



Nonlinear-Constrained Geographically Weighted Regression: Enabling Multidimensional Dynamics Analysis Using Monostatic Terrestrial Radar Interferometry at Helheim Glacier

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Abstract. Monostatic Terrestrial Radar Interferometry (TRI) provides an alternative approach for glacier one-dimensional velocity mapping when bistatic configurations are unavailable. However, existing monostatic methods typically rely on a single source of auxiliary information, limiting error suppression and reducing the accuracy of velocity retrieval. Here, we propose a nonlinear-constrained Geographically Weighted Regression (NL-GWR) framework that integrates TRI interferometric and amplitude-derived displacement observations with external velocity constraints under prior physical assumptions including short-term velocity-direction invariance and parallel flow to retrieve glacier velocity over Helheim Glacier in August, 2016. Validated against synchronous bistatic TRI measurements, NL-GWR achieves superior consistency metric between monostatic and bistatic measurements by mitigating quality deficiencies in amplitude-derived observations and alleviating the ill-conditioning inherent in conventional approaches. Leveraging this capability, we derive a 16-day velocity sequence at the Helheim Glacier terminus, where we identify short-term periodic variations in glacier velocity and strain rate potentially driven by both tidal forcing and meltwater processes. In addition, our analysis reveals a transition zone between the dominant controls on glacier motion located approximately 1–3 km upstream of the calving front. The proposed framework provides a practical pathway for overcoming the inherent information limitations of monostatic TRI in resource-constrained glacier kinematic studies and offers new opportunities for capturing and inferring the external forcing mechanisms driving changes at glacier terminus.

1 Introduction

Climate change has accelerated polar ice loss and sea-level rise, manifested by a substantial increase in ice-sheet melting rates and enhanced glacier velocities (Choi et al., 2021). Recent attention has increasingly focused on tipping points as a form of instability, whereby relatively small perturbations in environmental forcing exceed a critical threshold, triggering large-scale propagation and substantial, potentially irreversible changes in glacier dynamics (Hill et al., 2024; Kypke et al., 2026; Ochwat et al., 2025; Robinson et al., 2012). A typical manifestation occurs at the glacier terminus, where local changes in environmental forcing or geometry may trigger abrupt ice-shelf collapse, leaving the glacier terminus



insufficiently supported and potentially promoting marine ice-cliff instability, which can drive rapid inland glacier acceleration (Schlemm et al., 2022; Sun and Gudmundsson, 2026). This issue remains insufficiently characterized and represents one of the major uncertainties in future sea-level rise projections (Fox-Kemper et al., 2021). Neglecting such instabilities may result in an underestimation of future sea-level rise (Stokes et al., 2022). A key challenge is that the external forcing governing the dynamics of marine-terminating glacier terminus remains poorly resolved (Ochwat et al., 2025). Processes such as tides and meltwater input can evolve on hourly to daily timescales and may influence terminus stability, yet their short-term responses and spatial propagation remain difficult to resolve because of limited observational resolution, hindering the identification of initial triggers of glacier instability events (Amundson et al., 2020; Deakin et al., 2026). Therefore, observational techniques capable of providing higher spatiotemporal resolution are required to resolve these processes.

Terrestrial Radar Interferometry (TRI) has provided a new high-frequency observation approach for monitoring rapid glacier kinematic and topographical processes. Compared with conventional ground-based surveying methods and satellite remote sensing techniques, TRI is capable of continuously acquiring millimetre-scale surface deformation information remotely, while providing temporal sampling at the minute scale (Caduff et al., 2015; Wang et al., 2022). Initially, TRI was mainly applied to the landslides identification, rock slope monitoring, and other rapid surface deformation extraction (Noferini et al., 2007; Tarchi et al., 2003). Its application was subsequently extended to glaciers, where TRI provided new capabilities for observing ice-surface processes. Early glaciological applications of TRI primarily evaluated its capability for monitoring ice-surface motion and ice-marginal deformation. Continuous observations of a slowly moving debris-covered glacier tongue demonstrated that TRI could retrieve glacier velocity fields and elevation information from differential interferometric phases between different kinds of image pairs (Noferini et al., 2009). Subsequent studies showed that TRI could complement the limited temporal resolution of satellite SAR observations and capture rapid glacier velocity variations associated with dynamic events, such as glacier lake drainage (Riesen et al., 2011; Strozzi et al., 2009; Voytenko et al., 2012; Wehrlé et al., 2026). To date, TRI has been applied across a range of glaciological environments, including Iceland (Dixon et al., 2012), Greenland (Cassotto et al., 2021; Kim et al., 2024, 2025; Voytenko et al., 2015b, 2017; Xie et al., 2016, 2019), and European Alps (Sanz et al., 2025). In addition, research applications have evolved from direct radar measurements towards more comprehensive analyses, including elevation mapping (Wang et al., 2022; Xie et al., 2019), glacier surface rheological evolution (Cassotto et al., 2021) and multi-dimensional glacier velocity field retrieval based only on line-of-sight (LOS) measurements (Sanz et al., 2025; Voytenko et al., 2017).

The outstanding capability of TRI in capturing glacier surface morphological changes highlights its irreplaceable role in monitoring subtle processes on sub-daily timescales at glacier terminus. Public satellite-based velocity products derived from feature tracking or Differential Interferometric Synthetic Aperture Radar (D-InSAR) provide long-term observations of polar glacier changes, but remain constrained by long revisit intervals and susceptibility to temporal decorrelation or matching failure (Feng et al., 2023; Heid and Kääb, 2012; Wang et al., 2022; Zhang et al., 2026). These products typically have temporal baselines of at least the satellite revisit interval, often exceeding one day (Heid and Kääb, 2012; Kääb et al., 2016),



potentially limiting the detection and interpretation of events on sub-daily timescales. In contrast, the minute-level observation capability of TRI has enabled the identification of tide-modulated motion cycles within several kilometres of glacier terminus (Voytenko et al., 2015b), the detection and quantitative assessment of the geometric and dynamic characteristics of ice detachment and overturning processes (Xie et al., 2016), the observation of glacier velocity responses to subglacial drainage processes upstream (Dachauer et al., 2026), as well as detection of coherence loss caused by the rapid reorganization or movement of mélange occurring within minutes (Cassotto et al., 2021; Xie et al., 2019). These processes may exert threshold-like controls on calving-related instability at glacier terminus (Meng et al., 2025; Ochwat et al., 2025), highlighting the necessity of further applying TRI to investigate these mechanisms in greater depth.

With increasing demand for analysing the underlying kinematic and dynamic physical processes of glacier change, the requirements for velocity information derived from observations have become more sophisticated. Some glacier investigations generally require only a single scalar speed metric (Dachauer et al., 2026; Voytenko et al., 2012, 2015a, b). These one-dimensional measurements can identify glacier acceleration or deceleration, but a more complete understanding of its dynamics requires information on whether and where the ice undergoes extension or shear deformation, what types of forces act upon it and how it responds, and how these processes evolve and propagate in space and time (Alley et al., 2018). Strain rate and stress provide more direct indicators of ice deformation (Alley et al., 2018), ice-environment interaction forces (Meng et al., 2025) and the degree of ice weakening (Cassotto et al., 2021), offering critical insights into the glaciophysical mechanisms driving glacier terminus events. Strain rates and stress can be derived from only glacier motion measurements, but this requires at least two-dimensional velocity vectors and spatially distributed velocity information rather than one-dimensional or isolated point measurements (Alley et al., 2018; Nye, 1959). Regularly gridded two-dimensional velocity fields facilitate the calculation of spatial velocity gradients and strain rates (Alley et al., 2018). For TRI-based velocity retrieval, the requirement has consequently evolved from obtaining simple one-dimensional velocity magnitudes towards retrieving complete, geographically referenced velocity vectors.

TRI has undergone several technical developments towards multidimensional velocity retrieval. The sensitivity of TRI velocity measurements is primarily along the radar-to-target observation direction, known as the LOS direction, meaning that a single TRI system can only acquire one-dimensional interferometric velocity information (Caduff et al., 2015; Sanz et al., 2025; Voytenko et al., 2017). Bistatic TRI configurations can retrieve two-dimensional glacier velocity fields without requiring external information, although they still require the assumption of a horizontal glacier surface (Cassotto et al., 2021; Voytenko et al., 2017). However, due to resource and environmental constraints, monostatic TRI deployments are more commonly adopted. Under such configurations, interferometric measurements provide only one-dimensional speed information, requiring additional constraints or supplementary observations with unavoidable trade-offs. Existing monostatic approaches for obtaining complete two-dimensional velocity fields mainly follow two paradigms: (1) combining feature tracking to derive additional velocity components from TRI amplitude images (Kim et al., 2024, 2025), although the resulting velocity quality is substantially lower than that of interferometric measurements, thereby reducing the final accuracy (Hu et al., 2014); and (2) assuming a prior velocity direction (Voytenko et al., 2015b), while the corresponding



100 inversion is inherently ill-conditioned and requires the identification and removal of numerically unstable regions (Sanz et al., 2025). A method is therefore required to integrate heterogeneous observations and physical constraints while accounting for their differing uncertainties, enabling the robust retrieval of spatially complete multidimensional glacier velocity fields from a single TRI system.

To address the challenge of multidimensional velocity retrieval from monostatic TRI observations, this study proposes a
105 Nonlinear-Constrained Geographically Weighted Regression (NL-GWR) framework that, for the first time, mathematically incorporates glaciophysical priors into the regression approach to retrieve multidimensional glacier velocity fields from monostatic TRI observations. Under the assumptions of a horizontal glacier surface and short-term velocity-direction invariance, the framework achieves performance metrics comparable to conventional monostatic methods, demonstrating its ability to mitigate errors introduced through the incorporation of external data. Having established that these external
110 constraints do not degrade retrieval quality, we further obtain high-frequency observations of velocity and strain-rate variations over a two-week period at Helheim Glacier after analysing the effect of vertical tidal motion. This reveals distinct semidiurnal and diurnal signals, which are dynamically attributed to tidal forcing and diurnal meltwater processes, respectively. By overcoming the dimensionality limitations inherent to monostatic TRI observations, NL-GWR provides a practical approach for expanding TRI applications under resource-constrained conditions.

115 **2 Data and Data preprocessing**

2.1 TRI configuration and deployment

A GAMMA Portable Radar Interferometer (GPRI) was used in this study and hereafter referred to as TRI. The instrument is a Ku-band real-aperture radar system operating at a wavelength of 1.74 cm, capable of performing interferometric measurements and acquiring high-resolution intensity images (Strozzi et al., 2012; Voytenko et al., 2012; Werner et al., 2008,
120 2009). It provides sufficient sensitivity to resolve surface deformation at the millimetre scale. Each TRI system consists of three antennas, including one transmitter and two receivers. The three antennas are horizontally aligned and parallel to each other, with the two receiving antennas separated by a 25 cm vertical baseline. The antenna assembly is mounted on a rotatable frame, allowing 360° azimuthal scanning.

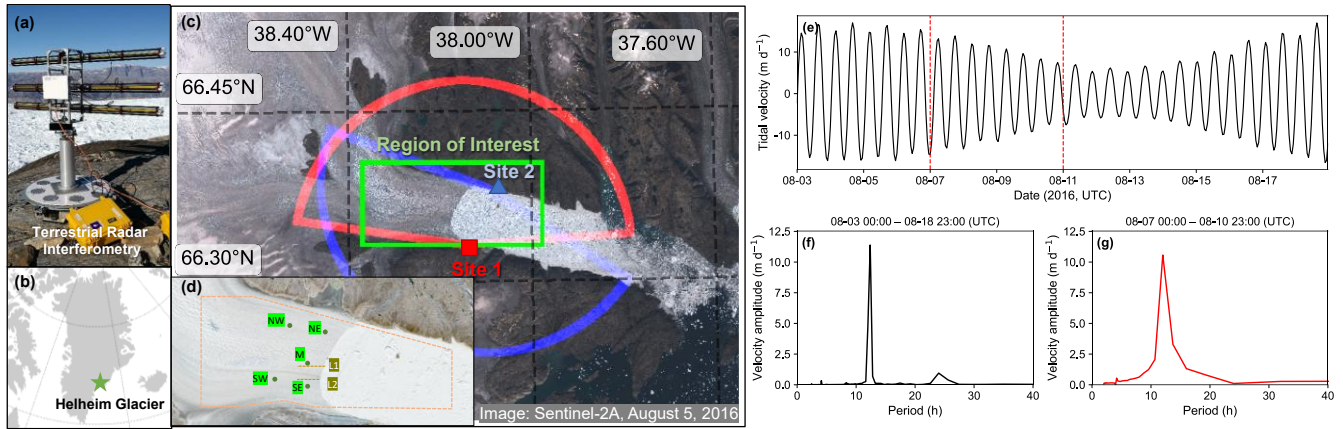


Figure 1 In-situ TRI deployment configuration and local environment, with the TRI instrument (a) positioned at two sites (Site1: red rectangular; Site2: blue triangle; c) on Helheim Glacier, East Greenland (b). The red, blue and green profiles denote the scan range of site1, site2, and the NL-GWR data processing area. Green points and yellow-green dashed lines in the enlarged Region of Interest panel in (d) denote sampling points and transects used in the Results section, while orange dashed lines outline the manually delineated glacier boundary. Panel (e) shows the forecasted tidal velocity from Tasilaq, ~100 km away from Helheim Glacier, while (f) and (g) denote the Fourier Amplitude Spectrum of such tidal velocity within the whole research time (f) and only the period that bistatic TRI configuration is available (g).

During the observation period, the TRI systems were installed on stable bedrock to ensure that no spatial displacement of the instruments occurred throughout the measurements. Therefore, no topographic phase correction was required during interferometric processing (Voytenko et al., 2017). The instrument is capable of 360° rotation. In this study, it was configured with an 8 ms frequency-modulated continuous-wave (FMCW) chirp to scan a range of approximately 16 km over a 160° azimuthal sector. The resulting TRI images have a nominal range resolution of approximately 0.75 m, which remains independent of range (Wang et al., 2022; Werner et al., 2008, 2009). In contrast, the azimuth resolution increases linearly with distance due to beam divergence, reaching approximately 7.5 m at a range of 1 km in our observation geometry (Strozzi et al., 2012; Voytenko et al., 2017).

Two TRI systems were deployed on opposite sides of the fjord downstream of the calving front of Helheim Glacier, eastern Greenland (Figure 1). According to the small GPS module mounted on the radar, the elevations of the two sites were 449.2 m and 299.8 m, respectively. These elevated locations provided unobstructed viewing geometries, reducing potential blockage by the glacier terminus and drifting icebergs (Voytenko et al., 2015a), whose elevations are typically below 300 m, while also maintaining accessibility. The two TRI systems were positioned near the calving front, with their 0° azimuth orientations directed towards the opposite side of the glacier. This deployment configuration maximised the radar coverage of the glacier surface, particularly the calving front, its upstream and downstream regions, providing optimal sensitivity for investigating processes associated with glacier terminus instability.

The two TRI stations operated during August 3, 00:00–August 10, 23:58 and August 7 00:00–August 18 23:58, 2016 (UTC), respectively, with a temporal sampling interval of 2 min. The overlapping observation period occurred between August 7



and August 10, 2016. In this study, 30 images acquired between August 7 18:00 and 18:58 UTC were selected for quality assessment, while the remaining observations were used for glacier surface velocity analysis.

2.2 TRI data preprocessing

2.2.1 Interferometric processing

155 Interferometric processing was applied to retrieve glacier surface velocity from the phase information of the electromagnetic signals acquired by the TRI systems. The GAMMA software package was used to generate interferograms from the single-look complex (SLC) images directly obtained from the TRI observations. First, the TRI data acquired at 2 min intervals were multilooked by averaging 12 pixels in the range direction to reduce speckle noise (Voytenko et al., 2015b, 2017). Subsequently, interferograms were generated from image pairs acquired by the same receiving antenna but at different acquisition times.

160 The generated interferograms were then adaptively filtered using a spatial window of 32 pixels to reduce phase noise (Goldstein and Werner, 1998). A coherence mask was applied to exclude areas with coherence coefficients below 0.4, thereby improving the reliability of the retrieved velocity measurements. Finally, phase unwrapping was performed using the minimum cost flow algorithm to obtain the continuous phase difference between consecutive acquisitions, denoted as $\Delta\varphi$.
165 The unwrapped phase $\Delta\varphi$ converts to LOS velocity v_r^{in} via (Voytenko et al., 2015a, 2017):

$$v_r^{\text{in}} = \frac{\Delta x_r^{\text{in}}}{\Delta t} = \frac{1}{\Delta t} \frac{\lambda_{\text{radar}}}{4\pi} \Delta\varphi, \quad (1)$$

where $\Delta t = 2$ min and $\lambda_{\text{radar}} = 1.74$ cm. LOS direction points from radar to measure points, with positive values indicating motion away from radar. Results were transformed from polar to Cartesian coordinates (10 m spatial resolution) using radar orientation angles of 2.5° and 153° , with iterative rotation refinement until bedrock features visually matched Sentinel-2A/B imagery (Voytenko et al., 2017). To reduce atmospheric noise (Gloor, 2023; Voytenko et al., 2012), interferometric pairs acquired within each one-hour period were averaged.

Two unwrapping anomalies were identified and addressed after coordinate transformation: (1) global 2π multiples detected and corrected using stable terrain velocity offsets, and (2) regional phase jumps from partial coherence loss. For the latter, we implemented an empirical correction: 30-pixel (300 m) grid block division was analyzed for anomaly data by hourly average; blocks exceeding 2π multiples were identified and corrected pixelwise, accommodating multiple local jump orders while minimizing misclassification from decorrelation or atmospheric noise (See details for both methods in Appendix A1).

2.2.2 Feature tracking-based velocity extraction

Feature tracking was applied to TRI-derived amplitude images to provide additional velocity component estimates. Image pairs with a temporal baseline of 3 days were processed using our Fourier-based subpixel normalized cross-correlation (NCC) algorithm (Zhang et al., 2026) at both TRI sites. This processing yielded additional LOS velocity (v_r^{ft}) and azimuth velocity



(v_a^{fl}) (Burton et al., 2018), where (v_a^{fl}) represents the velocity component perpendicular to the LOS direction within the horizontal plane and is defined as positive in the radar rotation direction.

Except for the first and last two days of each observation period, the amplitude-derived velocity for each day was retrieved from image pairs spanning from 00:00 of the previous day to 23:58 of the following day, corresponding to a nominal 3-day temporal baseline. For the first two days of each observation period, the image pairs were constructed using the earliest available acquisition and the acquisition 3 days later. Similarly, for the last two days, the latest acquisition and the acquisition 3 days earlier were paired. An exception was applied to the image pairs used for quality assessment. To ensure consistency between the two TRI sites, both stations used image pairs acquired between 7 August 18:00 and 10 August 19:00, although this configuration may introduce a small temporal offset.

2.3 Data for physical constraint

We selected several widely used external datasets as sources for specific physical constraints (Table 1). Sentinel-2A optical satellite observations were used to provide long-term velocity information for prescribing flow-direction constraints, while ArcticDEM was used to incorporate topographic information. The characteristics and applications of these datasets are described below:

Table 1 External data sources and their applications.

Source of Dataset	Date of Acquisition	Usage
Sentinel-2A	Aug 02, 2016	Feature tracking for long-term velocity
Sentinel-2A	Aug 19, 2016	Feature tracking for long-term velocity
ArcticDEM Stripe	Aug 02, 2016	Topography
ArcticDEM Mosaic	/	Fill gaps in ArcticDEM Stripe

2.3.1 Sentinel-2A/B

Sentinel-2A/B are multispectral imaging satellites developed and operated by the European Space Agency (ESA) under the Copernicus Programme, launched in 2015 and 2017, respectively. Each satellite carries a multispectral imager designed to acquire optical surface observations and monitor global surface changes (Drusch et al., 2012; Fernandez et al., 2013; Phiri et al., 2020). In this study, two Sentinel-2A Band 4 images acquired on 2 and 19 August 2016 were used to generate glacier surface velocity fields with a spatial resolution of 10 m following the methodology described in Zhang et al. (2026). The 17-day temporal baseline provides sufficient temporal averaging to reduce uncertainties in glacier velocity and the derived strain rates in the downstream region, thereby maintaining reliable physical continuity of the velocity field (Zhang et al., 2026). The derived velocity field was used as a prior reference for glacier flow direction.



205 2.3.2 ArcticDEM

ArcticDEM is a high-resolution digital elevation model dataset developed by the Polar Geospatial Center (PGC) and generated from high-resolution satellite stereo imagery using photogrammetric techniques (Porter and others, 2022, 2023). It provides detailed elevation information across the Arctic region for investigating surface topography and geophysical changes in polar environments (Wang et al., 2022). In this study, ArcticDEM was used to construct the surface elevation
 210 model of the study area and derive surface gradients and radar look angles. A DEM Strip acquired on 2 August 2016 was primarily used, while uncovered regions, mainly the mountain slopes on both sides downstream of the calving front, were supplemented using mosaic DEM products to ensure complete spatial coverage. The ArcticDEM data were further corrected to the radar reference datum using the elevation information provided by the radar record (Wang et al., 2022).

3 NL-GWR framework

215 3.1 Mathematical description of the computational framework

In this section, we explicitly formulate the observation system and prior constraints to establish the mathematical basis for the solution algorithm. The primary observation is the TRI interferometric LOS velocity, v_r^{in} . In a geographic coordinate system, its relationship with the glacier velocity components is expressed as:

$$v_{r,i}^{\text{in}} = v_x \cos \xi_i \sin \theta_i + v_y \sin \xi_i \sin \theta_i + v_z \cos \theta_i. \quad (2)$$

220 Here, v_x , v_y , and v_z denote the true eastward, northward, and vertically upward velocity components, respectively, which we will reconstruct in this study. The radar azimuth angle ξ_i is measured between the horizontal projection of the LOS direction and the eastward axis. The radar look angle θ_i is measured between the LOS direction and the vertically upward axis in the vertical plane containing the radar and the observation point. The LOS direction points from the radar towards the observation point. The subscript ($i = 1, 2$) identifies the two TRI stations (Site 1 and Site 2), each of which independently
 225 provides an interferometric LOS velocity observation.

In the approach proposed by Kim et al. (2024), offset tracking applied to pairs of TRI amplitude images at each station provides two additional velocity observations: the horizontal approximated LOS component v_r^{ft} , and the azimuth component v_a^{ft} . Their relationships with the horizontal glacier velocity components are:

$$\begin{cases} v_{r,i}^{\text{ft}} = v_x \cos \xi_i + v_y \sin \xi_i \\ v_{a,i}^{\text{ft}} = -v_x \sin \xi_i + v_y \cos \xi_i \end{cases} \quad (3)$$

230 Prior assumptions are frequently introduced, either explicitly or implicitly, in TRI-based glacier velocity studies to reconstruct multidimensional velocity fields or other scalar fields, such as speed, from a single interferometric velocity component. One common assumption is direction invariance, which states that glacier flow direction remains temporally fixed while its magnitude varies. This assumption appears in different forms, including fixed horizontal projection angles (Voytenko et al., 2015a, b) and topography-derived flow directions (Sanz et al., 2025). An implicit application is to align the



235 radar antenna with the glacier flow direction and interpret LOS velocity as glacier speed (Dachauer et al., 2026; Voytenko et al., 2012). Mathematically, this assumption can be expressed as:

$$\arctan \frac{v_y}{v_x} = \arctan \frac{v_y^{\text{ex}}}{v_x^{\text{ex}}}. \quad (4)$$

Here, v_x^{ex} and v_y^{ex} are independently obtained external velocity components. Under this assumption, the external data could represent mean glacier motion over a relatively long temporal baseline. In this study, we use a satellite-derived mean
 240 velocity field obtained from Sentinel-2A imagery using normalized cross-correlation (Sect. 2.3.1), with a temporal baseline exceeding 10 days.

If this equality constraint is adopted, we can transfer it into a functional condition, which can be explicitly written as:

$$g(v_x, v_y, d_x^{\text{ex}}, d_y^{\text{ex}}) = \frac{v_y}{v_x} - \frac{d_y^{\text{ex}}}{d_x^{\text{ex}}} = 0, \quad (5)$$

where $v_k^{\text{ex}} = v_k + d_k^{\text{ex}}$, ($k = x, y$). d_x^{ex} and d_y^{ex} denote the components of the difference between the external and target
 245 velocity vectors. In our numerical implementation, we multiply the constraint by $v_x d_x^{\text{ex}}$ and use the resulting division-free form:

$$g(v_x, v_y, d_x^{\text{ex}}, d_y^{\text{ex}}) = v_y d_x^{\text{ex}} - v_x d_y^{\text{ex}} = g_{\text{dire}}. \quad (6)$$

This formulation avoids division by zero and simplifies the gradient expression used in Eq. (15) in the following section.

Another kind of constraint with different physical meaning is based on surface topography, specifically the assumption that
 250 glacier flow is parallel to the ice surface (Sanz et al., 2025):

$$\mathbf{n}^T \cdot \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} -\frac{\partial H}{\partial x} & -\frac{\partial H}{\partial y} & 1 \end{pmatrix} \cdot \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = 0, \quad (7)$$

where $\mathbf{n}^T = \begin{pmatrix} -\frac{\partial H}{\partial x} & -\frac{\partial H}{\partial y} & 1 \end{pmatrix}$ is the unnormalized surface normal vector at the observation point, and H denotes surface elevation. For a horizontal glacier surface, both surface gradients vanish, and the constraint reduces to:

$$v_z = 0. \quad (8)$$

255 In our study area, ArcticDEM indicates that the surface gradients generally do not exceed 0.1 after suppressing the influence of surface crevasses using a 200-meter filter over our region of interest (Figure S1). Under the surface-parallel flow assumption, the corresponding vertical velocity is therefore generally less than 10% of the horizontal speed. We consequently adopt Eq. (8) as the default approximation in this study. This approximation may not hold in areas such as the mélange (Xie et al., 2019), floating portions of the glacier terminus (Friedl et al., 2020), and the grounding zone (Kim et al.,
 260 2024, 2025), where the surface topography may deform significantly from initial configuration. However, from the technical angle, the number of available equations is close to the number of unknowns, particularly under a monostatic configuration. Constraining one additional degree of freedom provides greater redundancy for estimating the remaining state variables. We discuss the effects of relaxing this approximation on the retrieved glacier velocities and clarify our treatment of the resulting



uncertainties in the following sections, with particular attention to their limited influence on the methodological assessment and the conclusions of the glacier dynamics analysis.

In summary, the complete observation and constraint structure can be assembled as:

$$\mathbf{y} = \begin{bmatrix} v_{r,1}^{\text{in}} \\ v_{r,1}^{\text{ft}} \\ v_{a,1}^{\text{ft}} \\ v_{r,2}^{\text{in}} \\ v_{r,2}^{\text{ft}} \\ v_{a,2}^{\text{ft}} \\ v_x^{\text{ex}} \\ v_y^{\text{ex}} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \cos \xi_1 \sin \theta_1 & \sin \xi_1 \sin \theta_1 & \cos \theta_1 & 0 & 0 \\ \cos \xi_1 & \sin \xi_1 & 0 & 0 & 0 \\ -\sin \xi_1 & \cos \xi_1 & 0 & 0 & 0 \\ \cos \xi_2 \sin \theta_2 & \sin \xi_2 \sin \theta_2 & \cos \theta_2 & 0 & 0 \\ \cos \xi_2 & \sin \xi_2 & 0 & 0 & 0 \\ -\sin \xi_2 & \cos \xi_2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ d_x^{\text{ex}} \\ d_y^{\text{ex}} \end{bmatrix}, \quad \mathbf{g} = [g_{\text{dire}}]. \quad (9)$$

Here, the $\mathbf{y}$ contains the TRI interferometric and amplitude-tracking velocity observations and we name it the observation vector. The external velocity components are also included because they are derived from independent observations with quantifiable measurement uncertainties. The $\mathbf{x}$, called state vector, contains the unknown quantities to be estimated. Its coordinate system components are the eastward, northward, and vertically upward glacier velocities. To incorporate the direction invariance assumption, the differences between the external and target horizontal velocities are introduced as auxiliary state variables and estimated jointly with the velocity components. A design matrix $\mathbf{A}$ describes the linear relationships between the observation vector and the unknown state variables. Relationships among the state variables that cannot be represented by this linear structure, i.e., Eq. (6), are incorporated into the nonlinear constraint function $\mathbf{g} = \mathbf{g}(\mathbf{x})$. This assembly does not contain any topography approximation.

Incorporating the topographic constraint introduced in Eq. (8) and ensuring compatibility of look angle term among the equations yields the following reduced system:

$$\mathbf{y} = \begin{bmatrix} v_{r,1}^{\text{in}} \\ v_{r,1}^{\text{ft}} \\ v_{a,1}^{\text{ft}} \\ v_{r,2}^{\text{in}} \\ v_{r,2}^{\text{ft}} \\ v_{a,2}^{\text{ft}} \\ v_x^{\text{ex}} \\ v_y^{\text{ex}} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \cos \xi_1 \sin \theta_1 & \sin \xi_1 \sin \theta_1 & 0 & 0 \\ \cos \xi_1 & \sin \xi_1 & 0 & 0 \\ -\sin \xi_1 & \cos \xi_1 & 0 & 0 \\ \cos \xi_2 \sin \theta_2 & \sin \xi_2 \sin \theta_2 & 0 & 0 \\ \cos \xi_2 & \sin \xi_2 & 0 & 0 \\ -\sin \xi_2 & \cos \xi_2 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} v_x \\ v_y \\ d_x^{\text{ex}} \\ d_y^{\text{ex}} \end{bmatrix}, \quad \mathbf{g} = [g_{\text{dire}}]. \quad (10)$$

The above two formulation systems describe the complete solution structure when observations from both TRI stations are available. When only a monostatic configuration is available, the system further reduces to:



$$\mathbf{y} = \begin{bmatrix} v_{r,i}^{\text{in}} \\ v_{r,i}^{\text{ft}} \\ v_{a,i}^{\text{ft}} \\ v_x^{\text{ex}} \\ v_y^{\text{ex}} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \cos \xi_i \sin \theta_i & \sin \xi_i \sin \theta_i & 0 & 0 \\ \cos \xi_i & \sin \xi_i & 0 & 0 \\ -\sin \xi_i & \cos \xi_i & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} v_x \\ v_y \\ d_x^{\text{ex}} \\ d_y^{\text{ex}} \end{bmatrix}, \quad \mathbf{g} = [g_{\text{dire}}]. \quad (11)$$

All the above observation and prior-constraint configurations can be represented within a unified mathematical formulation:

$$\mathbf{y} = \mathbf{Ax} + \boldsymbol{\epsilon}, \quad \mathbf{g}(\mathbf{x}) = \mathbf{0}. \quad (12)$$

285 Here, $\boldsymbol{\epsilon}$ denotes the vector of random errors associated with the components of $\mathbf{y}$. Because each component of the observation vector is directly derived from measurements, its corresponding error term has a clear physical interpretation. In the following section, we use the general formulation in Eq. (12) to represent the specific observation and constraint systems, i.e., Eqs. (9-11), and describe the solution algorithm, demonstrating its applicability to different system configurations.

3.2 Nonlinear-constrained solution algorithm

290 Several established numerical methods are available for solving constrained nonlinear systems. Here, we adopt a penalty formulation and solve the resulting optimization problem using Gauss–Newton iteration for Eq. (12). We first reformulate the constrained regression problem as an optimization problem, estimating the state vector $\mathbf{x}$ from the observation vector $\mathbf{y}$ by minimizing the following weighted least-squares objective:

$$\mathbf{x} = \arg \min_{\mathbf{x}} \mathcal{J}(\mathbf{x}) = \arg \min_{\mathbf{x}} ((\mathbf{y} - \mathbf{Ax})^T \mathbf{W} (\mathbf{y} - \mathbf{Ax}) + \lambda \|\mathbf{g}(\mathbf{x})\|^2), \quad (13)$$

295 where $\mathcal{J}(\mathbf{x}) = (\mathbf{y} - \mathbf{Ax})^T \mathbf{W} (\mathbf{y} - \mathbf{Ax}) + \lambda \|\mathbf{g}(\mathbf{x})\|^2$ is defined as the loss function of which the value the algorithm needs to minimize. To account for heterogeneous observation quality (Hu et al., 2014), a weight matrix $\mathbf{W}$ is introduced, including two parts: (1) prior confidence $\mathbf{W}_\sigma = 1/\text{Cov}(\mathbf{y})$, of which the covariance matrix can be inferred from observational data uncertainty analysis or adaptation methods (Hu et al., 2014), and (2) geographical structure introduced in Sect. 3.3. The nonlinear constraint $\mathbf{g}(\mathbf{x}) = \mathbf{0}$ is enforced through either penalty or Lagrange multiplier method, with the former adopted in
 300 this study, introducing a large penalty factor λ to the solution function. To preserve scale balance in the linear terms, the penalty method requires λ to be orders of magnitude larger than the linear term values (Sect. 4.1.2). The solution tends to satisfy $\mathbf{g}(\mathbf{x}) \approx \mathbf{0}$ but lacks strict enforcement. We also implemented a Lagrange multiplier approach (Text S1), which is another strategy designed for nonlinear optimization problem, but encountered convergence difficulties in this study.

Although introducing λ adds a parameter without a direct physical interpretation to the solving system, its use is supported
 305 by both theoretical and practical considerations for the problem addressed here. In TRI velocity retrieval, interferometric observations are commonly combined with prior constraints (Sanz et al., 2025; Voytenko et al., 2015a) rather than amplitude-derived velocity observations. However, conventional approaches based on these constraints can suffer from numerical instability (Sanz et al., 2025). Our objective is therefore to give the prior constraints as much weight as possible while retaining sufficient observational information to avoid numerically unstable solutions. Mathematically, we need to



310 seek a state vector $\mathbf{x}$ that makes $\|\mathbf{g}(\mathbf{x})\|^2$ as small as possible while keeping the observation residuals within acceptable bounds. The norm of $\mathbf{x}$ should also remain bounded at a physically reasonable magnitude. This objective motivates the use of a sufficiently large λ , rather than selecting it through an L-curve analysis to balance the observation misfit and the constraint penalty (De Rydt et al., 2019). In Sect. 4.1.2, we demonstrate that increasing λ yields the intended solution behaviour, and we provide the theoretical justification in Sect. 5.1.

315 We then employ the Gauss-Newton algorithm to solve Eq. (13) by transforming the optimization problem into finding stationary points. When the vector norm is defined as the 2-norm, we define a the following norm to simplify the expression:

$$r_\lambda(\mathbf{x}) = \begin{bmatrix} \sqrt{\mathbf{W}}(\mathbf{y} - \mathbf{A}\mathbf{x}) \\ \sqrt{\lambda}\mathbf{g}(\mathbf{x}) \end{bmatrix}, \quad \mathbf{x} = \arg \min_{\mathbf{x}} \|r_\lambda(\mathbf{x})\|^2. \quad (14)$$

Below, we employ the Gauss-Newton iterative method to solve this problem. This method essentially uses a first-order approximation of the nonlinear component, thereby transforming it into a linear problem. According to Taylor expansion:

$$320 \quad r_\lambda(\mathbf{x} + \Delta\mathbf{x}) \approx r_\lambda(\mathbf{x}) + \mathbf{J}_\lambda(\mathbf{x})\Delta\mathbf{x}, \quad \mathbf{J}_\lambda(\mathbf{x}) = \begin{bmatrix} -\sqrt{\mathbf{W}}\mathbf{A} \\ \sqrt{\lambda}\nabla\mathbf{g}(\mathbf{x})^T \end{bmatrix}, \quad (15)$$

where $\mathbf{J}_\lambda$ is the Jacobian matrix of the multivariate function r_λ , and $\nabla\mathbf{g}$ is the gradient vector of function $\mathbf{g}$. Thus, we have:

$$\|r_\lambda(\mathbf{x} + \Delta\mathbf{x})\|^2 \approx \|r_\lambda(\mathbf{x}) + \mathbf{J}_\lambda(\mathbf{x})\Delta\mathbf{x}\|^2 = r_\lambda(\mathbf{x})^T r_\lambda(\mathbf{x}) + 2r_\lambda(\mathbf{x})^T \mathbf{J}_\lambda(\mathbf{x})\Delta\mathbf{x} + \Delta\mathbf{x}^T \mathbf{J}_\lambda(\mathbf{x})^T \mathbf{J}_\lambda(\mathbf{x})\Delta\mathbf{x}. \quad (16)$$

To identify stationary points as candidates for minima of our loss function, we take the gradient of the above expression with respect to $\Delta\mathbf{x}$ and set:

$$325 \quad 0 = \nabla \|r_\lambda(\mathbf{x} + \Delta\mathbf{x})\|^2 \approx \nabla \|r_\lambda(\mathbf{x}) + \mathbf{J}_\lambda(\mathbf{x})\Delta\mathbf{x}\|^2 = 2\mathbf{J}_\lambda(\mathbf{x})^T r_\lambda(\mathbf{x}) + 2\mathbf{J}_\lambda(\mathbf{x})^T \mathbf{J}_\lambda(\mathbf{x})\Delta\mathbf{x}. \quad (17)$$

Here, we obtain the Gauss-Newton normal equation to calculate $\Delta\mathbf{x}$. In practice, a Levenberg-Marquardt damping factor μ_k can be introduced. Numerically, this reduces the likelihood of numerical instability by limiting excessively large updates. The complete iterative formula to calculate $\Delta\mathbf{x}$ in each step is:

$$(\mathbf{A}^T \mathbf{W} \mathbf{A} + \lambda \nabla\mathbf{g}(\mathbf{x}) \nabla\mathbf{g}(\mathbf{x})^T + \mu_k \mathbf{I}) \Delta\mathbf{x} = \mathbf{A}^T \mathbf{W}(\mathbf{y} - \mathbf{A}\mathbf{x}) - \lambda \mathbf{g}(\mathbf{x}) \nabla\mathbf{g}(\mathbf{x}), \quad (18)$$

330 where $\mathbf{I}$ is the identity matrix. This is a linear system that can be solved directly using matrix inversion algorithms. By setting an appropriate initial condition $\mathbf{x} = \mathbf{x}_0$ (we use regression solution ignoring nonlinear functions $\mathbf{g}(\mathbf{x})$ as this initial condition), we compute $\Delta\mathbf{x}$ from the above equation and update the solution via $\mathbf{x}_{k+1} = \mathbf{x}_k + \gamma\Delta\mathbf{x}$, where γ is relaxation factor used to avoid numerical instability. Convergence is achieved when the norm of $\Delta\mathbf{x}$ falls below a specified threshold τ_p , and the final $\mathbf{x}_{k+1}$ is output. We will also set a maximum iteration times M ; solutions that do not meet the convergence
 335 criterion within this limit are flagged as non-converged.

3.3 Geographically Weighted Processing

GWR leverages spatial autocorrelation by weighting neighboring observations according to geographic proximity (Brunsdon et al., 1998). For a measurement point at a local location $\mathbf{s}_j = (s^x, s^y)$, the geographical weight scalar for its neighbor $\mathbf{s}_i$ is:

$$w_j(\mathbf{s}_i) = K\left(\frac{d_{ji}}{h}\right), \quad d_{ji} = \|\mathbf{s}_j - \mathbf{s}_i\|, \quad (19)$$



with bandwidth h controlling spatial influence range and kernel function K defining distance decay. This study employs the Gaussian kernel:

$$K\left(\frac{d_{ji}}{h}\right) = \exp\left(-\frac{1}{2}\left(\frac{d_{ji}}{h}\right)^2\right). \quad (20)$$

Observations from neighbouring locations, which were previously used in separate pointwise inversions, are now jointly used to estimate the state vector at a central location. The pointwise optimization problem in Eq. (13) must therefore be extended to incorporate geographical weighting. With a diagonal prior observation weight matrix $\mathbf{W}_\sigma$, the linear term of loss function, $\mathcal{J}_L$, in Eq. (13) can be decomposed into contributions from observation components:

$$\mathcal{J}_L(\mathbf{x}) = (\mathbf{y} - \mathbf{A}\mathbf{x})^T \mathbf{W}_\sigma (\mathbf{y} - \mathbf{A}\mathbf{x}) = \sum_{n=1}^{\text{length}(\mathbf{y})} (y_n - \mathbf{A}_n \mathbf{x}) * w_{\sigma,n} * (y_n - \mathbf{A}_n \mathbf{x}) := \sum_{n=1}^{\text{length}(\mathbf{y})} \mathcal{J}_{L,n}(\mathbf{x}), \quad (21)$$

where n is the index of observation components at each location, y_n is the n -th component scalar of $\mathbf{y}$, $\mathbf{A}_n$ is the corresponding row of the design matrix, and $w_{\sigma,n}$ is its prior observation weight scalar. Geographical weighting is then introduced by summing each observation component over the neighbourhood of the target location $\mathbf{s}_j$:

$$\mathcal{J}_{L,n}(\mathbf{x}(\mathbf{s}_j)) = \sum_{i=1}^{N_i} [y_n(\mathbf{s}_i) - \mathbf{A}_n(\mathbf{s}_i) \mathbf{x}(\mathbf{s}_j)] * w_j(\mathbf{s}_i) * w_{\sigma,n}(\mathbf{s}_i) * [y_n(\mathbf{s}_i) - \mathbf{A}_n(\mathbf{s}_i) \mathbf{x}(\mathbf{s}_j)], \quad (22)$$

where N_i is the number of neighbouring locations included in the local regression. All neighbouring observations are thus fitted using the same local state vector $\mathbf{x} = \mathbf{x}(\mathbf{s}_j)$. This local optimization problem can be expressed in a matrix symbolic form by stacking the observations and their corresponding design matrices:

$$\mathcal{J}(\mathbf{x}) = (\mathbf{y}' - \mathbf{A}'\mathbf{x})^T \mathbf{W} (\mathbf{y}' - \mathbf{A}'\mathbf{x}) + \lambda \|\mathbf{g}(\mathbf{x})\|^2, \mathbf{y}' = \begin{bmatrix} \mathbf{y}(\mathbf{s}_1) \\ \vdots \\ \mathbf{y}(\mathbf{s}_{N_i}) \end{bmatrix} \in \mathbb{R}^{N_i \text{length}(\mathbf{y}) \times 1}, \mathbf{A}' = \begin{bmatrix} \mathbf{A}(\mathbf{s}_1) \\ \vdots \\ \mathbf{A}(\mathbf{s}_{N_i}) \end{bmatrix} \in \mathbb{R}^{N_i \text{length}(\mathbf{y}) \times \text{length}(\mathbf{x})}. \quad (23)$$

When the same prior observation weight matrix is used at all neighbouring locations, the combined weight matrix is calculated via the Kronecker product:

$$\mathbf{W} = \text{diag}(w_j(\mathbf{s}_1), \dots, w_j(\mathbf{s}_{N_i})) \otimes \mathbf{W}_\sigma \in \mathbb{R}^{N_i \text{length}(\mathbf{y}) \times N_i \text{length}(\mathbf{y})}. \quad (24)$$

This formulation retains the structure of the original optimization problem. Its linear observation term is equivalent to the geographically weighted sum of the observation residual objectives over the N_i neighbouring locations, evaluated using the common target state vector. Therefore, it does not affect our quantitative analysis of the loss function $\mathcal{J}(\mathbf{x})$ in other places.

3.4 Geographical Neighbour and Calculation Point Selection

Standard GWR's computational burden scales with data size (Eq. (23)). Given our high-resolution gridded data ($\sim 10^6$ points), we implement two efficiency measures

Spatial Pruning: Reference points $\mathbf{s}_i$ are included only within a discrimination radius r of the target point $\mathbf{s}_j$:

$$\|\mathbf{s}_j - \mathbf{s}_i\| = \sqrt{(s_j^x - s_i^x)^2 + (s_j^y - s_i^y)^2} \leq r. \quad (25)$$

Points beyond this circle, where spatial correlation decays, are excluded.



Sparse Regression Grid: Regression is performed only at every (p_x, p_y) -th pixel (stride), with results bilinearly
 370 interpolated to the original resolution. This trades modest spatial resolution loss for substantial efficiency gains.

It is noted that setting $r = 0$ and $(p_x, p_y) = (1, 1)$ reduces NL-GWR to nonlinear-constrained weighted regression (NL-WR)
 without geographic weighting.

Section 4.1.2 presents sensitivity analysis of geographically weighted hyperparameters $(h, p_{x/y} := p, r)$, where the three key
 parameters are set numerically equal ($h = r = p$) as the standard configuration when comparing their individual impacts.

375 Detailed analysis of this parameterization strategy is provided in Appendix A2.

3.5 Treatment of tidal effects

The topographic assumption described by Eq. (8) adopted in this study may neglect short-term vertical velocity component
 in floating regions near the glacier terminus, which potentially biases our physical analysis based on velocity extraction. In
 these regions, tidal changes in water level induce vertical ice-surface motion through adjustments in hydrostatic equilibrium,
 380 and this phenomenon has already been a general assumption in grounding zone detection researches (Friedl et al., 2020; Kim
 et al., 2024, 2025). Identifying the affected regions independently during our observation period is challenging because the
 grounding line can migrate substantially over a tidal cycle (Kim et al., 2024, 2025; Rignot et al., 2024; Sayag and Worster,
 2013; Zhong et al., 2023). Furthermore, whether the observed ice motion is predominantly horizontal or vertical is not
 immediately evident (Friedl et al., 2020). Previous studies suggest that the summer grounding line of Helheim Glacier may
 385 lie close to the calving front (Melton et al., 2022), which is supported by the expansion of a polynya near the glacier
 terminus observed during our TRI observations. We therefore adopt a velocity-decomposition approach (Zhong et al., 2023)
 to obtain a preliminary estimate of limited area of regions dominated by tide-induced vertical motion, commonly interpreted
 as floating regions, and assess the relative contributions of horizontal and vertical motion.

Under an elastic approximation (Sayag and Worster, 2013), ice-surface elevation responds approximately synchronously to
 390 tidal elevation changes over tidal timescales. We consequently assume that tidal effects are concentrated in velocity
 components at the characteristic tidal frequencies and decompose the velocity vector $\vec{v} = (v_x, v_y, v_z)$ as tide-related part
 $\vec{v}_{Tide}$ and the other part $\vec{v}_{Other}$:

$$\vec{v} = \vec{v}_{Tide} + \vec{v}_{Other}. \quad (26)$$

Tidal forecasted data from Tasiilaq, ~100 km away from Helheim Glacier, available from the Danish Meteorological
 395 Institute (Danish Meteorological Institute), yield the tidal amplitude spectrum shown in Figure 1e-g. Based on this spectrum,
 we approximate the tide-related velocity components, horizontal $v_{Tide,H} = \|(v_{Tide,x}, v_{Tide,y})\|$ and vertical $v_{Tide,V} = v_{Tide,z}$,
 using a single frequency corresponding to a period of approximately $T = 12.38$ hours:

$$\begin{cases} v_{Tide,H} = \alpha_H \sin(\frac{2\pi}{T}t + \varphi_H) \\ v_{Tide,V} = \alpha_V \sin(\frac{2\pi}{T}t + \varphi_V) \end{cases}. \quad (27)$$



Here, t is expressed as the observation time, while α and φ denote the amplitude and phase, respectively, obtained by
 400 applying a Fourier transform to the velocity or velocity-anomaly time series. The orthogonality of the Fourier basis allows
 the T-period component to be separated from components at other resolved frequencies. Under this single-frequency
 approximation, analyses of the other frequency components are therefore expected to be unaffected by tide-induced vertical
 motion.

Within this framework, the influence of tidal motion on velocity retrieval under the horizontal-flow assumption in Eq. (8) is
 405 expected to be concentrated in the 12.38 hours Fourier component. Unresolved vertical motion can introduce artefacts into
 the reconstructed horizontal velocity (Friedl et al., 2020). Here, we assume a phase difference of π between the two periodic
 velocity components, such that we can subtract the maximum amplitude of artifacts that might be contributed by vertical
 movement. We therefore estimate the minimum amplitude of the actual horizontal velocity variation $\alpha_{H,real}$ under bistatic
 configuration as:

$$410 \quad \min \alpha_{H,real} = \max\{\alpha_H - \alpha_V |\cot \theta^*|, 0\}. \quad (28)$$

Here, θ^* is the mean of the radar look angles at a given observation point for Sites 1 and 2. This average approximates the
 combined projection effects in the vertical plane, consistent with the equal weights generally assigned to the velocity
 observations from the two stations in the bistatic implementation. Because the monostatic observation and constraint system
 is close to exactly determined, its retrieved velocities are more susceptible to error-induced artefacts. Then, vertical motion is
 415 estimated using the bistatic configuration by relaxing the constraint on v_z and adopting the complete formulation in Eq. (9).
 We then extract the 12.38-hour component of vertical velocity through Fourier analysis to estimate the tide-related vertical
 motion within an acceptable uncertainty range.

Although the bistatic configuration used for the tidal analysis covers only part of the observation period, the tidal forcing
 exhibits a spectral energy distribution closely resembling that of the full period (Figure 1f-g). This interval can therefore be
 420 considered representative of the tidal forcing over the full observation period.

For the physical interpretation of the 12.38 h horizontal velocity component, we exclude regions where the amplitude of the
 corresponding vertical velocity component exceeds 2 m d^{-1} . This threshold identifies substantial vertical motion relative to
 the approximate TRI interferometric measurement uncertainty of 0.2 m d^{-1} , suggesting potentially floating ice where the
 adopted topographic constraint may be invalid.

425 3.6 Implementation Details

The results are presented in two parts. The first evaluates the proposed framework under the information limitations of a
 monostatic configuration. We use observations acquired during 7 August 18:00-19:00 to compare four approaches: (1)
 feature tracking combined with interferometry (FT-IN), (2) interferometry constrained by the direction-invariance
 assumption (DI; Voytenko et al., 2015a, b), (3) NL-WR, and (4) NL-GWR. The velocity field retrieved from bistatic TRI
 430 interferometric observations without auxiliary velocity information serves as the benchmark (Cassotto et al., 2021; Voytenko



et al., 2017). Both NL-WR and NL-GWR adopt the monostatic formulation in Eq. (11), incorporating the direction-invariance constraint and differing only in the spatial sampling and weighting settings described in Sect. 3.4. All methods, including the benchmark, assume a horizontal glacier surface as specified in Eq. (8), to ensure a consistent basis for comparison (Voytenko et al., 2015b). Furthermore, we set the 90° look angle to maintain consistency with the benchmark.

435 The tidal assessment and masking procedure described in Sect. 3.5 are therefore not applied at this stage, which focuses on consistency between the monostatic retrievals and the bistatic benchmark under these shared assumptions.

The second part uses all available TRI observations to construct a continuous glacier velocity time series over an extended period and infer the characteristics of the forcing acting on the Helheim Glacier terminus. Specifically, the bistatic formulation in Eq. (10) is used during the overlapping observation period of 7–10 August, while the monostatic formulation

440 in Eq. (11) is used when observations are available from only one station: only Site 1 configuration during 3–6 August and only Site 2 configuration during 11–18 August. Although the assumption in Eq. (8) is also adopted here, we incorporate the tidal analysis described in Sect. 3.5 to assess and limit the influence of potential tide-induced vertical motion on the retrieved velocities and their physical interpretation in Sects. 4.2.2 and 5.4.

Observation weights are assigned as follows: W_o component for v_r^{in} varies; others receive weights inversely proportional to

445 their variances from prior quality assessment (Zhang et al., 2026; Zheng et al., 2023). Specifically, stable terrain regions were extracted using Normalized Difference Snow Index (NDSI) (Wang et al., 2022), and the standard deviation of the observations within these regions was used to quantify measurement uncertainty (Zhang et al., 2026). Kernel smoothing was additionally implemented as an adaptive weighting strategy to provide an alternative estimation of observation uncertainty (Ramsay, 1991).

450 Spectral analyses of glacier velocity and strain rate were conducted described in Voytenko et al. (2015b) following the Fourier-based approach. Logarithmic strain rate was derived in this study using the transverse and longitudinal strain-rate calculation framework proposed by Alley et al. (2018) to improve the data quality of strain rate results (Alley et al., 2018; Zhang et al., 2026). Detailed configuration values for strain rate calculation parameters are provided in Table 2, with signal and physical meaning aligned with Alley et al. (2018). The other default non-physical parametric values adopted in this

455 study are listed in Table 2, unless explicitly modified for specific result analyses.

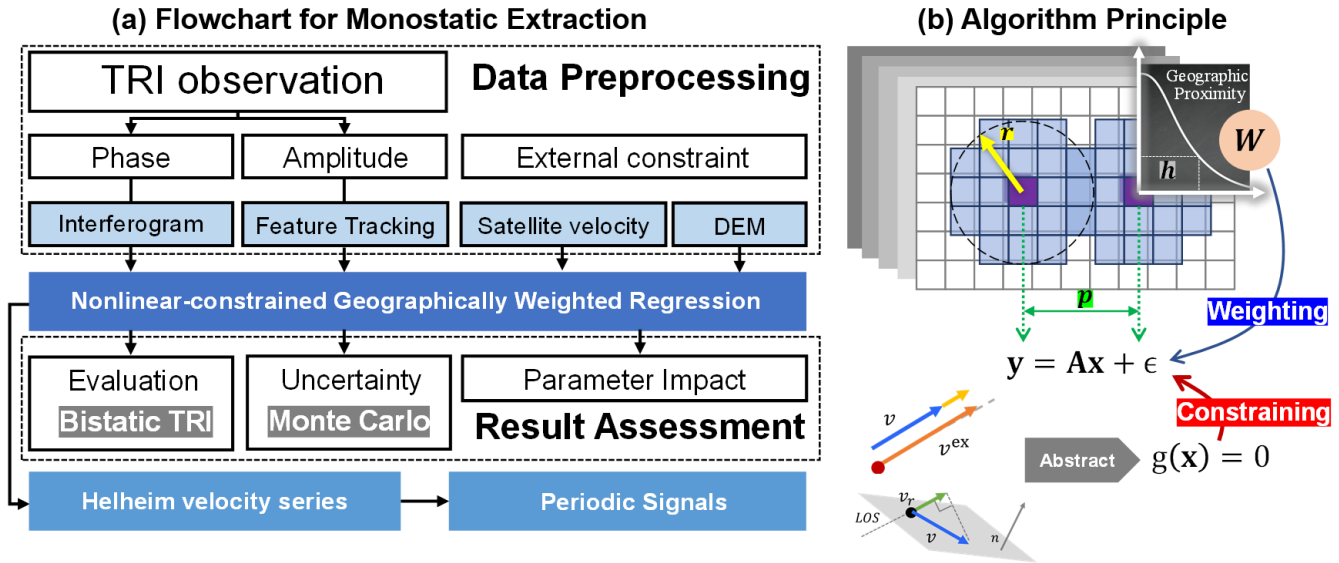


Figure 2 Flowchart for monostatic TRI velocity extraction (a) and schematic diagram illustrating the NL-GWR algorithm for monostatic TRI velocity extraction (b), where h , p , and r denote bandwidth, stride interval, discrimination radius.

Table 2 Parameter values used for NL-WR/NL-GWR in the main text and supplementary. For any NL-WR/NL-GWR operations conducted in the main text or appendices, if not otherwise specified, the default values are those listed above.

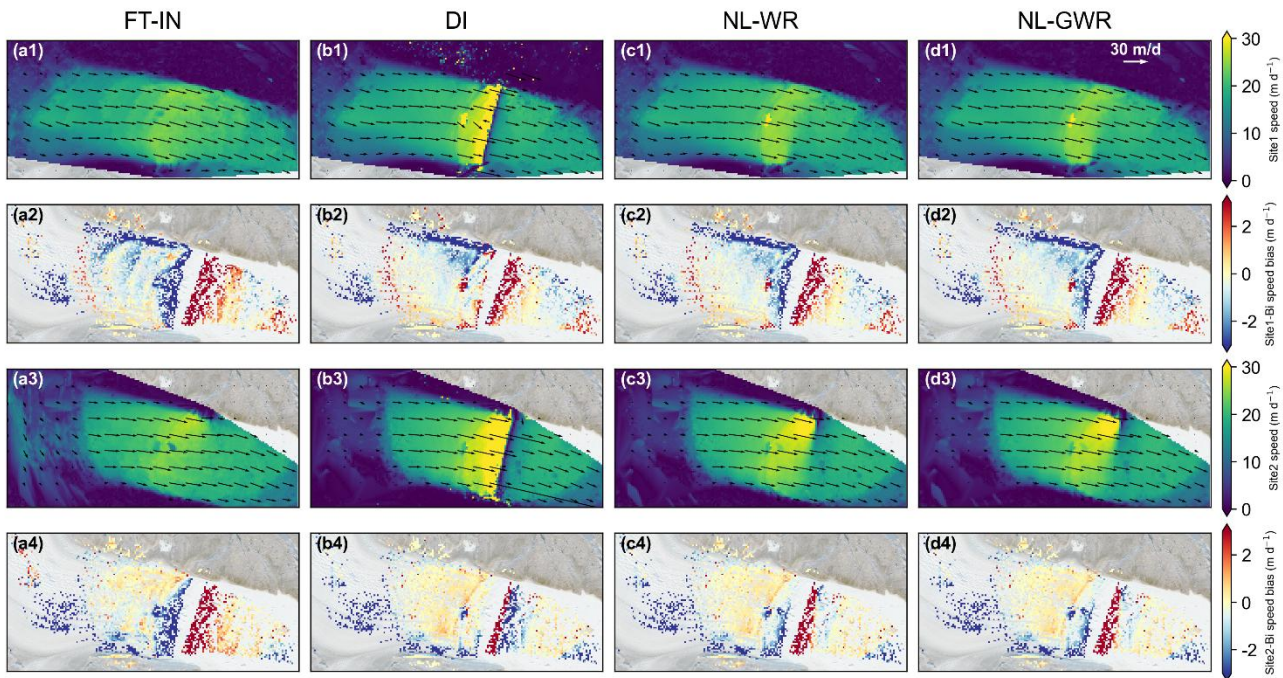
Symbol	Value	Brief Explanation
p	5 (NL-GWR) / 1 (NL-WR)	Sparse sampling interval for regression points
r	5 (NL-GWR) / 0 (NL-WR)	Discrimination radius for regression point sampling
h	5	GWR bandwidth
λ	10000	Penalty factor
μ_k	0	Levenberg-Marquardt damping factor
γ	0.5	Relaxation factor for nonlinear iteration
M	100	Maximum number of iterations for nonlinear iteration
τ_p	0.0001	Divergence criteria threshold
$\sigma(v_r^{\text{in}})$	0.2	Uncertainty for TRI Interferometric velocity ($m d^{-1}$)
$\sigma(v^{ft})$	2	Uncertainty for TRI amplitude-derived velocity ($m d^{-1}$)
$\sigma(v^{ex})$	0.5	Uncertainty for External Sentinel-2A derived velocity ($m d^{-1}$)
L_0	300	Original length for Logarithmic strain rate (m) (Alley et al., 2018)
$\Delta_\varepsilon t$	3	Increment of deformation time (d) (Alley et al., 2018)

4 Result

4.1 Quality assessment

4.1.1 Comparison with conventional monostatic approaches

465 NL-GWR/NL-WR achieves the best speed consistency with benchmark data without masking points with speed $> 30 \text{ m d}^{-1}$. Traditional methods exhibit substantially larger errors: FT-IN exceeds root mean square error (RMSE) of 3 m d^{-1} , and DI suffers numerical instability (Table 3). Data cleaning improves DI to highest consistency, surpassing NL-GWR/NL-WR, but discards substantial data. Spatially, FT-IN produces stair-step artifacts correlated with input feature-tracking observed discontinuities (Figure S1e), DI exhibits a characteristic cross-glacier line artifact perpendicular to velocity direction on the map, while NL-GWR/NL-WR generate smooth and continuous velocity fields (NL-GWR slightly blurrier, Figure 3). The accuracy improvement manifests primarily in azimuth velocity, independent from v_r^{in} . Traditional methods achieve LOS RMSE nearly 0 m d^{-1} . However, NL-WR exhibits LOS RMSE at least $10\times$ larger than traditional methods in exchange for lower azimuth RMSE, attaining lower speed RMSE, while NL-GWR further amplifies these pairwise trends.



475 **Figure 3** Speed extracted by monostatic methods from Site 1 (panels 1-2) and Site 2 (panels 3-4) radar data and their differences from bistatic TRI for: (a) FT-IN, (b) DI, (c) NL-WR, and (d) NL-GWR. Panels 1 and 3 show glacier velocity derived from different methods within the overlapping coverage of the two TRI stations. Panels 2 and 4 present the differences between each method and valid bistatic TRI measurements within the glacier area defined in Figure 1. Bistatic TRI data were pre-filtered by removing points with speed $> 30 \text{ m d}^{-1}$ or velocity direction differences $> 30^\circ$ from Sentinel-2A-derived velocities.

480 **Table 3** Statistical metrics of monostatic and bistatic TRI results (site 1/site 2). RMSE denotes the root mean square error against bistatic TRI velocity components (unit: m d^{-1}), evaluated for speed, LOS velocity, and azimuth velocity. (30) indicates threshold-

based cleaning by removing monostatic results with speed > 30 m d⁻¹. Bistatic TRI results were consistently pre-filtered as described in Figure 3.

Method	RMSE-speed	RMSE-LOS	RMSE-azimuth	Percentage of cleaned points
FT-IN	3.23/3.01	3e-3/3e-3	3.62/3.55	-
DI	474/4115	~0/~0	474/4115	-
NL-WR	2.82/2.63	0.05/0.09	3.18/2.98	-
NL-GWR	2.77/2.63	0.33/0.68	3.12/2.90	-
FT-IN (30)	3.23/3.01	3e-3/3e-3	3.62/3.55	0/0
DI (30)	2.62/2.15	~0/~0	3.03/3.59	2.02%/3.76%
NL-WR (30)	2.79/2.63	0.05/0.09	3.15/2.98	0.10%/0.06%
NL-GWR (30)	2.75/2.63	0.33/0.68	3.10/2.90	0.09%/0.06%

Selecting representative points on the glacier surface for comparison reveals that the velocity estimates from our approaches generally fall between those derived from FT-IN and DI, while being closer to DI. The results from NL-WR and NL-GWR are nearly identical, locating approximately along the line connecting the two conventional solutions and showing a clear tendency towards the DI estimates (Figure 4). The two conventional approaches also exhibit distinct error patterns. Their estimates can be located either on the same side of the bistatic TRI benchmark or on opposite sides, and the magnitude of their deviations from the benchmark does not show a consistent ranking. From the perspective of consistency with bistatic TRI observations, NL-WR and NL-GWR do not produce results inferior to those of conventional approaches. Moreover, when the two traditional methods exhibit deviations on opposite sides of the benchmark, the proposed methods can further achieve slightly improved agreement compared with either individual conventional approach (Figure 4b).

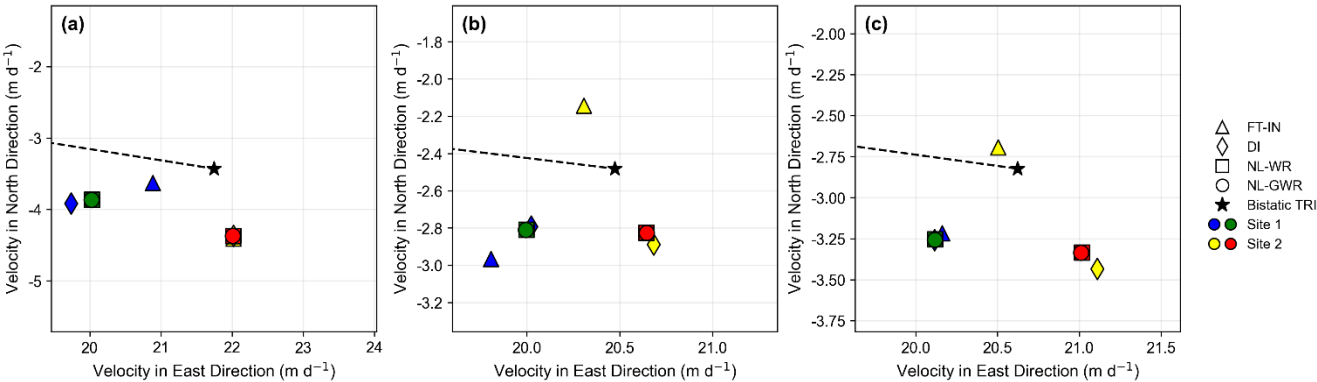
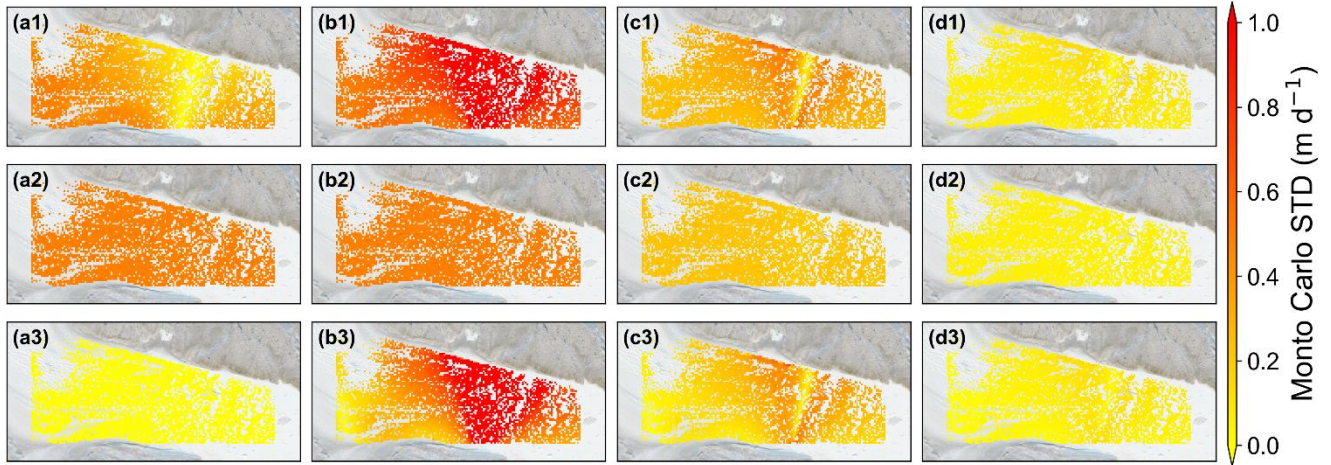


Figure 4 Vector of monostatic-derived and bistatic TRI glacier velocities, sampled at the NE (a), M (b), and NW (c) points shown in Figure 1. The bistatic TRI velocity vector (black star) is set as the origin, with the zero-direction indicated by a black dashed line. The x- and y-axis scales are identical. FT-IN (triangles) and DI (diamonds) are shown in blue (site 1) and yellow (site 2), while NL-WR (squares) and NL-GWR (circles) are shown in green (site 1) and red (site 2).

Monte Carlo simulations (See details in Appendix A3) confirm NL-GWR's superior noise suppression (Figure 5). With 0.5 m d⁻¹ synthetic noise added solely in v_r^{in} , FT-IN and NL-WR propagate ~ 0.5 m d⁻¹ disturbance, DI even shows > 1 m d⁻¹,



500 while NL-GWR constrains disturbance within 0.1 m d^{-1} . All methods exhibit azimuth disturbance patterns consistent with their speed performance.



505 **Figure 5** Spatial distribution of output standard deviations (STD) from monostatic techniques based on 500 Monte Carlo simulations (Appendix A3) using Site 1 data: (a) FT-IN, (b) DI, (c) NL-WR, and (d) NL-GWR. Subplot series 1–3 represent speed, LOS velocity, and azimuth velocity, respectively. Only interferometric velocity inputs are perturbed with Gaussian noise ($\sigma = 0.5 \text{ m d}^{-1}$).

4.1.2 Parametric Significance and Sensitivity

For the fluctuations inherently present in TRI interferometric LOS velocity measurements, NL-GWR preserves the overall variation trend while reducing the magnitude of relative deviations (Figure 6). The original two-minute TRI interferometric velocity observations during 18:00–19:00 exhibit irregular fluctuations with amplitudes reaching up to 1 m d^{-1} , which are observed at both TRI stations. After transformation into planar velocity components, the temporal variation pattern of the retrieved velocity magnitude remains highly consistent with that of the LOS velocity. The mean values of the two velocity series also become closely aligned, resulting in substantial overlap between the curves. However, the standard deviation is reduced by approximately $0.1\text{--}0.2 \text{ m d}^{-1}$.



Figure 6 Time series of 29 interferometric pairs with 2-minute temporal interval at two sites. Statistical analysis is performed on data at point M in Figure 1 to obtain LOS velocity mean values (a) and standard deviations (c) for each site; subsequently, NL-GWR is applied to interferometric data from each small-time segment separately, with other settings consistent with the main text, to obtain Speed mean values (b) and standard deviations (d) within the same region. Solid lines represent results from small time segments, while dashed lines represent temporal averages of the 29 datasets, i.e., the results using the data employed in the main text

Under equal parameter setting ($h = r = p$), introducing GWR reduces overall RMSE by $< 0.05 \text{ m d}^{-1}$, substantially smaller than method-level differences. Parameters increase monotonically raises LOS RMSE while azimuth RMSE initially drops then stabilizes; overall RMSE may decrease then increase (Figure 7) or rise directly (Figure S2). Beyond standard configuration, larger r yields marginal additional RMSE reduction only during azimuth RMSE's declining phase, with smaller effect size than monotonical increase. In addition, bandwidth minimally impacts RMSE except at very small stride.

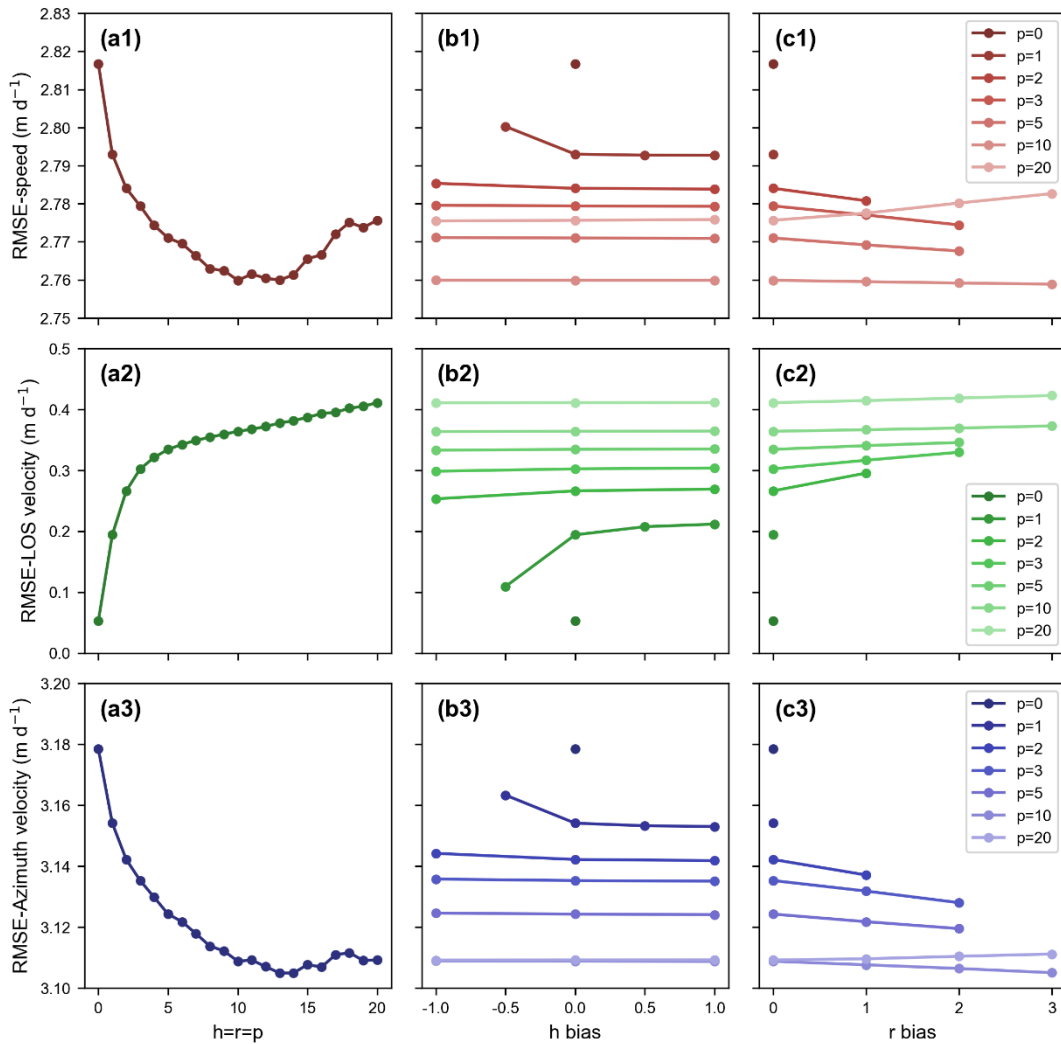


Figure 7 RMSE between NL-GWR output velocity (Site 1) and bistatic TRI versus different parameter settings: (a) simultaneous variation of equal parameters; (b) offsetting bandwidth h ; (c) offsetting discrimination radius r . Panel series 1–3 represent RMSE for speed, LOS velocity, and azimuth velocity, respectively.

The lower the prescribed global statistical uncertainty of the TRI interferometric velocity, the smaller the discrepancy between the retrieved velocity field and the bistatic TRI benchmark (Table 4). In the optimal case, the difference is even lower than the case when using observation uncertainty of 0.2 m d^{-1} . Increasing the prior uncertainty gradually shifts the solution towards non-interferometric approaches, i.e., methods relying only on TRI amplitude-derived velocity and the velocity direction invariance assumption. The velocity distribution also exhibits a transition from the NL-GWR solution toward the specific solution (Figure 8). The statistical inference approach based on adaptive method (Supplementary Text S2) does not achieve the highest consistency; instead, its performance is even lower than the case assuming an observational



uncertainty ten times larger. It also produces a velocity field with a similar spatial distribution to the large-uncertainty case, although with slightly enhanced smoothness.

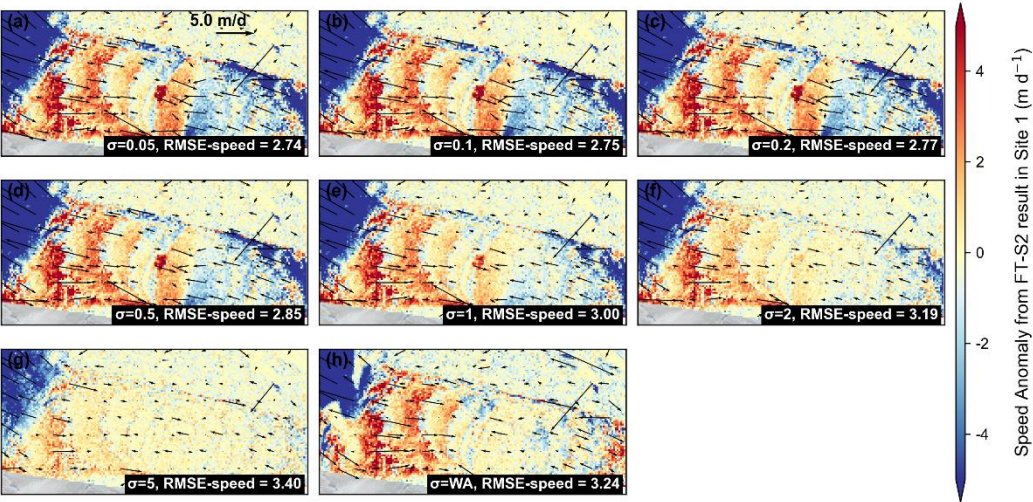


Figure 8 Velocity anomalies produced by different weighting schemes from site1, subtracting velocity obtained by applying NL-GWR using only TRI feature-tracking-derived velocity and Sentinel-2A-derived velocity information, including (a-g) under the condition that the prior uncertainties of TRI feature-tracking-derived velocity and Sentinel-2A-derived velocity components are set to the values adopted in the main text, the prior uncertainty of TRI interferometric LOS velocity is sequentially changed to (a) 0.05, (b) 0.1, (c) 0.2 (which is also the case adopted in the main text), (d) 0.5, (e) 1, (f) 2, (g) 5; (h) all weights are obtained using the adaptive weight selection method described in Supplementary Text S2. The unit of the RMSE-speed is m d^{-1} .

Table 4 Parametric impact of uncertainty for interferometric velocity on speed RMSE in site1. 0.2 is the default prior uncertainty gained by observation in our study. The unit of RMSE is m d^{-1} .

$W_{\sigma}(v_r^{in})$	0.05	0.1	0.2 *	0.5	1	2	5	WA	Inf
RMSE	2.74	2.75	2.77	2.85	3.00	3.19	3.44	3.24	3.64

The influence of the non-physical parameter λ involved only in the numerical optimization is limited in the quality assessment (Table 5). When $\lambda < 500$, variations in λ introduce differences of less than 10^{-3} in the consistency metric between the single-station and bistatic solutions. As λ increases, particularly around the value adopted in this study ($\lambda = 10000$), further increases in λ produce negligible changes in the consistency metric.

Table 5 Parametric impact of penalty factor λ on speed RMSE in site1. 10000 is the default penalty factor used in our study. The unit of RMSE is m d^{-1} .

λ	100	500	1000	5000	10000 *	50000	100000	1000000
RMSE	2.771029	2.770997	2.770993	2.770991	2.770991	2.770990	2.770990	2.770990

4.2 Time-series Velocity Analysis over Helheim Glacier

4.2.1 Velocity consistency

During the overlapping observation period of the two TRI stations (August 7-10), the velocity anomaly fields retrieved from the single-station and bistatic configurations show good consistency (Figure 9). The monostatic velocity fields derived under the direction-invariance and horizontal surface-parallel flow assumptions exhibit consistent spatial patterns after median averaging. However, they differ from the simultaneously retrieved NL-GWR bistatic velocities by up to approximately 1 $m d^{-1}$, with larger discrepancies occurring near the calving front. In contrast, after removing the 25-hour averaging result, the resulting velocity anomalies show very good consistency in their temporal variations (Figure 9c-f). Comparisons at multiple sampling locations demonstrate that both approaches successfully capture the temporal variations of velocity anomalies. The timing and amplitude of velocity peaks and troughs are highly consistent, with local differences of approximately 0.1 $m d^{-1}$, which are substantially smaller than the absolute bias of the variations ($\sim 0.5 m d^{-1}$). The 5-hour median filter does not alter the overall trend of the time series but only suppresses occasional sharp fluctuations lasting 1–2 hours.

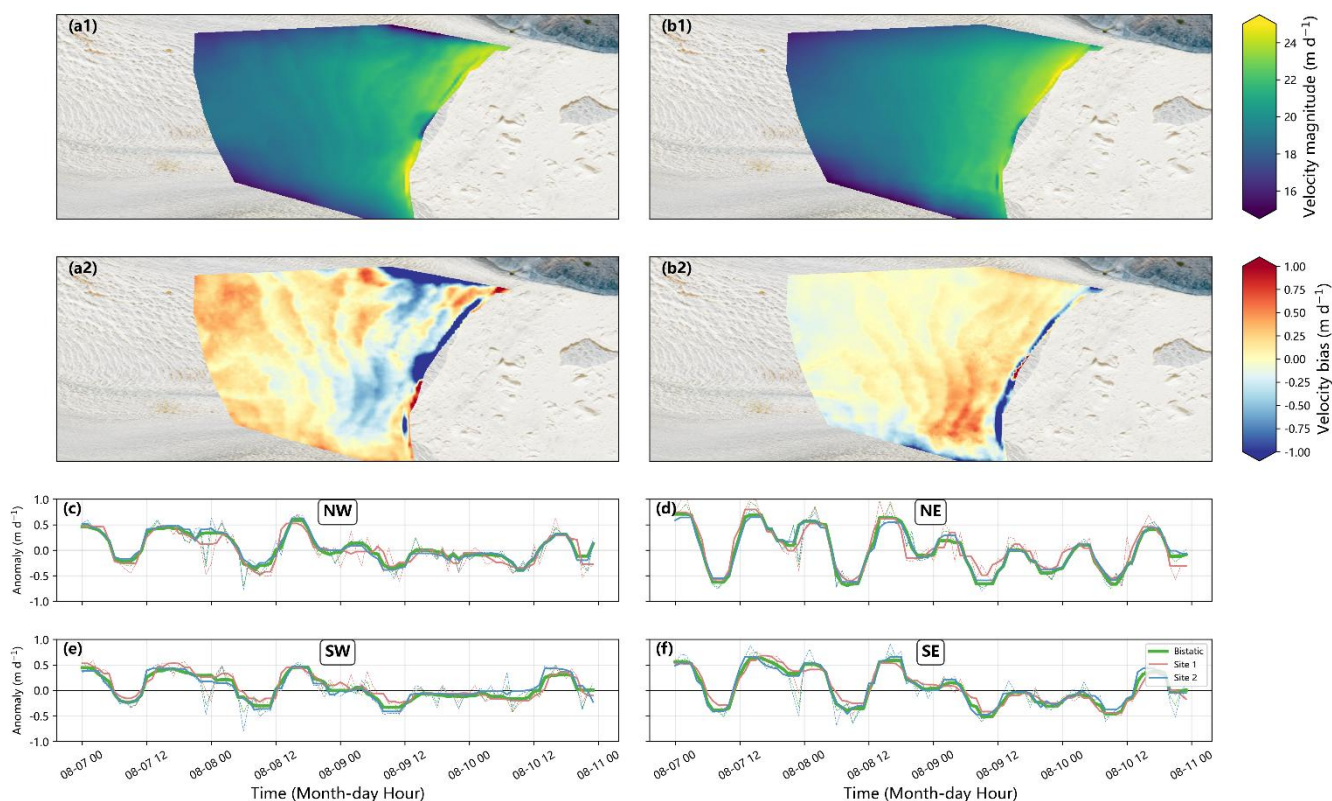


Figure 9 Comparison of monostatic and bistatic TRI velocity retrievals. Panels (a) and (b) show the velocity retrieved independently using NL-GWR at Site 1 (series 1) and Site 2 (series 2), respectively, together with their differences from the corresponding bistatic NL-GWR velocity. The velocity anomaly time series, original velocity subtracting 25-hour averaging result

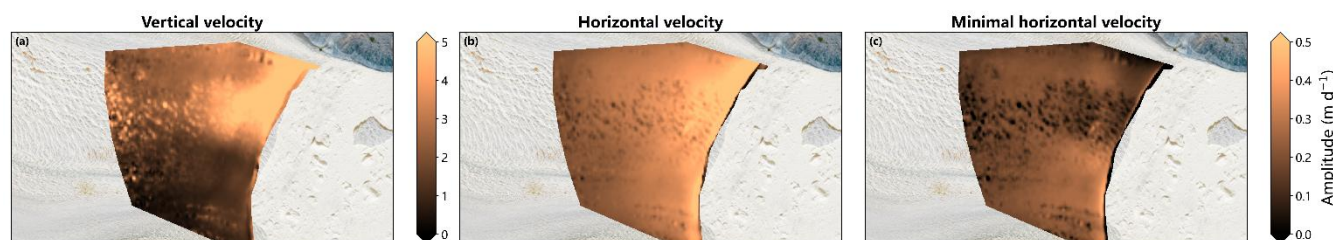


derived from single site and both sites from NL-GWR are compared at four sampling points in Figure 1: (c) NW, (d) NE, (e) SW, and (f) SE. Dashed and solid lines represent the original time series and the 5-hour median-filtered results, respectively.

575 4.2.2 Assessment of tidal effects

The amplitude of the periodic vertical velocity retrieved using the bistatic configuration without the topographic constraint exhibits distinctly different spatial patterns across the northern and southern parts of the glacier (Figure 10). In the north, the amplitude generally exceeds 3 m d^{-1} , with larger values near the calving front and a gradual decrease upstream. A similar upstream decrease occurs in the south, but the maximum amplitude near the terminus remains below approximately 2 m d^{-1} .

580 When projected onto the radar LOS direction, this southern vertical motion contributes approximately 0.2 m d^{-1} or less, comparable to the interferometric measurement uncertainty. Accounting for this potential vertical contribution still yields a substantial lower bound on the amplitude of periodic horizontal motion in the south, reaching 0.4 m d^{-1} , particularly near the calving front.



585 **Figure 10** Assessment of tidal vertical and horizontal velocities using the bistatic configuration. Panels show (a) the amplitude of vertical velocity variations at the tidal period retrieved using Eq. (9), (b) the amplitude of horizontal velocity variations at the same period retrieved using Eq. (10), and (c) the estimated lower bound on the amplitude of actual horizontal velocity variations obtained using Eq. (28).

4.2.3 Velocity periodic analysis

590 Fourier spectral analysis was applied to identify dominant horizontal periodic signals and two major spectral peaks were detected at approximately 12.38-hour and 23.93-hour periods. The 23.93-hour component exhibited the strongest energy, with an amplitude of up to 0.2 m d^{-1} , while the 12.38-hour component showed the second-largest energy, with an amplitude of approximately 0.15 m d^{-1} , both substantially exceeding the energy at other frequencies (Figure 11a).

The two periodic components exhibited distinct spatial propagation patterns in the two-dimensional point-wise Fourier analysis (Figure 11b-c). The amplitude of the 12.38-hour component was highest in the downstream region and gradually decreased upstream, with the strongest signal occurring at a period of 12.38 hours and reaching 0.3 m d^{-1} . In contrast, the 23.93-hour component showed a relatively uniform spatial distribution, maintaining a stable amplitude of approximately 0.2 m d^{-1} across the observation area. Using the region near the calving front as the phase reference, the 12.38-hour component exhibited nearly consistent phase along the glacier front, whereas the 23.93-hour component showed an opposite propagation pattern, with downstream regions lagging behind upstream regions.

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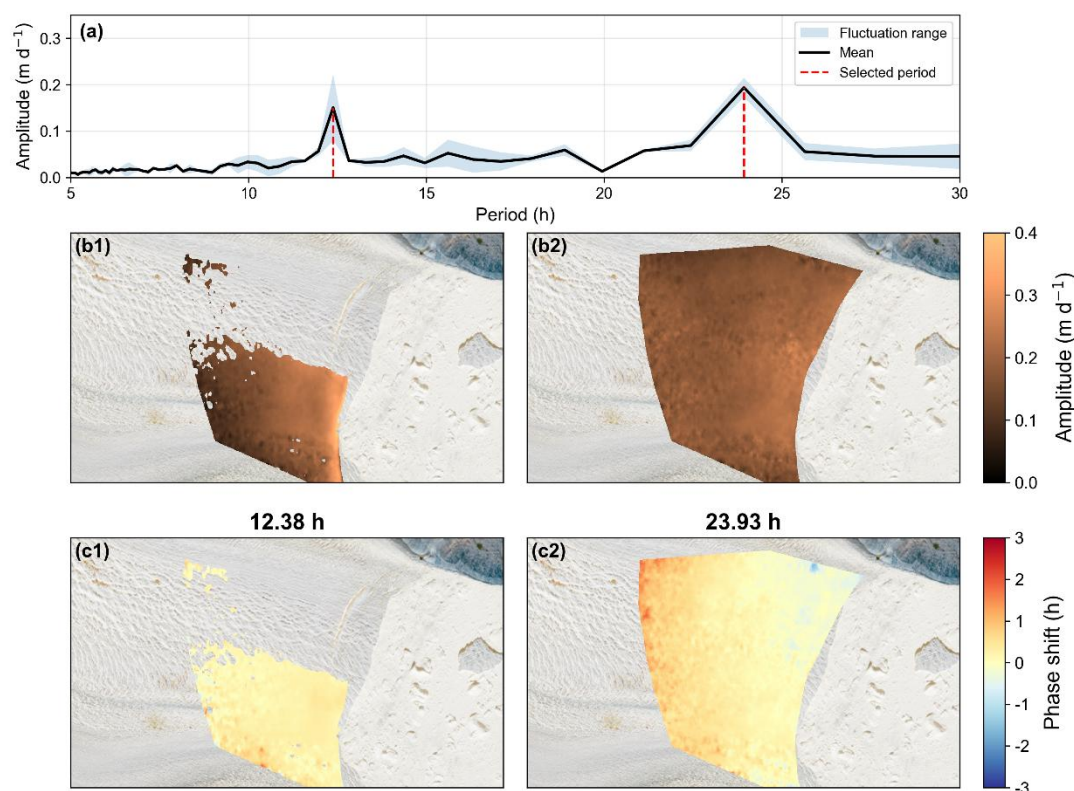


Figure 11 Fourier spectral analysis of glacier velocity anomaly magnitude. Panel (a) presents the combined spectral analysis results for the two southern sampling points (SW, and SE) shown in Figure 1 after applying a 5-hour median filter. The black dashed line represents the mean spectrum, and the blue shaded area indicates the range of spectral variability. The two red lines mark the periods of 12.38 hours and 23.93 hours from left to right. Panels (b–c) show the two-dimensional distributions of (b) periodic amplitude and (c) initial phase shift. Amplitudes correspond to the exact sampling frequencies of the Fourier transform, with periods of (1) 12.38 hours and (2) 23.93 hours. Phase shifts are referenced to the mean phase near the glacier terminus, which is set to zero, followed by phase unwrapping. For the 12.38-hour component, locations where the vertical velocity amplitude shown in Figure 10a exceeds 2 m d^{-1} were excluded.

Further analysis along the L1 and L2 transects revealed differences in the relative strength and propagation characteristics of the periodic signals (Figure 12). The 12.38-hour component generally reached its maximum amplitude near the glacier terminus and decreased towards the upstream region, whereas the 23.93-hour component maintained relatively stable amplitudes. The transition pattern varied laterally across the glacier. Along L1, near the glacier centre, the tidal and meltwater-related components maintained comparable amplitudes over an extended reach before the tidal amplitude decreased rapidly upstream. Along the more southerly L2, the tidal amplitude exhibited a pronounced peak approximately 300 m upstream of the calving front, followed by a transition zone farther upstream where two components maintained comparable amplitudes.

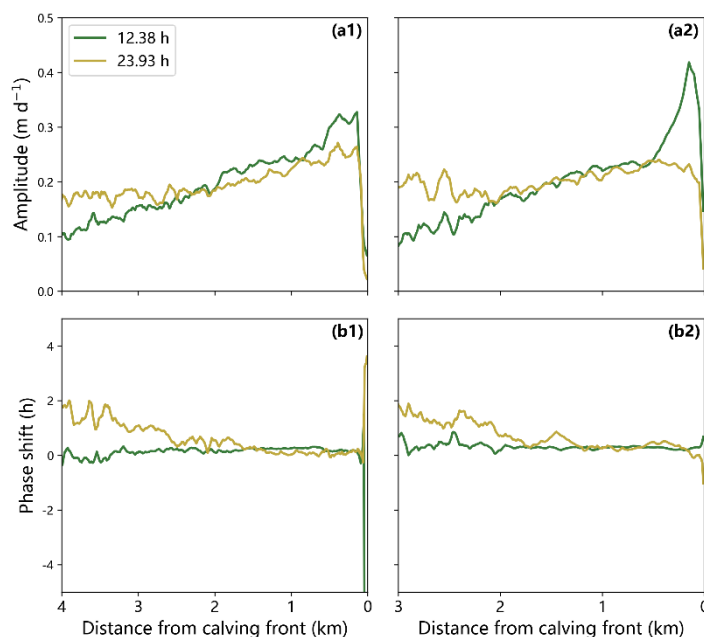
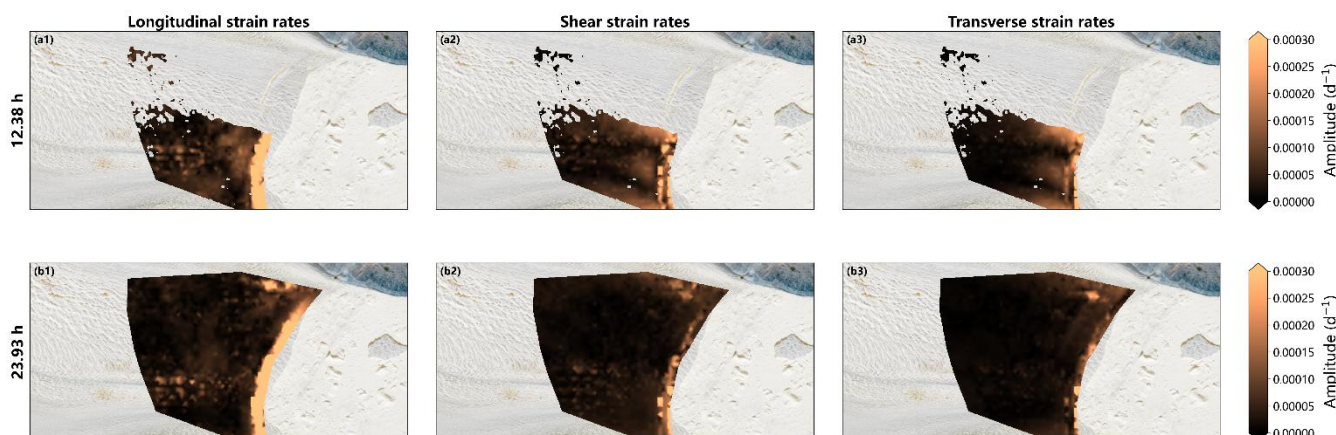


Figure 12 Profiles of periodic velocity anomaly amplitude and phase shift along transects. The distributions are shown along the two transects (1) L1 and (2) L2 indicated in Figure 1d, including (a) periodic amplitude and (b) initial phase shift. The phase terms were adjusted using the same reference-phase shifting procedure as in Figure 11.

4.2.4 Strain-rate analysis

Compared with velocity signals, strain-rate periodic signals exhibited fewer pronounced features but still showed distinct spatial variations (Figure 13). The 23.93-hour component showed limited periodic characteristics across the glacier, except for several anomalous peaks near the calving front, within the redundant range of logarithmic strain rate calculation. The 12.38-hour component generally exhibited weak signals within the glacier interior. However, enhanced periodic amplitudes were observed in the longitudinal strain rate within the region. The shear and transverse strain rates exhibited a rapid upstream decay pattern along the glacier flowline, extending from the calving front towards the interior glacier region.





630 **Figure 13 Distribution of periodic components in glacier surface strain rates. The amplitudes of periodic signals are shown for (a) longitudinal normal strain rate, (b) shear strain rate, and (c) transverse normal strain rate, including (1) the 12.38-hour period and (2) the 23.93-hour period. For the 12.38-hour component, locations where the vertical velocity amplitude shown in Figure 10a exceeds 2 m d^{-1} were excluded.**

5 Discussion

635 5.1 Advantages of the NL-GWR framework

5.1.1 Advantages of the nonlinear structure

A key advantage of our nonlinear loss function $\mathcal{J}(\mathbf{x})$ is its ability to favour the prescribed physical constraints while avoiding the numerical instability observed in the DI method. The direction-invariance constraint fixes only one degree of freedom of the three-dimensional velocity vector. When the remaining assumptions are inconsistent with the observations, 640 the retrieved components can therefore develop substantial artefacts. For example, imposing zero vertical velocity when the observations contain a non-negligible vertical contribution can bias the reconstructed horizontal motion, particularly when insufficient observational redundancy is available to identify the inconsistency. Under unfavourable viewing geometries, these artefacts can become large enough to produce numerical instability. In the DI method, this occurs particularly when the LOS direction is nearly perpendicular to the assumed flow direction, so that even a modest vertical contribution can 645 substantially affect the retrieval (Figure 3). Mathematically, this behaviour corresponds to an ill-conditioned inversion (Sanz et al., 2025).

The proposed nonlinear formulation introduces auxiliary residual components, d_x^{ex} and d_y^{ex} , into the reconstructed state vector $\mathbf{x}$, allowing adaptive control of the effective strength of the directional prior. Let $\mathbf{v}_H = (v_x, v_y)$, $\mathbf{d} = (d_x^{\text{ex}}, d_y^{\text{ex}})$ and $\mathbf{v}_{\text{ex}} = (v_x^{\text{ex}}, v_y^{\text{ex}})$ denote the horizontal velocity, the auxiliary residual vector, and the external horizontal velocity observation, 650 respectively. The external observation is represented by $\mathbf{v}_H + \mathbf{d}$, with a residual accounting for measurement uncertainty. Assuming equal and independent uncertainties σ_{ex} in the two external velocity components, and no error covariance with the TRI observations, the loss function in Eq. (13) can be expanded as:

$$\mathcal{J}(\mathbf{x}: [\mathbf{v}_H \ v_z \ \mathbf{d}]) = \mathcal{J}_{\text{TRI}}([\mathbf{v}_H \ v_z]) + \eta \|\mathbf{v}_{\text{ex}} - \mathbf{v}_H - \mathbf{d}\|^2 + \lambda (v_y d_x^{\text{ex}} - v_x d_y^{\text{ex}})^2. \quad (29)$$

Here, $\mathcal{J}_{\text{TRI}}$ is the weighted sum of squared residuals for the TRI interferometric and offset-tracking velocity observations, 655 and η is a combined weight constant related to prior uncertainty σ_{ex} and geographical weight. For a fixed $\mathbf{v} = [\mathbf{v}_H \ v_z]$, minimizing this loss function with respect to $\mathbf{d}$ yields:

$$\min_{\mathbf{d}} \mathcal{J}(\mathbf{x}) = \mathcal{J}_{\text{TRI}}(\mathbf{v}) + \frac{\eta \lambda}{\eta + \lambda \|\mathbf{v}_H\|^2} (v_y v_x^{\text{ex}} - v_x v_y^{\text{ex}})^2. \quad (30)$$

This expression shows that the effective weight of the directional constraint depends on the candidate velocity in the loss function. For nonzero $\mathbf{v}_H$, the denominator limits this weight to $\frac{\eta}{\|\mathbf{v}_H\|^2}$, even as λ increases to infinity. Consequently, a large



660 penalty factor enforces collinearity between $\mathbf{v}_H$ and $\mathbf{d}$ without imposing an arbitrarily large penalty on deviations from the observed external flow direction.

Alternatively, the directional constraint can be expressed in a linear form:

$$v_y v_x^{\text{ex}} - v_x v_y^{\text{ex}} = 0. \quad (31)$$

This allows the problem to be solved using linear regression, with an objective of the optimization function form:

$$665 \quad \mathcal{J}(\mathbf{x}) = \mathcal{J}_{TRI}(\mathbf{v}) + w_{\sigma,ex}(v_y v_x^{\text{ex}} - v_x v_y^{\text{ex}})^2. \quad (32)$$

In this formulation, the confidence assigned to the directional prior must be specified independently through $w_{\sigma,ex}$. Assigning a large value to strongly favour the prior does not provide the velocity-dependent adjustment present in the nonlinear formulation. Under unfavourable observation geometries, the resulting strong directional restriction can favour excessively large velocity estimates when fitting the LOS observations, retaining a risk of large-amplitude artefacts similar to those encountered in DI. By contrast, our nonlinear formulation limits the effective directional penalty while retaining the TRI observation term to constrain velocity magnitude.

670 The nonlinear formulation also provides flexibility for introducing more general physical constraints. For example, we change the strict direction equality rule into introducing an allowable angular difference $\Delta\psi$ (Mouginot et al., 2023). Such an inequality cannot be represented by simply appending a fixed equality row to an ordinary linear regression system, whereas it can be incorporated naturally through an appropriately defined constraint function:

$$675 \quad g_{dire}^*(\mathbf{x}) = \max \left\{ 0, \left| \arctan \frac{v_y}{v_x} - \arctan \frac{v_y^{\text{ex}}}{v_x^{\text{ex}}} \right| - \Delta\psi \right\}. \quad (33)$$

5.1.2 Advantages of observation and constraint setting

The aggregate performance metrics indicate that the framework extracts complementary information from the observations and physical priors. The improved speed RMSE results from a trade-off between LOS and azimuth velocity accuracy: both proposed methods exhibit increased LOS RMSE but reduced azimuth RMSE. This improvement depends on the physical applicability of the prior constraint, supported by the generally good agreement between DI and the bistatic benchmark after unstable DI retrievals are removed (Table 3).

685 The directional constraint also suppresses the large uncertainties associated with feature-tracking measurements (Figure S1). Owing to limitations of offset-tracking algorithms (Zhang et al., 2026), these measurements require comparatively long temporal baselines and remain substantially less precise than interferometric observations. Giving them influence comparable to that of the more reliable information could therefore obscure the accuracy and temporal-resolution advantages that motivate the use of TRI. In our framework, amplitude-derived velocities primarily contribute to the linear observation term, helping constrain solutions that would otherwise become unstable. Their influence is comparatively small elsewhere, where NL-GWR generally produces results close to those obtained directly from the directional prior (Figure 4).



690 5.1.3 Advantages of geographical weighting

Geographic weighting further reduces noise with rapid spatial variability and local anomalies through spatial averaging, leading to additional reductions in speed RMSE. However, this improvement reaches saturation when the stride exceeds a certain threshold (Figure 7), which is determined by the intrinsic quality of TRI measurements and the occurrence possibility of outliers. Beyond this threshold, information loss caused by overly sparse sampling outweighs the benefits of noise
 695 suppression. Notably, GWR preserves signals with low spatial variability (Figure 6), which may originate from genuine velocity variations or atmospheric noise effects (Gloor, 2023; Voytenko et al., 2012), thereby maintaining the completeness of the retrieved velocity information. Although this property may introduce the low quality of amplitude-derived observations and potentially unrealistic low-spatial-variability features from external constraints into the retrieved results, their influence is reduced when velocity anomalies or strain-rate differences are considered. Moreover, these error sources
 700 are predominantly low-frequency in time, and our analysis focuses on signals at daily and shorter timescales using Fourier transforms, further limiting their influence.

5.2 Parametric Selection Strategy

Our preference for the physical prior requires the constraint norm to be reduced sufficiently, motivating a sufficiently large penalty factor λ in Eq. (13). As shown by Eq. (30) in Sect. 5.1, increasing λ strengthens the effective directional constraint.
 705 However, its influence approaches a finite limit, while the TRI observation term continues to constrain the remaining velocity degrees of freedom. Specifically, for nonzero $\mathbf{v}_H$, the loss function after eliminating the auxiliary variables satisfies:

$$\lim_{\lambda \rightarrow \infty} \min_{\mathbf{d}} \mathcal{J}(\mathbf{x}) = \mathcal{J}_{TRI}(\mathbf{v}) + \frac{\eta}{\|\mathbf{v}_H\|^2} (v_y v_x^{\text{ex}} - v_x v_y^{\text{ex}})^2. \quad (34)$$

This limiting form provides a basis for the diminishing sensitivity of the reconstruction to further increases in λ . Consistent with this behaviour, our results further show that the retrieved velocities stabilize as λ increases, allowing a sufficiently large
 710 value to be selected ($\lambda > 500$ in this study) without substantially changing the reconstruction (Table 5). The observed plateau indicates that the remaining approximation associated with the finite penalty factor has little influence on the retrieved solution, supporting the robustness of the reconstruction to the choice of λ within the tested range.

Geographic parameters provide smaller accuracy improvements compared with the methodological developments, with their effects mainly reflected in the trade-off between LOS and azimuth velocity accuracy. Nevertheless, increasing h , r , and p
 715 appropriately can balance spatial resolution loss against suppression of rapid spatial variability error and computational efficiency (Table A1). Therefore, moderate parameter values (e.g., $h = r = p = 5$ in this study) are recommended.

For the selection of weighting strategies, statistically derived approaches, although convenient (Hu et al., 2014), do not necessarily guarantee optimal solutions (Figure 8). Observation-based uncertainty estimates provide improved validation performance but remain suboptimal (Table 4). This is because both measured and statistically inferred covariance matrices
 720 primarily characterize the statistical properties of regression residuals under the assumption of unbiased observations, while



systematic errors are not accounted for. Therefore, posterior adjustment using independent external evaluation data is recommended to obtain more reliable $\mathbf{W}_\sigma$.

5.3 Potential mechanisms underlying diurnal and semi-diurnal horizontal signals at Helheim Glacier terminus

The semi-diurnal signal, characterised by the strongest energy near the glacier terminus and rapid upstream attenuation, is likely dominated by tidal forcing. This spatial pattern suggests that the associated stress perturbation originates from boundary forcing, which is further supported by the spatial distribution of shear and longitudinal strain rates (stress-related indicators). A previous spectral analysis of LOS velocity observations at Helheim Glacier in 2013 also identified similar tidal signals and suggested that tidal energy reaches a maximum within a certain distance downstream of the glacier terminus before gradually decaying upstream (Voytenko et al., 2015b). Our observations along two transects reveal a similar delayed peak pattern (Figure 12). Because our observations are restricted to the glacier surface, they do not extend into the transition zone (Voytenko et al., 2015b), which was located within the mélange region. Therefore, we do not observe the characteristic phase-transition zone associated with mélange dynamics. The observed velocity variations may directly result from pressure changes induced by ocean tides, particularly when the mélange is not tightly coupled to the glacier front (Surawy-Stepney et al., 2024; Vieli et al., 2001). However, considering the potential role of mélange in exerting mechanical stresses on the glacier terminus (Burton et al., 2018; Meng et al., 2025) and the possibility that tidal forcing promotes mélange weakening and decoupling (Cassotto et al., 2021; Xie et al., 2019), the observed velocity variations may also reflect periodic changes in mélange buttressing caused by tidal modulation (Deakin et al., 2026).

The diurnal signal exhibits relatively uniform energy distribution throughout the downstream glacier region, while its phase shows a clear propagation pattern from upstream to downstream. This behaviour suggests that the signal may be associated with diurnal meltwater variability. The generally weak strain-rate response indicates that the mechanism responsible for this signal does not strongly modify the horizontal deformation field, implying that the forcing is likely dominated by vertical processes and remains relatively uniform in the horizontal direction. Our findings are consistent with previous TRI LOS acceleration analyses (Dachauer et al., 2026), which also identified meltwater-related periodicity. Furthermore, the observed phase offset between the meltwater-related cycle and the solar radiation cycle is approximately 4 hours (Dachauer et al., 2026). This interval is shorter than the 6.5-hour phase lag detected by GPS observations in inland regions away from the glacier terminus (Stevens et al., 2022), suggesting that the subglacial hydrological systems near the terminus and in the inland glacier may operate differently.

The most important implication of our observations is the simultaneous detection of the transition between semidiurnal and diurnal signals, suggesting a competition between tidal forcing and meltwater-driven processes at the glacier terminus. As shown in Figure 12, the transition between the dominant mechanisms occurs approximately 1–3 km upstream of the glacier terminus. This distance is substantially shorter than those inferred from observations and modelling at seasonal and longer timescales, which suggest that subglacial hydrology becomes dominant over terminus effects only approximately 14 km upstream (Sommers et al., 2024; Ultee et al., 2022). This discrepancy may reflect the scale-dependent variability of glacier



dynamics (Hanna et al., 2024). Moreover, our analysis effectively isolates the tidal signal from the broader, composite terminus effect, with tidal forcing representing only one component of the latter (Amundson et al., 2020; Deakin et al., 2026).

5.4 Potential implications of tide-induced vertical motion

The retrieved vertical motion suggests that the grounding line lies close to the calving front over parts of the Helheim Glacier terminus, consistent with the hydrographic observations (Melton et al., 2022). In the north, the results suggest a floating region extending several kilometres inland during at least 7–10 August. This interpretation is supported by the substantial reconstructed vertical motion and the near-zero lower bound on horizontal periodic motion after accounting for potential vertical-motion artefacts. However, this lower bound does not demonstrate the absence of periodic horizontal motion: the correction estimates only its minimum possible amplitude. To avoid ambiguity in the physical interpretation, we therefore exclude the northern region from the corresponding analysis in Sects. 4.2.3 and 4.2.4.

In contrast, the southern region exhibits no clearly resolved vertical motion, while the estimated horizontal periodic amplitude remains substantial even after accounting for potential vertical contamination. The actual horizontal amplitude may be larger than this lower-bound estimate. These results suggest that the grounding line may lie very close to the calving front in the south and that substantial periodic horizontal motion occurs there. This finding calls into question the assumption of negligible tide-related horizontal motion used in some grounding-zone analyses (Friedl et al., 2020). Where that assumption is applied, horizontal motion could be misinterpreted as vertical displacement, potentially leading to an overestimate of the grounding-zone extent.

5.5 Implications for glacier terminus research

The evolution of glacier terminus instability can be investigated directly from ice-flow dynamics and existing glacier models (Schlemm et al., 2022; Sun and Gudmundsson, 2026). However, fewer studies have focused on the initiation stage of instability, namely the onset of the first perturbation, which often involves crack propagation or local ice detachment (Ochwat et al., 2025). These short-lived and spatially limited disturbances are dynamically driven by changes in external forcing conditions (Cook et al., 2014; Deakin et al., 2026; Schlemm et al., 2022). While topographic controls have been extensively investigated (Ochwat et al., 2025; Schlemm et al., 2022; Ultee et al., 2022) due to their relative stability and data availability, rapidly varying and difficult-to-observe environmental factors, such as tidal and meltwater forcing discussed in this study, remain challenging to quantify.

Our study provides a resource-efficient approach for investigating the impacts of subtle environmental variations with free upgrade compared to conventional TRI observations. Instead of directly measuring tides or subglacial meltwater processes, which require additional observations or specialized instruments (Melton et al., 2022; Voytenko et al., 2015b), we retrieve complete velocity information and, particularly, strain-rate fields to infer their mechanical impacts. These results help facilitate the identification of mechanically sensitive regions and can provide valuable constraints for improving glacier



785 models through observation-based calibration. Future TRI studies can further exploit this framework to generate dynamic measurements and advance understanding of regional glacier instability mechanisms.

6 Conclusion

In this study, we develop a nonlinear-constrained Geographically Weighted Regression (NL-GWR) framework to retrieve multi-dimensional glacier velocity fields from monostatic TRI observations by integrating interferometric measurements, feature-tracking-derived velocities, and glaciophysical constraints. Compared with conventional monostatic approaches, NL-GWR achieves improved consistency with bistatic TRI observations while maintaining complete spatial and temporal coverage, demonstrating its ability to extract complementary information from heterogeneous observations and reduce uncertainties associated with individual measurement techniques. Applying this framework to Helheim Glacier enables long-term, high-frequency observations of both velocity and strain-rate variations, revealing distinct semidiurnal and diurnal dynamic signals near the glacier terminus. The identified spatial transition between tidal and meltwater-related controls highlights the importance of high-temporal-resolution observations for resolving short-term glacier instability processes that remain unresolved by conventional satellite-based observations.

The proposed framework provides a practical pathway for extending the capability of resource-limited monostatic TRI deployments and enables quantitative investigation of glacier dynamics beyond conventional velocity monitoring. The detection of competing tidal and hydrological influences demonstrates that glacier terminus responses are governed by multiple processes operating at different temporal and spatial scales, emphasizing the need to incorporate rapid spatiotemporal variability observations into future projections of glacier change and sea-level contribution. Future improvements may include the integration of additional physical constraints, selection of proper constraints in different areas, and more comprehensive multi-source observations, such as TRI DEM construction, to further improve velocity retrieval accuracy and advance mechanistic understanding of rapidly changing glacier systems.

Appendix A: Additional data processing details

A1 Phase-unwrapping error correction for interferometric velocity

The main text briefly summarized two types of phase-unwrapping errors and corresponding processing strategies. This section provides a detailed description of the manifestation of these errors and the design of the correction algorithms.

1) First type of unwrapping error: global 2π multiples

This error is characterized by spatially reasonable relative variation trends in the unwrapped phase difference (disregarding the second type of error), but with the entire field offset by an integer multiple of 2π , as illustrated in Figure A1. Detection of this error is relatively straightforward. Based on the assumption of stable terrain, where deformation detected should



815 approach zero, we note from Eq. (1) in the main text that the unwrapped phase difference should also be zero. Therefore, we
 employed the NDSI mask (Wang et al., 2022; Zhang et al., 2026) to extract rock outcrop regions and examined the mean
 value of the unwrapped phase differences over these areas. The magnitude of this mean value divided by 2π was then
 evaluated, as shown in Figure A1a. The corrected phase field was obtained by subtracting this integer multiple of 2π from
 the entire phase dataset, with the correction effect demonstrated in Figure A1b2 and Figure A1c2.

820 2) Second type of unwrapping error: regional phase jumps from partial coherence loss

This error manifests as localized, contiguous regions where the unwrapped phase difference differs from the surrounding
 values by multiples of 2π , as depicted in Figure A1c2. Our cleaning strategy employs blockwise anomaly analysis as a
 diagnostic aid. Statistical analysis reveals that these erroneous regions occur at relatively inconsistent locations, and the
 magnitude of phase jumps varies. Consequently, we averaged the unwrapped phase differences over time and computed the

825 anomaly results for each time step. If the median phase anomaly in a particular region at a specific time exceeds $\pm\pi$, this
 indicates the occurrence of local phase jumps in that region. However, measurement errors from the data source, chaotic
 phase-unwrapping results, and other non-phase-jump factors can also influence this detection process. Through empirical
 experimentation, we determined that using 30×30 pixel blocks and calculating the median of the overall anomaly for each
 block optimally satisfies both requirements: enabling detection of large-scale phase jumps while keeping other error sources

830 within reasonable bounds. For blocks flagged as containing phase jumps, pixel-by-pixel assessment was performed within
 each block to identify regions where anomalies corresponded to the block's phase-jump magnitude, and the phase differences
 in these regions were adjusted accordingly. The final correction effect is illustrated in Figure A1b3 and Figure A1c3.

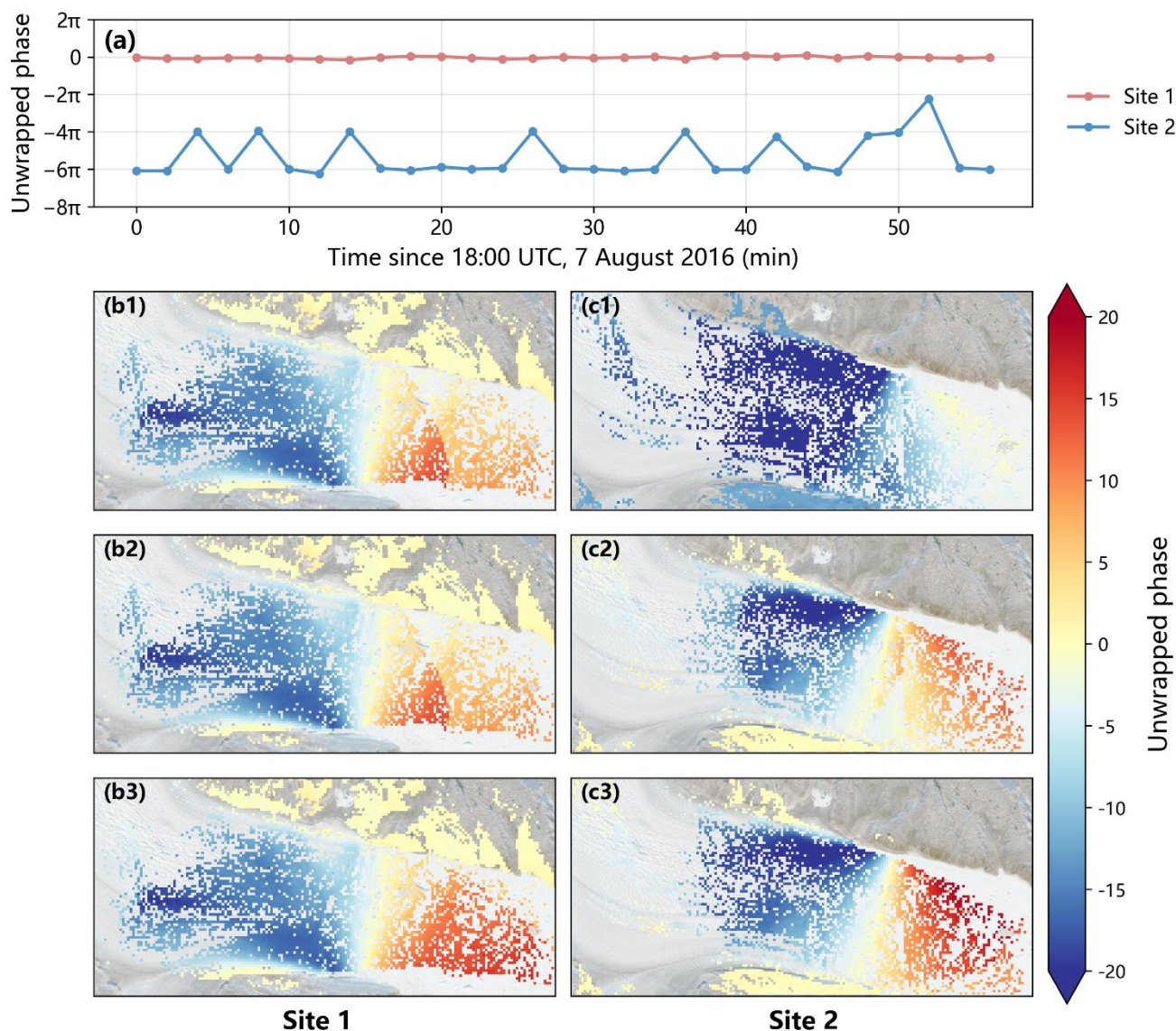


Figure A1: Illustration of phase unwrapping failures and corrections. Panel (a) shows the time series of spatially averaged uncorrected unwrapped phase over stable terrain. Panels (b) and (c) present schematic examples of phase-unwrapping correction for interferometric LOS velocity, using interferograms from Site 1 (18:00–18:02 on August 7, 2016) and Site 2 (18:08–18:10 on August 7, 2016), respectively. Subplot series (1) shows results after phase unwrapping and coordinate transformation without correcting phase anomalies; (2) presents results after removing the first type of unwrapping error; and (3) shows the final results after correcting the second type of error, namely regional phase jumps caused by partial coherence loss.

A2 Three-Parameter Numerically Equal Setting

This paper adopts the rule of setting the bandwidth h , discrimination radius r , and sampling padding (p_x, p_y) to be completely identical, as this rule can smooth out certain variable differences. First, it enables control of the approximate

equality of the total number of data points used in regression calculations. A larger discrimination radius indicates that more geographical reference data are used for regression at a single measurement point; while larger sampling padding indicates greater spacing between selected sampling points, reducing the total number of measurement points. The impact of these two rules on the total data usage Num is:

$$\text{Num}(r, p_{x/y}) \approx \frac{N_0}{p_{x/y}^2} \times \pi r^2$$

Therefore, when $r = p_{x/y} := p$ numerically, the number of data points used for regression is approximately the same across different parameter magnitude settings, depending only on the original remote sensing image pixel count N_0 . This eliminates the influence of information quantity on the method, allowing us to directly compare how different parameters alter the method's information extraction efficiency. At the same time, stable data usage facilitates comparison of how different parameters affect algorithmic efficiency.

Another advantage of having all three parameters identical lies in ensuring the invariance of the geographical correlation mathematical form. When $h = r$, the geographical weight for each geographical reference point used by a single measurement point is:

$$w_j(s_i) = K\left(\frac{d_{ji}}{h}\right) = K\left(\frac{d_{ji}}{r}\right)$$

Mathematically, this result indicates that the functional form of geographical weights is similar across sampling circles of different radii, and the function value at the outermost circle is always $K\left(\frac{r}{r}\right) = K(1) = \exp\left(-\frac{1}{2}\right)$. Specifically, the weight of geographical reference points will depend only on their relative position in the radius within the sampling circle, independent of the specific magnitude of parameters. Therefore, we can exclude the influence of geographical correlation and discuss the impact of other technical or data factors on results.

Notably, although the NL-GWR code characteristics ensure that the total number of data points is nearly consistent under identical three-parameter settings, actual experiments revealed that larger sparse sampling stride leads to faster code execution. The possible reason is that the main computational consumption of the algorithm occurs in the nonlinear iterative solving process, which is primarily determined by the number of $\mathbf{x}$, that is, the number of regression calculation points. Although the geographically weighted structure introduces more points, the computation for this structure does not consume significant resources, ultimately making the algorithm's time complexity primarily determined by the sparse sampling step size rather than remaining relatively unchanged. This result indicates that when trading off parameter magnitudes, as long as precision requirements are met, appropriately increasing the stride, i.e., overall parameter growth, is beneficial for improving computational efficiency as shown below:

Table A1: Computation time under different NL-WR/NL-GWR parameter selection schemes.

Setting	Average Computation Time (s)
NL-WR	105.05



$h=r=p=1$	108.15
$h=r=p=2$	31.80
$h=r=p=5$	9.89
$h=r=p=10$	8.67
$h=r=p=20$	8.78

Note: Tests were conducted using an 11th Gen Intel(R) Core(TM) i5-1135G7 @ 2.40GHz (2.42 GHz) processor with 16.0 GB RAM. The code for time measurement only includes assembling the computation matrix and performing the operations until convergence. Each configuration was tested 5 times, and the average value was taken. To avoid potential performance
 875 fluctuations caused by prolonged continuous computation, different parameter schemes were randomly shuffled during each round of testing.

A3 Monte Carlo Method

The idea of using Monte Carlo methods for uncertainty analysis is to perturb input variables of a system with random errors, repeat the experiment multiple times, and analyse the uncertainty range of output variables (Voytenko et al., 2015b). In
 880 scenarios where the system cannot be simply expressed using formulas, such as in this study where iterative solving is employed in the solution algorithm, Monte Carlo methods provide a simple and convenient experimental tool for error propagation analysis.

We focus primarily on the LOS velocity obtained from interferometric measurements, therefore only this velocity component is perturbed. In each iteration, we superimpose Gaussian noise with a standard deviation of 0.5 onto the two-
 885 dimensional LOS velocity data, while keeping the other four data components unchanged. Subsequently, velocity extraction is performed using the monostatic TRI method identical to the main text pattern, with a total of $N = 500$ iterations conducted. We then statistically analyze the pixel-wise standard deviation of the output speed, LOS velocity, and azimuth velocity across the 500 experiments.

A schematic diagram of the experiment is shown in Figure A2, illustrating the distribution of perturbed input data for a
 890 single pixel and the distribution of output results.

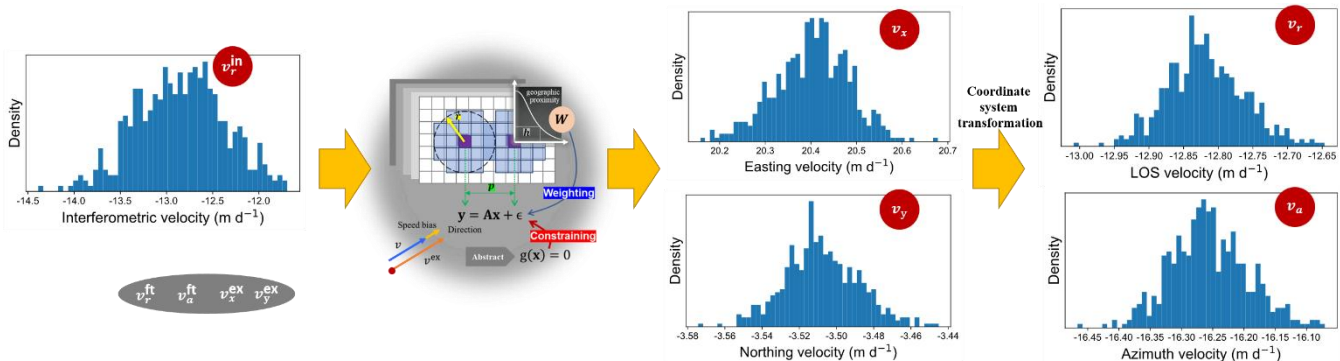




Figure A2: Schematic diagram of the Monte Carlo method, showing the perturbation process and results for a specific point.

Code and data availability

The code implementation of the proposed NL-GWR method is available at
895 https://github.com/RTC0Ritchie/NL_GWR_velocity. The datasets used for quality assessment are provided in the
preprocessData directory of the same repository. Sentinel-2A data used in this study are freely available from the Copernicus
Data Space Ecosystem (CDSE) operated by ESA: <https://dataspace.copernicus.eu>. ArcticDEM data used in this study are
provided by PGC at the University of Minnesota and are freely available at <https://www.pgc.umn.edu/data/arcticdem/>. Tidal
forecasted data is available from the Danish Meteorological Institute:
900 <http://www.dmi.dk/en/groenland/hav/tidevandstabeller-groenland/>. The complete Helheim Glacier velocity series dataset and
the original TRI observations can be provided from the corresponding author upon reasonable request.

Author contributions

CZ designed the methodology and algorithms, processed the data, and analyzed the results. XW initiated and led the study,
managed the data, and facilitated scientific discussions. YZ contributed to the interpretation and discussion of the results.
905 DMH and DH led the data acquisition campaign and provided guidance on interferometric processing. CZ prepared the
manuscript with contributions from all authors.

Competing interests

The authors have declared that none of the authors has any competing interests.

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