fig. 4a). As this case does not satisfy the reliability criteria defined in Sect. 3, no sub-resolution extrapolation is applied and $R_{\text{scaled}} = R_{\text{radar,circle 1km}}$. This

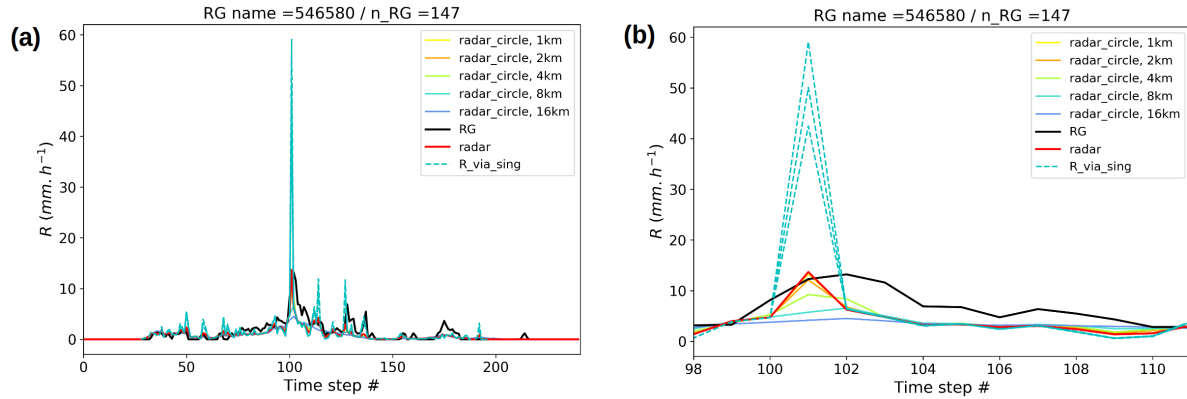


Figure 6. Comparison of rain-gauge and radar-derived rainfall for event #7 at rain gauge #147. (a) Temporal evolution over the complete event of the rain-gauge observation R_{RG} , original 1 km radar pixel R_{radar} , 1 km circular radar rainfall centred on the gauge $R_{\text{radar, circle 1km}}$, and locally scale-adjusted radar rainfall R_{scaled} . Panel (b) shows an enlarged period illustrating different local scaling configurations. The area in-between the dashed blue lines associated with R_{scaled} represents uncertainty propagated from the estimated local scaling exponent.

example illustrates the role of the reliability filtering introduced in Sect. 4.1: where the radar field contains insufficient evidence
 300 for a well-defined local scale dependence, the method retains the centred 1 km radar estimate rather than extrapolating a poorly constrained singularity.

The uncertainty associated with R_{scaled} also depends on the uncertainty of the retrieved $\gamma - H$. As shown by the uncertainty range in Fig. 6 (dashed light blue lines), the standard error of the fitted slope is propagated through Eq. 4 to quantify the uncertainty associated with the 50 m scale-adjusted rainfall. Because the transformation involves extrapolation over a scale
 305 ratio of 20, uncertainty in $\gamma - H$ can translate into appreciable uncertainty in R_{scaled} , particularly for less precisely estimated local scaling relationships.

Overall, the local singularity produces location- and time-dependent modifications to the radar rainfall field rather than a uniform correction. Positive and negative singularities respectively increase and decrease rainfall when extrapolated towards finer observational support, while no transformation is applied where the local scaling relationship is insufficiently constrained.
 310 The resulting R_{scaled} therefore contains greater local variability than the original 1 km radar estimate where reliable non-zero scale dependence is identified. These results address the second research question by demonstrating how locally retrieved scaling behaviour translates into explicit changes in radar rainfall towards finer observational support. Whether these scale-dependent modifications lead to closer agreement with the rain-gauge observations is examined separately in the following section.

315 4.3 Effect of local scaling on radar-rain-gauge comparison scores

We next examine whether the local scale transformation described above changes the apparent agreement between radar and rain-gauge observations. Figure 7 compares the three radar-derived rainfall estimates, R_{radar} , $R_{\text{radar, circle 1km}}$, and R_{scaled} ,

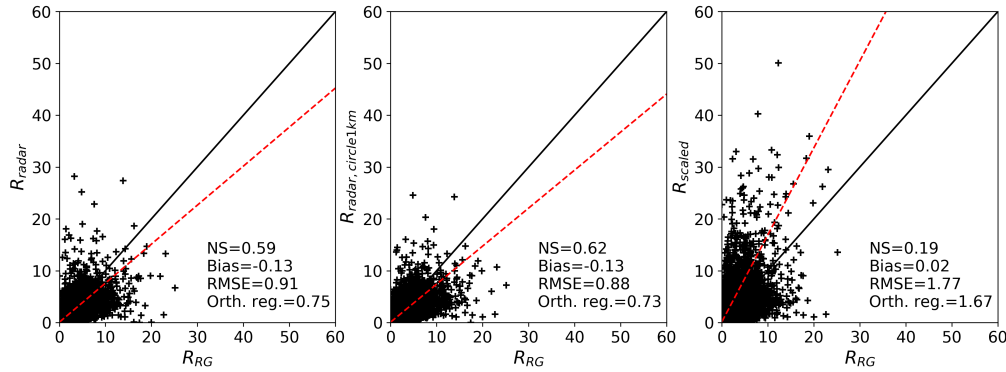


Figure 7. Scatter plots and associated verification scores (see Sect. 3.2) comparing the three radar-derived rainfall estimates with the rain-gauge observations R_{RG} for event #7, combining all rain gauges: (left) the original 1 km radar rainfall R_{radar} ; (middle) the 1 km circular radar rainfall centred on the gauge $R_{\text{radar,circle 1km}}$; and (right) the locally scale-adjusted radar rainfall R_{scaled} .

against R_{RG} for event #7, combining observations from all rain gauges. As defined in Sect. 3.2, R_{scaled} incorporates the local scale transformation where the retrieved singularity satisfies the reliability criteria; otherwise, $R_{\text{scaled}} = R_{\text{radar,circle 1km}}$.

320 The scatter distributions of R_{radar} and $R_{\text{radar,circle 1km}}$ are similar. Centring the 1 km radar sampling support on the rain-gauge location results in a small improvement in the comparison scores, indicating that the geometrical alignment between the radar sampling support and gauge location contributes to the observed radar-gauge differences, but only to a limited extent. In contrast, R_{scaled} exhibits a visibly wider scatter relative to R_{RG} , accompanied by poorer conventional verification scores. Thus, for event #7, extrapolating the locally observed scale dependence towards finer support does not bring the radar estimates
 325 systematically closer to the gauge observations.

To assess whether this behaviour is specific to event #7, Fig. 8 summarises the Nash-Sutcliffe efficiency (NS), root mean square error ($RMSE$), normalised bias ($Bias$), and orthogonal-regression slope ($Orth. reg.$) for all 10 events. The absolute level of radar-gauge agreement varies substantially among events. Considering first the conventional R_{radar} comparison, event #1 shows particularly poor agreement, with the lowest NS and among the least favourable values of the other scores,
 330 whereas events #5-8 generally exhibit better agreement. This event-to-event variability indicates that the magnitude and characteristics of radar-gauge disagreement are strongly storm dependent.

Despite these differences among events, the relative behaviour of the three radar representations is broadly consistent. The scores obtained using $R_{\text{radar,circle 1km}}$ remain close to those of R_{radar} , with generally small improvements when the radar sampling support is centred on the gauge. By contrast, incorporating the local scale transformation generally leads to poorer
 335 NS and $RMSE$ values and larger departures of $Bias$ and the orthogonal-regression slope from their respective optimal values. In particular, the increases in $Bias$ and $Orth. reg.$ indicate an overall shift towards larger R_{scaled} values relative to the gauge observations. This is consistent with the increased variability introduced by extrapolating locally identified scale dependence towards finer observational support.

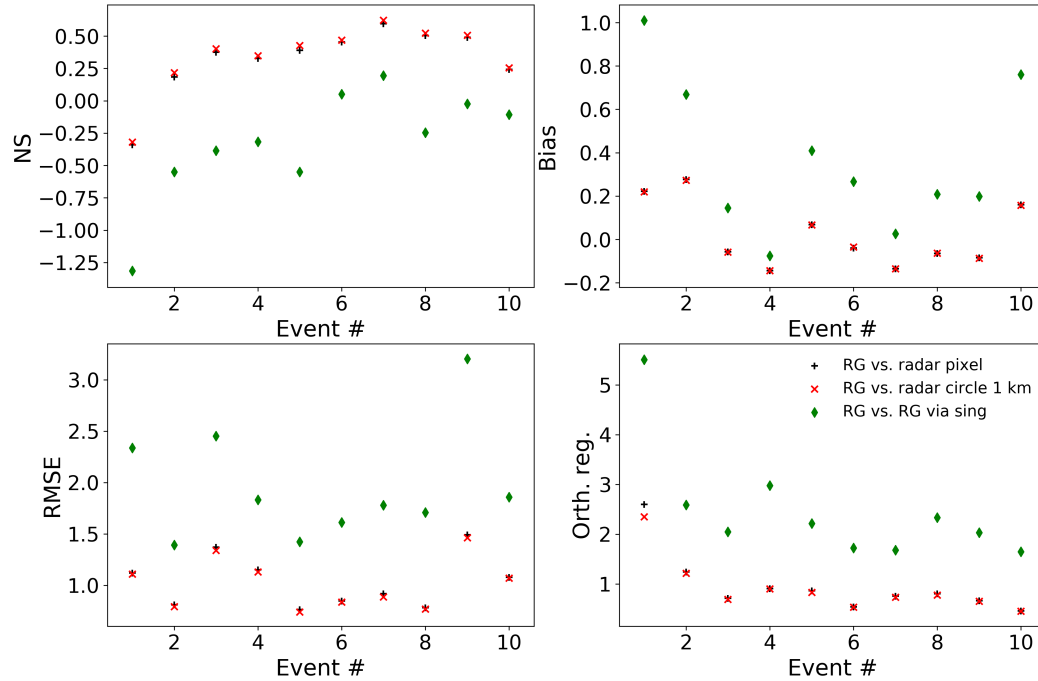


Figure 8. Verification scores for the three radar-derived rainfall estimates R_{radar} , $R_{\text{radar, circle 1 km}}$, and R_{scaled} against the rain-gauge observations R_{RG} , computed separately for each of the 10 rainfall events using all available rain gauges. The scores shown are Nash-Sutcliffe efficiency (NS), root mean square error ($RMSE$), normalised bias ($Bias$), and orthogonal-regression slope ($Orth. reg.$).

Overall, explicitly accounting for the locally observed scale dependence materially changes the radar-gauge comparison, but does not systematically improve agreement according to the conventional verification metrics used here. This result is important because improved radar-gauge agreement was not imposed as an objective of the scale transformation: the transformation acts only on the observational-support component of the comparison, based on the scale dependence identified from the radar rainfall field. The deterioration of conventional scores therefore raises a broader question of how radar-gauge differences should be interpreted when observational-support mismatch coexists with radar retrieval errors, gauge uncertainty, and unresolved rainfall variability. These implications are discussed in Sect. 4.4.

4.4 Implications for interpreting radar–gauge differences

The results above highlight an important distinction between reducing an observational-support mismatch and improving agreement between two observing systems. The local scale transformation explicitly addresses one source of difference between radar and rain gauges by expressing the radar rainfall towards a finer spatial support. Nevertheless, the conventional radar–



350 gauge verification scores do not systematically improve and generally deteriorate after the transformation. This result should not necessarily be interpreted as a failure of the local scaling approach. Rather, it indicates that observational-support mismatch represents only one component of the differences between radar and rain-gauge measurements.

In particular, the local singularity is derived from the spatial structure of the radar-observed rainfall field. Consequently, the transformation preserves, and may propagate towards finer scales, uncertainties already present in that field. These include
 355 errors in radar rainfall retrieval as well as spatial displacement or imperfect representation of the rainfall structure. For example, a positive local singularity indicates that the radar-observed rainfall becomes increasingly concentrated towards the gauge location at finer scales. Equation 4 will therefore increase the corresponding rainfall estimate. If the original radar rainfall at that location is already positively biased relative to the gauge, however, the scale transformation can increase rather than reduce their discrepancy. Similarly, an error in the spatial positioning of a rainfall feature may lead to a locally plausible
 360 singularity in the radar field that does not represent the rainfall structure actually sampled by the gauge. Local singularity therefore characterises the scale structure of the *radar-observed* rainfall field; it cannot by itself distinguish observational-scale mismatch from errors in the magnitude or spatial organisation of that field.

Uncertainty is also introduced by extending the observed scaling relationship below the radar resolution. The local exponent $\gamma - H$ is estimated from radar-resolved scales between 1 and 16 km, whereas R_{scaled} is evaluated at a target support of
 365 50 m. The transformation therefore assumes that the locally retrieved scaling behaviour persists below the smallest scale at which it is directly supported by the radar observations. Although the reliability criteria introduced in Sect. 4.1 restrict this extrapolation to cases with a well-defined scaling relationship over the observed range, they cannot demonstrate that the same relationship remains valid throughout the sub-kilometre range. The scale-adjusted rainfall should consequently be interpreted as an extrapolation conditional on the locally observed radar scaling behaviour, rather than as a reconstruction of the true
 370 rainfall at 50 m or at the physical sampling scale of the gauge.

These findings also have implications for the interpretation of conventional radar-gauge verification. Previous studies have demonstrated that unresolved small-scale rainfall variability affects the expected agreement between radar and gauge observations and that conventional theoretical optimum scores need not represent the appropriate target when the two instruments sample rainfall at different spatial supports (Gires et al., 2014). The present results provide a complementary local perspective.
 375 Explicitly introducing the scale dependence inferred from the surrounding radar field materially changes the radar rainfall estimates and their comparison with gauges, but does not systematically move the two measurements closer together. Hence, closer radar-gauge agreement should not in itself be interpreted as evidence that the observational-support problem has been resolved, just as poorer agreement after a scale transformation does not necessarily demonstrate that the transformation is physically inappropriate.

380 More generally, the observed radar-gauge discrepancy should be understood as the combined outcome of several processes, including radar retrieval uncertainty, rain-gauge measurement and sampling uncertainty, unresolved rainfall variability, and differences in observational support. These components cannot be uniquely separated by the present analysis. The value of the local-singularity framework is therefore primarily diagnostic: it provides a means of explicitly perturbing the observational support according to the locally observed multi-scale rainfall structure and examining how this component affects radar-gauge



385 comparison. The finding that this adjustment alone does not improve conventional verification scores suggests that the remaining sources of discrepancy are substantial and that observational scale should not be treated as the sole explanation for radar-gauge differences.

From this perspective, the principal result is not whether R_{scaled} achieves better conventional verification scores than the original radar rainfall. Instead, the analysis demonstrates that accounting explicitly for local scale dependence can substantially
 390 alter the apparent radar-gauge relationship, while closer agreement is constrained by other observational and retrieval uncertainties. Correcting one source of observational inconsistency therefore does not necessarily maximise agreement between two imperfect observing systems. Future work combining local scaling information with explicit representations of radar retrieval uncertainty, gauge sampling uncertainty, and independent high-resolution rainfall observations would be needed to further separate these contributions and to evaluate the validity of the extrapolated scaling behaviour below radar resolution.

395 5 Conclusions

This study investigated how differences in observational support affect the comparison between weather-radar and rain-gauge rainfall measurements, using local rainfall singularity as a scale-aware diagnostic framework. The local scaling exponent $\gamma - H$ was estimated independently at each rain-gauge location and time step from the multi-scale structure of the surrounding radar rainfall field. The analysis showed that locally identifiable scaling behaviour is present for a substantial proportion
 400 of the observations, with the 1–16 km range provides more consistent scaling relationships than those extending to larger spatial scales. Both positive and negative singularities were observed, reflecting whether rainfall intensity locally increases or decreases towards finer observational scales. Poor scaling fits occurred predominantly where the scale-dependent signal was weak, particularly when $\gamma - H$ was close to zero, or for some extreme singularities associated with very low rainfall intensities.

For sufficiently constrained local scaling relationships, the retrieved $\gamma - H$ was extrapolated from the radar-resolved scales
 405 towards a pragmatic target support of 50 m. The resulting scale transformation produced location- and time-dependent changes in radar rainfall rather than a systematic increase or decrease: positive singularities increased rainfall towards finer support, whereas negative singularities reduced it. Importantly, the 50 m rainfall estimates represent an extrapolation of scaling behaviour observed at radar-resolved scales rather than a direct reconstruction of rainfall at the physical sampling scale of a rain gauge. The approach therefore provides a means of explicitly investigating the observational-scale component of radar-gauge
 410 differences rather than recovering a 'true' point-scale rainfall value.

Accounting for this local scale dependence substantially altered the radar-gauge relationship, but did not systematically improve conventional verification scores. Instead, the scale-adjusted rainfall generally exhibited greater scatter and poorer agreement with the gauge observations. This finding should not be interpreted simply as a failure of the scaling approach, nor does it demonstrate that conventional radar-gauge verification is artificially optimistic. Rather, it highlights that observational-support
 415 mismatch is only one component of radar-gauge disagreement. Radar retrieval errors, gauge measurement and sampling uncertainty, spatial displacement of rainfall features, unresolved sub-pixel variability, and uncertainty associated with extrapolating



the scaling relationship can all contribute to the observed differences. Correcting one source of observational inconsistency therefore does not necessarily maximise agreement between two imperfect observing systems.

420 The main contribution of the local-singularity framework is consequently diagnostic. It provides a location- and time-specific means of perturbing radar observational support according to the multi-scale rainfall structure actually observed around each gauge and of examining how this component affects radar-gauge comparison. The results address the need to distinguish observational-support effects from other sources of uncertainty when interpreting radar-gauge verification statistics and when using gauges to evaluate or adjust radar rainfall products.

425 Several questions remain for future investigation. Most importantly, the persistence of locally retrieved scaling behaviour below the radar resolution should be evaluated using independent high-resolution rainfall observations, and potential scale breaks between radar-resolved and finer spatial scales should be explicitly characterised. A second extension is to formulate local singularity in a joint space-time framework, accounting for the intrinsic space-time nature of rainfall and the anisotropy between spatial and temporal scaling. Finally, the framework could be combined with explicit representations of radar retrieval and gauge sampling uncertainty to better separate their respective contributions to radar-gauge disagreement. Such developments 430 would also provide a stronger basis for revisiting gauge-based radar adjustment and merging methods in which differences in observational support are explicitly considered rather than implicitly absorbed into the adjustment.

Data availability. The Nimrod radar data can be accessed via CEDA (Centre for Environmental Data Analysis). The rain gauge records can be accessed via sending requests to the Environment Agency UK.

435 *Author contributions.* Underlying idea emerged from discussion between AG and LW. AG designed study and performed analysis. LW provided data. Both authors contributed to writing paper.

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