Atmospheric Chemistry and Physics

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Solar-tidal Phase and Amplitude in Middle-atmosphere Winds Investigated From an Adaptive Rational Neural Operator, two Reanalyses and Meteor-Radar Observations

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Author Comment (AC1): Reply on Referee Comment RC1

Open discussion of egusphere-2026-4972, Atmospheric Chemistry and Physics

Response to Anonymous Referee #1 (RC1)

Dear Referee,

We sincerely thank Referee #1 for the careful and constructive review and for recognising the observational constraint on reanalysis tidal amplitudes, the vector-wind evaluation, the architecture comparisons, the ablations and the explicit limitations. We agree that the mathematical interpretation of the learned poles and several aspects of the validation need clarification. Below we reproduce each comment, followed by our response and the changes we plan to make in the revised manuscript. Revised files cannot be uploaded during the open discussion, so all changes are described in the future tense. Section, equation, table and figure numbers refer to the discussion version. Supplement at the end of this document gives the notation table and the equations as implemented.

General comment:

"The manuscript addresses an important problem by connecting middle-atmosphere wind prediction with independent observational constraints on reanalysis tidal amplitudes. Its strengths include vector-wind evaluation, comparisons with several architectures, temporal-component ablations and explicit discussion of limitations. However, the mathematical interpretation of the learned poles and several aspects of validation require clarification. I recommend major revision."

Response:

We thank the referee for this summary and accept the recommendation of major revision. The revision will state the practical significance of the accuracy comparison, give a complete description of the discrete Takenaka–Malmquist (TM) layer as implemented and clarify what its poles represent, document the sample used for pole extraction and report statistics over the whole test period, add ATMNO tidal amplitudes and phases to the meteor-radar comparison, and rewrite the physical interpretation in terms of solar-tidal-period bands that comprise migrating and non-migrating contributions.

Comment 1:

"The scientific and practical significance of the improvement over the strongest baselines needs clearer justification. Table 1 reports wind speed RMSE values of 2.215, 2.323 and 2.333 m/s for ATMNO, Factformer and MWT, respectively. These correspond to absolute reductions of 0.108 and 0.118 m/s. The block-bootstrap intervals in Section 4.1 support a statistically detectable advantage, but they do not establish its practical importance. As far as I know, the velocity of the zonal wind field at 80 km is ~50 m/s. The significance of this performance improvement is unclear."

Response:

We agree that statistical detectability does not establish practical importance. The margins of 0.108 and 0.118 ms−1 (4.7 % and 5.1 %) are small, and the revised manuscript will say so explicitly.

On the reference scale, the climatological zonal wind near 80 km varies strongly with season and latitude: long-term radar observations at about 54° N and 69° N place the summer westward maximum near 75 km at 35–54 ms−1 (Jaen et al., 2023), whereas at Wuhan (30.6° N) the summer eastward wind at 80 km is below 10 m s−1 and winter eastward winds below 85 km reach 20–30 ms−1 (Zhao et al., 2005), so ~50 ms−1 characterises jet maxima rather than a typical 80 km wind over our domain. For a

3 h forecast the relevant reference is the 3 h change of the field: three-hour ERA5 persistence has a level-mean speed RMSE of 4.121 ms−1 (Sect. 4.1), so the MSE skill score relative to persistence is approximately 0.71 for ATMNO and 0.68 for both Factformer and MWT, and most of the skill is shared by the three strongest architectures. Against the meteor radars, all forecasts carry a wind-speed underestimate of 10–11 m s−1 inherited from ERA5 (Sect. 4.7), so differences of a few hundredths of a metre per second are not resolvable there.

In the revised manuscript we will present ATMNO as competitive with the strongest capacity-matched baselines; add skill scores relative to persistence and per-level ATMNO–baseline differences with block-bootstrap intervals to Sect. 4.1; report the mean and standard deviation of speed RMSE over multiple training seeds for ATMNO, Factformer and MWT under an identical training protocol; and add a paragraph on practical significance to Sect. 5.4.

Comment 2:

"The notation in Equations (2)-(6) should be defined more completely. Please specify the roles and dimensions of z, z_t, θ_t, α_k, p_k, c_k, ĉ_k and f_φ, including the summation ranges. Explain how 32 basis functions are used with only 16 time samples, since their sampled vectors cannot all be mutually orthogonal."

Response:

We agree, and in checking the equations against the code we found that Eqs. (4)–(6) do not describe the implementation exactly. The definitions as implemented are as follows. The TM layer acts independently on each of the 320 pooled spatial tokens (the 16 × 20 grid). For one token, $z_t \in \mathbb{R}^D$ with $D = 56$ is the real-valued feature vector at time step $t = 1, \ldots, T$, with $T = 16$ (48 h at 3 h), after a linear input projection; the layer is applied channel-wise. The sampling angle is $\theta_t = 2\pi(t - 1)/T$, so $\exp(i\theta_t)$ are T equally spaced points on the unit circle. The poles $\alpha_k \in \mathbb{C}$, $k = 1, \ldots, K$ with $K = 32$, are predicted once per token and per window by a two-layer perceptron applied to the temporal mean of z_t; its raw output $p_k = (p_k^{re}, p_k^{im}) \in \mathbb{R}^2$ is mapped to $\tanh(p_k^{re}) + i \tanh(p_k^{im})$ and then rescaled radially so that $|\alpha_k| \leq 0.95$ (Eq. 3). φ_k is the TM function of Eq. (2), with the Blaschke product over $j < k$. The implementation uses the real part of the sampled TM functions, $\Phi_{tk} = \mathrm{Re}\, \varphi_k(\exp(i\theta_t))$, a real $T \times K$ matrix. Eq. (4) is $c_k = \Sigma_{t=1}^{T} \Phi_{tk} z_t \in \mathbb{R}^D$, with no complex conjugate because Φ is real; Eq. (5) applies a real two-layer perceptron $f_\varphi: \mathbb{R}^D \to \mathbb{R}^D$, shared across all k, to each c_k; and Eq. (6) is $z_t^{TM} = \Sigma_{k=1}^{K} \Phi_{tk} \hat{c}_k$

$\in \mathbb{R}^D$. Eqs. (2)–(6) will be rewritten in this form with a notation table (Appendix A), and the overloading of θ (wind direction, sampling angle, pole angle, model parameters) and φ (basis functions, transform parameters) will be removed.

On orthogonality: the TM functions are orthonormal in the Hardy space $H^2(\mathbb{D})$ under the continuous inner product on the unit circle (Takenaka, 1925; Malmquist, 1926). Sampled at T = 16 points and reduced to real parts, the K = 32 vectors lie in a 16-dimensional space, have rank at most 16 and cannot be mutually orthogonal; discrete orthogonality of Malmquist–Takenaka systems holds only for special, pole-dependent node sets (e.g. Eisner and Pap, 2014), which equally spaced nodes do not satisfy. The 32 atoms therefore form a redundant dictionary, Eq. (4) yields analysis coefficients rather than expansion coefficients, and with $\hat{c}_k = c_k$ Eqs. (4) and (6) would apply $\Phi\Phi^T$, which is not the identity. Because f_φ and the downstream layers are trained end to end, this redundancy does not prevent the layer from learning a useful mapping, but individual coefficients are not interpretable. The sentence "the Blaschke product enforces orthonormality in the Hardy space" will be replaced by a statement of exactly this. We will also remove the implication that the TM basis resolves frequencies more finely than a Fourier basis over the same window: with 16 samples at 3 h the Nyquist period is 6 h and the 24, 12 and 8 h harmonics fall exactly on DFT bins 2, 4 and 6, and 32 poles cannot be identified from a single 16-sample window; the poles are predicted by a network trained across the whole training set.

On the interpretation of the poles: in this implementation the unit-circle coordinate is time, and the k-th sampled atom is essentially the k-th harmonic of the 48 h window, warped in time by the poles. Our checks with the extracted poles show that the frequency content of each atom is set by its index rather than by the pole angle, so the conversion period = $2\pi \cdot 3$ h/|arg α_k| of Sect. 3.5 does not hold for this construction. The pole angle instead marks a position within the input window: the poles with small angular spread lie at 29–47 h, i.e. within the last 20 h before the forecast origin. In the revised manuscript the periods of Table 4 and the frequency-locking wording of Sects. 4.3–4.5 and Supplement Sect. S6 will be withdrawn, the poles will be described as positions at which the layer concentrates temporal resolution. The tidal analyses of Sects. 4.6 and 4.7 are computed from the output fields and are unaffected. The revised Supplement will include dictionary diagnostics (singular values, condition number, mutual coherence) and the atom spectra on which this statement rests, together with an ablation over the number of poles (K = 8, 16 and 32) and a rerun of the Fourier-substitution ablation with a complete real Fourier basis.

Comment 3:

"The pole-extraction procedure requires further detail and a clearer justification of its analysis settings. Section 3.1 specifies a 16-step input sequence, described as a 48 h window. Section 3.5 pools pole statistics across 480 test windows and 153,600 spatiotemporal token locations. Please explain how these windows were selected, their temporal spacing and overlap, and their seasonal coverage."

Response:

The 153,600 token locations are 480 input windows × 320 spatial tokens of the 16 × 20 pooled grid; each token in each window receives one set of 32 poles from the first block. The 480 windows are the first 480 samples of the chronologically ordered test set, i.e. consecutive windows with a stride of one step (3 h): the window start times run from 09 July 2024 03:00 UTC to 07 September 2024 00:00 UTC, consecutive windows share 15 of their 16 steps, and by start date 431 windows fall in June–August and 49 in September. The sample therefore covers one late-summer period with strongly dependent windows, and the manuscript should have stated this.

We have now repeated the extraction over all 4311 test windows (09 July 2024 to 29 December 2025; by window start, 952 windows in DJF, 736 in MAM, 1167 in JJA and 1456 in SON). The circular-mean angles of the stable poles agree with the 480-window values to within 7°, vary between seasons by 3–11°, which is within their within-season circular spread of 5–21°, and are reproduced to within 0.2° by a non-overlapping subset of every sixteenth window (270 windows); seven-day block-bootstrap 95 % intervals for the circular means have half-widths of 0.5–1.1°. In the revised manuscript Sect. 3.5 will state the selection rule, stride, overlap, date range and seasonal counts and replace “spatiotemporal token locations” with “480 windows × 320 spatial tokens”; Table 4 and Fig. 5 will be replaced by the full-period statistics stratified by season, with the non-overlapping subset and the block-bootstrap intervals, reported as pole positions.

Comment 4:

"Figure 7(c, d) includes meteor-radar observations, represented by open symbols, alongside collocated ERA5 amplitudes. However, ATMNO-predicted tidal amplitudes and phases are not included in this comparison. Please add the corresponding model estimates for the same stations, observation times and altitude."

Response:

We agree, and we have computed them. For each station we took the ATMNO 3 h forecast at the ERA5 grid point nearest to the station at the 80 km level, aligned it with the radar series at the common valid times between 11 July and 31 December 2024 (1341 at Mohe, 783 at Beijing and 1323 at Wuhan), and fitted radar, ERA5 and ATMNO with the same least-squares model, an intercept, a linear trend and cosine–sine pairs at 24, 12 and 8 h, with seven-day block-bootstrap intervals. The ratio of ATMNO to ERA5 amplitude lies between 0.88 and 1.17 for 17 of the 18 station–component–harmonic combinations, the exception being the zonal terdiurnal harmonic at Beijing, where both amplitudes are below 0.7 m s−1 and their intervals overlap, and the local-solar-time phases of ATMNO and ERA5 agree within 0.35 h in all but that case. ATMNO therefore inherits the ERA5 deficit relative to the radars: on the common period ERA5 retains 18–33 % of the observed semidiurnal amplitude and 17–88 % of the diurnal amplitude, and ATMNO retains 20–31 % and 16–91 %, respectively. The semidiurnal phases of ERA5 and ATMNO agree with the radar phases within 0.3–2.3 h at all three stations, whereas the diurnal phases differ from the radar by 5–9 h at Beijing (v) and Wuhan (u and v) and agree within 2 h at Mohe.

In the revised manuscript Fig. 7c, d and Table 7 will show amplitudes and phases with intervals for radar, ERA5 and ATMNO, all computed on the common valid times of the stated period and from the single checkpoint used throughout, and Sect. 4.7 will note that the radar altitude is nominal, that meteor echo counts peak near 90 km (near 91 km at Wuhan; Zhao et al., 2005) so that 80 km lies near the lower edge of the echo distribution, and that the 80 km ERA5 level lies just below the model top at 0.01 hPa.

Comment 5:

"Physical interpretation. The discussion would benefit from a more systematic connection between the recovered spectral bands and atmospheric tidal generation and propagation. Sections 5.2 and 5.4 already acknowledge damping, non-migrating contributions and alternative sources of the 8 h signal. Latitude-independent periods alone cannot identify migrating tides without propagation or zonal-wavenumber information."

Response:

We agree. In an Earth-fixed frame, migrating and non-migrating solar tides share the periods 24/n h and differ in zonal wavenumber, so latitude independence of a period cannot distinguish them; our wording in Sects. 4.3, 4.4 and 5.4 and in the Conclusions went beyond what the evidence supports. In the revised manuscript these sections will refer to "solar-tidal-period bands (24, 12 and 8 h) comprising migrating and non-migrating contributions", and statements about these bands will rest on the harmonic analysis of the forecast and reanalysis fields (Sects. 4.6 and 4.7). We will add a subsection on tidal generation and propagation covering absorption of solar radiation by tropospheric water vapour and stratospheric ozone as the classical diurnal and semidiurnal sources (Chapman and Lindzen, 1970), tropospheric latent-heat release as a source of migrating and non-migrating diurnal and semidiurnal tides (Hagan and Forbes, 2002, 2003), direct solar forcing as the primary source of the migrating terdiurnal tide with contributions from nonlinear tide–tide and gravity-wave–tide interactions (Lilienthal et al., 2018; Lilienthal and Jacobi, 2019), and upward amplitude growth, trapped or evanescent diurnal modes at middle and high latitudes, dissipation and the reanalysis upper-level damping already discussed in Sect. 5.2; the station phases reported under Comment 4 will be discussed in this context. We will add the local-time phase of the 24, 12 and 8 h harmonics as a function of longitude within the regional domain for ERA5 and ATMNO, noting that migrating components have longitude-independent local-time phase and that the 78° longitude span (about 22 % of a latitude circle) makes this test qualitative, and we will expand the limitations accordingly.

We thank the referee again for comments that will substantially improve the clarity and rigour of the manuscript. After the discussion closes, we will submit the revised manuscript together with a point-by-point response indicating page and line numbers and a marked-up version.

Yang Wu, Qixiang Liao, Zhihui Li and Zheng Sheng, on behalf of all co-authors

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Supplement to the Reply on RC1 (AC1)

egusphere-2026-4972, *Solar-tidal phase and amplitude in middle-atmosphere winds investigated from an adaptive rational neural operator, two reanalyses and meteor-radar observations*

Yang Wu, Qixiang Liao, Zhihui Li and Zheng Sheng, on behalf of all co-authors

Notation of the Takenaka–Malmquist (TM) layer as implemented. This table accompanies our response to Comment 2. It describes the layer as it is implemented in the code, which differs in two respects from Eqs. (2)–(6) of the discussion version: the basis vectors are the real parts of the sampled TM functions, and the coefficients are real. The revised manuscript will present the equations in exactly this form. The period conversion of Sect. 3.5 of the discussion version is not reproduced here, because it does not hold for this construction (see reply to Comment 2).

Symbol	**Meaning**	**Type / dimension**	**Range or value**
T	input window length	integer	16 steps (48 h at 3 h)
K	number of poles / TM functions	integer	32
D	hidden width of one spatial token	integer	56
z_t	token feature at step t, after the linear input projection	real vector, $\mathbb{R}^D$	$t = 1, \ldots, T$
θ_t	sampling angle of step t	real	$\theta_t = 2\pi(t-1)/T$
$e^{i\theta t}$	sampling node on the unit circle	complex, modulus 1	T equally spaced nodes
$p_k = (p_k^{re}, p_k^{im})$	raw perceptron output for pole k	real pair, $\mathbb{R}^2$	unconstrained
α_k	k-th pole	complex	tanh-mapped, then rescaled radially so that $\lvert\alpha_k\rvert \le 0.95$; initial radius ≈ 0.5 at angle $2\pi(k-1)/K$
$\varphi_k(z)$	k-th TM function, Eq. (2), Blaschke product over $j < k$	complex function on the unit disk	$k = 1, \ldots, K$
Φ	sampled real basis matrix, Φ_{tk} = Re	real $T \times K$ matrix	rank ≤ 16

Symbol	Meaning	Type / dimension	Range or value
	$\varphi_k(e^{i\theta t})$		
c_k	analysis coefficient of atom k, Eq. (4)	real vector, $\mathbb{R}^D$	k = 1, …, K
f_φ	coefficient transform: two-layer perceptron, shared across all k	$\mathbb{R}^D \to \mathbb{R}^D$, parameters φ	—
$\hat{c}_k = f_\varphi(c_k)$	transformed coefficient, Eq. (5)	real vector, $\mathbb{R}^D$	k = 1, …, K
z_t^{TM}	reconstructed sequence, Eq. (6)	real vector, $\mathbb{R}^D$	t = 1, …, T
arg α_k	parameter angle of pole k; marks a position within the 48 h input window (see reply to Comment 2)	radians	position in window = 48 h · (arg α_k mod 2π)/2π
$\lvert\alpha_k\rvert$	modulus of pole k; sets how strongly the phase advance of the atom is concentrated near its angle	real	0 to 0.95

Equations as implemented (per spatial token, applied channel-wise):

$$\varphi_k(z) = \frac{\sqrt{1-|\alpha_k|^2}}{1-\bar{\alpha}_k z} \prod_{j<k} \frac{z-\alpha_j}{1-\bar{\alpha}_j z}, \qquad \theta_t = \frac{2\pi(t-1)}{T}, \qquad t = 1, \ldots, T, \quad k = 1, \ldots, K$$

$$\hat{\alpha}_k = \tanh\left(p_k^{\mathrm{re}}\right) + i \tanh\left(p_k^{\mathrm{im}}\right), \qquad \alpha_k = \hat{\alpha}_k \min\left(1, \frac{0.95}{|\hat{\alpha}_k|}\right)$$

$$\Phi_{tk} = \mathrm{Re}\, \varphi_k(e^{i\theta_t}), \qquad c_k = \sum_{t=1}^{T} \Phi_{tk}\, z_t, \qquad \hat{c}_k = f_\varphi(c_k), \qquad z_t^{\mathrm{TM}} = \sum_{k=1}^{K} \Phi_{tk}\, \hat{c}_k$$

Notes.

1. Φ is real, so Eq. (4) involves no complex conjugate, and c_k, $\hat{c}_k$ and z_t^{TM} are real.
2. The TM functions are orthonormal in the Hardy space $H^2(\mathbb{D})$ under the continuous inner product on the unit circle. After sampling at T = 16 nodes and taking real parts, the K = 32 columns of Φ form a redundant dictionary of rank at most 16 and are not mutually orthogonal. Eq. (4) therefore yields analysis coefficients, not expansion coefficients.
3. With all poles at the origin (the Fourier-substitution variant of Table 3), $\Phi_{tk} = \cos((k-1)\theta_t)$. At 16 equally spaced nodes these 32 columns contain only nine distinct vectors, $\cos(m\theta_t)$ with m = 0, …, 8, and no sine terms, so the matrix has rank 9.

4. At 3 h sampling the Nyquist period is 6 h; the 16-point window has a DFT bin spacing of 1/48 h^{-1} (22.5° per step), and the 24, 12 and 8 h harmonics fall on bins 2, 4 and 6 (45°, 90° and 135°).
5. The unit-circle coordinate θ_t is time, so the k-th sampled atom is essentially the k-th harmonic of the 48 h window, warped in time by the poles; its frequency content is set by its index k, and the pole angle marks a position within the window rather than a frequency.