Response to RC1 of egusphere-2026-4795

We want to thank the first referee of our paper for the review and the detailed comments. In the following, we address the given line comments (in blue) including the changes that we made to the manuscript:

l.143 In Eq.10 what does the subscript A in f^_A,i stand for?

The subscript "A" was referring to the scaling factor in Eq. (11), as the factor is related to the signal fraction "f_A" as in Eq. (13). Nevertheless, we renamed it to f_β to relate it directly to the extinction coefficient.

l.135 Eq.9 was first introduced by Breon and Goloub (1998), so it would be appropriate to cite that paper here.

We added the citation as suggested.

l.155 Eq.12 assumes that N = 3 without explicitly stating this, while in all previous equations N can be any number. Please, clarify.

Thanks for noting, with regard to the comment further below, we renamed the number of modes to "M", such that we changed the parameter in the respective equation to "M" as all the given equations hold in general for M modes.

ll.157-159 I don't understand Eq.13 at all. If all A_i are the same, then according to Eq.12 A_tot = N*A_i, and f_A,i = A_i/A_tot = 1/N and does not depend on i (while f^_A,i obviously does). Or what is "general scaling factor" _within_ A_i? Please, explain.

We tried to clarify the relevant section (including the renaming from the number of modes from "N" to "M") to

"As already observed by Alexandrov et al. (2015) from the application of the Rainbow Fourier Transform, the DSD can have a pronounced multimodal shape and decomposed the retrieved DSD into several single-mode gamma distributions. Assuming now that the DSD can be written as a superposition of M single-mode distributions $n(r) = \sum_{i=1}^{M} n_i(r)$, we can rewrite Eq. (5) as

$$\overline{P_{12}}(\theta) = \sum_{i=1}^{M} \frac{\int_0^\infty \sigma_{\text{sca}}(r) n_i(r) dr}{\int_0^\infty \sigma_{\text{sca}}(r) n(r) dr} \frac{\int_0^\infty \sigma_{\text{sca}}(r) P_{12}(r,\theta) n_i(r) dr}{\int_0^\infty \sigma_{\text{sca}}(r) n_i(r) dr} \equiv \sum_{i=1}^{M} \frac{\beta_{\text{ext},i}}{\beta_{\text{ext,tot}}} \overline{P_{12}^{i}} \equiv \sum_{i=1}^{M} f_{\beta,i} \overline{P_{12}^{i}}.$$

Hence, the observed scattered polarized radiation can be expressed in terms of a superposition of the polarized radiation scattered by the single distributions. The fraction that the individual size distribution contributes to the total signal $f_{\beta,i}$ equals hereby the fraction of the extinction

coefficient $\beta_{ext,i}$ to the total extinction coefficient $\beta_{ext,tot}$. Note that this holds in general for any arbitrary shape of the individual size distributions $n_i(r)$. However, assuming that the individual distributions obey the shape of single-mode gamma distributions, we introduce the fit function in the presence of multiple modes in accordance to Eq. (9) as

$$Q_{\text{fit}} = \sum_{i=1}^{M} A_i \cdot P_{12}^{i}\left[r_{\text{eff},i}, v_{\text{eff},i}\right](\theta) + B \cdot \cos^2(\theta) + C.$$

Comparing equations (10) and (11) suggests that the fit factors $A_i \propto f_{\beta,i}$, but they incorporate for the conversion between the phase matrix elements to the measured radiance Q, as A does in Eq.(9). For example, we do not convert the measured Q to the polarized reflectance. Further, the measured polarization can be reduced due to ice or aerosol particles within or above the cloud target. This allows to define a "signal fraction" f_A as the normalized fitted amplitude of each mode

$$f_{A,i} = \frac{A_i}{A_{\text{tot}}}, \quad A_{\text{tot}} = \sum_{i=1}^{M} A_i$$

where A_i refers to the fit parameter of Eq.(11). A_{tot} is hereby the normalization factor which ensures that $\sum_{i=1}^{M} f_{A,i} = 1$. Provided that the multiplicative scaling factor within A_i is the same for all modes, we get

$$f_{A,i} = \frac{A_i}{A_{\text{tot}}} \approx f_{\beta,i} = \frac{\beta_{\text{ext},i}}{\beta_{\text{ext,tot}}} = \frac{\tau_i}{\tau_{\text{tot}}},$$

where $f_{\beta,i}$ is the same factor as in Eq.(10) and τ_i and τ_{tot} denote the optical thickness of mode i and the total optical thickness, respectively. The assumption of a common multiplicative scaling factor is motivated by the saturation of the polarized cloud signal at sufficiently large liquid optical thicknesses, typically around 3–5 (Weber et al., 2025b). Then, the scaling factor may become less sensitive to further increases in optical thickness and is mainly determined by the conversion from polarized radiance (Q) to reflectance and external factors, such as depolarizing aerosol or ice particles above or within the clouds, respectively and the normalization of the phase functions. Further, we assume here that we observe a homogeneously mixed target, such that $\beta_{\text{ext},i}/\beta_{\text{ext,tot}} = \tau_i/\tau_{\text{tot}}$ holds."

ll.169-173 Denoting droplet number concentration by the same letter N that was already used for the number of DSD's modes is confusing. Please, chose different notation.

As already stated above, we changed the number of DSD modes to "M" and reserved "N" for the droplet number concentration.

1.185 Again yet another parameter is denoted by N. I suggest to use subscripts to avoid confusion.

Thanks for noting. We renamed the number of terms calculated in the Mie series to “N_l” referring to the Legendre Polynomials.

l.195 Lambda should be 1/(2 r_eff v_eff) to be consistant with Eq.6, since variable in Eq.21 is D = 2 r.

Thanks for noting, the correction was applied.

ll.256-262 It is not clear what "adding" another mode means. What information from the previous step is retained? If an actual bimodal distribution has, say, 5 um mode and 15 um modes the monomodal fit would return ~10 um mode, that should be split into two on the next step. But if results of the monomodal fit are not used, the bimodal unconstrained fit would be in the space of 6 parameters (3 for each mode) and is likely to be a very ill-posed problem requiring regularization (that is not included in the presented algorithm). Please, clarify.

From the first mode, the effective radius and variance are used as input, when another mode is added. However, it is not set as the final result, but we also allow the neighboring values in the LUT. Then, for every possible combination, the fit parameters (A_i and B and C) are jointly fitted again. This might seem to introduce a bias in the first mode at a first glance, but as already observed by Alexandrov et al. (2012) for a two-layer cloud, the single mode fit is tightly coupled to the dominant mode, and will not result in an average size distribution, which we also verified by using simple 1D radiative transfer simulations with regard to the posed question. Hence, with respect to the example of the reviewer, the retrieval of the first mode will derive reff ~5 µm, if the 5 µm mode is dominant or ~15 µm, when the 15 µm mode dominates the signal. Of course, if one of the modes is much weaker than the other, the RMSE of the fit might already be sufficiently low, such that only a single mode might be retrieved.

Therefore, adding a new mode is not an unconstrained global search over the full multimode parameter space. It is a sequential, locally restricted search. The local search windows limit the parameter space and thereby provide a practical regularization of the otherwise poorly constrained problem, although no explicit regularization term is included.

We tried to clarify the corresponding section to

“While the single mode fit function from Eq.(9) has five free parameters for the fit (A, B, C, reff, and veff), every additional mode in the fit function after Eq.(11) adds another three free parameters to the parameter space, because the background B and C is fitted only once. Hence, it would be ambiguous to search the whole lookup table (LUT) for each mode and we decided on a strategy, which subsequently adds modes to the fit and searches the LUT in three different stages. First, an independent one-mode search is performed within the multimode retrieval by performing a coarse search through the LUT, which keeps every sixth effective radius and effective variance values. Since the fit is much more sensitive to reff than veff, we look for the best fitting effective radii for the central effective variance of the kept values first. The results for the five best effective radius fits are then combined with all possible effective variances to find again the five best candidates. Then, the fit is refined in a secondary coarse search around these

best results from the first coarse search, which takes every second reff and veff in a window that includes the two neighbors of the coarsened LUT in both dimensions (reff and veff) around the found result into account. Again, the five best candidates from the second stage fit are kept, to perform a final search, which takes all candidates within a window of size two (±2 neighbors) around those results. Hereby, the result of the first mode found can be interpreted as the dominant mode generating the signal and does not resemble an average over the whole multimodal distribution, as already observed by Alexandrov et al. (2012a).

The result of the first mode serves as a starting point for sequentially adding up to a predefined maximum number of modes n_m. For the new mode, the fit strategy described above is repeated, while the results for the reff and veff of the single best fit from the preceding step are used as centers for local search windows. Hence, when adding a second mode, the possible reff and veff values of the coarsened LUT are combined with the neighboring values of the LUT of the best reff and veff of the first mode. For each combination, the fit parameters Ai, B, and C are refitted to minimize the objective function (here: RMSE), and the best result is kept to perform the secondary coarse search and the fine search with the newly added mode.

Modes are added until either the predefined maximum number of modes is reached or the newly obtained fit satisfies the RMSE threshold (currently 1.2) for all fitted color channels. A newly added mode is accepted only when the objective function does not worsen and the fitted mode amplitudes are positive for all fitted color channels. If the new fit worsens the objective for any of the color channels, the preceding best valid fit is retained.

Because different combinations of phase functions can produce similar angular radiance curves, the multimode inverse problem may admit several comparably good solutions. Hence, it should be mentioned that the result might describe a local minimum rather than the global minimum which could lead to errors in the resulting droplet size distributions. Further, the resulting solution is conditional on the LUT, search-window strategy, objective function, and positivity constraints. However, first applications of the current fit strategy show promising results, which are presented in Sec. 4."

Fig.9 Panel (a) is not mentioned in the caption.

We changed the second sentence in the caption to "Panel (a) shows the RGB image taken by the camera, while the cloud top height is projected to the image in panel (b)."

ll.448-450 and Figs.12-15 While initial RFT distribution may be noisy and include negative values, these artifacts are removed by mode decomposition. The satellite dataset from PACE HARP-2 instrument does not include the initial RFT curve but only mode decomposition parameters (currently for 2 modes). This also significantly reduces the data volume.

Thus, for "apple to apple" comparison I strongly recommend changing the RFT representation in Figs.12-15 with solid dark-blue curves representing mode decomposition results, while the initial curve may be depicted by light color possibly dashed (or entirely removed).

Actually, comparisons of multimodal-fit distributions with RFT mode decomposition results look quite good, so emphasizing them would only benefit the paper. Of coarse, due to the limited radius range RFT cannot see rain droplets.

We adjusted the representation of the lines in Figs. 12-15: the mode decomposition of the RFT is now shown in solid dark blue lines and the initial area distribution from the RFT is depicted in light dashed blue lines.

Also, RFT mode decomposition algorithm rejects any mode with fraction less then 5% (0.05), while the rain modes retrieved by the fitting technique have fractions around 0.01 and DSD maxima so small that they are visible only on the plot with logarithmic y-axis. However, the rainbow curves in (b) panels are consistent with presence of rain droplets. Are there in situ measurements demonstrating that such weak rain modes are real?

This is a fair question, but when it comes to droplet number concentrations it is quite expected that rain droplets are much fewer in numbers than cloud droplets. If one thinks of the liquid water content that is available to form rain droplets, it is limited by the liquid water content that originally forms the cloud droplets before precipitation occurs (or the ice water content for cold rain, but both fall back to the available water in the atmosphere). This can also be seen from Eq. (15): the droplet number concentration scales with LWC/(k*reff^3), thus, droplet numbers will be much lower at the same LWC. For example, looking at the often used parametrization of rain droplet size distributions by Marshall and Palmer (1948), an integration over the size distribution as given in Eq. (21) yields the total number concentration for radii between D_{min} and D_{max}

$$\mathrm{N}(D_{min} \leq D \leq D_{max}) = \frac{N_0}{\Lambda}(e^{-\Lambda D_{min}} - e^{-\Lambda D_{max}})$$

Hence, for example, a rain rate of R = 25 mm^{-1}h^{-1} results in about 0.003 droplets per cm^3 or 3000 droplets per m^3 if one integrates from 0.1 to 5 mm. On the other hand, typical cloud droplet number concentrations are more on the order of 50-1000 droplets per cm^3, dependent on the aerosol concentration (e.g., Martin et al., 1994). Martin et al. (1994) show also droplet size spectra of drizzling stratocumulus clouds (Fig. 13b), where the droplet number concentration is about 10^5 lower at 100 µm compared to 10 µm. Hence, those fractions are even smaller compared to the ones we observe in the examples. It should also be noted, that we show the area size distributions in panels c and d of Fig. 12-15, thus, they include a factor 100 between 10 µm and 100 µm.

Reference:

Martin, G. M., D. W. Johnson, and A. Spice, 1994: The Measurement and Parameterization of Effective Radius of Droplets in Warm Stratocumulus Clouds. *J. Atmos. Sci.*, **51**, 1823–1842, https://doi.org/10.1175/1520-0469(1994)051<1823:TMAPOE>2.0.CO;2.

Monomodal curves in Figs.12-15 are plotted in yellow and are barely seen on the white background. I suggest using green instead.

We changed the color of the single mode curves to green.

Response to RC2 of egusphere-2026-4795

We want to thank the second referee of our paper for the review and comments, which are addressed in detail in the following. The original comments are shown in blue, while our answers are given in black.

General Comments:

"A multimodal approach to derive cloud microphysics in deep convective clouds from polarized measurements of the cloudbow" presents a new algorithm capable of retrieving up to three cloud droplet size distribution modes over a large droplet size range. The algorithm was demonstrated using data from specMACS during the 2024 PERCUSSION air campaign. The multimodal retrievals were compared to results from the Rainbow Fourier Transfer (RFT) algorithm and generally show good agreement in the shorter radius range that RFT provides.

This paper is generally well-written and represents a substantial contribution to the field. I recommend it be accepted after clarifying questions are addressed and minor changes to improve the clarity of figures and mathematical formulae/notation are implemented.

Specific comments (by line)

1-20: The abstract doesn't address the cloud droplet effective variance which is also being retrieved in this algorithm. This is particularly valuable given the application to precipitating clouds.

We changed the abstract to include the following:
l. 2ff: "The retrieval allows up to three modes in the droplet size distribution (DSD) and hence, is situated between a single mode retrieval and the Rainbow Fourier Transform (RFT), which derives an arbitrary distribution." -> "The retrieval builds onto the existing single mode parametric approach, but allows up to three modes in the droplet size distribution (DSD), where each mode is described by a gamma distribution of effective radius r_{eff} and effective variance v_{eff}.

We further added the following in l. 10: "For the effective variance, increased uncertainties are found both for $r_{eff} \lesssim 3\mu m$ and $r_{eff} \gtrsim 200\mu m$. In between, the determination of the effective variance is also possible with an uncertainty of less than 50%."

24-26: Citation needed for "The cooling effect…and the size and shape of the cloud particles".

We added Chapter 8 of "An introduction to Atmospheric Radiation" (Liou, 2002) as a citation.

111: Equation 1 is the form in Bohren and Huffman. Although this book is cited, this should be made clear.

We removed the citations of Hansen and Travis and Grandy, as Bohren and Huffman is the relevant one following their notation, and included the citation of the relevant Section: "In general, for radiation traveling along the positive z-axis the scattered electric field vector $E_{sc} = (E_l,E_r)^T_{sc}$ is related to the incoming electric field vector $E_{in} = (E_l,E_r)^T_{in}$ by the scattering amplitude matrix (e.g., Bohren and Huffman, 1998, Sec. 3.2).."

143: It is unclear what A{A,i} is in equation 10. Please ensure that variables used in all equations are clearly defined upon the first instance of their use.

We are not entirely sure what the reviewer is referring to here, but we think that $f_{A,i}$ is meant. We changed the name to $f_{\beta,i}$, to better relate it to the extinction coefficient. The factor, however, was already explained in the manuscript in l. 145f and the explanatory sentence now states with the new notation "The fraction that the individual size distribution contributes to the total signal $f_{\beta,i}$ equals hereby the fraction of the extinction coefficient $\beta_{ext,i}$ to the total extinction coefficient $\beta_{ext,to}$ when absorption is negligible."

152: Do you have any insight on how the algorithm handles the change in slope of the polarized phase function due to, for example, ice contamination?

The algorithm is influenced in a similar manner as the single mode parametric retrieval. As long as the shape of the cloudbow/rainbow profile does not change, the retrieval results remain robust. Of course, when there is too much ice, within the same cloud volume or above, and the signal is too weak, the retrieval will be biased similar to the parametric fit. We verified that by simple 1D simulations, but will not show this in this paper as the information context would get too large.

155: Equation 3 limits the sum to 3 but this isn't really discussed in this section. It was mentioned in the abstract but should be stated clearly here.

Thanks for noting. That was a typo and changed to "M", since all the equations in that section hold in general for M modes. (The number of modes in the sum notations was changed to M, to keep "N" for the droplet number concentrations)

158: It's not clear to me where equation 13 comes from.

We tried to clarify the relevant section to

"As already observed by Alexandrov et al. (2015) from the application of the Rainbow Fourier Transform, the DSD can have a pronounced multimodal shape and decomposed the retrieved DSD into several single-mode gamma distributions. Assuming now that the DSD can be written as a superposition of M single-mode distributions $n(r) = \sum_{i=1}^{M} n_i(r)$, we can rewrite Eq. (5) as

$$\overline{P_{12}}(\theta) = \sum_{i=1}^{M} \frac{\int_0^\infty \sigma_{\text{sca}}(r) n_i(r) dr}{\int_0^\infty \sigma_{\text{sca}}(r) n(r) dr} \frac{\int_0^\infty \sigma_{\text{sca}}(r) P_{12}(r,\theta) n_i(r) dr}{\int_0^\infty \sigma_{\text{sca}}(r) n_i(r) dr} \equiv \sum_{i=1}^{M} \frac{\beta_{\text{ext},i}}{\beta_{\text{ext,tot}}} \overline{P_{12}^i} \equiv \sum_{i=1}^{M} f_{\beta,i} \overline{P_{12}^i}.$$

Hence, the observed scattered polarized radiation can be expressed in terms of a superposition of the polarized radiation scattered by the single distributions. The fraction that the individual size distribution contributes to the total signal $f_{\beta,i}$ equals hereby the fraction of the extinction coefficient $\beta_{ext,i}$ to the total extinction coefficient $\beta_{ext,tot}$. Note that this holds in general for any arbitrary shape of the individual size distributions $n_i(r)$. However, assuming that the individual distributions obey the shape of single-mode gamma distributions, we introduce the fit function in the presence of multiple modes in accordance to Eq. (9) as

$$Q_{\text{fit}} = \sum_{i=1}^{M} A_i \cdot P_{12}^{i}\left[r_{\text{eff},i}, v_{\text{eff},i}\right](\theta) + B \cdot \cos^2(\theta) + C.$$

Comparing equations (10) and (11) suggests that the fit factors $A_i \propto f_{\beta,i}$, but they incorporate for the conversion between the phase matrix elements to the measured radiance Q, as A does in Eq.(9). For example, we do not convert the measured Q to the polarized reflectance. Further, the measured polarization can be reduced due to ice or aerosol particles within or above the cloud target. This allows to define a "signal fraction" f_A as the normalized fitted amplitude of each mode

$$f_{A,i} = \frac{A_i}{A_{\text{tot}}}, \quad A_{\text{tot}} = \sum_{i=1}^{M} A_i$$

where A_i refers to the fit parameter of Eq.(11). A_{tot} is hereby the normalization factor which ensures that $\sum_{i=1}^{M} f_{A,i} = 1$. Provided that the multiplicative scaling factor within A_i is the same for all modes, we get

$$f_{A,i} = \frac{A_i}{A_{\text{tot}}} \approx f_{\beta,i} = \frac{\beta_{\text{ext},i}}{\beta_{\text{ext,tot}}} = \frac{\tau_i}{\tau_{\text{tot}}},$$

where $f_{\beta,i}$ is the same factor as in Eq.(10) and τ_i and τ_{tot} denote the optical thickness of mode i and the total optical thickness, respectively. The assumption of a common multiplicative scaling factor is motivated by the saturation of the polarized cloud signal at sufficiently large liquid optical thicknesses, typically around 3−5 (Weber et al., 2025b). Then, the scaling factor may become less sensitive to further increases in optical thickness and is mainly determined by the conversion from polarized radiance (Q) to reflectance and external factors, such as depolarizing aerosol or ice particles above or within the clouds, respectively and the normalization of the phase functions. Further, we assume here that we observe a homogeneously mixed target, such that $\beta_{\text{ext},i}/\beta_{\text{ext,tot}} = \tau_i/\tau_{\text{tot}}$ holds."

169-170: The variable N is already used for the droplet size distribution modes.

As noted above, we changed the number of droplet size modes to "M", such that the droplet number concentrations remains "N".

185: Here is another different use of N. The authors should take care with clarity of notation and avoid using the same variables in different ways.

We renamed that occurrence of "N" to N_l referring to the number of Legendre Polynomials that have to be calculated for the convergence of the Mie series.

193: Strange to switch to diameter when the rest of the paper consistently uses radius.

Here, we kept the diameter notation as the Marshall-Palmer distribution is written this way in most literature, but we added that D = 2r and the notation of the distribution with the radius as variable.

245-262: Presumably the best first fit would give the same result as the typical monomodal parametric retrieval. If subsequent modes are added to compensate and improve the fit, can't you get multiple solutions? Please clarify this section.

We agree that different multimode combinations can produce similar angular radiance curves and that the multimode inverse problem does not necessarily have a unique solution. This is also explicitly stated in the manuscript now. For the initial one-mode search, the coarse and secondary-coarse stages retain several promising candidates, and these candidate regions are refined in the subsequent search stages. When additional modes are introduced, the parameters of the previously fitted modes are initially centered on the best result from the preceding mode-addition step. The new mode is then searched over the coarsened LUT, and all amplitudes and background parameters are refitted for each candidate combination. The best candidate according to the selected objective function is retained, subject to the positivity and stopping criteria described in the manuscript.

Consequently, the reported multimode solution is the best solution found within the specified LUT, search-window, and sequential-search strategy. We have added this qualification to the revised description:

"While the single mode fit function from Eq.(9) has five free parameters for the fit (A, B, C, reff, and veff), every additional mode in the fit function after Eq.(11) adds another three free parameters to the parameter space, because the background B and C is fitted only once. Hence, it would be ambiguous to search the whole lookup table (LUT) for each mode and we decided on a strategy, which subsequently adds modes to the fit and searches the LUT in three different stages. First, an independent one-mode search is performed within the multimode retrieval by performing a coarse search through the LUT, which keeps every sixth effective radius and effective variance values. Since the fit is much more sensitive to reff than veff, we look for the best fitting effective radii for the central effective variance of the kept values first. The results for the five best effective radius fits are then combined with all possible effective variances to find again the five best candidates. Then, the fit is refined in a secondary coarse search around these best results from the first coarse search, which takes every second reff and veff in a window that includes the two neighbors of the coarsened LUT in both dimensions (reff and veff) around the found result into account. Again, the five best candidates from the second stage fit

are kept, to perform a final search, which takes all candidates within a window of size two (±2 neighbors) around those results. Hereby, the result of the first mode found can be interpreted as the dominant mode generating the signal and does not resemble an average over the whole multimodal distribution, as already observed by Alexandrov et al. (2012a).

The result of the first mode serves as a starting point for sequentially adding up to a predefined maximum number of modes n_m. For the new mode, the fit strategy described above is repeated, while the results for the reff and veff of the single best fit from the preceding step are used as centers for local search windows. Hence, when adding a second mode, the possible reff and veff values of the coarsened LUT are combined with the neighboring values of the LUT of the best reff and veff of the first mode. For each combination, the fit parameters Ai, B, and C are refitted to minimize the objective function (here: RMSE), and the best result is kept to perform the secondary coarse search and the fine search with the newly added mode.

Modes are added until either the predefined maximum number of modes is reached or the newly obtained fit satisfies the RMSE threshold (currently 1.2) for all fitted color channels. A newly added mode is accepted only when the objective function does not worsen and the fitted mode amplitudes are positive for all fitted color channels. If the new fit worsens the objective for any of the color channels, the preceding best valid fit is retained.

Because different combinations of phase functions can produce similar angular radiance curves, the multimode inverse problem may admit several comparably good solutions. Hence, it should be mentioned that the result might describe a local minimum rather than the global minimum which could lead to errors in the resulting droplet size distributions. Further, the resulting solution is conditional on the LUT, search-window strategy, objective function, and positivity constraints. However, first applications of the current fit strategy show promising results, which are presented in Sec. 4."

Figure 4: Figure 4 is not referenced in the text.

The figure was already referenced in the text in l. 267f: "The individual steps of the retrieval algorithm and the fit strategy are summarized in Fig. 4."

Figure 5b: There is artificial looking dark line at retrieved v_eff~0.025. Can you address what aspects of the algorithm might be causing that?

To begin with, we are really grateful that the reviewer noted this and pointed this out. We found out that this was actually related to an issue with the v_{eff} = 0.01 phase functions (the lowermost in our table) for r_{eff} > 200 µm, which propagated some "NaN" values in single wavelengths when integrating over the spectral response function of the cameras. Now, the dark line shifted to the lowermost effective variance bin, which relates to the insensitivity of large r_{eff} to v_{eff}, as can be seen from Fig. 2, such that the algorithm ends at one or the other end of the LUT. We verified, that particularly the large r_{eff} (approx. 500 µm and larger) contribute to that stripe, which is also why the wrong phase functions made the stripe move up one bin. We corrected this and reproduced the Figs. 2, 5, 6, and 10-15 with the correct data.

Further, we added the following to the description of Fig. 5b in the text:

"Further, it can be seen that the retrieved effective variances accumulate both in the smallest (v_{eff} = 0.01) and the largest (v_{eff} = 0.4) effective variance bins, which is mainly due to insensitivities of the phase functions for large effective radii as seen in Fig. 2."

Figure 7b: The RGB of DOLP is very dark and it's hard to see the cloudbow. The authors might consider additional image processing to make it more apparent.

Thanks for the comment. We processed the DOLP RGB such that the cloud/rainbow is better visible.

Figure 9a: Panel a in figure 9 is not mentioned in the caption.

We changed the second sentence in the caption to "Panel (a) shows the RGB image taken by the camera, while the cloud top height is projected to the image in panel (b)."

Figures 12-15: The yellow dashed line for single mode in panels c and d is very hard to see.

We changed the color to green.

569-570, 598,647: References that cite an entire textbook should be updated so they only cite the relevant sections/chapters.

We added the relevant sections in the citations within the text.

Technical corrections:

Please address the following grammar issues/ typos:

75: "…Atlantic before transferred to…"

Changed this to: "…Atlantic before being transferred to…"

227: "It exemplary shows the effect…"

Changed to "It illustrates the effect…"

294:" …might be caused because of hitting…"

Corrected to "..might be caused by reaching…"

337:" … but the sideward viewing geometry allows to observe rain…"

Changed this to "…but the sideward viewing geometry allows the observation of rain.."

573: Missing accent in "Bréon"

Applied the correction, thanks for noting!

Response to RC3 of egusphere-2026-4795

We want to thank the referee of our paper for the review and comments. Below are our responses (black) to the general and specific comments of the reviewer (in blue):

This study provides a new algorithm for polarimetric cloud retrievals, though it is based on the same physical basis as Bréon et al.'s parametric retrieval and Alexandrov et al.'s RFT retrieval. The authors describe the similarities and differences and emphasize the advantages of their approach over the others. The paper is well written and shows substantial potential for contributing to and improving the existing body of work. In short, great work. I think that, after addressing the questions and points raised, this work would be a excellent addition to the field.

That said, following are my general comments, followed by some line by line comments,

1. I'd rather see this algorithm described as an extension to parametric fitting. This is because, even in the "recipe" (Fig. 4), the extended, multi-mode retrieval is only performed when the single-mode parametric retrieval suggests a "suspicious quality assessment." This is not a criticism. In fact, I believe it's a smart way to improve existing methods. However, I'd suggest mentioning this appropriately in the abstract, i.e., noting that the multi-mode extension is applied when single-mode retrievals are unreasonable, and the method itself is extension of parametric retrievals, rather than a new technique between parametric and RFT.

   Thanks for the comment. We rephrased the beginning of the abstract to

   "We present a new method to derive droplet size distributions in clouds consisting of multiple modes using aircraft based polarized measurements of the cloudbow. The retrieval builds onto the existing single mode parametric approach, but allows up to three modes in the droplet size distribution (DSD), where each mode is described by a gamma distribution of effective radius r_{eff} and effective variance v_{eff}. While the DSDs of shallow cumulus or stratiform clouds are often well described by single mode distributions, this does not hold in the presence of drizzle or rain and the measured polarized radiance is not accurately described by a single mode fit. We show that allowing multiple modes in those cases not only improves the fit significantly but also shows good agreement to the Rainbow Fourier Transform (RFT), which derives an arbitrary distribution."

2. Another important missing part, for me, is how important angular resolution is for performing such retrievals up to very large CER values. When the CER is large, cloudbow features become very narrow. It would be a natural, practical question to ask what the minimal angular resolution would need to be to perform such 3-mode retrievals. I couldn't find any information about the angular resolution of specMACS used here. In addition, a

sensitivity study on the importance of viewing-angle resolution would greatly strengthen the work.

We agree, that the angular resolution might play an important role in resolving particularly the rainbow of larger droplets and performed a first simple analysis based on simulations. While the retrieval of the cloud mode (r_{eff} = 10 µm) works reasonably well for angular resolutions up to about 2° (dependent on the optical thickness ratio between cloud and rain droplets), the retrieval of the rain mode becomes less accurate for angular resolutions less than about 1°. Since this paper, however, deals rather with the general implementation of the multimode retrieval and the demonstration of its capability, we leave a full sensitivity study for future work. Nonetheless, we clarified the section about the resolution of specMACS in the introduction and added the angular resolution on which the retrieval is performed:

"Prior to the EUREC4A campaign the instrument has been extended by two downward looking polarization resolving Phoenix 5.0 MP Polarization Model cameras (Pörtge et al., 2023). In 2024, two more sideward-facing polarization cameras were integrated, providing an overall field of view of about 91° x 220° (along x across track) for the observation of deep convective clouds reaching flight altitudes during the PERCUSION campaign. Each camera covers a field of view of about 91° x 78° and has a resolution of 2448 x 2048 pixels (along x across track, Weber et al., 2024). Moreover, the cameras provide simultaneous measurements at a frame rate of 5−8Hz resulting in a high temporal resolution. Together, this allows to aggregate the scattered polarization signal from cloud targets at a spatial resolution of about 100 x 100m and an angular resolution of 0.3° (Pörtge et al., 2023)."

3. Another point I'd argue is that polarimeters only see the top of the clouds. So, is it possible to see big droplets at such high levels? I agree with the point made (around line 356) that it is possible to see raindrops from the side with sufficient angular resolution, or that this kind of method would generally be useful for satellite retrievals. However, in this particular case study shown in Fig. 8, I can't shake the feeling that it's unclear whether we can really see those large droplets in the regions where Fig. 10 shows multi-mode retrievals. Perhaps you could check how the stereographic cloud-top retrievals shown in Fig. 9 (b) can be overlaid on Fig. 8(b), along the track where radar observations are available, and then show the multi-mode retrievals from Fig. 10 on the same figure. In this way, if the multi-mode retrievals coincide with regions where larger droplets are closer to the identified cloud top, it would strengthen the argument that the algorithm is correctly identifying and performing multi-mode retrievals where large droplets exist near the cloud top.

Thank you for this thought. We added a new figure to the manuscript (now Fig. 12), which relates the retrieval results around the radar beam to the radar reflectivity and the optical thickness measured by the WALES lidar, which was also operated onboard the aircraft

during the PERCUSION campaign. We plot the timeline of the effective radius results for the different modes and the corresponding heights of the observed cloud targets, which shows that specMACS indeed penetrates to the areas with enhanced radar reflectivity, while the heights still coincide with the optical thickness measurement of the lidar, i.e. the mean height within the cone is around an optical thickness of one or where the lidar is attenuated.

We added the following description of the figure to the manuscript:

"A remaining question to consider is whether large droplets can really be seen at the higher cloud tops as indicated by Fig. 10 and Fig. 9b. To this end, Fig. 12 combines the cloud targets from specMACS within the 1° cone around the radar beam with the radar measurements and optical thickness measurements of the WAter vapor Lidar Experiment in Space (WALES; Wirth et al., 2009; Wirth and Groß, 2025). In panel (a) we show the radar reflectivity as in Fig. 8b, but this time overlayed with the cloud top heights (mean, minimum, and maximum) of the retrieved targets within the cone. The heights of the specMACS cloud targets are partly well below the radar heights (excluding the cirrus above), i.e. towards the upper left of the deeper convective cloud, which indicates that specMACS is sensitive to targets deeper within the cloud. The optical thickness at 532nm measured by WALES is shown in panel (b) together with the corresponding cloud target heights, illustrating that the mean height of the retrieval points from specMACS is well in line with the attenuation of the lidar at optical thicknesses around one. Thus, this might be evidence that specMACS penetrates into the areas where the radar shows an enhanced reflectivity and large droplets might be present as the optical thickness above is low. In panel (c), we show the corresponding effective radii derived for the up to three modes. The dominant mode depicted in red shows targets with values up to about 100µm around 13:50 UTC, exactly where the mean of the specMACS cloud target heights coincides with the higher radar reflectivity."

line 5: "Since current look-up tables ..." At this point readers would have no clue what meant by "current" here. I'd suggest avoid doing that.

We changed the respective sentence to "The parametric retrieval fits polarized phase functions derived by Mie theory to the measured polarized radiance, such that, for the retrieval of large droplets, new Mie calculations were performed to include effective radii up to 1000 µm."

line 40: "phase tables" or "phase function table" ??

We clarified the wording and changed "phase table(s)" to "phase functions table(s)" throughout the whole manuscript.

line 106: "Theoretical basics" or "Theoretical basis"?

We renamed the section to "Theoretical framework".

line 324: ".., while there is hardly any *overestimation* for ..."

The missing “overestimation” was set in.

line 397: Why do the cloud top height uncertainties matter here? Is it because all observations are mapped to cloud top before performing retrievals? If so, I think this would be a major pint worth mentioning explicitly.

The cloud top height is important for the aggregation of the signal, i.e. if one follows a cloud target through multiple images to derive the angularly resolved cloudbow signals. If the cloud top heights were wrong, the aggregated signal would be wrong and potentially contaminated by signals from different cloud parts or the ocean surface. We extended the existing sentence to clarify the role of the cloud top heights:

“A reason for this might be other uncertainties in the aggregation of the measured signal, e.g. due to the development of the cloud, convective movement, which is not captured in the mean wind from ERA5 data used in the single mode retrieval (Pörtge et al., 2023), or potential uncertainties in the cloud top height, which are needed for the correct identification of cloud targets in subsequent images.”

line 443: "... RFT is its current implementation only retrieves radii up to 100 um .." Wouldn't it be possible to extend RFT to those larger CER values, so that we could see whether RFT can also handle them?

Yes, in principle one could extend the RFT to larger radii, however, as we argue in the introduction (l. 90ff. of the original preprint), the current implementation of the RFT is based on an equally spaced database (0.5 µm in radius at an angular resolution of 0.2°), such that a huge database would be required. Further, the oscillations in the phase functions of the individual phase functions increase for larger radii such that it would need to be tested, whether an angular resolution of 0.2° would still be sufficient. Another point that would need to be addressed, are the corrections introduced in Alexandrov et al. (2012) for the original transformation with radii up to 100 µm. Altogether, we agree that it would be very interesting to extend the RFT to larger radii, and will leave it to future work to investigate whether it would be possible.