



A fast parameterization for polarized Monte Carlo radiative transfer simulations of the cloudglint

Anna Weber¹ and Bernhard Mayer¹

¹Meteorologisches Institut, Ludwig-Maximilians-Universität München, Munich, Germany

Correspondence: Anna Weber (Weber.Ann@physik.uni-muenchen.de)

Abstract. Ice crystals in cirrus clouds can orient themselves depending on their size, shape, and the hydrodynamical conditions. Ice crystal orientation affects the radiative impact of clouds and can introduce biases in retrievals of cloud properties. Horizontally oriented ice crystals such as plates, for example, specularly reflect sunlight and produce the so-called cloudglint. This paper introduces a parameterization of horizontally oriented ice crystals for Monte Carlo radiative transfer simulations, which is based on the treatment of reflection by a statistically rough ocean surface. The parameterization includes polarization and allows for comparably fast simulations of the cloudglint without having to store large, multidimensional tables of scattering phase functions. Oriented ice crystals are approximated by plates with infinitely extended top and base faces without sides and a Gaussian orientation angle distribution. The parameterization includes external reflections as well as an internal reflection and transmission through the ice crystal. Multiple scattering between different ice crystals is accounted for. The parameterization was validated by a comparison with cloudglint simulations of the raytracing algorithm CrystalTrace. Deviations between CrystalTrace and the parameterization for large solar zenith angles and large aspect ratios can be explained by the influence of the crystal sides, which are neglected in the parameterization. Possible applications of the parameterization include, for example, the detection of even small amounts of (oriented) ice crystals from polarized observations of the cloudglint or the improvement of retrievals of cloud properties.

1 Introduction

Evidence of oriented ice crystals in clouds was already found by Ono (1969). The riming properties inferred from cloud samples indicated preferred orientations of ice crystals depending on their shape. Platt (1978) found unexpectedly strong backscatter and low depolarization ratios in groundbased lidar observations, which could only be explained by oriented ice crystals. The possible horizontal orientation of planar ice crystals was confirmed by the study of the fall attitudes of ice crystals with photographs of light pillars performed by Sassen (1980). Falling ice crystals orient themselves to provide maximum resistance to the air. Their exact orientation depends on crystal size, shape, and the hydrodynamical conditions including turbulence, which destabilizes the alignment (Klett, 1995). While plates align horizontally with top and base facing up- and downwards, the preferred alignment of columns are sideward looking top and base faces.

Some optical phenomena can only be explained by the presence of oriented ice crystals. Horizontally oriented surfaces, e.g. from horizontally oriented plates in cirrus clouds, specularly reflect sunlight and produce the so-called cloudglint. Thus,



ice crystal orientation affects the radiative impact of cirrus clouds. Takano and Liou (1989a) found that oriented ice crystals can increase the cloud albedo by up to 40%. Chepfer et al. (1999) investigated the occurrence of specular reflection signals in data from the POLDER satellite and found oriented ice crystals in 40% of the observed cirrus clouds. In addition, cloudglint observations from the DSCOVR satellite were used to study ice crystal orientation (Marshak et al., 2017; Kostinski et al., 2026).

In radiative transfer simulations, ice crystals in clouds are usually assumed to be randomly oriented. Due to symmetries, the scattering phase matrix then reduces to a matrix with only six independent non-zero components and depends only on the scattering angle (van de Hulst, 1981). In contrast to randomly oriented particles, the scattering phase matrix for oriented ice crystals generally has 16 non-zero elements and depends on the incident and scattered directions as well as on the ice crystal orientation. For Mie scattering or randomly oriented ice crystals, where the scattering phase function depends on the scattering angle only, phase functions can be tabulated and scattered photon directions be sampled from the tables. However for oriented ice crystals, the tables would be multidimensional (depending on two incoming and two outgoing angles and on the crystal orientation) which requires considerably more storage space and makes the sampling of photon directions much more challenging. Retrievals of cloud optical properties usually assume randomly oriented ice crystals, which can introduce biases in the retrievals of cloud optical properties (Masuda and Ishimoto, 2004). At the same time, existing retrievals of ice crystal orientation applied single scattering approximations (e.g. Bréon and Dubrulle, 2004; Noel and Chepfer, 2004).

A common method for simulating the single scattering properties of oriented ice crystals is the geometric raytracing method, which was developed by Wendling et al. (1979) and applied to calculate the scattering phase functions or matrices of horizontally oriented ice crystals by e.g. Rockwitz (1989) and Borovoi et al. (2008). Other methods for computing the single scattering properties of oriented ice crystals include the finite-difference time domain method (e.g. Chen et al., 2006) and the T-matrix method (e.g. Baran et al., 2001). A database of the single scattering properties of horizontally oriented ice crystals is available from Saito and Yang (2019).

Takano and Liou (1989b) developed a radiative transfer model based on the adding method, which includes oriented ice crystals. Moreover, horizontally oriented ice crystals were included into a Monte Carlo radiative transfer model by Zhou et al. (2012) to simulate lidar signals. In addition, Prigarin (2009) and Mu et al. (2021) performed unpolarized simulations of optically anisotropic clouds with a Monte Carlo radiative transfer model. Besides, the geometric raytracing method was combined with Monte Carlo radiative transfer methods by Forster et al. (2025) in the CrystalTrace algorithm. In contrast to the aforementioned methods, this combined method allows for simulations of oriented ice crystals without having to store precomputed scattering phase functions. But it is computationally expensive for simulations with high angular resolution, needed for example to resolve narrow features like the cloudglint, and does not include polarization.

In this paper, a parameterization for fast Monte Carlo radiative transfer simulations of the cloudglint including polarization is presented. The parameterization is introduced in Sect. 2. Oriented ice crystals are approximated by plates with infinitely extended top and base faces. The parameterization includes external reflections at the crystal top as well as an internal reflection at the base, and transmission through both faces of the ice crystal. The local estimate technique is used to speed up radiance calculations. In addition, multiple scattering of photons by several different randomly or horizontally oriented ice crystals is

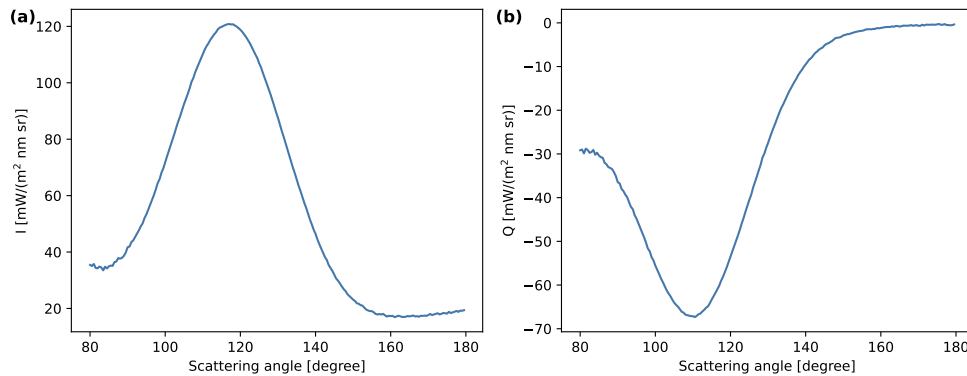


Figure 1. Cross section of a sunglint simulation along the scattering plane for a wavelength of 550nm, a solar zenith angle of 30° and a near surface wind speed of 5m/s. Panel (a) shows the total intensity I and panel (b) the Q component of the Stokes vector which describes linear polarization.

accounted for. Simulations of oriented ice crystals with the parameterization can be performed without having to store large, multidimensional tables of (polarized) scattering phase functions. The parameterization was implemented in the library for radiative transfer libRadtran (Mayer and Kylling, 2005; Emde et al., 2016) with the Monte Carlo solver MYSTIC (Mayer, 2009), which includes polarization (Emde et al., 2010) and spherical geometry (Emde and Mayer, 2007). It works in forward and backward mode. Moreover, the parameterization was validated by comparing simulation results to simulations performed with CrystalTrace in Sect. 3. Finally, the results are summarized in the conclusion.

2 Parameterization description

The basic idea of the parameterization is to treat specular reflection on horizontally oriented ice crystals similar to the reflection on a statistically rough ocean surface introduced by Cox and Munk (1954a, b). In the Monte Carlo approach, a photon reaching the surface can be reflected isotropically and weighted with the probability for reflection into the randomly chosen direction which is given by the bidirectional polarization distribution function (BPDF) respectively the ocean reflection matrix. An ocean reflection matrix describing the scattering on a surface with a Gaussian distribution of surface slopes including polarization in the Kirchhoff approximation was introduced by Tsang et al. (1985), implemented by Mishchenko and Travis (1997), and adapted for the parameterization. Figure 1 shows an example sunglint simulation using the ocean reflection matrix. the sunglint is visible as a broad maximum in the absolute intensity and is strongly polarized. More generally, the probability for reflection at a plane interface is given by the Fresnel coefficients for the respective incident angle and increases for large incident angles, see Fig. 2.

Whenever a photon is scattered on its path in MYSTIC, the scatter type is determined first. If the scattering is by an ice cloud, the photon can be scattered by oriented ice crystals with a probability of C_{oriented} , where C_{oriented} is the fraction of oriented

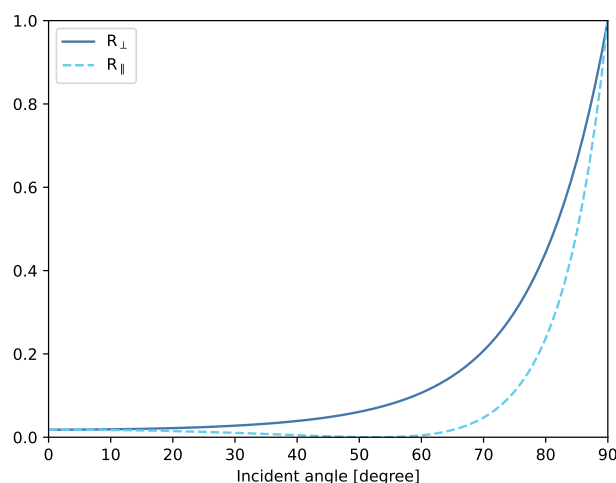


Figure 2. Parallel and perpendicular components of the Fresnel power reflection coefficients as a function of incident angle for a refractive index of 1.3 (as for ice at 550nm).

80 crystals. Otherwise, it is scattered by randomly oriented ice crystals, which is the default in MYSTIC. Typical oriented crystal fractions are below 10 % (Bréon and Dubrulle, 2004).

In case the photon is scattered by oriented ice crystals, it is scattered isotropically and weighted according to a BPDF, similarly to the treatment of reflection on a rough ocean surface. In general, specular reflection can occur on any horizontally oriented surface, and plates as well as columns can have preferred horizontal orientations. However, an analysis of Noel and
85 Chepfer (2010) showed that cloudglint producing cirrus clouds are mostly found at temperatures between -30 to -10 °C, where plates are the dominant habit (Bailey and Hallett, 2009; Takahashi, 2014). In addition, the total area of horizontally oriented surfaces is larger for plates than for columns. Therefore, oriented ice crystals were approximated by a plate with infinitely extended parallel top and base faces as indicated in Fig. 3.

When scattered by an oriented crystal, a photon can either be externally reflected at the top of the crystal (a), be transmitted
90 through the top and internally reflected at the base (b), or transmitted through the top followed by another transmission through the base of the ice crystal (c). The different interaction types and the concept of the parameterization are shown in Fig. 3 and described in more detail in the following Sect. 2.1 to 2.3. What happens is explicitly decided by drawing a random number and using importance sampling (Marchuk et al., 1980): Reflected photons are scattered isotropically and weighted with the BPDF for either internal or external reflection, similarly to the treatment of surface reflection. The photon direction remains
95 unchanged by transmission, so the scattered direction is set to the incident direction for this case. If polarization is considered, each photon is described by a Stokes vector. The transformation of the Stokes vector at an oriented scattering event is given by the external reflection, internal reflection, and transmission matrices for the randomly drawn interaction and scattered direction. In contrast to the reflection on the ocean surface, upward propagating photons can also be reflected back by the base of another oriented ice crystal.

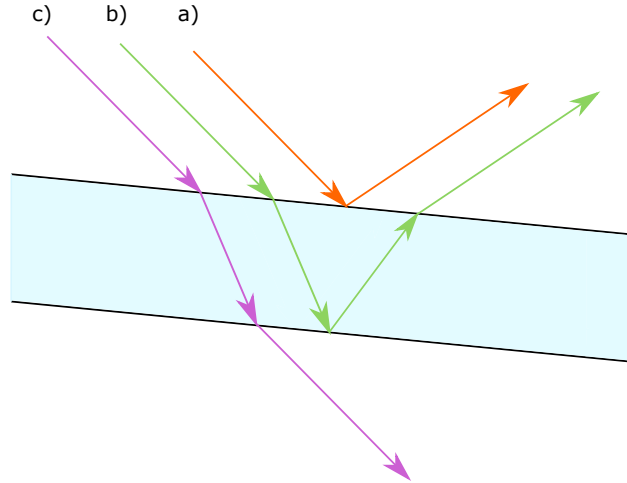


Figure 3. Interaction types included in the parameterization: a) external reflection, b) internal reflection, c) transmission.

100 In order to speed up radiance calculations, the local estimate technique was used and extended to account for oriented ice
crystals in the parameterization, which will be described in more detail in Sect. 2.4. At every interaction, the probability that
the photon is scattered into the direction of the detector and the extinction along the path to the detector are calculated. Here,
the extinction from the interaction point to the detector includes the contribution by transmission through horizontally oriented
ice crystals, which can be described by a delta-function in forward direction. To account for this, the extinction is reduced by
105 the contribution of transmission through oriented ice crystals.

2.1 External reflection

The simplest interaction is the external reflection at the crystal top. This case can be treated analogously to the reflection at a
rough ocean surface with the ocean reflection matrix in the Kirchhoff approximation using the stationary phase approximation,
which is given in the appendix of Mishchenko and Travis (1997), equations 62-86. The ocean reflection matrix includes
110 the Mueller reflection matrix, describing the transformation of the Stokes vector by reflection, as well as the probability for
scattering from a given incident to a given reflected direction, which is related to the distribution of surface slopes. For reflection
on oriented ice crystals, the refractive index of water has to be replaced by the refractive index of ice. The distribution of surface
slopes corresponds to the distribution of ice crystal orientation and is described by a Gaussian distribution (Mishchenko and
Travis, 1997):

$$115 \quad p\left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) = \frac{1}{2\pi s^2} \exp\left(-\frac{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}{2s^2}\right). \quad (1)$$

The mean square surface slope s^2 of the Gaussian distribution of surface slopes is calculated from the width of the angular
distribution of crystal orientations σ_β :

$$s^2 = \tan(\sigma_\beta)^2. \quad (2)$$



β is the deviation angle from the horizontal, often also referred to as tilt angle. σ_β is the width of the Gaussian distribution of tilt angles or the orientation angle corresponding to the mean surface slope, sometimes also called characteristic tilt angle. Typical widths of the angular distribution of the cloudglint were found to be in the order of 1° (Noel and Sassen, 2005). The total matrix describing the external reflection is given by (Mishchenko and Travis, 1997):

$$\mathbf{R}_{\text{ext}} = \frac{1}{4 \cos \theta_i \cos \theta_r} \frac{1}{2s^2} \frac{\|\mathbf{k}_d\|^4}{k_{dz}^4} \exp \left(-\frac{k_{dx}^2 + k_{dy}^2}{2k_{dz}^2 s^2} \right) \mathbf{R}(\cos \theta_i, \cos \theta_r, \phi_r - \phi_i), \quad (3)$$

where θ_i , ϕ_i and θ_r , ϕ_r are the zenith and azimuth angles of the incident and reflected direction. $\mathbf{k}_d = (k_{dx}, k_{dy}, k_{dz})^T$ is the local surface normal for which the incident and reflected photon direction form a specular reflection, and $\mathbf{R}$ is the Stokes reflection matrix containing the Fresnel coefficients for this specular reflection (called $\mathbf{M}$ in Mishchenko and Travis (1997)). The prefactor includes the probability of the surface slope corresponding to the local surface normal and converts the matrix into a BPDF.

2.2 Internal reflection

As already discussed, cloudglints are mainly produced by oriented plates. Such a plate crystal consists of parallel top and base faces as well as side faces. A photon that is scattered by an oriented plate can not only be reflected at the top but also be transmitted through the top or sides and internally be reflected until it is transmitted again. If a photon is, for example, transmitted through the top, internally reflected at the base, and transmitted through the top again, it will also contribute to the cloudglint signal. In the parameterization, the plate crystal is approximated by infinitely extended top and base faces. Thus, the impact of the crystal sides is neglected. To account for reflections within the crystal, a single internal reflection is considered. For small zenith angles, the reflectivities according to Fresnel are small, and higher-order multiple reflections within the crystal do not contribute much (see Fig. 2). For larger zenith angles, the contribution of multiple reflections increases. However, a real crystal is not infinitely extended, and higher-order reflections might already happen outside the crystal.

The total interaction consists of three individual interactions: transmission followed by internal reflection and another transmission, which can all be described by different transmission and reflection matrices. A combined matrix for all three interactions can be obtained similarly to the external reflection matrix in the Kirchhoff approximation. The total interaction is symmetric, such that the local surface normal for which the incident and randomly chosen scattered direction form a specular reflection for the combined matrix is the same as for the external reflection and the internal reflection matrix can be computed with:

$$\mathbf{R}_{\text{int}} = \frac{1}{4 \cos \theta_i \cos \theta_r} \frac{1}{2s^2} \frac{\|\mathbf{k}_d\|^4}{k_{dz}^4} \exp \left(-\frac{k_{dx}^2 + k_{dy}^2}{2k_{dz}^2 s^2} \right) \mathbf{T}_2 \mathbf{R} \mathbf{T}_1. \quad (4)$$

$\mathbf{T}_2$, $\mathbf{R}$, and $\mathbf{T}_1$ are the Stokes matrices which contain the Fresnel coefficients for the geometry of specular reflection at a surface with the local surface normal $\mathbf{k}_d$ and depend on incident and reflected directions. $\mathbf{T}_1$ and $\mathbf{T}_2$ describe the transmission from air to ice and ice to air, respectively, and $\mathbf{R}$ the internal reflection on the ice-air interface at the base of the plate. The formula for the Stokes transmission matrix $\mathbf{T}$ describing the transformation of the Stokes vector for a transmission from air to ice or



150 ice to air can be found in Chowdhary (1999) or Tsang and Kong (2001) and the formula for $\mathbf{R}$ is the same as for the external reflection with reversed refractive indices. The photon directions after the transmission through the top can be calculated from the incident directions and the local surface normal using Snell's law.

2.3 Transmission

A photon transmitted through the top can again be transmitted at the base and not only be internally reflected. This becomes
155 important for larger optical depths and oriented crystal fractions, when multiple scattering by different ice crystals becomes more probable. The transmission through the top and base leaves the propagation direction of the photon unchanged. Thus, a photon transmitted through a first crystal and reflected at a second one can also contribute to the cloudglint. Therefore, transmission through an ice crystal was included in the parameterization.

The total transmission is described by the product of the transmission matrices for the first transmission from incident to
160 refracted direction at the air-ice interface and the second transmission from refracted direction back to the incident direction at the ice-air interface:

$$\mathbf{T} = \mathbf{T}_2 \mathbf{T}_1. \quad (5)$$

The matrix $\mathbf{T}_1$ is equal to $\mathbf{T}_2$ with reversed incident and refracted directions and reversed refractive indices. For the transmission case, the ice crystals were assumed to be exactly horizontally oriented, which should have only little impact on the results since
165 the transmission coefficients $(1 - R)$ change only slowly with angle except for very flat incidence, as can be seen in Fig. 2.

2.4 Consideration of transmission in the local estimate

To speed up radiance simulations in MYSTIC, the local estimate technique is used (Marshak and Davis, 2005). The probability that the photon is scattered into the direction of the detector and reaches the detector without becoming extinct can be calculated at every scattering point (see Fig. 4). The probability that a photon travels from its current location to the detector without
170 becoming extinct according to Bouguer-Lambert-Beer's law is $\exp(-\tau_{\text{ext,loc}})$, where $\tau_{\text{ext,loc}}$ is the extinction optical depth along the path from the photon location to the detector. In the case of oriented ice crystals, a photon that is scattered into the direction of the detector can afterwards be transmitted through one or more ice crystals and still reach the detector. However, the probability for forward scattering respectively transmission through an oriented ice crystal is described by a delta function. To account for this, the extinction optical depth $\tau_{\text{ext,loc}}$ for the local estimate was reduced by the contribution of transmission
175 through an oriented ice crystal:

$$\tau'_{\text{ext,loc}} = \tau_{\text{ext,loc}} - c_{\text{oriented}} \tau_{\text{sca,ic}} \sum_{n=1}^{\infty} p_n (c_{\text{oriented}} \tau_{\text{sca,ic}}) T_{\text{eff}}^{2n}. \quad (6)$$

In total, $\tau'_{\text{ext,loc}}$ is the extinction for the local estimate, including the transmission through oriented ice crystals. $c_{\text{oriented}} \tau_{\text{sca,ic}}$ is the optical depth of oriented ice crystals with the fraction of oriented ice crystals c_{oriented} and the optical depth for scattering on ice clouds $\tau_{\text{sca,ic}}$. $T_{\text{eff}} = \frac{1}{2}(T_{\perp} + T_{\parallel})$ is the effective Fresnel power transmission coefficient for transmission through a single

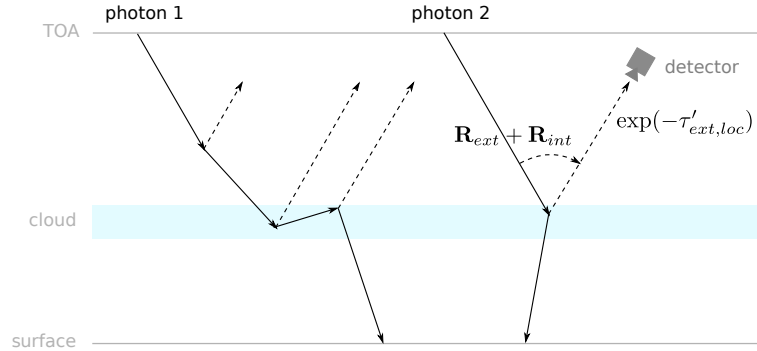


Figure 4. Basic principle of the local estimate technique for a one-dimensional atmosphere containing oriented ice crystals. Solid lines denote the photon path and dashed lines the local estimate.

180 interface, such that T_{eff}^2 describes the probability for transmission through both interfaces of an ice crystal, neglecting the change of polarization through transmission. To include polarization here, T_{eff} would have to be replaced by the transmission Mueller matrix and Stokes vectors be transformed accordingly as discussed in Sect. 3. As for the transmission case, the ice crystals are assumed to be horizontally oriented and the incident angles therefore given by the zenith angle of the detector. Finally, p_n is the probability for scattering exactly n times for a given optical depth τ :

$$185 \quad p_n(\tau) = \frac{1}{n!} \tau^n \exp(-\tau). \quad (7)$$

A derivation of this equation can be found in the Appendix.

The complete local estimate for oriented ice crystals is given by the sum of

$$\exp(-\tau'_{\text{ext,loc}}) / \cos \theta_d \cdot (\mathbf{R}_{\text{ext}} + \mathbf{R}_{\text{int}}) \cdot w \mathbf{I} \quad (8)$$

at every scattering event of a photon, which is described by $w\mathbf{I}$. The probability that the photon is scattered into the direction of the detector is included in the reflection matrices $\mathbf{R}_{\text{external}}$ or $\mathbf{R}_{\text{internal}}$. Since polarization is considered in the parameterization and reflection changes the polarization, the Stokes vector $\mathbf{I}$ has to be transformed by the reflection matrix of the randomly chosen interaction. In the unpolarized case, the reflection matrices would be replaced by the Fresnel power reflection coefficients. The extinction along the path from the photon location to the detector is given by $\tau'_{\text{ext,loc}}$. Thus, the factor $\exp(-\tau'_{\text{ext,loc}})$ is the probability that the photon reaches the detector from its current location. w is the photon weight, which is for example used for the importance sampling, and the factor $\cos \theta_d$ (where θ_d is the detector zenith angle) is needed due to the projected area in the definition of the radiance.

3 Results and discussion

In this section, simulation results of the cloudglint parameterization are analyzed and discussed. The parameterization does not include crystals sides. Ice crystals are assumed to consist of parallel top and base only. To analyze the impact of the crystal

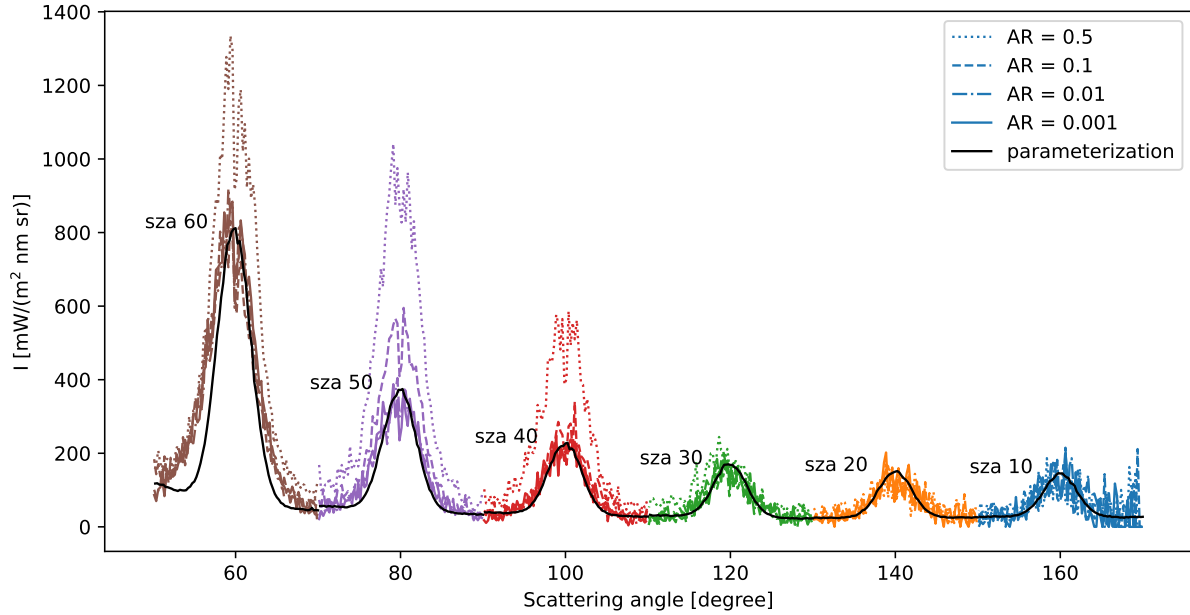


Figure 5. Comparison of the cloudglint parameterization (black) and CrystalTrace (colored) for different solar zenith angles and aspect ratios. The optical depth for all simulations was 1, the angular distribution width $\sigma_\beta = 1^\circ$, the wavelength 550nm, and the oriented crystal fraction 0.05. In addition, the CrystalTrace simulations were for 100% plates, while for the parameterization smooth plates from Yang et al. (2013) were used. Both, simulations with CrystalTrace and the parameterization were performed with the same angular resolution of 0.25° .

sides on the cloudglint and validate the parameterization, cloudglint simulation results of the parameterization were compared to raytracing simulations with CrystalTrace. CrystalTrace is a raytracing algorithm introduced by Forster et al. (2025), which combines the raytracing method for single scattering on oriented ice crystals with Monte Carlo radiative transfer methods. It is included in the Monte Carlo solver MYSTIC in libRadtran and allows the simulation of different mixtures of ice crystals with various shapes and orientations. Whenever a photon hits an ice crystal on its path through the atmosphere, it enters the crystal and is traced through the crystal with the raytracing method until it escapes and continues its journey through the atmosphere.

Cloudglint profiles obtained with the new parameterization were compared to raytracing simulations with CrystalTrace for different ice crystal aspect ratios and solar zenith angles in Fig. 5. Here, the aspect ratio is defined as $AR = \frac{L}{2a}$, where a is the radius of the top and L the length of the side face of a plate. Typical aspect ratios of around 0.2 to 0.3 and maximum dimensions of about 100 to $150\mu\text{m}$ were observed for plates in cirrus clouds by Um et al. (2015). Common dimensional relationships give aspect ratios of 0.1 to 0.2 for plates with the same size and decreasing aspect ratios with increasing size of the plates (Pruppacher and Klett, 1997). Since CrystalTrace does not include polarization, only the I-component of the Stokes vector was compared to the results from CrystalTrace.

Generally, the simulations with both CrystalTrace and the parameterization in Fig. 5 show good agreement. The agreement is better for smaller aspect ratios and smaller solar zenith angles. This is expected, since less photons encounter the crystal sides



215 in these cases and the relative importance of the crystal sides is therefore smallest. The increasing amplitudes of the cloudglint
with increasing solar zenith angle reflect the dependence of the Fresnel reflection coefficients on the incident angle (see Fig. 2).
The differences between the simulations for different aspect ratios with CrystalTrace in Fig. 5 indicate that the influence of the
crystal sides for different aspect ratios and solar zenith angles is very complex. For example, for larger solar zenith angles, the
cloudglint maximum is most pronounced at larger aspect ratios, whereas the differences between the simulations for different
220 aspect ratios are small for small solar zenith angles, which is the result of the complex influence of the crystal sides. At
larger solar zenith angles, increased reflection on the crystal sides and multiple scattering lead to increasing amplitudes of the
cloudglint for the larger aspect ratios.

The shape of the simulated cloudglints is Gaussian in the case of the parameterization and close to Gaussian also for CrystalTrace. Noel and Sassen (2005) found good agreement between their simple Gaussian model for the cloudglint shape and
225 observations. The full width at half maximum of the cloudglint simulations performed with the parameterization in Fig. 5
is around $2 \cdot 2\sqrt{2\log 2}\sigma_\beta$, as expected from the definition of the Gaussian distribution of orientation angles for the parameterization. The second factor 2 comes from the specular reflection. In summary, there are only small deviations between the
cloudglint parameterization and CrystalTrace, which can be explained by the influence of the crystal sides and consequently
depend on the ice crystal aspect ratio and the solar zenith angle.

230 At the same time, the simulations with the parameterization are significantly faster compared to CrystalTrace, when narrow
features like the cloudglint are computed with high angular resolution, since CrystalTrace simulations can only be performed
in forward mode. While this is reasonable if the complete sky radiance distribution is to be calculated, it is inefficient if
only some viewing angles are to be calculated. In contrast, the scattering angles of interest can be computed individually and
independently with the introduced parameterization. The simulations with the parameterization and CrystalTrace in Fig. 5 have
235 the same angular resolution of 0.25° . Depending on the solar zenith angle, the simulations with the parameterization needed
about 50 to 80 s on an Intel® Xeon(R) W-2245 CPU @ 3.90 GHz processor for the scattering angle range of 20° , performing
the simulations of the individual angles one after another. The standard deviations of the simulated radiances were around 2-3%.
On the other hand, the simulations with CrystalTrace took between 8 and 15 min for the different solar zenith angles on the same
processor. However, the mean standard deviation of the simulated radiances was around 10-50% for CrystalTrace, which is on
240 average a factor of about 10 larger than the standard deviation of the simulations with the parameterization. In order to achieve
the same accuracy with CrystalTrace, the number of photons would have had to be increased by a factor of $10 \cdot 10 = 100$, since
the relative standard deviation for Monte Carlo radiative transfer is proportional to $1/\sqrt{N}$, where N is the number of photons
(Mayer, 2009). Thus, CrystalTrace simulations with the same standard deviation as the simulations with the parameterization
would have taken about 800 to 1500 min, which is approximately 1000 times slower than the parameterization.

245 There are various options how the parameterization could be further improved in the future. First, polarization could be
considered in the local estimate. So far, the transmission through oriented ice crystals is accounted for by the effective Fresnel
power transmission coefficient in equation 6. In reality, transmission through ice crystals also changes the polarization. This
could be included by sampling the number of scatterings on oriented ice crystals between the current position of the photon
and the detector and multiplying the Stokes vectors with the total transmission matrix for every scattering. Moreover, instead



250 of assuming a horizontal orientation for the transmission in Sect. 2.3, the orientation distribution of ice crystals could also be included in the transmission matrices, similar to the reflection matrices. An orientation angle deviating from the horizontal leads to small changes of the incident angles and, therefore, also to small changes of the transmission coefficients. To account for this, the refracted direction could be chosen randomly and the total transmission matrix be weighted with the probability for the surface slope, which forms a specular transmission for this direction, similar to the treatment of reflection. However, this
255 would potentially increase the computation time, since many photons will be refracted into non-specular directions and thus have very small weights. In addition, the error due to the assumption of a horizontal orientation is small for typical small tilt angles, since the transmission coefficients change only slowly except for flat incidence (see Fig. 2). If cloudglint simulations are computed for the near-infrared wavelength range instead of the visible, absorption within an ice crystal becomes important. This could be included by reducing the photon weight, similar to the general treatment of absorption in the atmosphere.

260 It turned out, that there is the risk of spikes in the simulation results from the parameterization due to the very narrow probability distribution for specular reflection at horizontally oriented ice crystals, which can cause large photon weights in the local estimate. Typical widths of the angular distribution of crystal orientations are on the order of 1° (Noel and Sassen, 2005). Similar problems are also known for specular reflection at rough ocean surfaces with small wind speeds. Convergence could be increased and spikes suppressed by either using delta scaling or a variance reduction method. However, the former
265 is problematic for multiple scattering, while the development and implementation of the latter is very complex. The variance reduction method VROOM by Buras and Mayer (2011) suppresses spikes, which are caused by the strong forward scattering peak in the scattering phase function for ice clouds. However, by including horizontally oriented ice crystals that can specularly reflect radiation, there are two problematic directions with very narrow peaks, one in the transmission direction and the other one in the specular direction. This makes it very hard to optimize and reduce spikes and requires a new and extended variance
270 reduction method. To deal with the spikes, several runs of the simulations and an outlier detection can be performed. Even then, the parameterization is still significantly faster than CrystalTrace for simulations of narrow features like the cloudglint with a high angular resolution.

4 Conclusions

In this work, a parameterization for polarized Monte Carlo radiative transfer simulations of the cloudglint, which is formed
275 by scattering on horizontally oriented ice crystals, was introduced. The presented cloudglint parameterization allows very fast simulations of cloudglints compared to the raytracing algorithm CrystalTrace by Forster et al. (2025) and additionally includes polarization. The computation time is significantly lower if only a small angular range is simulated with high resolution, as for the cloudglint. In addition, it is not necessary to store large, multidimensional look-up tables of scattering phase functions for oriented ice crystals, as in other approaches. The mayor interaction types of solar radiation with an ice crystal, external
280 reflection, one internal reflection, and transmission, are accounted for. Moreover, the parameterization includes multiple scattering on different ice crystals. A Gaussian distribution of surface slopes respectively ice crystal orientation was assumed in the parameterization.



The parameterization was validated by a comparison of cloudglint simulations with the parameterization to simulations with CrystalTrace. There was generally good agreement between both methods. Small deviations between cloudglints simulated with CrystalTrace and with the parameterization can be explained by the neglect of the crystal sides in the parameterization. Horizontally oriented ice crystals are approximated by plates with infinitely extended top and base faces in the parameterization. The impact of the crystal sides on the cloudglint is very complex and depends on solar zenith angle, aspect ratio, and possible multiple scattering effects. Since the parameterization neglects the crystal sides, it is not possible to simulate other features caused by oriented ice crystals, such as subparhelia or subparhelic circles, besides the simulation of the cloudglint.

Possible applications of the parameterization include a retrieval of ice crystal orientation from polarized observations of the cloudglint. The observation of the cloudglint indicates the presence of horizontally oriented surfaces. The width of the cloudglint is related to the characteristic tilt angle of the ice crystals relative to the horizontal, while the amplitude of the cloudglint depends on the fraction of oriented ice crystals. In general, retrievals of cloud optical properties can be biased due to the assumption of randomly oriented ice crystals in most models. This bias can be reduced by including the presented parameterization of oriented ice crystals in the simulations used for the retrievals.

Code availability. The cloudglint parameterization and CrystalTrace are part of the open source version of the libRadtran library for radiative transfer and can be accessed at Mayer et al. (2026) (DOI: <https://doi.org/10.5281/zenodo.21935470>). However, we recommend obtaining the latest version of the library from <http://www.libradtran.org> (libRadtran, 2026).

Appendix A: Derivation of scattering probability

The equation for p_n , the probability for scattering exactly n times for a given optical depth τ , can be obtained as follows. The probability for zero scatterings corresponds to the transmission probability and is given by Bouger-Lambert-Beer's law

$$p_0(\tau) = \exp(-\tau). \quad (\text{A1})$$

Moreover, the probability for scattering once is determined by the probability for transmission from $\tau = 0$ to τ' , where the scattering takes place, and for transmission from τ' to τ integrated over all possible scattering locations τ' :

$$p_1(\tau) = \int_0^\tau \exp(-\tau') \exp(-(\tau - \tau')) d\tau' = \int_0^\tau \exp(-\tau) d\tau' = \tau \exp(-\tau). \quad (\text{A2})$$

Similarly, the probability that a photon is scattered twice can be calculated:

$$p_2(\tau) = \int_0^\tau \int_0^{\tau''} \exp(-\tau') \exp(-(\tau'' - \tau')) \exp(-(\tau - \tau'')) d\tau' d\tau'' = \int_0^\tau \int_0^{\tau''} \exp(-\tau) d\tau' d\tau'' = \frac{1}{2} \tau^2 \exp(-\tau). \quad (\text{A3})$$



Probabilities for larger numbers of scatterings can be obtained accordingly by inserting an additional integration for each scattering. This leads to the equation for the scattering probability $p_n(\tau)$:

$$310 \quad p_n(\tau) = \frac{1}{n!} \tau^n \exp(-\tau). \quad (\text{A4})$$

Author contributions. AW and BM developed the concept of the parameterization. AW implemented the parameterization and wrote the manuscript with input from all co-authors.

Competing interests. The authors declare that they have no conflict of interest.

315 *Acknowledgements.* We would like to thank Linda Forster for help and explanations concerning CrystalTrace. In addition, we thank Claudia Emde for valuable explanations concerning polarization in general and in MYSTIC. This research was supported by the German Research Foundation (DFG) within the project SPP 1294 under project number 442667104.



References

- Bailey, M. P. and Hallett, J.: A Comprehensive Habit Diagram for Atmospheric Ice Crystals: Confirmation from the Laboratory, AIRS II, and Other Field Studies, *Journal of the Atmospheric Sciences*, 66, 2888 – 2899, <https://doi.org/10.1175/2009JAS2883.1>, 2009.
- 320 Baran, A. J., Francis, P. N., Havemann, S., and Yang, P.: A study of the absorption and extinction properties of hexagonal ice columns and plates in random and preferred orientation, using exact T-matrix theory and aircraft observations of cirrus, *J. Quant. Spectrosc. Radiat. Transfer*, 70, 505–518, 2001.
- Borovoi, A. G., Burnashov, A. V., and Oppel, U.: Scattering matrices for horizontally oriented ice crystals, *Journal of Quantitative Spectroscopy and Radiative Transfer*, 109, 2648–2655, <https://doi.org/10.1016/j.jqsrt.2008.06.002>, 2008.
- 325 Bréon, F.-M. and Dubrulle, B.: Horizontally Oriented Plates in Clouds, *Journal of the Atmospheric Sciences*, 61, 2888 – 2898, <https://doi.org/10.1175/JAS-3309.1>, 2004.
- Buras, R. and Mayer, B.: Efficient unbiased variance reduction techniques for Monte Carlo simulations of radiative transfer in cloudy atmospheres: The solution, *Journal of Quantitative Spectroscopy and Radiative Transfer*, 112, 434–447, <https://doi.org/10.1016/j.jqsrt.2010.10.005>, 2011.
- 330 Chen, G., Yang, P., Kattawar, G. W., and Mishchenko, M. I.: Scattering phase functions of horizontally oriented hexagonal ice crystals, *Journal of Quantitative Spectroscopy and Radiative Transfer*, 100, 91–102, <https://doi.org/10.1016/j.jqsrt.2005.11.029>, VIII Conference on Electromagnetic and Light Scattering by Nonspherical Particles, 2006.
- Chepfer, H., Brogniez, G., Goloub, P., Bréon, F. M., and Flamant, P. H.: Observations of horizontally oriented ice crystals in cirrus clouds with POLDER-1/ADEOS-1, *Journal of Quantitative Spectroscopy and Radiative Transfer*, 63, 521–543, [https://doi.org/10.1016/S0022-4073\(99\)00036-9](https://doi.org/10.1016/S0022-4073(99)00036-9), 1999.
- 335 Chowdhary, J.: Multiple Scattering of Polarized Light in Atmosphere-Ocean Systems, Ph.D. thesis, Columbia University, 1999.
- Cox, C. and Munk, W.: Measurement of the Roughness of the Sea Surface from Photographs of the Sun's Glitter, *J. Opt. Soc. Am.*, 44, 838–850, <https://doi.org/10.1364/JOSA.44.000838>, 1954a.
- Cox, C. and Munk, W.: Statistics of the sea surface derived from sun glitter, *Journal of Marine Research*, 13, 198–227, 1954b.
- 340 Emde, C. and Mayer, B.: Simulation of solar radiation during a total eclipse: a challenge for radiative transfer, *Atmospheric Chemistry and Physics*, 7, 2259–2270, <https://doi.org/10.5194/acp-7-2259-2007>, 2007.
- Emde, C., Buras, R., Mayer, B., and Blumthaler, M.: The impact of aerosols on polarized sky radiance: model development, validation, and applications, *Atmospheric Chemistry and Physics*, 10, 383–396, <https://doi.org/10.5194/acp-10-383-2010>, 2010.
- Emde, C., Buras-Schnell, R., Kylling, A., Mayer, B., Gasteiger, J., Hamann, U., Kylling, J., Richter, B., Pause, C., Dowling, T., and Bugliaro, L.: The libRadtran software package for radiative transfer calculations (version 2.0.1), *Geoscientific Model Development*, 9, 1647–1672, <https://doi.org/10.5194/gmd-9-1647-2016>, 2016.
- 345 Forster, L., Weber, A., and Mayer, B.: CrystalTrace: a Monte Carlo raytracing algorithm for radiative transfer in cirrus clouds with oriented ice crystals, *Atmospheric Measurement Techniques*, 18, 7853–7863, <https://doi.org/10.5194/amt-18-7853-2025>, 2025.
- Klett, J. D.: Orientation Model for Particles in Turbulence, *Journal of Atmospheric Sciences*, 52, 2276 – 2285, [https://doi.org/10.1175/1520-0469\(1995\)052<2276:OMFPIT>2.0.CO;2](https://doi.org/10.1175/1520-0469(1995)052<2276:OMFPIT>2.0.CO;2), 1995.
- 350 Kostinski, A., Marshak, A., and Várnai, T.: Constraining orientation statistics of ice crystals in clouds with observations from deep space, *Frontiers in Remote Sensing*, Volume 6 - 2025, <https://doi.org/10.3389/frsen.2025.1705235>, 2026.
- libRadtran: libRadtran - Homepage, <https://www.libradtran.org>, last access: 14 august 2026, 2026.



- Marchuk, G. I., Mikhailov, G. A., Nazaraliev, M. A., Darbinjan, R. A., Kargin, B. A., and Elepov, B. S.: The Monte Carlo Methods in
355 Atmospheric Optics, Springer, Berlin, Heidelberg, <https://doi.org/10.1007/978-3-540-35237-2>, 1980.
- Marshak, A. and Davis, A.: 3D Radiative Transfer in Cloudy Atmospheres, Springer, iSBN-13 978-3-540-23958-1, 2005.
- Marshak, A., Várnai, T., and Kostinski, A.: Terrestrial glint seen from deep space: Oriented ice crystals detected from the Lagrangian point,
Geophysical Research Letters, 44, 5197–5202, <https://doi.org/10.1002/2017GL073248>, 2017.
- Masuda, K. and Ishimoto, H.: Influence of particle orientation on retrieving cirrus cloud properties by use of total and polarized reflectances
360 from satellite measurements, J. Quant. Spectrosc. Radiat. Transfer, 85, 183–193, 2004.
- Mayer, B.: Radiative transfer in the cloudy atmosphere, EPJ Web of Conferences, 1, 75–99, <https://doi.org/10.1140/epjconf/e2009-00912-1>,
2009.
- Mayer, B. and Kylling, A.: Technical note: The libRadtran software package for radiative transfer calculations - description and examples of
use, Atmospheric Chemistry and Physics, 5, 1855–1877, <https://doi.org/10.5194/acp-5-1855-2005>, 2005.
- 365 Mayer, B., Emde, C., Kylling, A., Gasteiger, J., Jakub, F., and Weber, A.: libRadtran - library for radiative transfer - version 2.0.6-cloudglint,
<https://doi.org/10.5281/zenodo.21935470>, 2026.
- Mishchenko, M. I. and Travis, L. D.: Satellite retrieval of aerosol properties over the ocean using polarization as well as intensity of reflected
sunlight, J. Geophys. Res., 102, 16 989–17 013, <https://doi.org/10.1029/96JD02425>, 1997.
- Mu, Q., Kablukova, E. G., Kargin, B. A., and Prigarin, S. M.: Monte Carlo simulation of halos in the crystal clouds, Journal of Physics:
370 Conference Series, 2099, 012 067, <https://doi.org/10.1088/1742-6596/2099/1/012067>, 2021.
- Noel, V. and Chepfer, H.: Study of Ice Crystal Orientation in Cirrus Clouds Based on Satellite Polarized Radiance Measurements, Journal of
the Atmospheric Sciences, 61, 2073 – 2081, [https://doi.org/10.1175/1520-0469\(2004\)061<2073:SOICOI>2.0.CO;2](https://doi.org/10.1175/1520-0469(2004)061<2073:SOICOI>2.0.CO;2), 2004.
- Noel, V. and Chepfer, H.: A global view of horizontally oriented crystals in ice clouds from Cloud-Aerosol Lidar and Infrared Pathfinder
Satellite Observation (CALIPSO), Journal of Geophysical Research, 115, D00H23, <https://doi.org/10.1029/2009JD012365>, 2010.
- 375 Noel, V. and Sassen, K.: Study of planar ice crystal orientations in ice clouds from scanning polarization lidar observations, Journal of
Applied Meteorology, 44, 653–664, 2005.
- Ono, A.: The Shape and Riming Properties of Ice Crystals in Natural Clouds, Journal of Atmospheric Sciences, 26, 138 – 147,
[https://doi.org/10.1175/1520-0469\(1969\)026<0138:TSARPO>2.0.CO;2](https://doi.org/10.1175/1520-0469(1969)026<0138:TSARPO>2.0.CO;2), 1969.
- Platt, C. M. R.: Lidar Backscatter from Horizontal Ice Crystal Plates, Journal of Applied Meteorology and Climatology, 17, 482 – 488,
380 [https://doi.org/10.1175/1520-0450\(1978\)017<0482:LBFHIC>2.0.CO;2](https://doi.org/10.1175/1520-0450(1978)017<0482:LBFHIC>2.0.CO;2), 1978.
- Prigarin, S. M.: Numerical simulation of halos in crystal clouds by Monte Carlo method, Russian Journal of Numerical Analysis and Math-
ematical Modelling, 24, 481–493, <https://doi.org/doi:10.1515/RJNAMM.2009.031>, 2009.
- Pruppacher, H. R. and Klett, J. D.: Microphysics of Clouds and Precipitation, Kluwer Academic Publishers, Dordrecht, 2., rev. and enl. ed.
edn., 1997.
- 385 Rockwitz, K.-D.: Scattering properties of horizontally oriented ice crystal columns in cirrus clouds. Part 1, Appl. Opt., 28, 4103–4110,
<https://doi.org/10.1364/AO.28.004103>, 1989.
- Saito, M. and Yang, P.: Oriented Ice Crystals: A Single-Scattering Property Database for Applications to Lidar and Optical Phenomenon
Simulations, Journal of the Atmospheric Sciences, 76, 2635 – 2652, <https://doi.org/10.1175/JAS-D-19-0031.1>, 2019.
- Sassen, K.: Remote Sensing of Planar Ice Crystal Fall Attitudes, Journal of the Meteorological Society of Japan. Ser. II, 58, 422–429,
390 https://doi.org/10.2151/jmsj1965.58.5_422, 1980.



- Takahashi, T.: Influence of Liquid Water Content and Temperature on the Form and Growth of Branched Planar Snow Crystals in a Cloud, *Journal of the Atmospheric Sciences*, 71, 4127 – 4142, <https://doi.org/10.1175/JAS-D-14-0043.1>, 2014.
- Takano, Y. and Liou, K.-N.: Solar Radiative Transfer in Cirrus Clouds. Part I: Single-Scattering and Optical Properties of Hexagonal Ice Crystals, *Journal of Atmospheric Sciences*, 46, 3 – 19, [https://doi.org/10.1175/1520-0469\(1989\)046<0003:SRTICC>2.0.CO;2](https://doi.org/10.1175/1520-0469(1989)046<0003:SRTICC>2.0.CO;2), 1989a.
- 395 Takano, Y. and Liou, K.-N.: Solar Radiative Transfer in Cirrus Clouds. Part II: Theory and Computation of Multiple Scattering in an Anisotropic Medium, *Journal of Atmospheric Sciences*, 46, 20 – 36, [https://doi.org/10.1175/1520-0469\(1989\)046<0020:SRTICC>2.0.CO;2](https://doi.org/10.1175/1520-0469(1989)046<0020:SRTICC>2.0.CO;2), 1989b.
- Tsang, L. and Kong, J. A.: *Scattering of Electromagnetic Waves - Advanced Topics*, John Wiley, New York, 2001.
- Tsang, L., Kong, J. A., and Shin, R. T.: *Theory of Microwave Remote Sensing*, John Wiley, New York, 1985.
- 400 Um, J., McFarquhar, G. M., Hong, Y. P., Lee, S.-S., Jung, C. H., Lawson, R. P., and Mo, Q.: Dimensions and aspect ratios of natural ice crystals, *Atmospheric Chemistry and Physics*, 15, 3933–3956, <https://doi.org/10.5194/acp-15-3933-2015>, 2015.
- van de Hulst, H.: *Light Scattering by Small Particles*, Dover Books on Physics, Dover Publications, 1981.
- Wendling, P., Wendling, R., and Weickmann, H. K.: Scattering of solar radiation by hexagonal ice crystals, *Appl. Opt.*, 18, 2663–2671, <https://doi.org/10.1364/AO.18.002663>, 1979.
- 405 Yang, P., Bi, L., Baum, B. A., Liou, K.-N., Kattawar, G. W., Mishchenko, M. I., and Cole, B.: Spectrally Consistent Scattering, Absorption, and Polarization Properties of Atmospheric Ice Crystals at Wavelengths from 0.2 to 100 μm , *Journal of the Atmospheric Sciences*, 70, 330 – 347, <https://doi.org/10.1175/JAS-D-12-039.1>, 2013.
- Zhou, C., Yang, P., Dessler, A. E., Hu, Y., and Baum, B. A.: Study of Horizontally Oriented Ice Crystals with CALIPSO Observations and Comparison with Monte Carlo Radiative Transfer Simulations, *Journal of Applied Meteorology and Climatology*, 51, 1426 – 1439, <https://doi.org/10.1175/JAMC-D-11-0265.1>, 2012.
- 410