



1 **A Physics-Constrained Bayesian Transformer**
2 **framework for reliable probabilistic flood forecasting**
3 **across contrasting hydrological regimes**

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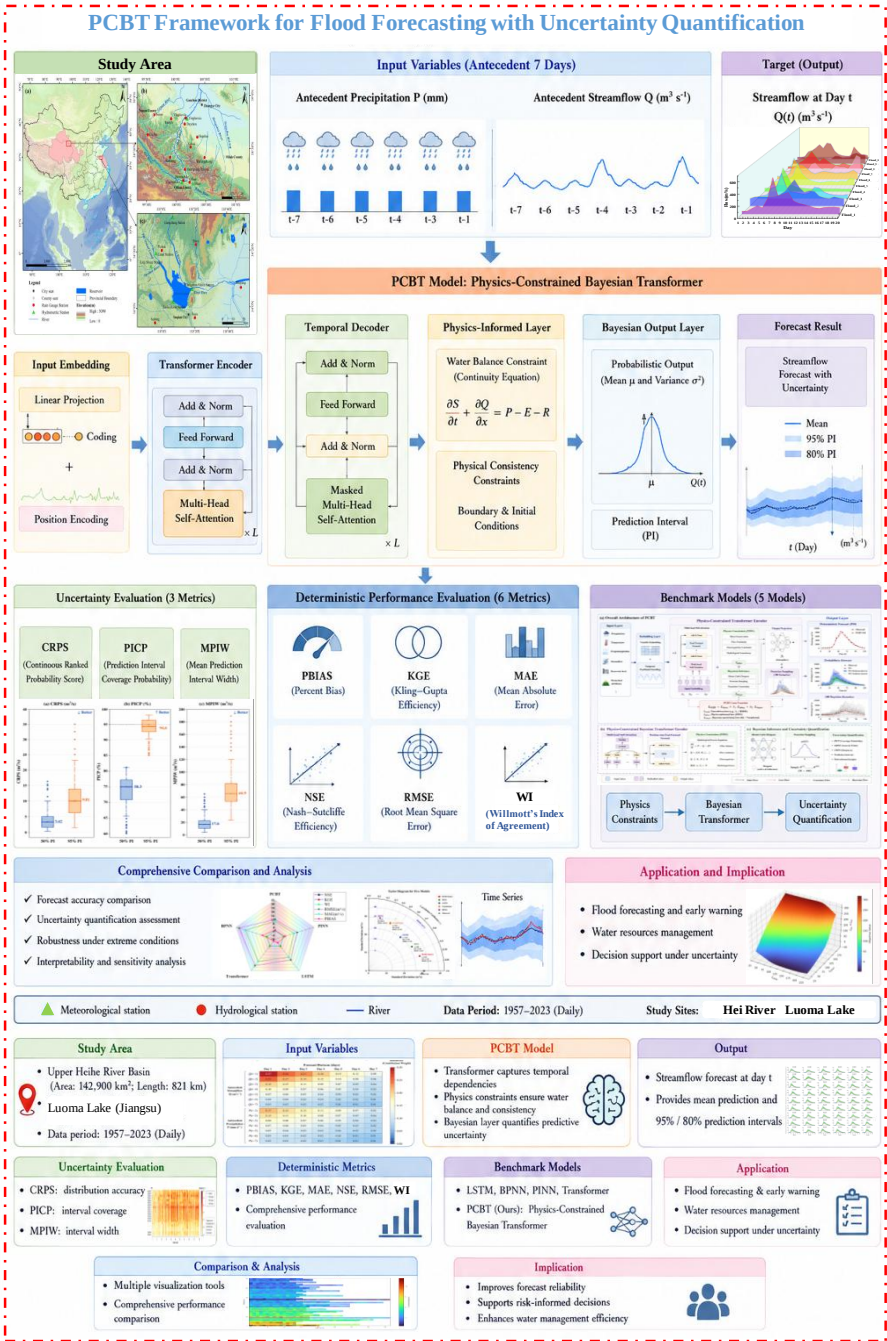
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11 **Graphical abstract:**

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Abstract: Flood forecasting remains challenging due to the nonlinear nature of rainfall–runoff processes, limited observations of extreme events, and inherent uncertainties associated with hydrological prediction. Although deep learning models have demonstrated strong predictive capability, their purely data-driven structures often lack physical consistency and provide limited information regarding forecast reliability. This study develops a Physics-Constrained Bayesian Transformer (PCBT) framework that integrates temporal representation learning, hydrological physical constraints, and Bayesian uncertainty quantification for probabilistic flood forecasting. The proposed framework combines a Transformer encoder to capture long-range dependencies within antecedent precipitation and streamflow sequences, physics-informed constraints based on water balance and flow continuity principles to regulate hydrologically feasible predictions, and a Bayesian inference module to characterize predictive uncertainty through probabilistic forecasting. The framework was evaluated using 50 independent flood events from two hydrologically contrasting watersheds in China: the mountainous Upper Hei River Basin and the lowland Luoma Lake Basin. The two basins represent distinct runoff generation mechanisms and provide a rigorous test of model robustness under heterogeneous hydrological conditions. Results demonstrate that PCBT effectively reproduces flood hydrograph dynamics and improves both deterministic accuracy and probabilistic reliability compared with Long Short-Term Memory (LSTM), Transformer, Physics-Informed Neural Network (PINN), and Backpropagation Neural Network (BPNN) benchmarks. Across independent testing events, PCBT achieved Nash–Sutcliffe Efficiency values exceeding 0.85 while



36 providing well-calibrated prediction intervals for uncertainty representation. The
37 Bayesian component successfully captured increased uncertainty during rapid rising
38 stages and flood peaks, whereas physical constraints improved the consistency between
39 predicted and observed hydrological responses. This study demonstrates that
40 integrating deep temporal learning, hydrological knowledge, and probabilistic
41 inference provides an effective pathway toward reliable flood forecasting under diverse
42 hydrological regimes. The proposed framework offers a transferable approach for
43 uncertainty-aware flood prediction and supports risk-informed water resources
44 management under changing environmental conditions.

45 **Keywords:** Probabilistic flood forecasting; Physics-informed machine learning;
46 Bayesian Transformer; Prediction uncertainty; Hydrological constraints; Contrasting
47 basins

48 1. Introduction

49 Flood forecasting is a fundamental component of hydrological prediction and water
50 resources management because accurate forecasts provide critical support for flood
51 mitigation, reservoir operation, emergency response, ecological protection, and
52 sustainable water allocation (Montanari et al., 2013; Blöschl et al., 2019; Shen, 2018).
53 Under the combined influences of climate change, intensified human activities, and
54 increasing hydrological variability, extreme flood events have become more frequent
55 and severe in many regions worldwide (IPCC, 2021; Gupta et al., 2014; Muñoz-Castro
56 et al., 2026). Reliable flood forecasting therefore plays a crucial role in reducing flood-
57 related losses and improving watershed resilience. However, flood generation and



58 routing processes are governed by complex nonlinear interactions among precipitation,
59 antecedent moisture conditions, watershed characteristics, and channel hydraulics.
60 These interactions introduce considerable uncertainty into hydrological forecasting and
61 make accurate prediction of flood evolution and peak discharge particularly challenging
62 ([Gupta et al., 2014](#); [Kratzert et al., 2018](#)).

63 To address these challenges, data-driven approaches have been increasingly adopted
64 in hydrological forecasting. Unlike conceptual hydrological models, machine learning
65 algorithms can automatically learn nonlinear relationships directly from observations
66 without requiring explicit descriptions of physical processes ([Chen et al., 2025a](#); [Liao
67 et al., 2025](#); [Solanki et al., 2025](#)). Early studies mainly relied on backpropagation neural
68 networks (BPNNs), support vector machines (SVMs), and random forest models for
69 rainfall–runoff simulation ([Kumar et al., 2026](#); [Amini et al., 2023](#)). More recently, deep
70 learning techniques have significantly advanced hydrological forecasting by improving
71 the representation of nonlinear temporal dynamics. Among them, Long Short-Term
72 Memory (LSTM) networks have demonstrated remarkable capability in capturing long-
73 term temporal dependencies and have become one of the most widely used deep
74 learning models in hydrology ([Gegenleithner et al., 2025](#); [Khatun et al., 2023](#); [Zhu et
75 al., 2025b](#)). Numerous studies have reported that LSTM-based models outperform
76 traditional conceptual models and shallow machine learning methods in streamflow
77 forecasting, rainfall–runoff simulation, and flood prediction tasks ([Zou et al., 2023](#);
78 [Chen et al., 2022](#); [Mosaffa et al., 2026](#)). For instance, [Gegenleithner et al. \(2025\)](#)
79 demonstrated that LSTM-based hybrid forecasting frameworks substantially



80 outperformed conventional ARIMA models in operational flood forecasting,
81 particularly for long lead-time predictions. This finding suggests that deep learning
82 architectures possess superior capabilities for extracting long-term temporal
83 dependencies from hydrological time series. [Chen et al. \(2022\)](#) proposed a ConvLSTM
84 framework by integrating convolutional neural networks and LSTM networks to jointly
85 extract spatial and temporal features from hydrological data. Their results demonstrated
86 that incorporating spatial information significantly improved flood arrival time and
87 peak discharge predictions compared with conventional time-series models. [Zhu et al.](#)
88 [\(2023\)](#) proposed an optimized model based on LSTM networks for early warning and
89 forecasting of ponding in urban drainage systems. This approach can quickly identify
90 and locate areas of ponding while maintaining high predictive accuracy. [Zhu et al.](#)
91 [\(2025b\)](#) proposed a hybrid hydrological forecasting framework that integrates the Xin-
92 an-jiang (XAJ) model and LSTM networks to simultaneously capture physical
93 processes and nonlinear relationships. By combining PSO and Bayesian optimization
94 for parameter calibration, Bayesian inference for uncertainty quantification, and SHAP
95 for model interpretability, the framework significantly improved forecasting accuracy,
96 reliability, and probabilistic prediction capability.

97 Recently, Transformer architectures have emerged as a promising alternative for
98 hydrological time-series forecasting ([Saravana et al., 2026](#)). Originally developed for
99 natural language processing, Transformers employ self-attention mechanisms to
100 capture long-range dependencies without relying on recurrent structures ([Ambika et al.,](#)
101 [2025; Gao et al., 2024](#)). Compared with LSTM models, Transformers can process long



102 sequences more efficiently and better represent complex interactions among multiple
103 hydrometeorological variables (Pölz et al., 2024; He et al., 2026). Their ability to model
104 long-term hydrological memory has led to growing applications in streamflow
105 forecasting, rainfall–runoff simulation, and regional hydrological prediction (Arsenault
106 et al., 2023; Ling et al., 2026). For instance, Fadillah et al. (2025) proposed a hybrid
107 LSTM–Transformer framework that improved flood forecasting accuracy by jointly
108 capturing local and global temporal dependencies, outperforming conventional
109 machine learning models. Xia et al. (2025) proposed a spatiotemporal Pathformer
110 model that improved multi-step flood forecasting by effectively capturing nonlinear
111 spatiotemporal dependencies, outperforming both LSTM and Transformer models.
112 Ambika et al. (2025) developed a Transformer-based FutureTST model for streamflow
113 forecasting that jointly utilizes historical and future hydrometeorological information.
114 The model achieved robust performance over extended forecast horizons and
115 outperformed traditional process-based hydrological models. Nevertheless, despite
116 their strong predictive capability, Transformer-based models remain purely data-driven.
117 Model training depends entirely on historical observations, and the resulting predictions
118 are not explicitly constrained by hydrological principles (Liu et al., 2026).
119 Consequently, these models may generate physically inconsistent outputs, especially
120 under extrapolation conditions, sparse-data scenarios, or extreme flood events where
121 observations are limited (Pan et al., 2026; Küllahcı et al., 2026).

122 To improve physical consistency, Physics-Informed Neural Networks (PINNs) have
123 recently attracted considerable attention in scientific machine learning (Raissi et al.,



2019). PINNs incorporate governing equations directly into neural network training by embedding physical residuals into the loss function, thereby constraining model solutions within physically feasible regions. This framework has achieved remarkable success in fluid dynamics, groundwater modeling, environmental systems, and hydrological simulations (Raissi et al., 2019; Yang et al., 2026; Virupaksha et al., 2026). For instance, Virupaksha et al. (2026) investigated continuous-time, discrete-time, and time-decomposition PINNs for transient groundwater flow modeling. Their results demonstrated that the discrete-time PINN achieved superior accuracy and training efficiency, reducing computational costs while improving performance under complex dynamic conditions. Jia et al. (2025) developed a multi-scale hybrid learning framework that integrates PINNs, neural operators, and convolutional neural networks for hydrodynamic transport modeling. By combining multi-scale feature extraction with hard physical constraints, the framework improved prediction accuracy, robustness, and physical consistency in complex spatiotemporal systems. In hydrology, PINNs have demonstrated advantages in improving model robustness, reducing dependence on large training datasets, and enhancing extrapolation capability under data-scarce conditions (Feng et al., 2022; Read et al., 2019). Recent studies have applied PINNs to flood forecasting, water-level prediction, and rainfall-runoff simulation, showing that physical constraints can significantly improve model interpretability and hydrological consistency (Wei et al., 2026; Lu et al., 2025; Tian et al., 2025). For instance, Wei et al. (2026) proposed a Physics-Informed Gated Transformer framework that integrates soft hydrological constraints, gated residual



146 networks, and variance-constrained learning to improve flood forecasting in data-scarce
147 catchments. The model effectively reduced phase-lag errors and enhanced robustness
148 to noisy inputs, outperforming conventional LSTM, HBV, and standard Transformer
149 models in both forecasting accuracy and spatial generalization. [Lu et al. \(2025\)](#)
150 proposed a PINN framework that integrates physically guided hyper-parameter
151 selection and data augmentation to improve lake water level forecasting. The model
152 effectively addressed data imbalance under extreme conditions and achieved higher
153 accuracy and computational efficiency than conventional deep learning and
154 hydrodynamic models. [Tian et al. \(2025\)](#) enhanced PINNs for solving two-dimensional
155 shallow water equations by integrating dimensional transformation and neuron-wise
156 adaptive activation functions. Their results showed that a hybrid primitive–conservative
157 formulation improved simulation accuracy and stability, highlighting the potential of
158 PINNs for physically consistent flood modeling.

159 Despite these advances, several limitations remain unresolved. Firstly, most existing
160 PINN applications employ fully connected feedforward neural networks as the
161 backbone architecture. Although such structures effectively enforce physical
162 constraints, they are not specifically designed to learn long-range temporal
163 dependencies embedded in flood evolution processes ([Chen et al., 2025a](#)).
164 Consequently, their ability to represent complex temporal dynamics remains limited.
165 Second, most Transformer-based hydrological studies focus primarily on prediction
166 accuracy while neglecting hydrological consistency. As a result, physically unrealistic
167 fluctuations may occur in predicted hydrographs, particularly during flood peaks and



168 recession periods (Zhu et al., 2025c; Adibfar et al., 2025; Liu et al., 2026). Thirdly,
169 uncertainty quantification remains largely overlooked in existing physics-informed
170 flood forecasting studies (Kohanpur et al., 2023). Most models provide deterministic
171 forecasts only, making it difficult to assess prediction reliability and support risk-based
172 decision making (Xu et al., 2022; Zhang et al., 2025). Reliable uncertainty estimation
173 is particularly important for operational flood forecasting because management
174 decisions often depend not only on forecast accuracy but also on the confidence
175 associated with predictions (Xu et al., 2022; Han et al., 2017). Nevertheless, most
176 existing deep learning models generate deterministic forecasts and provide limited
177 information regarding predictive uncertainty. To overcome this limitation, Bayesian
178 inference has been increasingly incorporated into hydrological forecasting frameworks.
179 For instance, Zhu et al. (2025a) proposed a Bayesian multi-model ensemble framework
180 for probabilistic groundwater level forecasting. The framework integrates multiple
181 categories of input variables, including autocorrelation, meteorological, hydrological,
182 and anthropogenic factors, to capture the lagged effects and driving mechanisms
183 governing groundwater dynamics.

184 Therefore, three critical challenges remain in current flood forecasting research: (1)
185 effectively capturing long-term temporal dependencies in flood evolution. (2) ensuring
186 physical consistency throughout the prediction process, and (3) quantifying predictive
187 uncertainty in a physically meaningful manner. To date, few studies have
188 simultaneously addressed these challenges within a unified framework. Existing
189 approaches generally focus on either temporal representation learning, physics-



190 informed modeling, or uncertainty quantification individually, while comprehensive
191 integration of these components remains limited.

192 To address these gaps, this study proposes a Physics-Constrained Bayesian
193 Transformer (PCBT) framework for probabilistic flood forecasting. The proposed
194 framework integrates Transformer-based temporal feature extraction, physics-informed
195 hydrological constraints, and Bayesian probabilistic inference within a unified
196 architecture. Specifically, the Transformer encoder is employed to extract high-
197 dimensional temporal representations from antecedent precipitation and streamflow
198 sequences and to capture long-range hydrological dependencies. Physical constraints
199 derived from water balance principles and flow continuity relationships are then
200 incorporated into the loss function through a physics-informed learning strategy,
201 ensuring that model predictions remain consistent with hydrological processes.
202 Furthermore, a Bayesian output layer is introduced to quantify predictive uncertainty
203 and generate probabilistic forecasts in the form of prediction intervals.

204 The proposed framework is evaluated using 50 independent flood events from two
205 hydrologically contrasting watersheds: the Upper Hei River Basin, which is
206 characterized by mountainous runoff processes, and the Luoma Lake Basin, which
207 represents a typical lowland lake–river system. Comparative analyses are conducted
208 against PINN, Transformer, LSTM, and BPNN models using deterministic and
209 probabilistic evaluation metrics. The objectives of this study are threefold: (1) to
210 develop a physically consistent probabilistic flood forecasting framework by
211 integrating Transformer learning, hydrological constraints, and Bayesian inference; (2)



212 to investigate the contribution of physical constraints and uncertainty quantification to
213 forecasting performance; and (3) to evaluate the robustness and transferability of the
214 proposed framework across distinct hydrological regimes. The findings are expected to
215 provide a new pathway toward reliable, interpretable, and uncertainty-aware flood
216 forecasting for water resources management and flood risk mitigation.

217 **2. Methodology**

218 **2.1. Overview**

219 The proposed PCBT framework integrates Transformer-based temporal feature
220 learning, physics-informed hydrological constraints, and Bayesian uncertainty
221 quantification within a unified flood forecasting architecture. This design enables
222 simultaneous improvements in predictive accuracy, hydrological consistency, and
223 probabilistic forecast reliability. Initially, historical precipitation and streamflow
224 observations are processed to construct antecedent input sequences, which capture the
225 temporal evolution of hydrological processes. These sequences are encoded through a
226 Transformer network to extract long-range temporal dependencies, while hydrological
227 constraints derived from water balance principles are incorporated to reduce physically
228 unrealistic predictions. A Bayesian output layer is subsequently applied to estimate
229 predictive uncertainty and generate probabilistic forecasting intervals, providing a
230 comprehensive representation of forecast reliability.

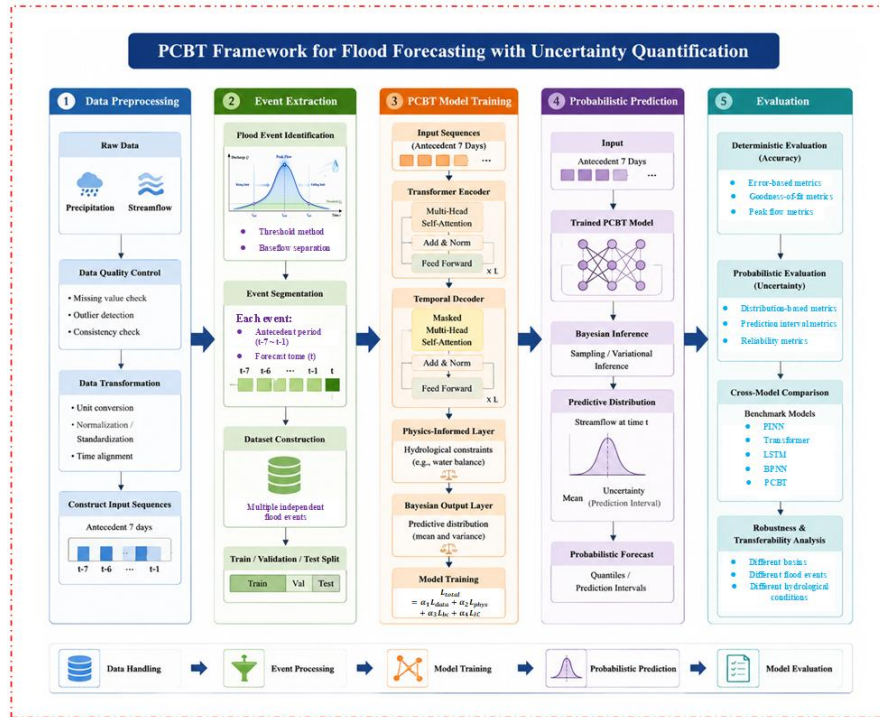


Fig. 1. Research Methodological Framework.

Fig. 1 illustrates the overall technical route of the proposed PCBT framework.

Historical precipitation and streamflow records are first processed to construct the input feature set. The PCBT framework then combines transformer-based temporal representation learning with physically informed constraints to improve predictive consistency and hydrological realism. A Bayesian output layer is incorporated to estimate predictive distributions and quantify forecast uncertainty. Model performance is evaluated using both deterministic metrics: Nash–Sutcliffe Efficiency (NSE), Kling–Gupta Efficiency (KGE), Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Willmott Index of Agreement (WI), and Percent Bias (PBIAS), and probabilistic metrics: Continuous Ranked Probability Score (CRPS), Prediction Interval Coverage Probability (PICP), and Mean Prediction Interval Width (MPIW).



244 Finally, the proposed framework is compared with several benchmark models to assess
245 forecasting accuracy, uncertainty representation, robustness, and practical applicability
246 for flood forecasting and water resources management.

247 **2.2. PCBT Framework**

248 **2.2.1 Architecture of the PCBT framework**

249 The proposed PCBT framework is developed to provide physically consistent and
250 uncertainty-aware flood forecasting by integrating Transformer-based sequence
251 learning, hydrological process constraints, and Bayesian probabilistic inference within
252 a unified architecture. Unlike conventional deep learning models that rely solely on
253 data-driven representations, PCBT explicitly incorporates hydrological knowledge into
254 the learning process to improve physical realism and generalization capability. The
255 framework consists of three major components: an input representation module for
256 extracting antecedent hydrometeorological information, a physics-constrained
257 Transformer encoder for learning nonlinear temporal dependencies while enforcing
258 hydrological consistency, and a Bayesian output layer for probabilistic prediction and
259 uncertainty quantification. Through the synergistic integration of these components,
260 PCBT simultaneously enhances forecasting accuracy, interpretability, and predictive
261 reliability under diverse hydrological conditions. The overall architecture of the
262 proposed framework is presented in [Fig. 2](#).

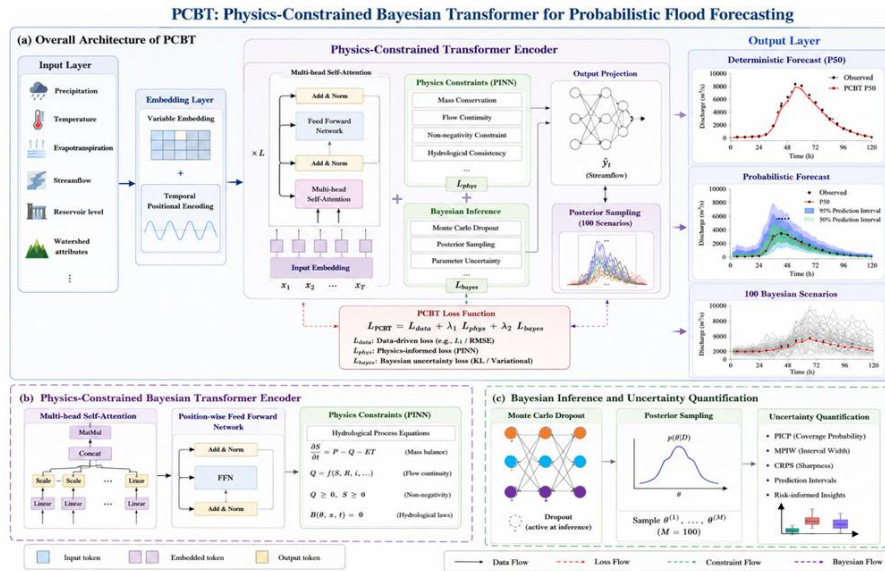


Fig. 2. Overall architecture of the proposed PCBT. (a) The complete framework of PCBT, (b) the internal structure of the physics-constrained transformer encoder, and (c) the Bayesian inference module and uncertainty quantification process.

As shown in Fig. 2, the framework follows a sequential modeling strategy from hydrometeorological input representation to deterministic prediction and probabilistic uncertainty estimation. Historical precipitation, discharge, and watershed-related information are first transformed into embedded feature representations and subsequently processed through a physics-constrained Transformer encoder. Physical constraints derived from hydrological principles are incorporated during training to ensure physically realistic predictions. Bayesian inference is then employed to estimate predictive distributions and generate uncertainty bounds. The resulting framework produces both deterministic forecasts and probabilistic prediction intervals, enabling comprehensive assessment of forecast accuracy and reliability.

(1) Input Layer

The input layer consists of key hydrological variables, including precipitation and



279 discharge. The input data are organized into time-series sequences to preserve temporal
280 dependencies, enabling the model to capture the evolution of hydrological processes
281 under varying rainfall–runoff conditions.

282 (2) Physics-Constrained Transformer Encoder

283 The encoder comprises multi-head self-attention and a feedforward network,
284 modeling complex temporal dependencies by progressively stacking multiple layers.
285 Concurrently, physical information is introduced via a PINN module, incorporating the
286 continuity equation for flow, water balance constraints, and boundary conditions to
287 ensure that predictions adhere to hydrological physical laws. This design enables the
288 model to maintain physical consistency and interpretability while learning long-term
289 dependencies.

290 (3) Bayesian Inference & Uncertainty Quantification

291 At the output layer, PCBT uses Bayesian inference to sample model parameters,
292 generating multi-scenario (Monte Carlo Sampling) forecast results. Using these
293 forecasts, the model calculates uncertainty metrics such as the prediction interval,
294 CRPS, PICP, and MPIW to systematically quantify the uncertainty in flow forecasts.

295 (4) Output Layer

296 The output layer provides both deterministic and probabilistic forecasts. The
297 deterministic forecast displays the median flow rate, while the probabilistic forecast
298 reflects the uncertainty range through multi-scenario simulations. This design allows
299 users to simultaneously obtain the most likely value of the flow rate and its confidence
300 interval, providing a reliable basis for flood early warning and water resource



301 management.

302 (5) Performance Evaluation and Comparative Analysis

303 The framework includes uncertainty assessment metrics (CRPS, PICP, MPIW) and
304 deterministic performance metrics (NSE, KGE, MAE, RMSE, PBIAS, WI). It
305 compares these metrics against various benchmark models (LSTM, BPNN, PINN,
306 Transformer) to systematically evaluate the model's performance across different river
307 basins and flood events.

308 **2.2.2. PINN Network**

309 PINN is a class of models that integrates physical constraints into data-driven neural
310 networks ([Wong et al., 2026](#)). During training, the system's physical equations, initial
311 conditions, and boundary conditions are embedded into the loss function as constraints,
312 enabling the model to maintain physical consistency while fitting observed data.
313 Typically, a multi-layer feedforward neural network is used as a function approximator.
314 Physical residuals are computed via automatic differentiation and combined with data
315 errors to form the total loss function, thereby optimizing the network parameters and
316 achieving good generalization performance with limited training data.

317 The PINN model accepts spatio-temporal inputs, including hydrological variables
318 such as discharge and precipitation, as well as other environmental factors. These inputs
319 are first preprocessed through a neural network module, which consists of fully
320 connected layers with nonlinear activation functions (tanh, Swish, or ReLU) to extract
321 high-dimensional feature representations ([Jia et al., 2025](#)). The physics-driven module
322 integrates governing hydrological equations, automatic differentiation for derivatives,



323 and physical residuals to enforce constraints such as continuity, mass conservation, and
324 boundary conditions. These physically driven outputs are incorporated into the loss
325 function, which combines multiple terms: data-driven loss (L_{data}), physical residual loss
326 (L_{phys}), and boundary and initial condition loss (L_{bc} and L_{IC}). These terms are weighted
327 to form the total loss (L_{total}). During training, the optimizer iteratively updates the model
328 parameters via backpropagation. When L_{total} converges below a preset threshold, the
329 trained model outputs the predicted variables (e.g., flow) along with associated
330 uncertainty quantification results (prediction intervals, Bayesian posterior samples).
331 This design enables the PINN model to learn simultaneously from both data and
332 physical laws, ensuring prediction accuracy while maintaining consistency with
333 hydrological principles. See [Fig. S1](#) in [Supplementary material](#).

334 **2.2.3. Transformer**

335 [Transformer](#) adopts an encoder-decoder architecture to learn nonlinear rainfall-
336 runoff relationships from historical hydrometeorological time series ([Miao et al., 2026](#)).
337 [Through its self-attention mechanism](#), the Transformer can effectively capture long-
338 term temporal dependencies and complex interactions among multiple variables,
339 thereby better characterizing flood evolution processes and providing more accurate
340 runoff forecasts for the target time period ([Fig. S2](#) in [Supplementary material](#)):

341 (1) Data Entry and Preprocessing

342 Historical hydrological time series include multi-source information such as flood,
343 precipitation, and temperature. After standardization, the data is fed into the model, and
344 the encoder learns feature representations at different time steps. An independent fully



connected layer without activation functions is used to map the input feature vector X_t
 $\in R^{m \times n}$ to a high-dimensional embedding space, thereby enhancing the model's
 representational capacity. This “embedding layer” operates on a per-time-step (e.g. one
 day) basis, using sine and cosine functions to encode temporal position information, as
 shown in the following formula:

$$E(p, 2i) = \sin \frac{p}{10000 \frac{2i}{d_m}} \quad (1)$$

$$E(p, 2i + 1) = \cos \left(\frac{p}{10000 \frac{2i}{d_m}} \right) \quad (2)$$

where E represents the position code; p represents the time slot position; i represents
 the unique index; and d_m represents the dimension vector.

(2) Encoder

The multi-head attention mechanism is used to capture the interrelationships between
 key time steps in a time series. This mechanism maps input features to multiple
 attention subspaces and computes the weights of each attention head in parallel. For
 each attention head, it generates query (Q), key (K), and value (V) vectors separately.
 Subsequently, the outputs from each attention head are concatenated and transformed
 via a linear transformation to obtain the final attention representation, thereby enabling
 the model to automatically learn and effectively capture complex spatiotemporal
 dependencies among multiple variables:

$$A(Q, K, V) = \text{softmax} \left(\frac{QK^T}{\sqrt{d_k}} \right) V \quad (3)$$

where $A(Q, K, V)$ represents the attention weight; the vectors Q , K , and V contain
 complete input sequences of multi-factor information such as flow, water level, and



366 precipitation, which are used to calculate the correlation weights between elements in
367 the sequence; d_k denotes the dimension of the key vector; and T denotes the number of
368 time periods.

369 (3) Decoder

370 The features extracted by the encoder are mapped to the predicted flood sequence to
371 enable multi-step flood forecasting. The decoder generates daily flood forecast
372 sequences for use in short-term early warning or water resources management.

373 2.2.4. Bayesian Uncertainty Quantification

374 The Bayesian component of the proposed PCBT framework is designed to explicitly
375 quantify predictive uncertainty by representing model outputs as probability
376 distributions rather than deterministic values. This probabilistic formulation enables the
377 model to characterize uncertainty arising from parameter estimation, model structure,
378 and the inherent stochasticity of hydrological processes.

379 According to Bayes' theorem, the posterior distribution of network parameters θ
380 conditioned on the training dataset D can be expressed as

$$381 \quad p(\theta|D) = \frac{p(D|\theta)p(\theta)}{p(D)} \quad (4)$$

382 where $p(\theta)$ denotes the prior distribution of model parameters, $p(\theta|D)$ is the
383 likelihood function; and $p(D)$ is the marginal likelihood.

384 The posterior distribution $p(\theta|D)$ combines prior knowledge with observational
385 information and forms the basis for probabilistic prediction.

386 Let $y = \{y_1, y_2, \dots, y_N\}$ denote the observed streamflow series and $\hat{y} = f_\theta(x)$
387 represent the model output given the input features x and network parameters θ . The



388 posterior predictive distribution is then defined as

$$389 \quad p(y|x, D) = \int p(y|x, \theta)p(\theta|D)d\theta \quad (5)$$

390 where $p(y|x, D)$ represents the likelihood of observations conditioned on the model
391 parameters and $p(\theta|D)$ denotes the posterior parameter distribution.

392 Because exact evaluation of Eq. (5) is computationally intractable for deep neural
393 networks, Monte Carlo (MC) Dropout is adopted as an efficient variational Bayesian
394 approximation. During inference, dropout layers remain active, allowing repeated
395 stochastic forward passes through the network. This procedure can be interpreted as
396 sampling from an approximate posterior distribution of model parameters (Zhu et al.,
397 2025a).

398 To approximate this integral efficiently, Monte Carlo dropout is employed, sampling
399 M sets of parameters $\{\theta^{(m)}\}_{m=1}^M$ during inference. The posterior predictive mean and
400 variance are estimated as:

$$401 \quad \hat{\mu}(x) = \frac{1}{M} \sum_{m=1}^M f_{\theta^{(m)}}(x) \quad (6)$$

$$402 \quad \hat{\sigma}^2(x) = \frac{1}{M} \sum_{m=1}^M (f_{\theta^{(m)}}(x) - \hat{\mu}(x))^2 \quad (7)$$

403 These estimates form the basis of probabilistic prediction intervals, e.g., the 95%
404 prediction interval:

$$405 \quad PI_{95\%}(x) = [\hat{\mu}(x) - 1.96\hat{\sigma}(x), \hat{\mu}(x) + 1.96\hat{\sigma}(x)] \quad (8)$$

406 The Bayesian module allows explicit quantification of uncertainty arising from both
407 model structure and parameter estimation. Generate multiple prediction samples at each
408 time step to provide robust statistics for CRPS, PICP, and MPIW. This enables rigorous
409 evaluation of the probabilistic performance of the PCBT framework under complex



410 hydrological conditions.

411 **2.3. Loss Function**

412 **2.3.1. Construction of loss function**

413 In flood forecasting, peak discharge serves as the core predictive variable, and its
414 modeling and prediction are carried out using the PCBT framework. This framework
415 employs automatic differentiation techniques to accurately calculate the residuals of the
416 governing partial differential equations and combines them with observation errors to
417 form a comprehensive loss function. This loss function is used as the optimization
418 objective and is solved using NSGA-II. During the data-driven learning process, the
419 model strictly adheres to physical laws, providing clear physical constraints for the
420 prediction results, thereby ensuring that flood simulations are both accurate and
421 physically consistent.

422 (1) Data-driven loss (L_{data})

423 Measurements were made using the mean square error between observed data
424 (including precipitation, flow, and other monitoring data) and the PINN model
425 prediction:

$$426 \quad L_{data} = \frac{1}{N} \sum_{i=1}^N |\hat{Q}_i - Q_i|^2 \quad (9)$$

427 where: $\hat{Q}_i$ is the simulated value of the flow (m^3/s); Q_i is the observed value of the
428 flow (m^3/s); L_{data} represents the loss function for the flood; N is the number of flood
429 samples.

430 (2) Physical constraint loss (L_{phys})

431 River discharge is assumed to follow a simplified kinematic-wave-based continuity



432 formulation under the assumption of gradually varied flow:

$$433 \quad L_{flow} = \frac{1}{N} \sum_{i=1}^N \left| \frac{\partial h(x_i, t)}{\partial t} + c \frac{\partial Q(x_i, t)}{\partial x} \right|^2 \quad (10)$$

434 where Q denotes discharge (m^3/s), x is spatial position, t is time (h), and c represents
 435 the wave propagation velocity. This constraint enforces physically consistent and
 436 smooth propagation of flood waves along the river channel.

437 To further ensure hydrological realism at the basin scale, discharge dynamics are
 438 constrained by a simplified rainfall–runoff balance:

$$439 \quad L_{rr} = \frac{1}{N} \sum_{t=1}^N (Q_t - (Q_{t-1} + P_t - \alpha Q_{t-1}))^2 \quad (11)$$

440 where P_t denotes basin-average precipitation and $\alpha \in [0, 1]$ represents a runoff loss
 441 coefficient accounting for infiltration, evapotranspiration, and subsurface losses. This
 442 formulation provides a simplified but physically interpretable representation of
 443 catchment water balance.

444 The total physical constraint loss is formulated as:

$$445 \quad L_{phys} = \lambda_1 L_{flow} + \lambda_2 L_{rr} \quad (12)$$

446 where λ_1 and λ_2 are weighting coefficients balancing river routing dynamics and
 447 rainfall–runoff consistency.

448 (3) Boundary condition loss (L_{bc})

449 The boundary loss is calculated by comparing the simulated boundary conditions
 450 with the actual boundary conditions. The boundary loss function is calculated as follows:

$$451 \quad Q_{min} \leq Q \leq Q_{max} \quad (13a)$$

$$452 \quad \begin{cases} L(Q_{min}) = \frac{1}{N} \sum_{i=1}^N |Q_{pred}(y_i) - Q_{obs}(y_i)| \\ L(Q_{max}) = \frac{1}{N} \sum_{i=1}^N |Q_{pred}(y_i) - Q_{obs}(y_i)| \end{cases} \quad (13b)$$

$$453 \quad L_{bc} = L(Q_{min}) + L(Q_{max}) \quad (13c)$$



454 where $L()$ denotes the boundary loss; $Q_{\text{pred}}(y_i)$ represents the flood at the boundary
455 (m^3/s); $Q_{\text{obs}}(y_i)$ represents the observed flood at the boundary (m^3/s); Q_{min} and Q_{max} are
456 the minimum and maximum values of the boundary flood (m^3/s), respectively.

457 (4) Initial condition loss (L_{IC})

458 The initial flow is guaranteed by the following loss:

$$459 \quad L_{\text{IC}} = \frac{1}{N} \sum_{i=1}^N |Q_{\text{pred}}(x_i, t_0) - Q_{\text{obs}}(y_i, t_0)|^2 \quad (14)$$

460 where t_0 is the initial time; Q_{pred} represents the predicted flood (m^3/s); Q_{obs} represents
461 the observed flood (m^3/s); and L_{IC} denotes the loss function for flood, respectively.

462 (5) The total loss function is the weighted sum of the above losses, and is calculated
463 as follows:

$$464 \quad L_{\text{total}} = \alpha_1 L_{\text{data}} + \alpha_2 L_{\text{phys}} + \alpha_3 L_{\text{bc}} + \alpha_4 L_{\text{IC}} \quad (15)$$

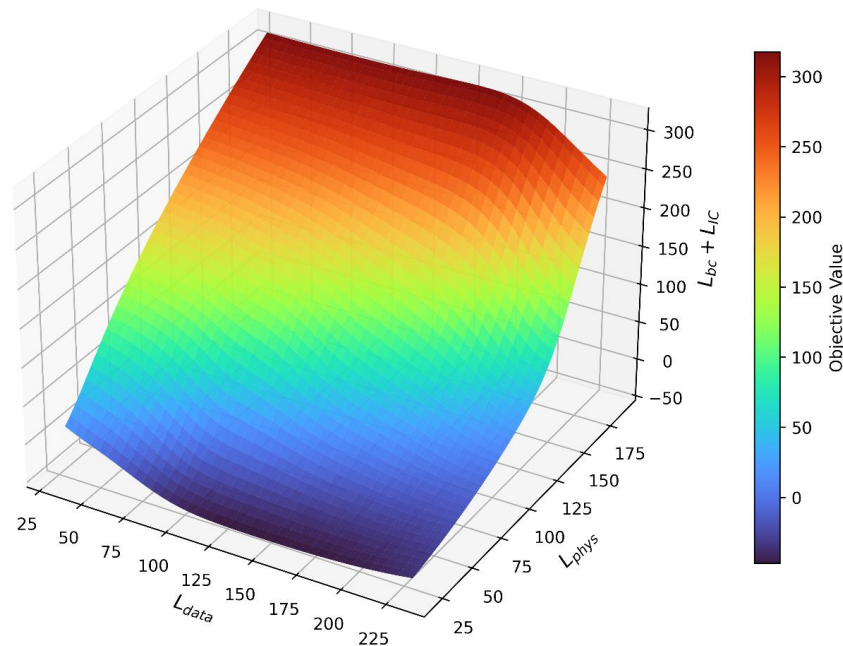
465 where α_1 , α_2 , α_3 , and α_4 are weight coefficients representing the importance of each loss
466 term, which can be tuned using cross-validation.

467 **2.3.2. Loss function solution**

468 The workflow of the NSGA-II algorithm for optimizing the total loss function of the
469 PCBT framework is shown in Fig. S3 in [Supplementary material](#). Firstly, the model
470 and loss components (L_{data} , L_{phys} , L_{bc} , L_{IC} , and L_{total}) are defined, and an initial
471 population is randomly generated and evaluated. Secondly, through iterative
472 evolutionary operations including selection, crossover, and mutation, offspring are
473 generated and combined with the parent population. Non-dominated sorting and
474 crowding distance calculation are then applied, and the best individuals are retained to
475 form the next generation (Deng et al., 2025; Chen et al., 2025b). This process repeats



476 until the maximum number of generations is reached, at which point optimal solutions
 477 are obtained, from which the final parameter set is selected according to practical
 478 performance criteria. The results balance the trade-off between predictive accuracy and
 479 physical consistency, and are used for peak discharge forecasting.



480
 481 **Fig. 3.** Pareto-smoothed loss surface of PCBT predictions.

482 **Fig. 3** illustrates the Pareto-smoothed objective surface of the PCBT loss function,
 483 revealing the nonlinear trade-off between L_{data} and L_{phys} during NSGA-II optimization.
 484 The surface demonstrates stable convergence and highlights the set of non-dominated
 485 solutions from which the final parameter configuration is selected, ensuring a balance
 486 between predictive accuracy and physical consistency.

487 **2.4. Input selection and parameter determination**

488 To ensure the physical consistency and reproducibility of the proposed PCBT, a
 489 systematic framework was developed for input feature selection and antecedent-



490 window determination. Candidate predictors, including precipitation and streamflow
491 variables, were first identified according to hydrological processes governing rainfall–
492 runoff transformation. Pearson correlation analysis (PCC) and cross-correlation
493 functions (CCF) were then employed to quantify the relationships between predictors
494 and target streamflow, thereby determining the effective antecedent period for model
495 inputs (Ahmad et al., 2025). In addition, autocorrelation (ACF) analysis of streamflow
496 series was used to characterize the hydrological memory of the basin and to constrain
497 the maximum length of the input sequence.

498 Based on the identified hydrological memory, multiple combinations of antecedent
499 windows and forecasting horizons were constructed and used as inputs to the PCBT
500 framework. The Transformer encoder was adopted to capture long-range temporal
501 dependencies among hydrometeorological variables, while physics-informed
502 constraints were incorporated into the learning process to enforce hydrological
503 consistency. A Bayesian output layer was further introduced to characterize predictive
504 uncertainty and generate probabilistic forecasts in the form of prediction intervals.

505 **2.4.1. Variable Filtering**

506 The selection of key forecasting variables was based on hydrological process analysis
507 and statistical correlation analysis (SCA) (Karakoyun et al., 2025). Firstly, candidate
508 variables were selected based on the hydrological mechanisms of the watershed,
509 including precipitation, air temperature, evapotranspiration, and prior flood.
510 Subsequently, PCC is used to quantitatively assess the linear correlation between each
511 variable and flood, and CCF is employed to analyze their lag response characteristics,



512 thereby identifying key input variables and their effective lag terms.

513 (1) PPC

$$514 \quad \gamma_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}} \quad (16)$$

515 where x represents the candidate meteorological variable; y represents flood; γ_{xy}
 516 represents the correlation coefficient. When $\gamma_{xy} > \gamma$, the variable is considered to have a
 517 significant effect.

518 (2) Construction of the lagged variable

519 Building the input based on correlation analysis:

$$520 \quad x_t^{(k)} = x_{t-k} \quad (17)$$

521 Denotes the k -day lagged input.

522 **2.4.2. Determination of delay**

523 The maximum lag time reflects the length of a watershed's hydrological memory and
 524 is determined jointly using ACF and the rainfall-flood CCF. When the correlation
 525 decays to a level outside the range of statistical significance, the corresponding lag time
 526 is defined as the effective memory length of the system.

527 (1) ACF

$$528 \quad R_{xx}(k) = \frac{\sum_{t=1}^{n-k} (x_t - \bar{x})(x_{t+k} - \bar{x})}{\sum_{t=1}^n (x_t - \bar{x})^2} \quad (18)$$

529 (2) CCF

$$530 \quad R_{xy}(k) = \frac{\sum_{t=1}^{n-k} (x_t - \bar{x})(y_{t+k} - \bar{y})}{\sqrt{\sum_{t=1}^n (x_t - \bar{x})^2} \sqrt{\sum_{t=1}^n (y_t - \bar{y})^2}} \quad (19)$$

531 (3) Definition of Maximum Delay

$$532 \quad L_{max} = \max \{k | R_{xx}(k) > R_c \text{ or } R_{xy}(k) > R_c\} \quad (20)$$

533 where R_c represents the 95% confidence interval threshold. When the correlation falls



below the significance level, it is considered that the hydrological influence has diminished. $L_{\max}$ represents the hydrological memory length of the watershed.

2.4.3. Joint Optimization of Lag-Horizon

Based on the determined hydrological memory length, various combinations of maximum lag time and lead time are established, and a joint sensitivity analysis is conducted under physical constraints to determine the optimal parameter configuration.

(1) Parameter Set Definition

$$L \in \{L_1, L_2, \dots, L_m\}, H \in \{H_1, H_2, \dots, H_n\} \quad (21)$$

(2) Physical constraints

$$H \leq L \quad (22)$$

Predictions must not exceed the system's memory capacity.

(3) Joint optimization of the objective function

$$\min_{L,H} J(L, H) = \omega_1 RMSE + \omega_2 (1 - NSE) + \omega_3 |PBIAS| + \omega_4 (1 - KGE) \quad (23)$$

(4) Definition of an optimal solution

$$(L^*, H^*) = \operatorname{argmin} J(L, H) \quad (24)$$

The optimal lag-horizon combination is selected by minimizing the multi-objective loss function under physical constraints. This framework ensures that input selection and temporal parameters are jointly constrained by hydrological theory and data-driven optimization, improving both interpretability and predictive robustness of the Trans-PINN model.

2.5. Model Training

In this study, a PCBT framework was developed for flood forecasting. Model



parameters were optimized by minimizing a composite loss function that simultaneously incorporates data-driven errors and physical constraints, thereby improving predictive accuracy while ensuring adherence to hydrological laws. During training, the Transformer encoder was employed to extract high-dimensional temporal features and capture long-range dependencies among hydrometeorological variables, enhancing the model's representation of rainfall–runoff dynamics. The dataset was divided into training, validation, and testing subsets for parameter learning, hyperparameter tuning, and evaluation of generalization performance, respectively. Model parameters were iteratively updated through gradient-based optimization to minimize discrepancies between predicted and observed streamflow.

2.6. Comparative Analysis

2.6.1. Comparison Model

To comprehensively evaluate the performance of the proposed PCBT, four benchmark models, including PINN, Transformer, LSTM, and BPNN, were selected for comparison. Among them, LSTM is widely used for hydrological forecasting due to its capability to capture long-term temporal dependencies ([Fig. S4 in Supplementary material](#)), whereas BPNN represents a conventional data-driven approach for learning nonlinear input–output relationships ([Fig. S5 in Supplementary material](#)). Transformer and PINN were further included to assess the contributions of attention-based temporal feature extraction and physics-informed learning, respectively.

To further investigate the contribution of individual model components, an ablation analysis was conducted by comparing the full PCBT framework with its simplified



578 variants. Specifically, the standalone Transformer model was used to evaluate the
579 contribution of physical constraints, whereas the PINN model was adopted to assess
580 the benefits of the Transformer-based temporal representation and Bayesian uncertainty
581 quantification. By systematically removing or simplifying key modules, the
582 independent and synergistic effects of the Transformer architecture, physics-informed
583 constraints, and Bayesian inference were quantified. Through benchmark comparisons
584 and ablation experiments, the effectiveness, robustness, and physical consistency of the
585 proposed PCBT framework, as well as the necessity and complementarity of its
586 constituent components, were comprehensively evaluated.

587 **2.6.2. Performance Metrics**

588 To evaluate the predictive performance of the proposed PCBT, six widely used
589 deterministic metrics, including RMSE, MAE, PBIAS, NSE, KGE, and WI, were
590 employed. Detailed definitions and discussions of these metrics can be found in [Shi et](#)
591 [al. \(2025\)](#).

592 In addition to deterministic evaluation, probabilistic forecasting performance was
593 assessed to quantify predictive uncertainty. Three widely adopted uncertainty metrics,
594 namely the CRPS, PICP, and MPIW, were used to evaluate the reliability, sharpness,
595 and overall quality of probabilistic forecasts. The mathematical definitions of these
596 metrics are presented as follows ([Zhu et al., 2025a](#)):

597 (1) CRPS is used to comprehensively evaluate the discrepancy between the
598 probability forecast distribution and observed values, and it simultaneously reflects
599 both the accuracy of the forecast and its ability to characterize uncertainty. CRPS is



defined as the integral of the squared difference between the forecast cumulative distribution function and the observed cumulative distribution function:

$$CRPS(F, y) = \int_{-\infty}^{+\infty} [F(x) - H(x - y)]^2 dx \quad (25a)$$

where $F(x)$ is the cumulative distribution function (CDF) of the predictor variable; y is the observed value; $H(\cdot)$ is the Heaviside step function:

$$H(x - y) = \begin{cases} 0, & x < y \\ 1, & x \geq y \end{cases} \quad (25b)$$

The smaller the CRPS value, the closer the predicted distribution is to the observed value, and the better the probability prediction performance.

(2) The PICP is used to evaluate the coverage of the prediction interval for the true observed values; it is defined as the proportion of observed values that fall within the prediction interval:

$$PIPC = \frac{1}{N} \sum_{i=1}^N c_i \quad (26a)$$

where:

$$c_i = \begin{cases} 1, & L_i \leq y_i \leq U_i \\ 0, & otherwise \end{cases} \quad (26b)$$

where N represents the sample size; y_i is the i -th observation; L_i and U_i denote the lower and upper bounds of the prediction interval, respectively; c_i is the coverage indicator function.

The closer the PICP is to the preset confidence level (e.g., 95%), the higher the reliability of the prediction interval.

(3) The MPIW is used to measure the width of the prediction interval, i.e., the range of uncertainty in the prediction results, and is defined as:

$$MPIW = \frac{1}{N} \sum_{i=1}^N (U_i - L_i) \quad (27)$$



622 where U_i is the upper bound of the prediction interval; L_i is the lower bound of the
623 prediction interval; N is the sample size.

624 The smaller the MPIW, the narrower the prediction interval and the more
625 concentrated the prediction results. However, an MPIW that is too small may result in
626 reduced coverage; therefore, the quality of the prediction interval should be evaluated
627 in conjunction with the PICP.

628 **3. Case Study**

629 **3.1. Study Area**

630 As shown in Fig. 4, the upper reaches of the Hei River (Fig. 4a) basin are located on
631 the northeastern edge of the Qinghai-Tibet Plateau. It is a typical mountainous inland
632 river basin characterized by rugged terrain, significant elevation variations, uneven
633 spatial distribution of precipitation, and the combined effects of snowmelt and rainfall.
634 The mechanisms underlying flood formation in the basin are complex, and its
635 hydrological processes exhibit marked nonlinearity and spatiotemporal heterogeneity
636 (Xue et al., 2025). The Luoma Lake basin (Fig. 4b) is situated in the plain region of the
637 Huai River basin in eastern China and is a typical plain lake basin. The terrain in this
638 area is flat, with a dense river network and significant water storage and regulation by
639 the lake. The flood evolution process is heavily influenced by the connectivity between
640 rivers and lakes as well as hydraulic regulation. Floods are primarily driven by heavy
641 rainfall, and the runoff process is more gradual compared to that in mountainous basins.

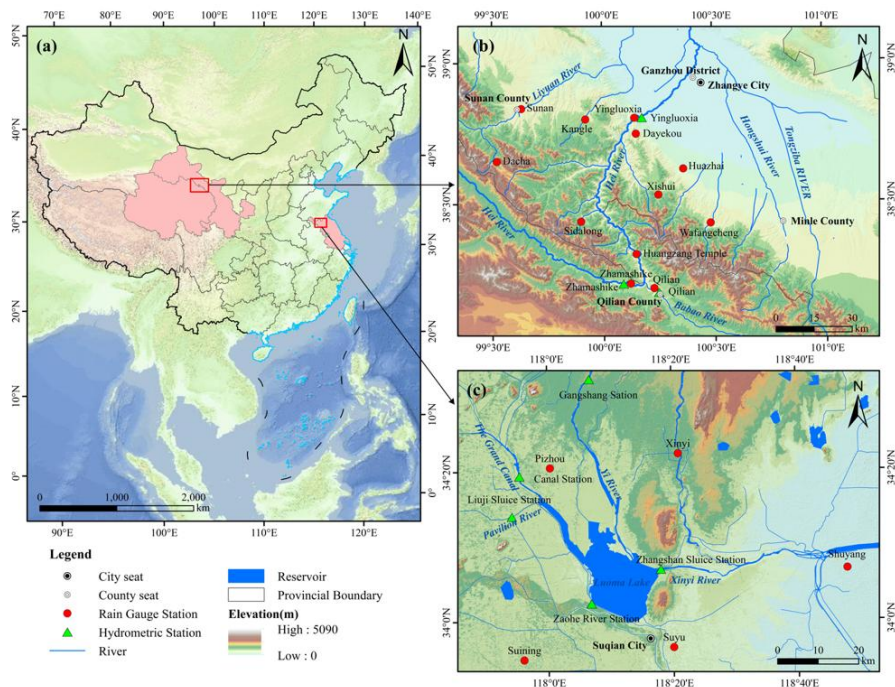


Fig. 4. Study area map (a) is China, (b) is the Upper Hei River, and (c) is the Luoma Lake.

Selecting the upper Hei River basin and the Luoma Lake basin as study areas is of great significance. These two areas represent two distinct and significantly different hydrological environments: a mountainous river basin and a plain lake basin, respectively. The former is characterized by rapid runoff, strong topographic control, and complex runoff generation and discharge mechanisms, while the latter exhibits significant river-lake regulation effects and slow flood propagation processes. By conducting experiments in these two representative basins, we can systematically evaluate the applicability and stability of the PCBT framework under varying topographic conditions, climatic backgrounds, and flood formation mechanisms, thereby verifying its cross-regional generalizability. This design not only aids in assessing the model's predictive accuracy but also validates the robustness of the



655 physical constraint mechanism and the Bayesian uncertainty quantification method
656 across different hydrological environments, providing a basis for the model's broader
657 application in other basins.

658 **3.2. Data Sources**

659 Hydrometeorological data were collected from two watersheds with distinctly
660 different hydrological characteristics (namely, the upper Hei River and the Luoma Lake)
661 to evaluate the performance and generalizability of the proposed PCBT framework. For
662 the upper Hei River, daily discharge observations from three representative
663 hydrological stations (including Yingluoxia, Qilian, and Zhamashike) were obtained
664 from the Hei River Administration (<http://www.hhglj.org/>). Corresponding
665 precipitation records were obtained from 12 meteorological stations distributed
666 throughout the basin. The observation period spanned from January 1, 1957, to
667 December 31, 2023 (<https://weather.cma.cn/>), providing a long-term, continuous record
668 for flood analysis and model development.

669 For the Luoma Lake, observational data from five key stations were obtained from
670 the Huai River Basin Hydrological Yearbook (<http://www.hrc.gov.cn/>), including three
671 inflow stations (Gang-Shang, Canal, and Liu-Ji Sluice) and two outflow control stations
672 (Zao-He and Zhang-Shan Sluice). Meteorological data (including precipitation and
673 temperature) were obtained from five national meteorological stations within and
674 around the basin (<https://weather.cma.cn/>). To characterize the basin's topography, the
675 30-meter resolution ASTER Global Digital Elevation Model (ASTER GDEM) was
676 obtained from the NASA (<https://www.nasa.gov/>) and the Geospatial Data Cloud



platform. This elevation model data was used to extract topographic information and support hydrological analysis (<https://www.gscloud.cn/>).

To ensure a rigorous evaluation of model performance, individual flood events were first extracted from the continuous streamflow record using a threshold-based event separation approach. A flood event was identified when streamflow exceeded a predefined threshold Q_{th} , and consecutive exceedances were grouped into a single event. Following this procedure, a total of 50 flood events were identified from the historical streamflow records. Flood events were extracted using a peak-over-threshold approach, in which the thresholds were defined based on the 90-th percentile of daily discharge for each basin. This resulted in threshold values of $120 \text{ m}^3/\text{s}$ for the Upper Hei River Basin and $1000 \text{ m}^3/\text{s}$ for the Luoma Lake Basin. To ensure event independence, a minimum inter-event time of 3~5 days was applied between successive flood events.

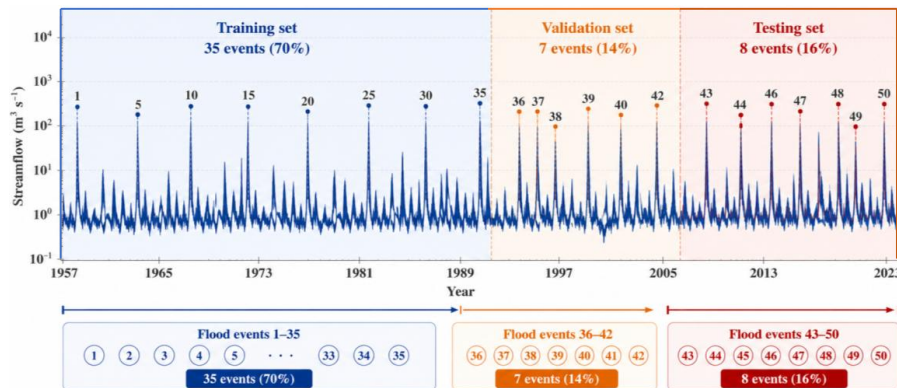


Fig. 5. Dataset partitioning based on flood events. (*Note:* Flood events are identified based on the separation criterion ($Q \geq Q_{th}$) and arranged in chronological order)

Fig. 5 illustrates the temporal distribution of the extracted flood events and the corresponding dataset partitioning strategy. To preserve the chronological characteristics of hydrological processes and avoid information leakage, the events



695 were divided sequentially into training, validation, and testing subsets. Specifically, the
696 first 35 flood events (70%) were used for model training, the subsequent 7 events (14%)
697 were used for hyperparameter tuning and model selection, and the remaining 8 events
698 (16%) were reserved for independent testing. The chronological partitioning strategy
699 enables the model to be evaluated under realistic forecasting conditions, where future
700 flood events are predicted using information derived solely from historical observations.
701 Furthermore, the selected events span a wide range of hydrological conditions and flood
702 magnitudes, providing a robust basis for assessing the predictive skill and
703 generalization capability of the proposed PCBT framework.

704 It was preprocessed using a normalization method. The normalization formula is as
705 follows:

$$706 \quad X_s = \frac{x - x_{\min}}{x_{\max} - x_{\min}} \quad (28)$$

707 Where x and X_s represent the flow values before and after normalization, respectively;
708 $x_{\max}$ and $x_{\min}$ represent the maximum and minimum values of the flow data, respectively.

709 **4. Results**

710 The proposed PCBT framework was implemented for 24-h-ahead flood forecasting
711 under contrasting hydrological conditions. The model was driven by antecedent
712 hydrometeorological inputs, including precipitation, streamflow, and derived watershed
713 response features constructed using a sliding time window to capture short- and long-
714 term temporal dependencies. These inputs were embedded into the Transformer-based
715 encoder to extract sequential hydrological dynamics. Specifically, Section 4.1 examines
716 the extraction of flood processes and the sensitivity of input variables. Section 4.2

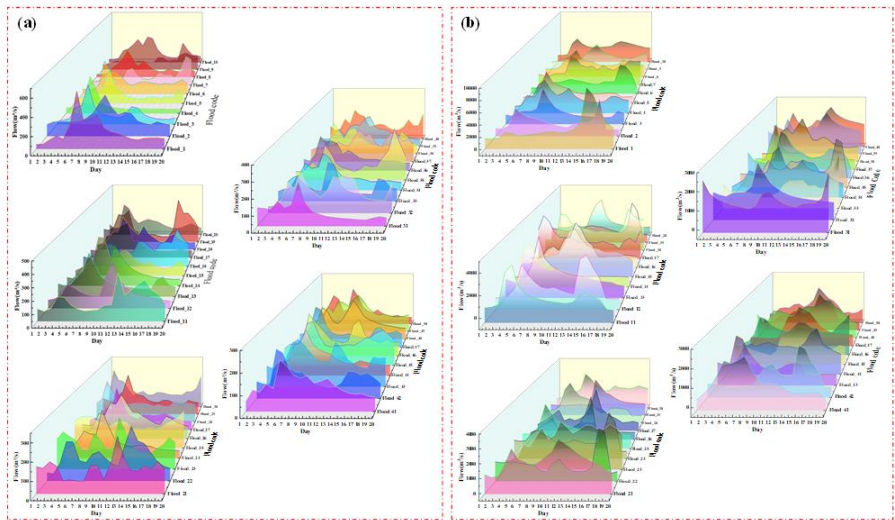


717 evaluates the predictive performance of PCBT in the Hei River and Luoma Lake basins,
718 and Section 4.3 investigates model comparison across five baseline models using six
719 evaluation metrics, providing a comprehensive assessment of accuracy, stability, and
720 uncertainty quantification.

721 **4.1. Extraction of flood processes and the sensitivity of input variables**

722 **4.1.1. Extraction of flood processes**

723 Fig. 6 presents the observed hydrographs of the 50 flood events in Hei river and
724 Luoma lake used in this study. To improve visualization, the events are grouped into
725 five subsets, each containing ten representative floods. Considerable variability can be
726 observed in both flood magnitude and hydrograph shape across the events. Peak
727 discharges range from moderate to extreme flood conditions, while the timing, rising
728 limb, recession characteristics, and flood duration exhibit substantial differences.



729 **Fig. 6.** Composite visualization of 50 observed flood hydrographs in the upper Hei River (a) and
730 Luoma Lake (b).
731

732 Several events are characterized by rapid increases in discharge followed by sharp



733 recessions, indicating short-duration and high-intensity runoff responses. In contrast,
734 other events display broader hydrographs with prolonged recession periods, reflecting
735 stronger catchment storage effects and delayed hydrological responses. Multiple
736 hydrographs also exhibit secondary peaks, suggesting the influence of successive
737 precipitation events or complex runoff generation processes.

738 The large diversity in flood magnitude and temporal dynamics demonstrates the
739 highly nonlinear nature of the rainfall-runoff relationship within the study basin. Such
740 heterogeneity provides a rigorous benchmark for evaluating model robustness and
741 generalization capability. By including flood events spanning a wide range of
742 hydrological conditions, the dataset enables a comprehensive assessment of the
743 proposed PCBT framework under both ordinary and extreme flood scenarios. The
744 observed hydrographs reveal rapid runoff responses and pronounced peak variability,
745 which are typical characteristics of mountain catchments where precipitation events are
746 strongly controlled by topography and exhibit substantial spatial heterogeneity.

747 **4.1.2. Input Sensitivity and Antecedent Information Analysis**

748 Fig. 7 presents the sensitivity weights of antecedent hydrometeorological variables
749 under different forecasting horizons. Antecedent streamflow consistently exhibited
750 greater predictive importance than antecedent precipitation across all forecast horizons.
751 The highest contribution was associated with one-day antecedent discharge (Q_{t-1}), with
752 a sensitivity weight of 0.27 for one-day-ahead forecasting. Variable importance
753 decreased progressively with increasing lag time, indicating that recent hydrological
754 conditions dominate flood predictability in the basin.

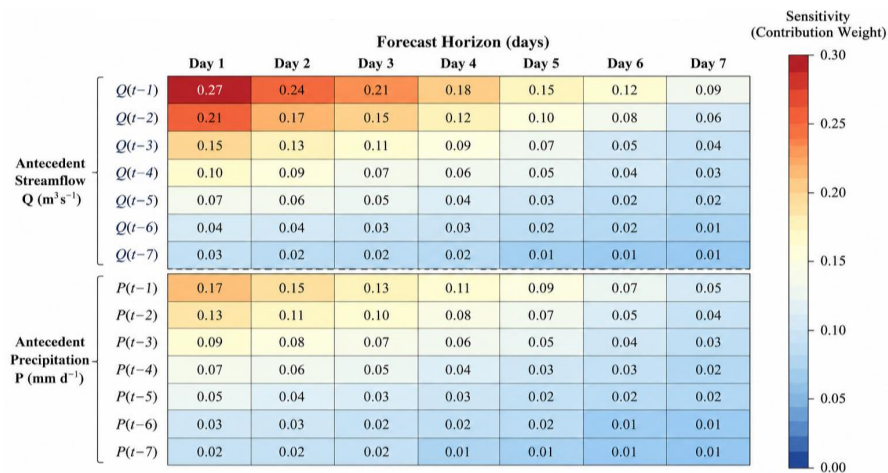


Fig. 7. Sensitivity weights of antecedent hydrometeorological variables across different forecast horizons.

The influence of antecedent precipitation was comparatively weaker but remained important for short-term forecasting. The highest precipitation contribution was observed for P_{t-1} , whereas precipitation information beyond four antecedent days provided limited additional predictive skill. Furthermore, the overall sensitivity of all predictors declined as forecast horizon increased from one to seven days, reflecting the gradual loss of hydrological memory and increasing forecasting uncertainty. These results support the selection of antecedent discharge and precipitation sequences as model inputs and confirm the dominant role of recent streamflow conditions in flood forecasting within the study basin.



767 **Table 1**
 768 *Architecture and training settings of the proposed PCBT model*

Module	Configuration
Input variables	$P(t-1:t-7), Q(t-1:t-7)$
Sequence Length	7
Embedding dimension	64
Number of encoder layers	2
Number of decoder layers	2
Multi-head attention	4 heads
Feed-forward network	256 neurons
Activation function	GELU
Bayesian inference	Variational Bayes
Monte Carlo samples	100
Physics-informed constraints	Water balance + continuity equation
Optimizer	Adam
Learning rate	1×10^{-3}
Batch size	32
Epochs	200
Early stopping	20 epochs

769 **Table 1** summarizes the architecture and training configuration of the proposed
 770 PCBT framework. Antecedent precipitation and discharge observations from the
 771 previous seven time steps, $P(t-1:t-7)$ and $Q(t-1:t-7)$, are used as model inputs. The
 772 Transformer network adopts an embedding dimension of 64 with two encoder layers,
 773 two decoder layers, and four attention heads. A feed-forward network containing 256
 774 neurons and the GELU activation function is employed for nonlinear feature extraction.
 775 Probabilistic prediction is achieved through variational Bayesian inference with 100
 776 Monte Carlo realizations. Physical constraints are incorporated through the water



777 balance and continuity equations. Model optimization is conducted using the Adam
778 optimizer with a learning rate of 1×10^{-3} and a batch size of 32. Training proceeds for
779 a maximum of 200 epochs, while early stopping with a patience of 20 epochs is used to
780 improve generalization performance and avoid overfitting.

781 **4.2. PCBT forecasting performance**

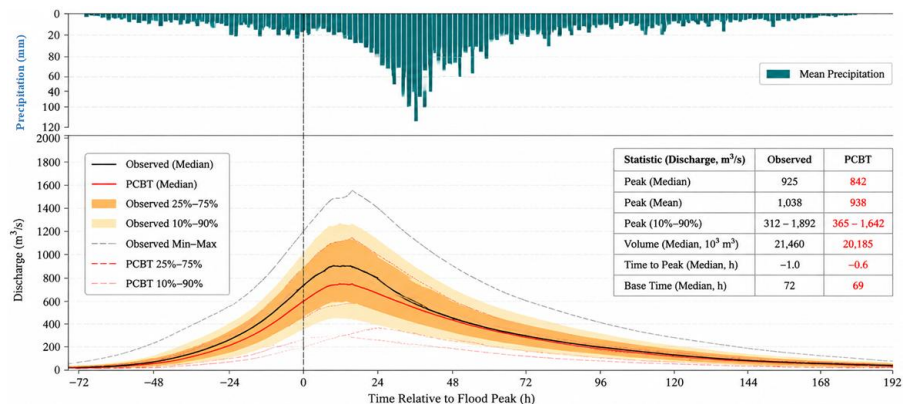
782 A clear hydrological contrast is observed between the Upper Hei River and Luoma
783 Lake basins. The Upper Hei River, as a mountainous semi-arid catchment, is
784 characterized by steep terrain, rapid runoff generation, and short response time, leading
785 to sharp hydrograph peaks and limited storage effects. In contrast, Luoma Lake
786 represents a lowland lake–river system with strong attenuation and routing effects,
787 resulting in smoother hydrographs and delayed peak responses. These differences lead
788 to distinct error structures across basins, where peak-related errors dominate in the
789 Upper Hei River, while volume and recession biases are more pronounced in Luoma
790 Lake. The contrasting rising and recession limb dynamics further reflect differences in
791 runoff generation mechanisms and catchment storage regulation. The results
792 demonstrate that basin hydrological characteristics play a key role in shaping model
793 performance and uncertainty behavior.

794 **4.2.1. The Upper Hei River basin**

795 Fig. 8 presents the composite rainfall–runoff characteristics of 50 flood events in the
796 Upper Hei River Basin with time aligned at the observed flood peak (0 h). The results
797 show a pronounced pre-event rainfall accumulation, driving a rapid rise in discharge
798 followed by a relatively smooth recession. Across all events, the observed median peak



799 discharge reaches 925 m³/s, while the PCBT model reproduces a slightly lower median
800 peak of 842 m³/s, indicating a modest underestimation of extreme flood magnitude.
801 Importantly, the model successfully captures the overall hydrograph shape and
802 variability, with the observed 10~90% discharge range (312~1892 m³/s) largely
803 enclosed by the predicted interval (365~1642 m³/s), demonstrating good uncertainty
804 representation. In addition, key event-scale statistics such as runoff volume
805 (21,460×10³ m³ observed vs. 20,185×10³ m³ simulated) are well preserved, confirming
806 that PCBT maintains physical consistency while achieving reliable probabilistic flood
807 forecasting.



808
809 **Fig. 8.** Composite rainfall runoff characteristics and corresponding PCBT predictions of 50 flood
810 events in Hei River. (*Note:* Time is aligned, so 0 hours is the peak observed time for each event)

811 **Fig. 9** presents the probabilistic flood forecasting results of the PCBT framework for
812 50 independent flood events in the Hei River. The observed hydrographs are shown as
813 blue dots, while the PCBT median predictions (P_{50}) are indicated by red lines. The 50%
814 prediction interval ($P_{25} \sim P_{75}$) and 95% prediction interval ($P_{2.5} \sim P_{97.5}$) are shaded in
815 orange and light blue, respectively. The figure illustrates that the PCBT framework can
816 accurately capture both the central tendency and uncertainty of flood peaks across a



range of events, demonstrating its reliability and robustness for hydrological forecasting in the Hei River Basin.

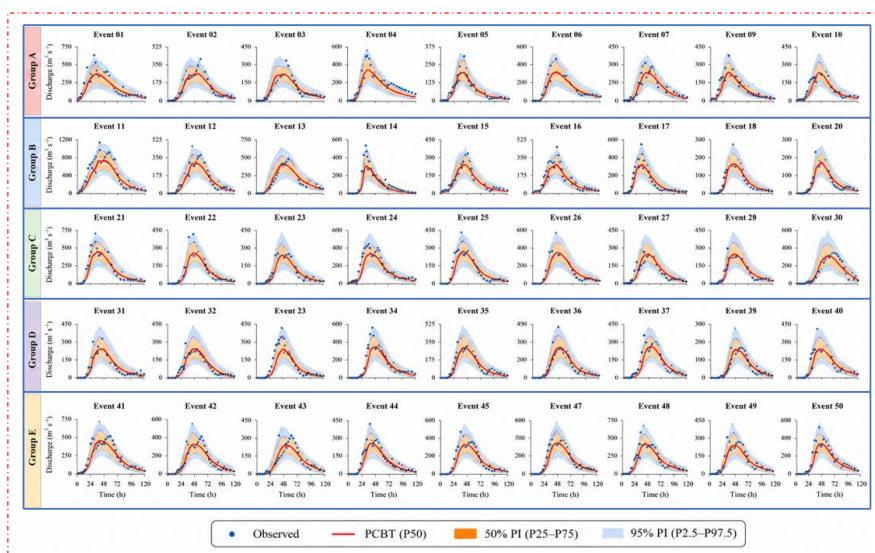


Fig. 9. Probabilistic flood forecasting results of the PCBT framework for 50 independent flood events.

Fig. 10 summarizes the probabilistic forecasting performance of the PCBT framework for 50 flood events in the Hei River Basin. The predictive uncertainty exhibits clear temporal variability among different flood events (Figure 10a). In general, uncertainty is relatively low during the recession stage of flood hydrographs but increases substantially around the rising limb and peak discharge periods, indicating that the model appropriately reflects the higher uncertainty associated with rapid hydrological responses and extreme flow conditions.

The boxplots in **Fig. 10b** further quantify the uncertainty characteristics under the 50% and 95% prediction intervals. The median CRPS values are approximately 3.62 m³/s and 9.81 m³/s for the 50% and 95% Prediction intervals, respectively, suggesting that the predictive distributions remain close to the observed flows despite the increased



833 uncertainty range. The median PICP increases from 75.3% under the 50% PI to 94.0%
834 under the 95% PI, indicating that the observed streamflow is successfully captured
835 within the prediction intervals for most flood events. Notably, the 95% PI coverage
836 approaches the theoretical confidence level, demonstrating good calibration of the
837 Bayesian uncertainty estimates. Meanwhile, the median MPIW increases from 17.6
838 m^3/s to 66.9 m^3/s , reflecting the expected trade-off between interval coverage and
839 interval sharpness.

840 Fig. 10c presents the event-wise distributions of forecasting metrics across the 50
841 flood events. The median MAE and RMSE values are approximately 7~8 m^3/s and 8~10
842 m^3/s , respectively, indicating relatively small forecasting errors compared with the
843 magnitude of observed flood peaks. The PBIAS values are centered around zero,
844 suggesting the absence of systematic overestimation or underestimation. Furthermore,
845 the NSE and WI values are concentrated near 0.95 and 1.00, respectively, confirming
846 the strong agreement between simulated and observed hydrographs. Although a few
847 extreme events exhibit larger forecasting errors, the overall metric distributions remain
848 highly concentrated, demonstrating the robustness and generalization capability of the
849 PCBT framework across diverse hydrological conditions.

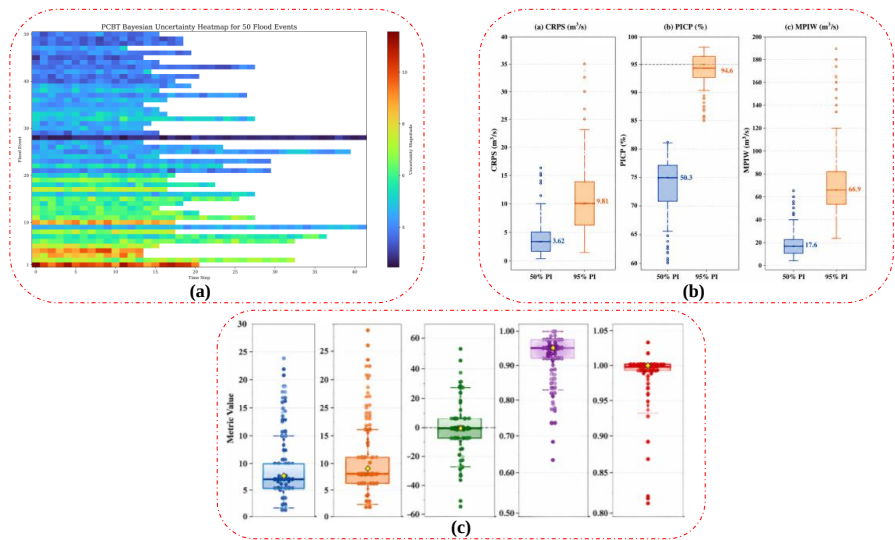


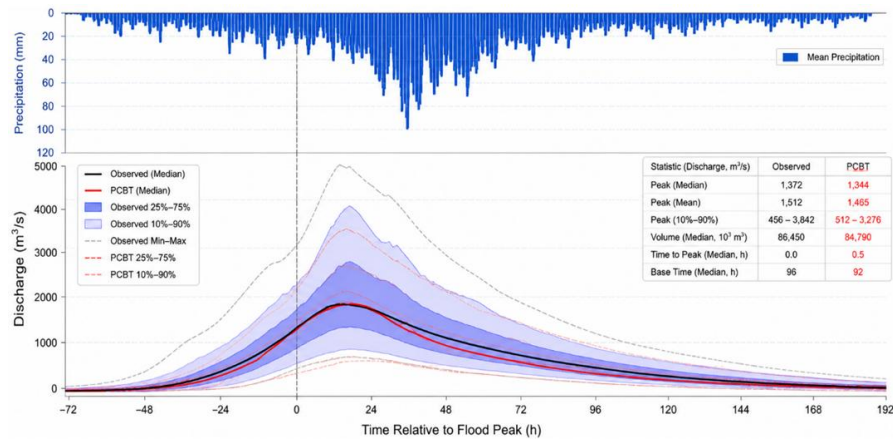
Fig. 10. Probabilistic performance of the PCBT model for 50 flood events in the Hei River. (a) Heatmap of predictive uncertainty across flood events and forecast lead times. (b) Boxplot comparison of CRPS, PICP and MPIW under 50% and 95% prediction intervals. (c) Distribution of event-wise uncertainty statistics across the 50 flood events.

4.2.2. Luoma lake basin

Fig. 11 presents the composite rainfall–runoff characteristics of 50 flood events in the Luoma Lake watershed along with corresponding PCBT framework predictions, serving as an independent evaluation dataset. The time series are aligned such that 0 h corresponds to the observed flood peak for each event. Observed median discharge is indicated by the solid black line, with the 25~75% and 10~90% quantile ranges shaded in dark and light blue, respectively. PCBT predictions are shown in red, with 50% and 90% prediction intervals depicted as dark and light red shaded areas. Key hydrograph statistics, including peak discharge (median: 1372 m³/s; mean: 1512 m³/s), volume (median: 86,450×10³ m³), time to peak (median: 0 h), and base time (median: 96 h), are summarized in the inset table, alongside PCBT predictions (peak median: 1344 m³/s; peak mean: 1465 m³/s; volume median: 84,790 ×10³ m³; time to peak: 0.5 h; base time:



867 92 h).



868
869 **Fig. 11.** Composite rainfall runoff characteristics and corresponding PCBT predictions of 50 flood
870 events in Luoma Lake.

871 **Fig. 11** highlights the model’s ability to reproduce both central tendencies and event-
872 wise variability of flood hydrographs in a lowland lake environment, providing a
873 benchmark comparison for the Hei River upstream case. The inclusion of Luoma Lake,
874 a representative plain-lake watershed, complements the mountainous Hei River basin,
875 enhancing the generalizability and robustness assessment of the PCBT framework
876 across contrasting hydrological regimes.

877 **Fig. 12** presents the probabilistic streamflow forecasting results of the PCBT
878 framework for 50 flood events in the Luoma Lake Basin. As shown in **Fig. 12a**, the
879 flood events exhibit substantial variability in magnitude, with peak discharges ranging
880 from approximately 20 to 400 m³/s. The training (35 events), validation (7 events), and
881 testing (8 events) datasets cover both low- and high-flow conditions, providing a
882 comprehensive basis for model evaluation. Across all events, the median predictions
883 closely follow the observed stream-flows, indicating that the PCBT framework



884 successfully captures the temporal evolution and peak characteristics of flood
885 hydrographs.

886 The prediction intervals further demonstrate the reliability of the probabilistic
887 forecasts. For most flood events, the observed peak discharges are contained within the
888 corresponding 90% Prediction intervals, suggesting that the Bayesian component
889 effectively characterizes predictive uncertainty. The prediction intervals remain
890 relatively narrow for moderate floods but become wider for larger events, reflecting the
891 increased uncertainty associated with extreme hydrological conditions. Notably, the
892 model maintains satisfactory predictive performance in the testing period, indicating
893 strong generalization capability and robustness under unseen flood events.

894 Fig. 12b illustrates the spatial–temporal distribution of predictive uncertainty across
895 all flood events and forecast horizons. The uncertainty exhibits clear event-to-event
896 variability, with larger uncertainty generally corresponding to periods of high
897 streamflow. Several major flood events occurring between flood IDs 15~35 show
898 broader prediction intervals, indicating stronger nonlinear rainfall–runoff responses and
899 more complex hydrodynamic processes. In contrast, low-magnitude events are
900 associated with relatively small uncertainty ranges, suggesting that the model is more
901 confident when predicting stable flow conditions.

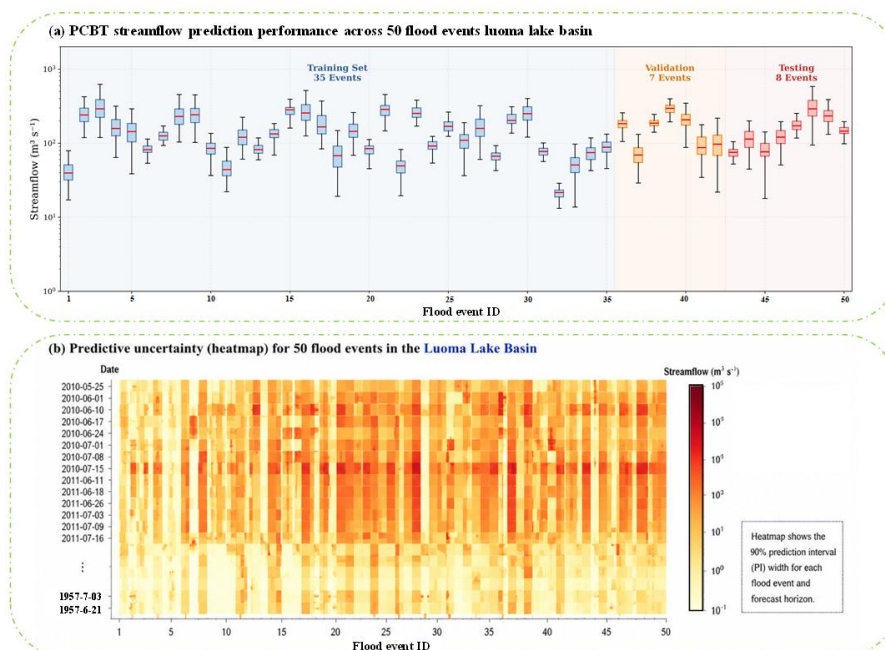


Fig. 12. Probabilistic streamflow prediction and uncertainty analysis of 50 flood events in the Luoma Lake Basin using PCBT.

Fig. 13 presents the deterministic performance of the PCBT framework for 50 flood events in the Luoma Lake, with the dataset partitioned into training (events 1~35, 70%), validation (events 36~42, 14%), and testing (events 43~50, 16%) sets. Six evaluation metrics are reported for each flood event: NSE, KGE, WI, RMSE, MAE, and PBIAS. The median NSE and KGE values for the testing set are 0.86 and 0.84, respectively, indicating strong predictive skill. The WI remains high at 0.91, and RMSE and MAE medians are 31.2 m³/s and 23.9 m³/s, respectively. PBIAS shows a slight underestimation tendency with a median of -1.9%. These results demonstrate that PCBT reliably captures the hydrograph dynamics of multiple flood events, with minimal bias and high accuracy across the training, validation, and testing periods.

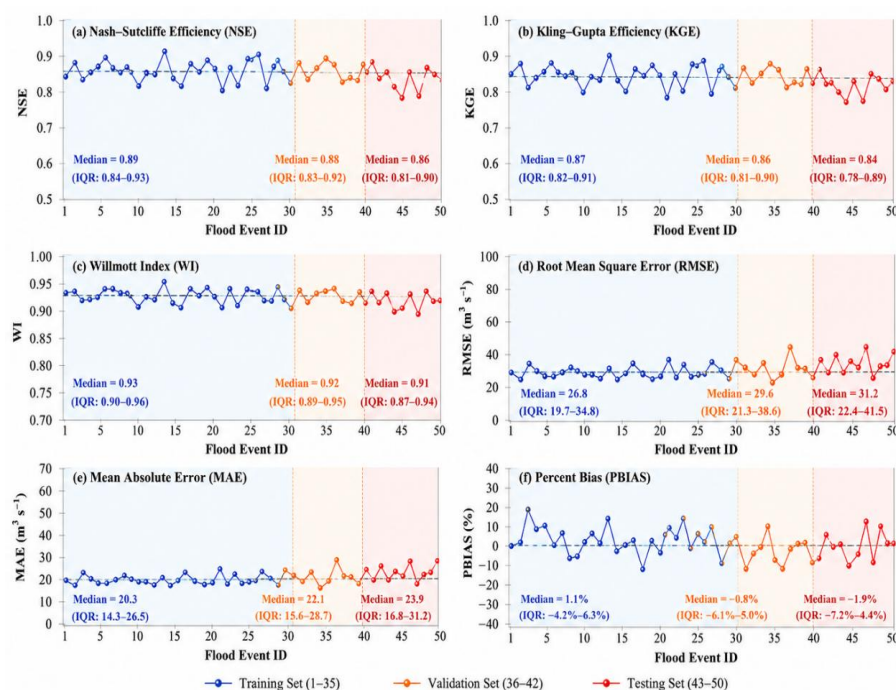


Fig. 13. Performance of PCBT for 50 Flood Events in the Luoma Lake.

Fig. 14 illustrates the distribution of residuals (observed minus predicted streamflow)

for the 50 flood events in the Luoma Lake. The residuals are approximately symmetrically distributed around zero, with most falling within $\pm 200 \text{ m}^3/\text{s}$, indicating that the PCBT framework provides unbiased predictions across the dataset. Extreme residuals are rare, confirming the model's robustness in capturing both moderate and extreme flood events. Collectively, Figs. 13 and 14 highlight the PCBT framework's capability to accurately reproduce observed flood dynamics and maintain consistent predictive performance across a large set of independent events.

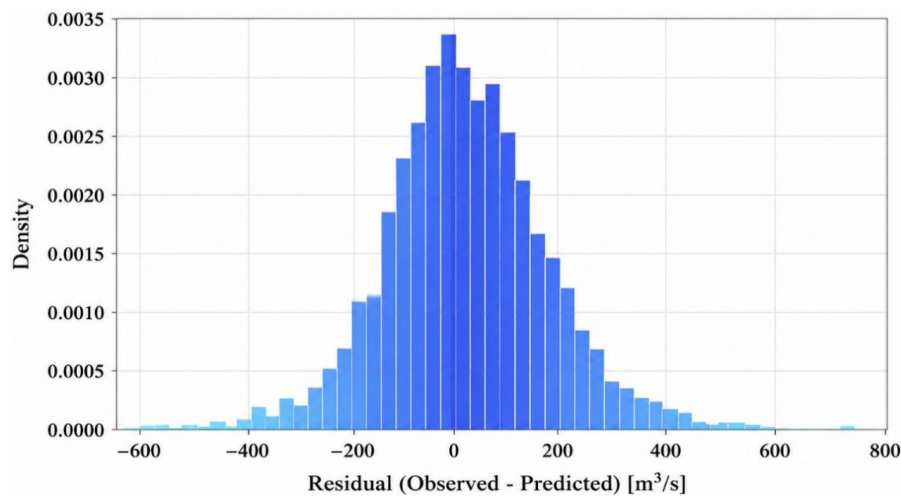


Fig. 14. Error Distribution of PCBT Predictions Luoma Lake Basin.

4.2.3. Overall Performance of PCBT Across Different Datasets

Table 2 summarizes the predictive performance of the PCBT framework across the training, validation, and testing datasets in the Upper Hei River and the Luoma Lake Basin. For the Upper Hei River Basin, the median NSE values were 0.91, 0.85, and 0.86 for the training, validation, and testing datasets, respectively. The corresponding median WI values were 0.91, 0.88, and 0.84. Median RMSE increased from 32.6 m³/s in the training dataset to 50.5 m³/s in the testing dataset, while median MAE increased from 22.7 to 33.6 m³/s. Median PBIAS remained within -7.2%, indicating a slight tendency toward underestimation but no systematic prediction bias. The interquartile ranges of NSE (0.84~0.93, 0.82~0.92, and 0.80~0.92) suggest relatively stable performance across the 50 flood events.



938 **Table 2**

939 *Event-wise deterministic performance across training, validation, and testing periods*

Watershed	Dataset	Median NSE	25%- 75% IQR	10%- 90% IQR	Median WI	Median RMSE (m ³ /s)	Median MAE (m ³ /s)	Median PBIAS
Upper Hei River	Training set	0.91	0.84- 0.93	0.82- 0.95	0.91	32.6	22.7	-3.6%
	Validation set	0.85	0.82- 0.92	0.80- 0.94	0.88	41.8	28.9	-5.8%
	Testing set	0.86	0.80- 0.92	0.78- 0.95	0.84	50.5	33.6	-7.2%
Luoma Lake	Training set	0.89	0.81- 0.91	0.79- 0.93	0.93	26.8	20.3	1.1%
	Validation set	0.88	0.79- 0.89	0.77- 0.91	0.92	29.6	22.1	-0.8%
	Testing set	0.86	0.77- 0.87	0.75- 0.90	0.91	31.2	23.9	-1.9%

940 For the Luoma Lake Basin, median NSE values are 0.89, 0.88, and 0.86 for the
 941 training, validation, and testing datasets, respectively. Median WI remained
 942 consistently high at 0.93, 0.92, and 0.91. Median RMSE values were 26.8, 29.6, and
 943 31.2 m³/s, while median MAE values were 20.3, 22.1, and 23.9 m³/s. The corresponding
 944 median PBIAS values were 1.1%, -0.8%, and -1.9%, indicating that the model
 945 reproduced flood volumes with negligible systematic bias. Compared with the Upper
 946 Hei River Basin, forecasting errors in the Luoma Lake Basin were generally lower and
 947 exhibited smaller variations among different datasets.

948 Overall, the PCBT framework maintained NSE values greater than 0.85 and WI
 949 values greater than 0.84 across all datasets and both basins. The relatively small
 950 differences between training, validation, and testing performance indicate good
 951 generalization capability and limited overfitting. The consistent performance in both
 952 the mountainous Upper Hei River Basin and the low-relief Luoma Lake Basin further



953 demonstrates the robustness and transferability of the proposed framework under
954 contrasting hydrological conditions.

955 **Table 3** presents the uncertainty quantification of PCBT predictions for 50 flood
956 events in the Upper Hei River and Luoma Lake basins, evaluated using three key
957 probabilistic metrics: CRPS, PICP, and MPIW. For the Upper Hei River, the CRPS is
958 $0.093 \pm 0.011 \text{ m}^3/\text{s}$, PICP is $91.6 \pm 1.1\%$, and MPIW is $656 \pm 45 \text{ m}^3/\text{s}$, indicating high
959 predictive reliability and well-calibrated uncertainty bounds. In the Luoma Lake basin,
960 the CRPS is slightly higher at $0.118 \pm 0.009 \text{ m}^3/\text{s}$, PICP is $90.8 \pm 1.0\%$, and MPIW is
961 narrower at $360 \pm 40 \text{ m}^3/\text{s}$, reflecting more compact prediction intervals due to lower
962 hydrological variability. These results demonstrate that the PCBT framework produces
963 robust probabilistic forecasts, with calibrated uncertainty ranges that capture observed
964 flood variability across both basins.

965 **Table 3**
966 *Comparison of uncertainty quantification metrics for PCBT in two basins*

Basin	CRPS	PICP(%)	MPIW(m ³ /s)
Upper Hei River	0.093±0.011	91.6±1.1	656±45
Luoma Lake	0.118±0.009	90.8±1.0	360±40

967 **4.3. Performance Comparison of Five Models Across Different Basins**

968 All benchmark models, including PINN, LSTM, Transformer, and BPNN, are
969 evaluated under a consistent experimental setting to ensure a fair comparison.
970 Specifically, all models are trained and tested using identical input variables
971 (precipitation and discharge), the same flood event segmentation strategy, and the same
972 training–validation–testing data splits. In addition, hyperparameters of all benchmark
973 models are tuned within comparable search budgets to avoid advantages arising from



974 uneven optimization effort. Under this controlled setup, the proposed PCBT framework
975 is systematically compared against the baseline models using both deterministic and
976 probabilistic evaluation metrics.

977 [Fig. 15](#) presents a comprehensive comparison of the predictive performance of five
978 models in the upper Hei River. The radar chart summarizes six evaluation metrics,
979 including NSE, KGE, WI, RMSE, MAE, and PBIAS, while the Taylor diagram
980 provides an integrated assessment of correlation coefficient, standard deviation, and
981 centered RMSE. The PCBT framework exhibits the best overall performance among
982 all competing methods. In the Taylor diagram, PCBT achieves the highest correlation
983 coefficient ($R = 0.98$), the lowest RMSE ($13.24 \text{ m}^3/\text{s}$), and a standard deviation ($SD =$
984 $35.34 \text{ m}^3/\text{s}$) that closely matches the observed variability. By comparison, PINN yields
985 $R = 0.93$ and $RMSE = 14.58 \text{ m}^3/\text{s}$, while LSTM, Transformer, and BPNN show lower
986 correlations of 0.88, 0.79, and 0.74, respectively, together with substantially larger
987 RMSE values. The radar chart further demonstrates that PCBT attains the highest NSE,
988 KGE, and WI values and the lowest error-related metrics, indicating its superior
989 capability in reproducing both flood magnitude and temporal dynamics. These results
990 suggest that the integration of physical constraints, Transformer-based temporal
991 learning, and Bayesian uncertainty quantification substantially improves flood
992 forecasting performance in the mountainous Hei River.

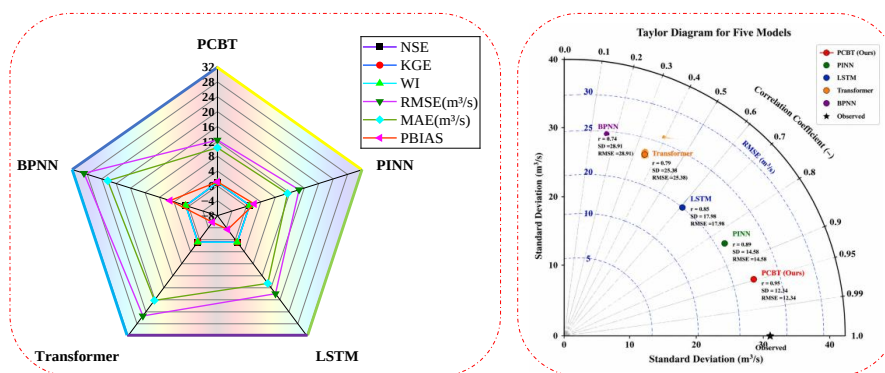


Fig. 15. Comprehensive evaluation of the predictive performance of five models using six evaluation indicators in the upper Hei River.

Fig. 16 shows the corresponding model comparison for the Luoma Lake. Similar

patterns can be observed across all evaluation metrics. According to the Taylor diagram, PCBT achieves $NSE = 0.95$, $KGE = 0.94$, $RMSE = 15.24 \text{ m}^3/\text{s}$, $MAE = 14.92 \text{ m}^3/\text{s}$, and $PBIAS = 1.1\%$, outperforming all benchmark models. The correlation coefficient reaches approximately 0.98, and the model standard deviation is closest to observed value, indicating a strong ability to reproduce the variability of flood processes. In contrast, the PINN model achieves $NSE = 0.89$ and $KGE = 0.86$, whereas LSTM, Transformer, and BPNN exhibit progressively lower predictive skill. The radar chart confirms that PCBT consistently provides the highest efficiency metrics and the lowest prediction errors. The performance gains are particularly evident for RMSE and MAE, demonstrating improved simulation of flood peaks and recession limbs.

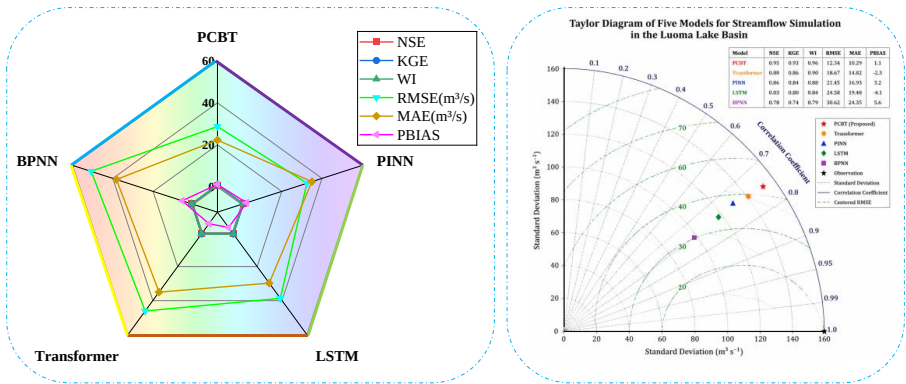


Fig. 16. Comprehensive evaluation of the predictive performance of five models using six evaluation indicators in the Luoma Lake.

A comparison between Figs. 15 and 16 reveals that PCBT maintains superior performance in two hydrologically contrasting environments. The upper Hei River Basin represents a mountainous watershed characterized by rapid runoff concentration and strong topographic controls, whereas the Luoma Lake Basin is a low-relief plain–lake system with relatively slow hydrological responses. The consistent dominance of PCBT across both basins indicates that the proposed framework is capable of capturing complex rainfall–runoff relationships under diverse hydrological conditions, highlighting its robustness, transferability, and general applicability for flood forecasting.

Table 4 summarizes the deterministic performance of PCBT and benchmark models across two contrasting basins using six evaluation metrics (NSE, KGE, WI, RMSE, MAE, and PBIAS). Overall, PCBT consistently achieves the best performance in both basins. In the Upper Hei River Basin, it attains NSE/KGE/WI of 0.95/0.93/0.96 and the lowest errors (RMSE = 12.3 m³/s, MAE = 10.3 m³/s), outperforming PINN, LSTM, Transformer, and BPNN. A similar ranking pattern is observed in the Luoma Lake Basin,



where PCBT also yields the highest accuracy ($NSE = 0.90$, $KGE = 0.88$, $WI = 0.91$) and lowest error magnitudes. Relative to PINN, the second-best model, PCBT improves NSE by 6.7% and 5.9% in the two basins, while reducing RMSE by 15.8% and 13.6%, respectively. These results indicate that PCBT more effectively captures nonlinear rainfall–runoff dynamics and maintains robust predictive skill across heterogeneous hydrological conditions.

Table 4

Detailed quantitative performance metrics of flood forecasting models across two hydrologically contrasting basins, providing numerical basis for Figures 14–15

Watershed	Model	NSE	KGE	WI	RMSE (m^3/s)	MAE (m^3/s)	PBIAS (%)
Upper Hei River	PCBT	0.95	0.93	0.96	12.3	10.3	1.1
	PINN	0.89	0.88	0.92	14.6	11.4	2.2
	Transformer	0.79	0.74	0.82	25.4	20.2	-5.8
	LSTM	0.85	0.83	0.86	17.9	14.5	-3.4
	BPNN	0.74	0.70	0.79	28.9	22.4	5.1
Luoma Lake	PCBT	0.90	0.88	0.91	45.2	38.6	2.0
	PINN	0.85	0.82	0.87	52.3	45.0	3.5
	Transformer	0.75	0.72	0.78	72.1	63.0	-6.5
	LSTM	0.80	0.78	0.82	60.8	52.4	-4.0
	BPNN	0.71	0.68	0.75	80.5	70.2	6.0

5. Discussion

5.1. Physical consistency and predictive skill of the PCBT framework

The results obtained from the Upper Hei River and Luoma Lake basins demonstrate that the PCBT framework provides accurate and physically consistent flood forecasts across a wide range of hydrological conditions. Compared with PINN, LSTM, Transformer, and BPNN models, PCBT consistently achieved the highest NSE, KGE, and WI values while maintaining the lowest RMSE and MAE values (Table 4). In the Upper Hei River, PCBT achieved an NSE of 0.95 and an RMSE of $12.3 m^3/s$, representing improvements of 6.7%, 11.8%, 20.3%, and 28.4% over PINN, LSTM,



1043 Transformer, and BPNN, respectively. Similar advantages were observed in the Luoma
1044 Lake, where NSE reached 0.90 and RMSE decreased to 45.2 m³/s.

1045 The superior performance of PCBT can be attributed to the synergistic integration of
1046 Bayesian learning, Transformer-based sequence modeling, and physics-informed
1047 constraints. Traditional data-driven models such as LSTM and Transformer primarily
1048 rely on statistical relationships embedded in historical observations (Jia et al., 2021;
1049 Shen et al., 2018). Although these models effectively capture temporal dependencies,
1050 they often struggle to maintain physical consistency during extreme flood events. In
1051 contrast, the physics-constrained component embedded in PCBT explicitly
1052 incorporates hydrological conservation principles into the optimization process.
1053 Consequently, the model is capable of learning both data-driven patterns and physically
1054 meaningful runoff dynamics simultaneously.

1055 The event-based results shown in Fig. 9 further support this interpretation. Most
1056 observed hydrographs are well enclosed within the 95% prediction intervals, while the
1057 median predictions accurately reproduce flood rising limbs, peak timing, and recession
1058 processes. This indicates that the physical constraints effectively prevent unrealistic
1059 predictions during rapidly changing flow conditions. The residual distribution shown
1060 in Figure 14 is approximately symmetric around zero, suggesting that systematic bias
1061 has been substantially reduced and that prediction errors are primarily associated with
1062 random hydrological variability rather than model structural deficiencies.

1063 Another important finding is that predictive skill remains relatively stable across the
1064 training, validation, and testing datasets. As shown in Table 2, NSE values in the Upper



1065 Hei River decrease only from 0.91 to 0.86 between training and testing periods, while
1066 Luoma Lake exhibits a similar decline from 0.89 to 0.86. The limited reduction in
1067 performance indicates that the proposed framework successfully avoids severe
1068 overfitting and maintains strong generalization capability when applied to unseen flood
1069 events.

1070 **5.2. Uncertainty quantification and reliability of probabilistic forecasts**

1071 Reliable uncertainty estimation is essential for operational flood forecasting because
1072 decision makers require information regarding both expected flow conditions and
1073 associated prediction risks. The Bayesian component of PCBT enables direct
1074 quantification of predictive uncertainty through Monte Carlo sampling, generating
1075 probabilistic forecasts rather than single deterministic predictions (Willard et al., 2022).

1076 The uncertainty evaluation metrics presented in Table 3 indicate that the probabilistic
1077 forecasts are both reliable and informative. For the Upper Hei River basin, the mean
1078 PICP reaches 91.6%, while the CRPS remains low at 0.093. Similar results are obtained
1079 in Luoma Lake, where PICP equals 90.8% and CRPS equals 0.118. These values
1080 suggest that the generated prediction intervals successfully capture most observed flood
1081 responses while maintaining relatively sharp forecast distributions.

1082 The uncertainty heatmaps and boxplots shown in Figs. 10 and 12 provide additional
1083 insights into forecast behavior. Prediction uncertainty generally increases around flood
1084 peaks and during rapid rising stages, reflecting the increased hydrological complexity
1085 associated with intense runoff generation and routing processes. Conversely,
1086 uncertainty remains relatively low during recession periods when streamflow dynamics



1087 are more stable. Such behavior is hydrologically reasonable because runoff generation
1088 mechanisms become increasingly nonlinear under high-flow conditions.

1089 The comparison between 50% and 95% prediction intervals further demonstrates the
1090 robustness of the probabilistic framework. Although wider prediction intervals
1091 naturally produce larger MPIW values, the corresponding increase in PICP confirms
1092 improved coverage of extreme events. This trade-off between sharpness and reliability
1093 is a common characteristic of probabilistic forecasting systems and indicates that the
1094 Bayesian component effectively balances forecast confidence and uncertainty
1095 representation.

1096 **5.3. Superiority and Cross-Basin Transferability of the PCBT** 1097 **Framework**

1098 Recent developments in hydrological forecasting have increasingly emphasized the
1099 integration of physical laws, deep sequence learning, and probabilistic inference.
1100 PINNs improve physical consistency but often suffer from training instability and
1101 limited uncertainty representation (Raissi et al., 2019; Read et al., 2019). Transformer-
1102 based models excel at capturing long-range temporal dependencies in hydrological
1103 sequences (Kratzert et al., 2022; Muñoz-Castro et al., 2026), yet may generate
1104 physically inconsistent predictions under extreme events. Bayesian deep learning
1105 enhances predictive reliability through explicit uncertainty quantification (Kendall and
1106 Gal, 2017; Zheng et al., 2024), but typically lacks physical constraints.

1107 In contrast, the proposed PCBT framework unifies these three paradigms within a
1108 single architecture, enabling complementary interaction between temporal



1109 representation learning, physical consistency enforcement, and probabilistic inference.
1110 This integration leads to consistent improvements in both accuracy and reliability.
1111 Specifically, in the Upper Hei River basin, PCBT achieved $NSE = 0.95$ and $RMSE =$
1112 $12.3 \text{ m}^3/\text{s}$, outperforming PINN ($0.89, 14.6 \text{ m}^3/\text{s}$), LSTM ($0.85, 17.9 \text{ m}^3/\text{s}$), Transformer
1113 ($0.79, 25.4 \text{ m}^3/\text{s}$), and BPNN ($0.74, 28.9 \text{ m}^3/\text{s}$). Similar improvements were observed
1114 in Luoma Lake ($NSE = 0.90, RMSE = 45.2 \text{ m}^3/\text{s}$), confirming that the coupled
1115 framework effectively reduces structural prediction errors while preserving
1116 generalization ability. Meanwhile, PICP values above 90% and relatively low CRPS
1117 further indicate that PCBT achieves a favorable balance between reliability and
1118 sharpness, which remains a key challenge in probabilistic flood forecasting.

1119 Beyond predictive accuracy, the framework also demonstrates strong robustness
1120 across contrasting hydrological regimes. The Upper Hei River basin is characterized by
1121 mountainous terrain and rapid runoff generation, while Luoma Lake represents a
1122 lowland lake–river system with pronounced storage and routing effects. Despite these
1123 differences, PCBT consistently maintains high performance ($NSE > 0.90$ and > 0.86 in
1124 the two basins, respectively), indicating that the model is not overly dependent on basin-
1125 specific statistical patterns. This robustness is primarily attributed to the incorporation
1126 of hydrological constraints, which embed physically universal laws into the learning
1127 process and enhance generalization capability across watersheds (Feng et al., 2022).
1128 Comparative analyses in Figs. 15 and 16 further confirm that PCBT remains closest to
1129 the ideal reference in both radar and Taylor space, particularly under complex flood
1130 conditions with sharp peaks and rapid hydrograph variation. Overall, these results



1131 demonstrate that PCBT provides a unified, physically consistent, and uncertainty-aware
1132 framework that significantly advances both predictive performance and cross-basin
1133 transferability in flood forecasting.

1134 **5.4. Limitations and future research directions**

1135 Although the proposed framework demonstrates promising performance, several
1136 limitations remain. Firstly, the current model primarily incorporates water balance and
1137 continuity constraints. Additional hydrodynamic information, such as momentum
1138 equations, channel routing processes, and lake storage dynamics, may further improve
1139 physical realism and predictive accuracy ([Muñoz-Castro et al., 2026](#)).

1140 Secondly, the study focuses on historical flood events from two representative basins.
1141 While the results demonstrate encouraging transferability, broader evaluations across
1142 different climatic regions, watershed scales, and hydrological regimes are still required.
1143 Future studies should investigate the framework's applicability in highly regulated river
1144 systems, arid watersheds, and regions experiencing nonstationary climate conditions
1145 ([Reichstein et al., 2019](#); [Muñoz-Castro et al., 2026](#)).

1146 Thirdly, uncertainty estimation currently relies on Bayesian inference and Monte
1147 Carlo sampling. Although effective, the computational cost increases with the number
1148 of samples and forecasting horizons. Future work may explore more efficient
1149 probabilistic learning strategies, including ensemble physics-informed models and
1150 diffusion-based uncertainty estimation techniques.



1151 5. Conclusion

1152 Flood forecasting remains challenging because of the strong nonlinearity of rainfall–
1153 runoff processes, watershed heterogeneity, and uncertainties associated with extreme
1154 flood events. To address these challenges, this study developed a Physics-Constrained
1155 Bayesian Transformer (PCBT) framework that integrates Transformer-based sequence
1156 learning, physics-informed constraints, and Bayesian probabilistic inference within a
1157 unified architecture. The framework was evaluated using 50 independent flood events
1158 from two hydrologically contrasting watersheds, namely the Upper Hei River Basin
1159 and the Luoma Lake Basin. The major conclusions are as follows.

1160 (1) The proposed PCBT framework successfully achieved physically consistent and
1161 highly accurate flood forecasting by integrating Transformer learning with hydrological
1162 constraints. Across both study basins, PCBT consistently outperformed PINN,
1163 Transformer, LSTM, and BPNN models in deterministic forecasting performance. In
1164 the Upper Hei River Basin, PCBT achieved NSE, KGE, and WI values of 0.95, 0.93,
1165 and 0.96, respectively, while RMSE and MAE were reduced to 12.3 m³/s and 10.3 m³/s.
1166 In the Luoma Lake Basin, the corresponding values were 0.90, 0.88, 0.91, 45.2 m³/s,
1167 and 38.6 m³/s. The results indicate that combining data-driven temporal feature
1168 extraction with physical constraints effectively improves the representation of complex
1169 rainfall–runoff dynamics and enhances hydrological consistency in flood prediction.

1170 (2) The incorporation of physics-informed constraints and Bayesian uncertainty
1171 quantification substantially improved forecasting reliability and predictive robustness.
1172 Comparative analyses demonstrated that the physics-constrained architecture reduced



1173 physically unrealistic predictions and enhanced flood peak reproduction compared with
1174 purely data-driven models. Meanwhile, Bayesian inference enabled explicit
1175 characterization of predictive uncertainty, yielding CRPS values of 0.093 ± 0.011 and
1176 0.118 ± 0.009 for the Upper Hei River and Luoma Lake, respectively. The
1177 corresponding PICP values reached 91.6% and 90.8%, indicating that the generated
1178 prediction intervals effectively captured observed streamflow variability and flood peak
1179 uncertainty. These findings confirm that the integration of physical constraints and
1180 probabilistic inference contributes substantially to both forecast accuracy and reliability.

1181 (3) The proposed framework exhibited strong robustness and transferability across
1182 distinct hydrological regimes. Despite pronounced differences in topography, climate
1183 conditions, runoff-generation mechanisms, and storage characteristics between the
1184 mountainous Upper Hei River Basin and the lowland lake–river system of Luoma Lake,
1185 PCBT maintained stable predictive performance throughout all flood events. Median
1186 NSE values remained above 0.85 across training, validation, and testing datasets, while
1187 residual distributions were centered near zero and approximately symmetric. These
1188 results demonstrate that the framework possesses strong generalization capability and
1189 can be effectively transferred across watersheds with contrasting hydrological
1190 characteristics.

1191 Overall, the results demonstrate that the synergistic integration of Transformer-based
1192 temporal learning, physics-informed hydrological constraints, and Bayesian
1193 uncertainty estimation provides a reliable, interpretable, and uncertainty-aware solution
1194 for flood forecasting. The proposed PCBT framework offers a promising pathway for



1195 improving operational flood forecasting, water resources management, and flood risk
1196 mitigation under changing hydroclimatic conditions.

1197 **Author Contributions**

1198 **Mingrui Shi:** Writing – Original Draft, Writing – Review & Editing,
1199 Conceptualization, Methodology, Software, Visualization, Formal analysis, Data
1200 Curation. **Hongyuan Fang:** Conceptualization, Project administration, Investigation,
1201 Resources, Funding acquisition, Validation, Supervision.

1202 **Conflict of Interest**

1203 The contact author has declared that none of the authors has any competing interests.

1204 **Data Availability Statement**

1205 Data will be made available on request.

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