



Theory of Recession Based on Concepts from Percolation Theory

Allen G. Hunt¹, Sonali Mohan¹, Yongqiang Zhang², Witold F. Krajewski³, Qingyun Duan⁴

¹Department of Physics, Wright State University, Dayton 45435 USA

²Institute of Geographic Science and Natural Resources Research, CAS, Beijing 100101 China

5 ³University of Iowa, Department of Civil & Environmental Engineering, Iowa City, 52242 USA

⁴State Key Laboratory for Water Disaster Prevention, Hohai University, Nanjing, 210029 China

Correspondence to: Allen G. Hunt (allen.hunt@wright.edu)

10 **Abstract.** Using the definition of recession as the decline in streamflow following a rainfall event peak and before the next peak (provided evapotranspiration may be neglected), recession defines the relationship between catchment storage and outflow. The present analysis addresses the limiting processes absent in existing recession analysis, which are nonetheless treatable in the language of percolation theory. This includes: a) a universal decline in hydraulic conductivity as the percolation threshold is approached, and b) a universal decline in accessible moisture content beyond the simple decline in
 15 overall moisture content. The process of addressing these two time-dependent phenomena generates the basic structure of recession response: 1) linear response for an average of multiple events, 2) the corresponding time frame variation, 3) the specific characteristics of its non-linear response, specifically, its convexity. The present approach results in an absolute value of the non-linearity that is still underestimated as compared to results published in the literature

1 Introduction

20 Recession of river flow following precipitation has traditionally been interpreted as describing the relationship between subsurface storage and streamflow, the terms left over in the water balance, when effects of both precipitation and evapotranspiration can be neglected. Recession is addressed through a wide range of hypotheses, as well as interpretations, and characterized as a “subject in disarray” (Stewart, 2014). Disarray, like non-Fickian dispersion (Cushman and O’Malley, 2015), may not be anomalous in hydrology. Understanding recession, with difficulties entering from heterogeneity, relief,
 25 seasonal variation, and multiplicity of processes, is further complicated by the different means used to extract meaningful hydrologic (e.g. Troch et al. 2013), or geomorphologic, (e.g., Biswal and Marani; 2010) information from the data. Historically popular, a method developed by Brutsaert and Nieber (1977), has been termed a “cloud of points” analysis. This method uses data from all the hydrographs of a particular catchment simultaneously by plotting each individual value of $\log|dQ/dt|$ vs. its associated $\log Q$, where Q is the runoff. Using the entire data set simultaneously, incorporates a tacit
 30 assumption of a special relevance for each individual value of the streamflow, rather than a relevance of the specific sequence of values leading up to it. Other authors (e.g., Tashie et al. 2019; 2020) emphasize the exact runoff decay in any individual event, for which the same variables are analyzed. Such events tend to traverse the overall “cloud of points” at a steeper slope than that derived from the entire cloud simultaneously. Thus, each given curve through the cloud of points tends to yield a larger parameter describing its recession slope (implying a stronger non-linear process) than the assemblage



35 of all the recession curves together would indicate. As in many cases in hydrology, the particular history does matter, if for nothing else than predicting the continuing behavior of the event in question.

The above discussion illuminates the existing dichotomy in the value of “cloud of points” vs. event-based analyses of recession. Specifically, such analyses may reveal distinct characteristics of a catchment, or the process of recession. For
40 example, the tendency for cloud of points analysis to generate linear, or near linear, recession behavior with consistent time scales of ca. 10 days to 60 days (Fenicia et al. 2006; Chen and Krajewski, 2015; Kleidon and Savenjie, 2017; Gao et al. 2022; Brutseart, 2008; Bogaart et al. 2016) across wide ranges of length scale and climate regimes may reveal universal tendencies of the evolution of catchments (e.g., Kleidon and Savenije, 2017; Mohan and Hunt, 2026), while the particular curve may reveal specific characteristics of the associated catchment. The greater similarities of the cloud of points results
45 demand an explanation within the same framework as the rather strongly non-linear event-based recession results that as a rule exhibit convexity, Tashie et al. 2020, rather than concavity of response. As Tashie et al. (2020) state, linear responses tend to universality, while linearity may be violated in many different ways.

1.1 Changing Perceptual Models of the Recession Curve

Ever since continuous discharge measurements started to become more widespread it has been recognized that the recession
50 curve is an expression of storage depletion in a catchment. The early analyses of Hall (1918) showed how there was a tendency for individual event recessions to exhibit an initial nonlinearity, but become more consistent with time, allowing him to superimpose multiple event recession curves into what he called the “Basic Curve” (now more commonly called the “Master Recession Curve”) that could then be used as a means of separating “surface runoff” from “percolation discharge”. Recessions were therefore thought to be the result of subsurface drainage of storage. Hall drew the analogy of such
55 subsurface drainage with discharge from a tank or reservoir, providing a basis for a mathematical description of the process. In fact, an equation for describing the drainage to a stream channel of saturated storage in soil and rock on a hillslope had been available since Boussinesq (1877). In Boussinesq’s first approximation of his 1877 paper, he considered a form of Darcy’s law under the Duperoy-Forchheimer assumption that hydraulic potential in the saturated profile at any point is uniform. Combining an expression for flux at a point with the continuity equation gives the Boussinesq equation:

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$$\frac{dh}{dt} = \frac{k}{\theta_f} \left[\cos \alpha \frac{d}{dx} \left(\frac{dh}{dx} \right) + \sin \alpha \left(\frac{dh}{dx} \right) \right] + \frac{r}{\theta_f} \quad (1)$$

where k is the saturated hydraulic conductivity, r is the rainfall rate, α is the slope angle; θ_f is the drainable porosity, $h(x,t)$ is the height of the water table, x is distance from the base of the slope and t is time. This is a nonlinear parabolic equation that is not easy to solve analytically. Boussinesq (1877) simplified the problem by assuming that where the slope α is
65 significant, the differential term $\frac{d}{dx} \left(\frac{dh}{dx} \right)$ will be negligible, and the equation reduces to a kinematic wave equation, which



allows a simple solution for recessions from any initial flux. In the simplest case this reduces to the equation for the outflow from a linear storage:

$$Q = Q_o e^{-t/t_0} \quad (2)$$

70 where Q_o is an initial flow and t_0 is a time constant. This equation has been widely used in the analysis of baseflow recessions. It has also been frequently used as a representation for recession curves from catchments and aquifers, even though Boussinesq himself and Maillet (1905) recognized that it was not always a good fit to observed data. Boussinesq (1904) suggested adding a constant, or representing the resulting form as a sum of exponentials with different time constants or fitting the double exponential:

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$$Q = Q_o e^{-\left(\frac{t}{t_0}\right)^n} \quad (3)$$

A similar suggestion was also made by Robert Horton, recognizing that there may be multiple storages of water supplying water to the recession (Horton, 1933). He states that he had first used this in 1904, in the analyses of Horton (1914). The multiple source idea then allows the disaggregation of recession curves into segments, which have been sometimes
 80 interpreted in terms of different flow processes. Barnes (1939; 1944) for example used breaks of slope on a log-linear plot to separate “storm-seepage”, which he later called “intermediate flow” or “interflow”, from “base-flow”.

For a horizontal bed, the kinematic approximation clearly does not work because only the first term of Eq. (1) survives, and in Boussinesq’s second approximation of 1904, he provided a specific analytical solution for drainage under the assumption
 85 that all flow lines are horizontal and for an initial water table described by an inverse beta function in a domain of finite length. This solution has the simple form:

$$Q = Q_o/[at - 1]^2 \quad (4)$$

The recession problem has continued to intrigue mathematically inclined hydrologists ever since with continued explorations
 90 of analytical solutions to the Boussinesq equation (e.g. Bartlett and Porporato, 2018). Notable reviews were published by Hall (1968), Tallaksen (1995) and Rupp and Selker (2009). What is common to all these solutions derived from the Boussinesq equation however is a perceptual model that is based on the drainage of a single effective saturated zone to a river channel with an assumption of zero recharge (which was neglected even in the case of the multiple components of Barnes) and zero evaporation losses, that might affect the shape of recession curves when discharges are low and the
 95 demands of plants may be high. The heterogeneity of saturated zone storage across a catchment resulting from topography



and irregularities of hydraulic conductivity profiles in soils and regolith are also usually neglected (but see Harmon et al., 2009; Clark et al., 2009).

Thus, there is potential for an alternative perceptual model and theoretical framework for the representation of recession
100 curves, including initial nonlinearity effects, through percolation theory. Recession is then viewed as the gradual breakdown
of continuous subsurface pathways in the vadose and saturated zones. The initial non-linearity of recession follows as the
approach to the percolation transition in near surface relatively rapidly fluctuating storage reservoirs, when the medium
moisture content approaches the threshold for connectivity (Sahimi, 2025). At this point, flow rates start to diminish
significantly, while an increasing fraction of water-filled pores becomes cut off from flow paths (Stauffer and Aharony,
105 1994). This allows a theoretical derivation of the recession curve. Further, this description is based on a time-independent
formulation of recession that was shown to be consistent with the linear recession analyses of the cloud of points (Mohan
and Hunt, 2026). Thus, we exploit fundamental results from percolation theory to illustrate a fundamental contrast between
perceptual models that explicitly incorporate time-dependent modeling and those that do not. This contrast is, in a broader
sense, in line with an increased emphasis on “memory” in hydrologic modeling (Famani et al. 2025; Gharari and Razavi,
110 2018; Beven et al., 2026).

More importantly, however, we will be employing the explicitly fractal structure of percolation theory (Sahimi, 2023) which
we describe in the language of the critical path for percolation (Hunt and Sahimi, 2017), for water transport through an
unsaturated zone. This theoretical approach unites (in a time-independent format) the tendency for recession in a cloud of
115 points analysis to be nearly linear, but in a time-dependent format for recession to be highly non-linear for individual events.
Using global data descriptions (Blöschl and Sivapalan, 1995) for unsaturated flow rates on the order of microns per second
and sources (Pelletier et al. 2016) for depths of unconsolidated sediments on the order of meters, the time-independent
approach generates the observed time scales of linear analysis on the order of weeks, including the shorter time scales of
mountain catchments (Zecharias and Brutsaert, 1998), while the latter generates from theory both the time constants and the
120 tendency for individual rainfall events to generate strongly non-linear and convex recession curves.

2 Theoretical Framework

2.1 General Recession Analysis

125 The proposition that, following a precipitation event, streamflow decays as a power law, i.e., $dQ/dt = -aQ^b$ (with a and b
parameters to be found by experiment or predicted by theory) traces back to Brutsaert and Nieber (1977), who suggested that



analysis using this framework could lead to better understanding of catchment hydrology. For the particular case that $b = 1$, known as the linear model, $Q = Q_0 \exp [-at]$. In this case, $1/a$ is the time constant, i.e., the time when the runoff has declined to a magnitude of $1/e$ times the initial value. All other values of b lead to a power-law decay in streamflow, for which a representative time does not precisely exist, although it is possible to define times that help compare the rates of recessions in distinct catchments (Bogaart et al. 2016) or quantify effects of climate change through systematic changes in the values of the parameters. Finding a framework within which actual data for recession can be understood has become a highly sought goal of hydrologists since Brutsaert and Niebert (1977).

2.2 Simplest (Euclidean) Approach and Conceptual Basis

By showing that actual stream velocities have essentially no influence on recession curves, Chen and Krajewski (2015) were able to establish that the primary influence on low flow recession curves is subsurface flow. Adding to this the insights regarding the (lack of) relevance of groundwater flow parameters (McDonnell et al. 2014; 2015), eliminates the specific horizontal dimensions of the catchment from relevance. Accordingly, the volume of the catchment vadose zone water storage capacity as measured by its area A , porosity ϕ , and depth to the water table, d , must be relevant for the storage, while its depth d , porosity ϕ and large-scale hydraulic conductivity, k define a time scale. Thus, within this entirely Euclidean framework, one can write for recession processes,

$$Q = kA = \frac{k}{d}V = \frac{k}{\phi d}S \quad (5)$$

where Q is the outflow, and the storage is not simply the volume, but its product with the porosity, ϕ . From Eq. (5)

$$\frac{dQ}{dt} = \frac{k}{\phi d} \frac{dS}{dt} = -\frac{k}{\phi d}Q \quad (6)$$

The result is compatible with the linear recession model; an equivalent result was obtained by Zecharias and Brutsaert (1988) from linearization of the Boussinesq equation. For future reference, the quotient k/ϕ represents the velocity v of vertical flow through this vadose zone; thus, d/v represents an approximate time scale, regardless of the (non)linearity of the recession curve. In the case of the linear model, it represents an e-folding time, while the relationship is more complicated in the case that a non-linear model is preferred (Bogaart et al. 2016). Using typical values of vadose zone flow rates in the microns per second presented by Blöschl and Sivapalan (1995) and typical thicknesses of unconsolidated sediments (globally) in regions without dramatic relief measured in the meters (Pelletier et al. 2016), values in accord with extracted time scales from the linear models cited above in the range of a few days to a few weeks are obtained.



2.2 2.3 Adding Non-Euclidean Characteristics of Percolation Theory Without Memory

Mohan and Hunt (2026) demonstrated that the linear model could also be derived from a very simple theoretical result from percolation theory, verified in simulations, Kunz and Souillard, 1978). This result is that the area of a percolation cluster with outflow boundaries into streams is universally proportional to its volume (i.e., storage). The relationship is equally valid in two dimensions, with the perimeter proportional to the area surrounded by the perimeter. Mohan and Hunt (2026) did not specify the proportionality constant, as no guidance was given to its value by Kunz and Souillard (1978). Thus, we can write, $A \propto V$. The numerical coefficient missing from percolation theory, of order 1, is not particularly relevant here, but the coefficients resulting from recession analysis are important. Using $Q = kA$ and $S = \emptyset V$, leads to the (mismatched in units) linear recession model,

$$\frac{dQ}{dt} \propto -\frac{k}{\emptyset} Q \quad (7)$$

Given its otherwise exact correspondence with Eq. (6), it seems logical that the missing length scale in the denominator should be d . This simple time scale was not derived in Mohan and Hunt (2026). However, such a percolation cluster, if it defines the hydraulic conductivity of the medium as well, must be very near the percolation threshold as defined by critical path analysis. The implication, if the same range of time scales is indeed appropriate as is given in the description of Eq. (6), is that the critical paths of percolation theory represent the most efficient continuous flow pathways through the medium. These pathways are assumed initially saturated – and the part that controls flow near the outlet remains saturated throughout the recession. However, that fact conceals the important process that the path is becoming sparser and more tortuous, leaving clumps of fully saturated pores disconnected as it drains the medium, important for the next section.

As seen and as has been reported (Mohan and Hunt, 2026) Eq. (7) generates the linear recession model in common use for the last 40 years since the prominent publication of Brutsaert and Nieber (1978). Upon addition of the factor d in the denominator, it also includes the otherwise missing time scale, known to be compatible with a wide range of reported data. However, its implication of fractal geometry of flow paths and outlet boundaries is compatible with modern hydrologic understanding of catchments. Figure 1 shows a conceptual, or visual, representation of the basis of our theoretical model.

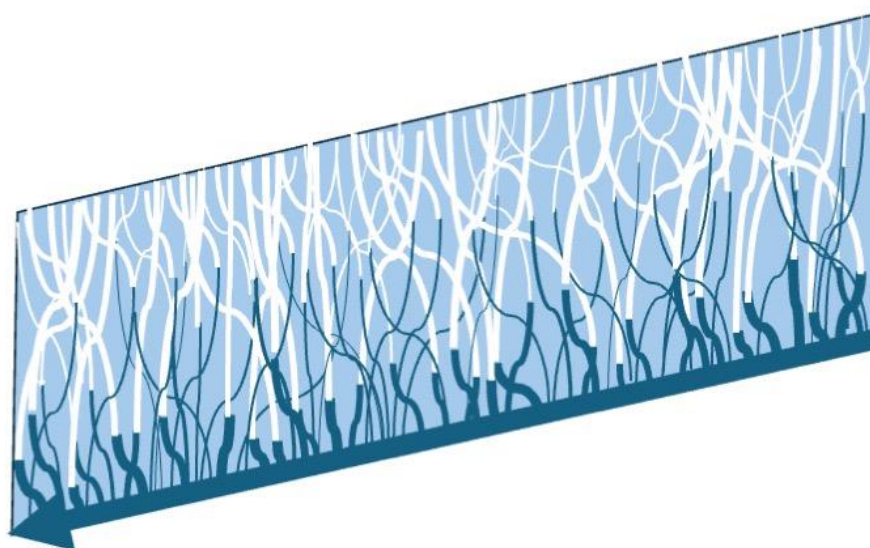


Figure 1. Schematic picture of basis for percolation-based calculation of recession characteristics. White parts of flow paths have drained. Darker portions are still water-filled. Slope effects suggested are not yet incorporated. The proportionality of volumes of percolation clusters to their surface areas holds for all times on a given recession curve and for all recession curves.

2.4 Adding Memory (Time-dependence)

To add the effect of memory, we use the percolation theoretical expressions for the dependence of the hydraulic conductivity on total vadose zone moisture content θ as well as the fraction of the water-filled pore space accessible to an infinite cluster (i.e., what can be removed by flow). This becomes an effective storage *and* it is history-dependent. In fact, it likely provides the missing physics of Farmani et al. (2025) in their discussion of hysteresis, as it has already been proven (Hunt, 2004) to yield core-scale hysteresis without use of tunable parameters, as foreseen by Heiba et al. (1982).

It will be convenient in the following analysis to represent Q in terms of the hydraulic conductivity and the initial cross-sectional area represented by saturated flow paths, rather than directly in terms of precipitation. Although, this does not give us a direct way to represent different hydrographs for different values of P , it may be inferred that larger storm events (in wetter seasons) will tend to generate larger areas with saturated subsurface stormflow connections to streams. Although the



200 discussion here does not fully answer the related question of how best to construct a rainfall-runoff model, it is sufficient for an understanding of the somewhat simpler subject of recession.

The fraction of water-filled vadose zone that is connected to an infinite cluster at total moisture content θ is proportional to $(\theta - \theta_t)^{0.41}$ in three dimensions, with θ_t its critical value for percolation. Note that the relevant pore-size distribution is
 205 certainly not known in preferred flow paths in the field; thus the hydraulic conductivity may be better written simply as the universal expression (e.g., Sahimi, 2025) $K_S(\theta - \theta_t)^2$, where K_S relates to the relevant value for the (saturated) preferential flow path. This value is assumed, when the number of flow paths per unit area is averaged over, to generate the hydraulic conductivity of the medium. Accordingly, the coupled dynamic percolation equations for S and Q are given as:

$$210 \quad S = \theta(\theta - \theta_t)^{0.41} Ad \quad (8)$$

$$Q = AK_S(\theta - \theta_t)^2 \quad (9)$$

The dynamic of these equations is represented by the fact that the moisture content θ has a time dependence. For the time derivative we must therefore now employ the chain rule for the quantity S in terms of θ , i.e., $dS/dt = (dS/d\theta)(d\theta/dt)$.

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The two factors in Eq. (8) and Eq. (9) are next incorporated into Eq. (5), including the factor d . Note that we will be changing variables from $(\theta - \theta_t)$ to y , but the differentials of these two variables are equal, since the threshold moisture θ_t is considered constant:

$$220 \quad \frac{dS}{dt} = [Ad(\theta - \theta_t)^{0.41} + 0.41Ad\theta(\theta - \theta_t)^{-0.59}] \frac{d(\theta - \theta_t)}{dt} = -Q = -AK_S(\theta - \theta_t)^2 \quad (10)$$

Equation (10) simplifies to,

$$[Ad(y)^{0.41} + 0.41Ad(y + \theta_t)(y)^{-0.59}] \frac{d(y)}{dt} = -AK_S(y)^2 \quad (11)$$

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where the quantity $\theta - \theta_t$ is written as y . The factor $y + \theta_t$ is prevented from diverging by the θ_t and therefore plays no role in the predicted value of b . Thus, it may be approximated as either its initial value $\emptyset$ (our choice), or by its final value, θ_t :

$$[d(y)^{0.41} + 0.41d\emptyset(y)^{-0.59}] \frac{d(y)}{dt} = -K_S(y)^2 \quad (12)$$



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Defining y/ϕ as z allows the above equation to be rewritten as

$$\left[z^{0.41} + \frac{0.41}{\phi} z^{-0.59} \right] \frac{d(z)}{dt} = - \frac{\phi^{0.59} K_S}{d} z^2 \quad (13)$$

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Here, the resulting time constant has a more complicated dependence on porosity ϕ , but still depends on d/K_S as in Eq. (6) and in Eq. (7), despite the fact that we are treating a preferential flow in highly non-Euclidean geometry. However, it must also be understood that the result for the hydraulic conductivity (expressed earlier as k) is always dominated by the preferred flow paths. Also, K_S differs slightly from its usual definition in that it here refers to a hydraulic conductivity as a function of saturation of the portion of the medium comprising the dominant flow paths.

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Consider now the overall behavior of the derived recession curve. At the beginning, when z is nearly 1, the first term on the left-hand side of Eq. (13) is larger than the second term. The result from carrying out the subsequent analysis on the first term is,

$$\frac{dz}{dt} \approx -z^{1.59} \quad (14)$$

which produces a value of the exponent b of 1.3. However, it is nearly impossible to saturate an entire vadose zone of a catchment (or even its predominant flow trajectories) and this value of b is unlikely to be measured for an individual event, if this theoretical approach is appropriate and sufficient. At the limit of large times, where z approaches zero, the second term of Eq. (13) is infinitely larger than the first, and

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$$\frac{dz}{dt} \approx -z^{2.59} \quad (15)$$

The result for b in this case is 1.8.

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The conclusion is that, as time increases, the value of b will increase from somewhat larger than 1.3 to 1.8, a convex curvature in the description of Tashie et al. (2020). An average power across such a curvature would, however, be less than 1.8. The reason why the initial power will normally be somewhat larger than 1.3 is because most hydrographs will not start from near 100% saturation. Note that, in rapidly draining soils, the period of recession may be predominantly characterized by the exponent 1.8, since the first days can be excluded from the analysis (Tashie et al. 2020), thus leading to the conclusion that larger exponents are found in rapidly draining (e.g., mountain) soils and humid climates.

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We have repeated the calculation for the two-dimensional case (flow dominated by mud cracks, fractures, etc.), where the exponent 0.41 is replaced by $5/36$ and the power 2 for the conductivity is replaced by 1.3. The results are insignificantly different, with b increasing from a minimum value of 1.24 to a maximum of 1.9. Omission of the accessibility factor in this case leads to a value of $b = 1.24$ with no convexity.

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In Fig. 2 we show simultaneous depictions of representative hydrograph, moisture drawdown, and recession curve obtained from Eq. (12) numerically using the exponents for three dimensions. Here, the initial Q was obtained, without specifying catchment depth, from the product of the catchment area A and the saturated hydraulic conductivity which, in a linear and Euclidean model would imply a fully saturated surface layer throughout the catchment and does not directly reflect variable precipitation, nor different values of a residual moisture, etc. It is found that the larger value of the hydraulic conductivity produces: 1), a larger initial Q , 2) a shorter time period of recession, 3) a stronger non-linearity, 4) greater convexity. In fact, the two $\log(-dQ/dt)$ vs. Q curves would actually cross each other, if plotted on the same graph. Taken together, these results are compatible with conservation of mass.

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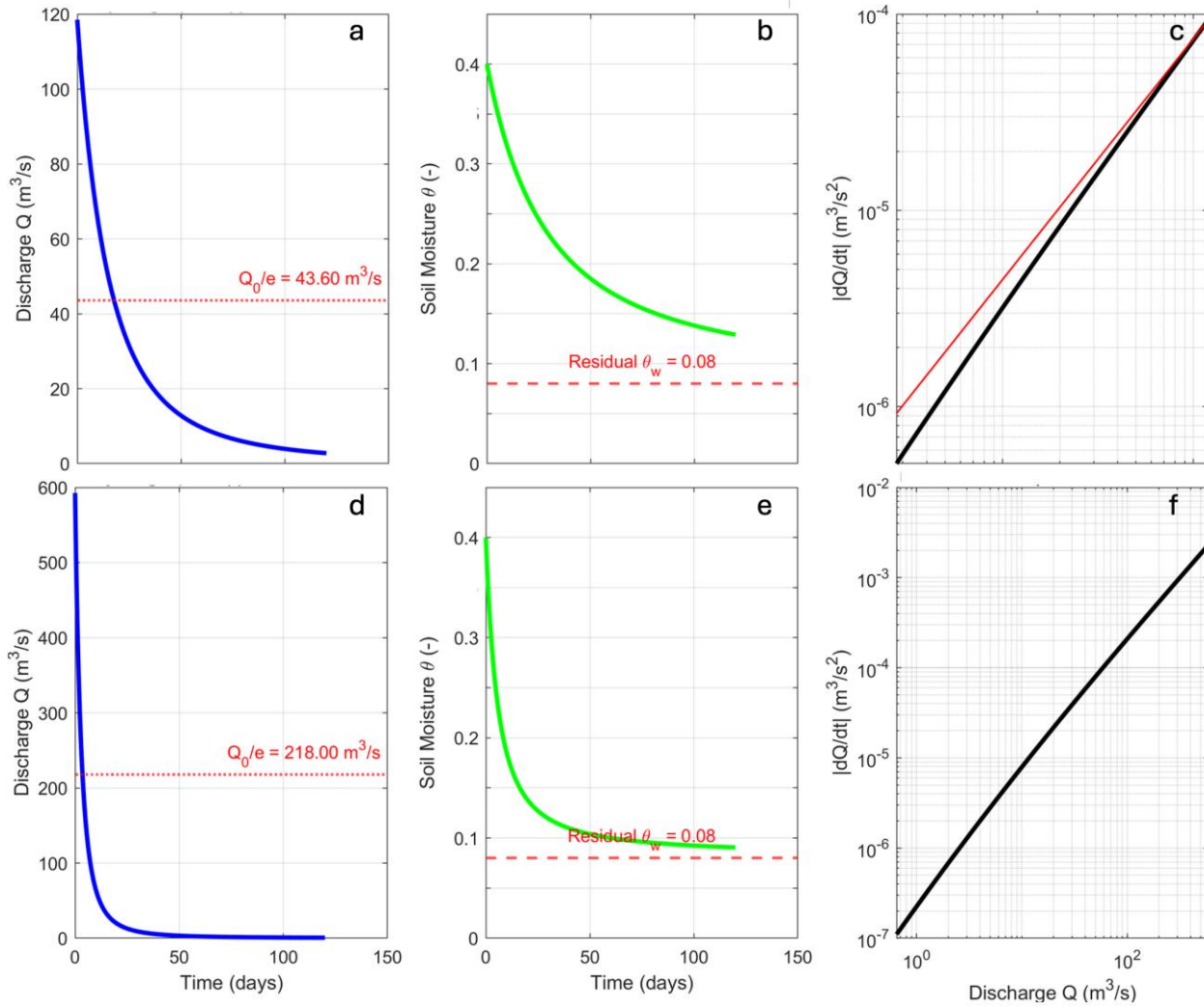


Figure 2. Representations of the hydrograph (a), the moisture drawdown (b), and the recession characteristics (c) ($\log(|dQ/dt|)$ as a function of $\log Q$) for a catchment using in Eq. (9) representative preferential flow $K_s = 1.16 \mu\text{m s}^{-1}$ for depth $d = 1\text{m}$, area $A = 1000\text{km}^2$, and critical (“residual”) moisture content of 0.08, thus 20% of the porosity of 0.4, (d), (e), and (f), same graphs and same parameters except for the saturated hydraulic conductivity, which is increased to $5.8 \mu\text{m s}^{-1}$. This combination of constants generates $118 \text{ m}^3 \text{ s}^{-1}$ and $590 \text{ m}^3 \text{ s}^{-1}$ for the initial Q values appearing in Fig. 2a and 2d, respectively. Here, effects of d are subsumed into the constant K_s , i.e., the magnitude of what is reported as K_s is actually K_s/d .

The average of the two saturated hydraulic conductivity values used in Fig. 2 is near the median flow velocity of somewhat more than $3 \mu\text{m s}^{-1}$ in the vadose zone and in the groundwater represented at the time scale of about 1 month on the Stommel



diagram of Blöschl and Sivapalan (1995). In (c), “recession slope characteristics,” the red line is a depiction of a value of b ca. 1.3, the expected initial slope, in order to reveal the increasing slope with decreasing Q . Note that the average slope of the curve in (f) is near 1.6, in conformance with expectations for a curved hydrograph with (negative) exponent increasing from a magnitude of around 1.3 to near 1.8.

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By comparing Fig (2c) with Fig. (2f), it is apparent that, in our theoretical treatment, more highly permeable soils produce a recession more obviously convex. Increasing K by a factor 5 leads to a decrease in the associated time scale of the same magnitude. Because d is folded into K (as K/d) decreasing the unsaturated zone thickness by the same factor of 5 (e.g., in mountain catchments) would have the identical effect.

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It may be noted that, if the accessibility factor is ignored, the result for b for recession in 3D is 1.5. In that case, the percolation model would yield (if not addressing the time dependence at all) $b = 1$, but including the time dependence, $b = 1.5$ (no curvature in either case). Although these results are the same limiting values as obtained from using the Boussinesq equation in the limit of long times (Troch et al. 2013; Tashie et al., 2020), the underlying concept is quite different.

300 3 Discussion

3.1 Focus on Individual Recession Curves

Consider in detail the continental scale summary of Tashie et al. (2020) with 150,000 individual recession curves from on the order of 1,000 different catchments. Notably, 99% of all their catchments exhibited convex recession curves most of the time (but not all the time!). Comparison with that data set suggests that our values of b are a significant underestimation of their median value of 2.31. Nevertheless, their cloud of points (with most values between 1 and 1.5) median value is larger than 1 as well, at 1.32. Similarly, Bogaart et al. (2016) found that 50% of their individual catchments (cloud of points determination) were between 1.1 and 1.4. Values of $b > 1$ have also been interpreted in terms of multiple relevant time scales, or other measures of heterogeneity at larger scales than those considered here. If additional factors which we have not addressed can change the slope of the recession averages from 1 to 1.32, it is at least plausible that individual recession curves could be affected similarly. An increase in our calculated range of b values by 32% would bring them from an initial 1.72 to a final value 2.38, still an underestimation, but much closer to the observed data summary. In any case, the present theoretical treatment does not generate the late-time cross-over to the nearly vertical slopes often seen in individual recession curves of both Tashie et al. (2019) and Tashie et al. (2020). Tashie et al. (2019) attribute the cause to the influence of evapotranspiration. In the percolation-based picture related to hydraulic conductivity, however, the rapid decline of K with decreasing moisture content near the end of the recession curve would make it equally difficult for plants to exploit any

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remaining soil moisture deep in the profile, as well. Thus, the explanation of Tashie et al. (2019) in terms of plant behavior does not appear to be compatible with our framework. A second potential explanation that we tested was the arrival of the drying front at the base of the vadose zone (in Fig. 1, when the average drained portions in white reach the bottom). Such a drying front advance corresponds to bond invasion percolation with trapping (Sheppard et al. 1999). However, when we apply the relevant 3D percolation exponent (1.46) pertaining to the downward migration of the drying front together with the larger hydraulic conductivity value compatible with Fig. 2 (and a depth of 1m), our calculations yield a time scale of about 75 days, which appears to be too long for Figure 2d, though by less than an order of magnitude. If we use the 2D percolation exponent of 1.22 that describes advance of the drying front, we find a time scale of 7 days, which appears to be too short. In order for the 2D predictions (with actual recession exponent behavior nearly the same as in 3D) to be relevant, linear structures such as convergence zones in surface topography, or fractures in the bedrock would have to be guiding the recession. Alternatively, certain correlations can reduce exponents related to tortuosity or backbone dimensionalities (applied in wetting and drying front advance), and thus the associated time scales. Overall, we cannot yet claim that we account quantitatively for entire individual recession curves – a potential subject for additional research.

3.2 Non-Euclidean (Percolation) Model: Time-independent Average vs. Time-dependent Events

Equation (12), which generated Fig. 1, preserves the structure of the time-independent analysis insofar as the roles of S and Q are concerned, i.e., both are proportional to the relevant area, A . In the particular hydrographs chosen, we let A be the area of the watershed and an initial moisture content $\theta = \emptyset$, which would generate, for our recession model, a maximum possible subsurface contribution to the recession. (Use, instead, of field capacity as the beginning point of recession would substitute ca. 80% saturation for 100%, a negligible quantitative change, would be compatible with omitting the first few days of declining streamflow from the recession, as reported by Tashie et al., 2020). However, for smaller precipitation, or more localized maximum rainfall rates, a range of initial values of A will be possible. Every recession curve, however, scales with the initial value of A , because the initial S , and both initial Q and initial dQ/dt are all proportional to the same area, as in Mohan and Hunt (2026). In this simplified picture, the initial points (Q , $-dQ/dt$) of all the recession curves will tend to line up along the same linear model, progressing downward and to the left, with smaller contributing area (e.g., lighter precipitation), nevertheless, each with initial slopes near 1.3, increasing towards 1.8. In any case, the overall relevance of the linear model in relating individual recession events providing steeper slopes that pass through a region of $\log Q$ vs. $\log |dQ/dt|$ space which would approximately generate an average linear recession model, is explicitly preserved, as shown in Fig. 3.

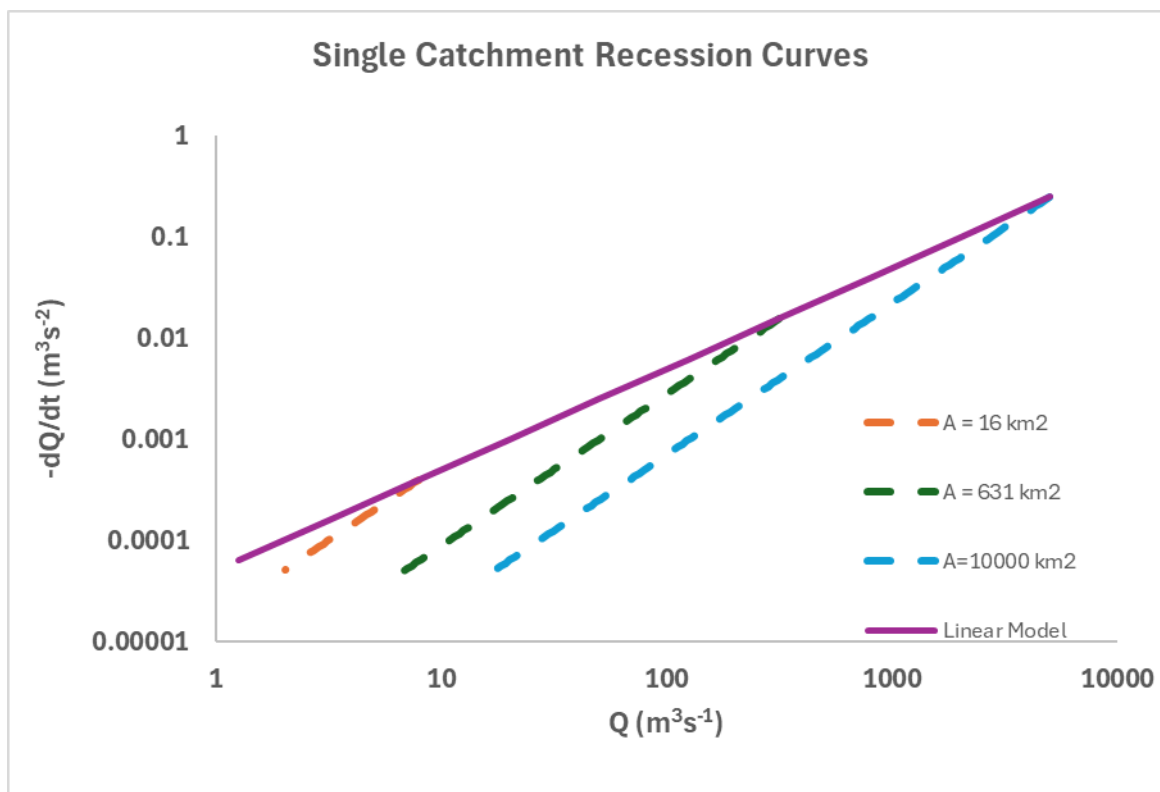


Figure 3. Representation of hydrographs for a catchment with characteristics similar to that of Fig. 2, but with a range of contributing areas that includes at the far right a hydrograph appropriate for wetting the full catchment, but progressively smaller wetted areas progressing downward to the left, with the leftmost hydrograph corresponding to a wetted fraction of about 3%. Here, each individual recession is approximated using the average slope of -1.5 .

Except for the very late recession period, which we do not yet quantitatively integrate into predictions, the results of Fig. 3 are in close visual correspondence with the Piscataquis River recession curves highlighted as examples in Figures 2 – 4 of Tashie et al. (2020). Although the scale of variability shown in Fig. 3 is greater than in Tashie et al. (2020), one can simply view a smaller portion of Fig. 3 in greater detail, in order to replicate a large fraction of their data.

4 Conclusions

We presented a theory of recession that yields a predicted value at least for the upper bounds (see Fig. 3) of the “cloud of points” determinations of the exponent b equal to 1 (the linear model), but the result $b = 1.8$ in individual recessions in the limit at later time scales together with an increasing magnitude of b with increasing time or decreasing flow. The increasing magnitude of b arises from approach to a percolation transition, when flow from the surface through the vadose zone to saturated zones loses connectivity. The distinction between the two predictions was a result of tracking the temporal effects



of drying of the unsaturated zone, not merely following the universal proportionality between percolation cluster surface area and percolation volume. This distinction is a further reminder of the importance of “memory” in the field of hydrology. The “perceptual model” for this theoretical approach was the decay of loading of horizontal saturated zone drainage through the temporal decay of the continuous flow pathways in the system. The mathematical framework for the time-dependent recession was supplied by the universal theory of percolation. In this picture, both the rates of flow (affecting directly Q) and the connectivity of accessible storage (affecting S) were accounted for. An equivalence between a simple geometrical model of the medium and the fractal continuous flow pathways was drawn, allowing time scales to be predicted approximately from the Euclidean geometry. The resulting recession time scales were between 10 and 30 days, closely related to, but somewhat smaller than, typically reported times of 10 to 60 days, although these time scales are likely somewhat too large when compared with the individual recession curves of Tashie et al. (2020). However, numerical solution of Eq. (12) revealed a strong dependence of the time scale on the hydraulic conductivity, with the time scale for flow to fall by a factor of $1/e$ dropping from about 20 days to about 4 days with the five-fold increase for K_S chosen. Since our estimates of K_S effectively fold in a contribution from the thickness of the unconsolidated medium overlying bedrock (in Eq. 12, the combination K/d appears), such a dramatic shortening of time scales is likely compatible with the much shorter time scales in humid mountain catchments observed by Zecharias and Brutsaert (1988). The particular very late-time characteristic of nearly vertical recession curves is not yet reproduced, though we do not rule out the possibility of its general compatibility with the percolation model as resulting from the arrival of the drying front at the base of the unsaturated zone.

Use of the accessibility function from percolation theory in only the drying phase (recession) is incomplete for rainfall-runoff modeling. Drawing on the analogy for core-scale experiments (Hunt, 2004), for which the entry of water into the medium is delayed until all the pores allowed for water actually percolate generates a counterclockwise hysteresis loop. It is likely that incorporation of the accessibility factor in wetting as well will lead to a delay in wetting and thus a verifiable additional memory effect in accord with observations of catchment hysteresis. Such a result would then provide an additional important contribution to the “missing physics” referred to by Farmani et al. (2025).

Code and data availability

No data were accessed or used. AI generated code was used to solve Eq. (13) numerically and construct Figure 2. There was no other use of AI.

Supplement link

No supplement exists or is needed.



Team list

Assembled authors contributing to this publication have, as a group, never before collaborated.

Author contributions

395 AH and SM originated model framework. WK, and QD strengthened framework, provided background and made contact with existing models and interpretation. YZ provided equation solutions and graphics. AH wrote first draft. All authors contributed to revisions.

Competing interests

No competing interests to declare.

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Review statement

410 The review statement will be added by Copernicus Publications listing the handling editor as well as all contributing referees according to their status anonymous or identified.



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