

Response to RC2 of egusphere-2026-4795

We want to thank the second referee of our paper for the review and comments, which are addressed in detail in the following. The original comments are shown in blue, while our answers are given in black.

General Comments:

“A multimodal approach to derive cloud microphysics in deep convective clouds from polarized measurements of the cloudbow” presents a new algorithm capable of retrieving up to three cloud droplet size distribution modes over a large droplet size range. The algorithm was demonstrated using data from specMACS during the 2024 PERCUSSION air campaign. The multimodal retrievals were compared to results from the Rainbow Fourier Transfer (RFT) algorithm and generally show good agreement in the shorter radius range that RFT provides.

This paper is generally well-written and represents a substantial contribution to the field. I recommend it be accepted after clarifying questions are addressed and minor changes to improve the clarity of figures and mathematical formulae/notation are implemented.

Specific comments (by line)

1-20: The abstract doesn't address the cloud droplet effective variance which is also being retrieved in this algorithm. This is particularly valuable given the application to precipitating clouds.

We changed the abstract to include the following:

l. 2ff: “The retrieval allows up to three modes in the droplet size distribution (DSD) and hence, is situated between a single mode retrieval and the Rainbow Fourier Transform (RFT), which derives an arbitrary distribution.” -> “The retrieval builds onto the existing single mode parametric approach, but allows up to three modes in the droplet size distribution (DSD), where each mode is described by a gamma distribution of effective radius r_{eff} and effective variance V_{eff} .”

We further added the following in l. 10: “For the effective variance, increased uncertainties are found both for $r_{\text{eff}} \lesssim 3\mu\text{m}$ and $r_{\text{eff}} \gtrsim 200\mu\text{m}$. In between, the determination of the effective variance is also possible with an uncertainty of less than 50%.”

24-26: Citation needed for “The cooling effect...and the size and shape of the cloud particles”.

We added Chapter 8 of “An introduction to Atmospheric Radiation” (Liou, 2002) as a citation.

111: Equation 1 is the form in Bohren and Huffman. Although this book is cited, this should be made clear.

We removed the citations of Hansen and Travis and Grandy, as Bohren and Huffman is the relevant one following their notation, and included the citation of the relevant Section: “In general, for radiation traveling along the positive z-axis the scattered electric field vector $E_{sc} = (E_i, E_r)^T$ is related to the incoming electric field vector $E_{in} = (E_i, E_r)^T$ by the scattering amplitude matrix (e.g., Bohren and Huffman, 1998, Sec. 3.2)..”

143: It is unclear what $A\{A, i\}$ is in equation 10. Please ensure that variables used in all equations are clearly defined upon the first instance of their use.

We are not entirely sure what the reviewer is referring to here, but we think that $f_{A,i}$ is meant. We changed the name to $f_{\beta,i}$, to better relate it to the extinction coefficient. The factor, however, was already explained in the manuscript in l. 145f and the explanatory sentence now states with the new notation “The fraction that the individual size distribution contributes to the total signal $f_{\beta,i}$ equals hereby the fraction of the extinction coefficient $\beta_{ext,i}$ to the total extinction coefficient $\beta_{ext,to}$ when absorption is negligible.”

152: Do you have any insight on how the algorithm handles the change in slope of the polarized phase function due to, for example, ice contamination?

The algorithm is influenced in a similar manner as the single mode parametric retrieval. As long as the shape of the cloudbow/rainbow profile does not change, the retrieval results remain robust. Of course, when there is too much ice, within the same cloud volume or above, and the signal is too weak, the retrieval will be biased similar to the parametric fit. We verified that by simple 1D simulations, but will not show this in this paper as the information context would get too large.

155: Equation 3 limits the sum to 3 but this isn't really discussed in this section. It was mentioned in the abstract but should be stated clearly here.

Thanks for noting. That was a typo and changed to “M”, since all the equations in that section hold in general for M modes. (The number of modes in the sum notations was changed to M, to keep “N” for the droplet number concentrations)

158: It's not clear to me where equation 13 comes from.

We tried to clarify the relevant section to

“As already observed by Alexandrov et al. (2015) from the application of the Rainbow Fourier Transform, the DSD can have a pronounced multimodal shape and decomposed the retrieved DSD into several single-mode gamma distributions. Assuming now that the DSD can be written as a superposition of M single-mode distributions $n(r) = \sum_{i=1}^M n_i(r)$, we can rewrite Eq. (5) as

$$\overline{P_{12}}(\theta) = \frac{\sum_{i=1}^M \int_0^\infty \sigma_{sca}(r) n_i(r) dr}{\int_0^\infty \sigma_{sca}(r) n(r) dr} \frac{\int_0^\infty \sigma_{sca}(r) P_{12}(r, \theta) n_i(r) dr}{\int_0^\infty \sigma_{sca}(r) n_i(r) dr} \equiv \sum_{i=1}^M \frac{\beta_{ext,i}}{\beta_{ext,tot}} \overline{P_{12}^i} \equiv \sum_{i=1}^M f_{\beta,i} \overline{P_{12}^i}.$$

Hence, the observed scattered polarized radiation can be expressed in terms of a superposition of the polarized radiation scattered by the single distributions. The fraction that the individual size distribution contributes to the total signal $f_{\beta,i}$ equals hereby the fraction of the extinction coefficient $\beta_{ext,i}$ to the total extinction coefficient $\beta_{ext,tot}$. Note that this holds in general for any arbitrary shape of the individual size distributions $n_i(r)$. However, assuming that the individual distributions obey the shape of single-mode gamma distributions, we introduce the fit function in the presence of multiple modes in accordance to Eq. (9) as

$$Q_{fit} = \sum_{i=1}^M A_i \cdot P_{12}^i[r_{eff,i}, v_{eff,i}](\theta) + B \cdot \cos^2(\theta) + C.$$

Comparing equations (10) and (11) suggests that the fit factors $A_i \propto f_{\beta,i}$, but they incorporate for the conversion between the phase matrix elements to the measured radiance Q , as A does in Eq.(9). For example, we do not convert the measured Q to the polarized reflectance. Further, the measured polarization can be reduced due to ice or aerosol particles within or above the cloud target. This allows to define a "signal fraction" f_A as the normalized fitted amplitude of each mode

$$f_{A,i} = \frac{A_i}{A_{tot}}, \quad A_{tot} = \sum_{i=1}^M A_i$$

where A_i refers to the fit parameter of Eq.(11). A_{tot} is hereby the normalization factor which ensures that $\sum_{i=1}^M f_{A,i} = 1$. Provided that the multiplicative scaling factor within A_i is the same for all modes, we get

$$f_{A,i} = \frac{A_i}{A_{tot}} \approx f_{\beta,i} = \frac{\beta_{ext,i}}{\beta_{ext,tot}} = \frac{\tau_i}{\tau_{tot}},$$

where $f_{\beta,i}$ is the same factor as in Eq.(10) and τ_i and τ_{tot} denote the optical thickness of mode i and the total optical thickness, respectively. The assumption of a common multiplicative scaling factor is motivated by the saturation of the polarized cloud signal at sufficiently large liquid optical thicknesses, typically around 3–5 (Weber et al., 2025b). Then, the scaling factor may become less sensitive to further increases in optical thickness and is mainly determined by the conversion from polarized radiance (Q) to reflectance and external factors, such as depolarizing aerosol or ice particles above or within the clouds, respectively and the normalization of the phase functions. Further, we assume here that we observe a homogeneously mixed target, such that $\beta_{ext,i}/\beta_{ext,tot} = \tau_i/\tau_{tot}$ holds."

169-170: The variable N is already used for the droplet size distribution modes.

As noted above, we changed the number of droplet size modes to " M ", such that the droplet number concentrations remains " N ".

185: Here is another different use of N. The authors should take care with clarity of notation and avoid using the same variables in different ways.

We renamed that occurrence of “N” to N_l referring to the number of Legendre Polynomials that have to be calculated for the convergence of the Mie series.

193: Strange to switch to diameter when the rest of the paper consistently uses radius.

Here, we kept the diameter notation as the Marshall-Palmer distribution is written this way in most literature, but we added that $D = 2r$ and the notation of the distribution with the radius as variable.

245-262: Presumably the best first fit would give the same result as the typical monomodal parametric retrieval. If subsequent modes are added to compensate and improve the fit, can't you get multiple solutions? Please clarify this section.

We agree that different multimode combinations can produce similar angular radiance curves and that the multimode inverse problem does not necessarily have a unique solution. This is also explicitly stated in the manuscript now. For the initial one-mode search, the coarse and secondary-coarse stages retain several promising candidates, and these candidate regions are refined in the subsequent search stages. When additional modes are introduced, the parameters of the previously fitted modes are initially centered on the best result from the preceding mode-addition step. The new mode is then searched over the coarsened LUT, and all amplitudes and background parameters are refitted for each candidate combination. The best candidate according to the selected objective function is retained, subject to the positivity and stopping criteria described in the manuscript.

Consequently, the reported multimode solution is the best solution found within the specified LUT, search-window, and sequential-search strategy. We have added this qualification to the revised description:

“While the single mode fit function from Eq.(9) has five free parameters for the fit (A , B , C , r_{eff} , and v_{eff}), every additional mode in the fit function after Eq.(11) adds another three free parameters to the parameter space, because the background B and C is fitted only once. Hence, it would be ambiguous to search the whole lookup table (LUT) for each mode and we decided on a strategy, which subsequently adds modes to the fit and searches the LUT in three different stages. First, an independent one-mode search is performed within the multimode retrieval by performing a coarse search through the LUT, which keeps every sixth effective radius and effective variance values. Since the fit is much more sensitive to r_{eff} than v_{eff} , we look for the best fitting effective radii for the central effective variance of the kept values first. The results for the five best effective radius fits are then combined with all possible effective variances to find again the five best candidates. Then, the fit is refined in a secondary coarse search around these best results from the first coarse search, which takes every second r_{eff} and v_{eff} in a window that includes the two neighbors of the coarsened LUT in both dimensions (r_{eff} and v_{eff}) around the found result into account. Again, the five best candidates from the second stage fit

are kept, to perform a final search, which takes all candidates within a window of size two (± 2 neighbors) around those results. Hereby, the result of the first mode found can be interpreted as the dominant mode generating the signal and does not resemble an average over the whole multimodal distribution, as already observed by Alexandrov et al. (2012a).

The result of the first mode serves as a starting point for sequentially adding up to a predefined maximum number of modes n_m . For the new mode, the fit strategy described above is repeated, while the results for the r_{eff} and v_{eff} of the single best fit from the preceding step are used as centers for local search windows. Hence, when adding a second mode, the possible r_{eff} and v_{eff} values of the coarsened LUT are combined with the neighboring values of the LUT of the best r_{eff} and v_{eff} of the first mode. For each combination, the fit parameters A_i , B , and C are refitted to minimize the objective function (here: RMSE), and the best result is kept to perform the secondary coarse search and the fine search with the newly added mode.

Modes are added until either the predefined maximum number of modes is reached or the newly obtained fit satisfies the RMSE threshold (currently 1.2) for all fitted color channels. A newly added mode is accepted only when the objective function does not worsen and the fitted mode amplitudes are positive for all fitted color channels. If the new fit worsens the objective for any of the color channels, the preceding best valid fit is retained.

Because different combinations of phase functions can produce similar angular radiance curves, the multimode inverse problem may admit several comparably good solutions. Hence, it should be mentioned that the result might describe a local minimum rather than the global minimum which could lead to errors in the resulting droplet size distributions. Further, the resulting solution is conditional on the LUT, search-window strategy, objective function, and positivity constraints. However, first applications of the current fit strategy show promising results, which are presented in Sec. 4."

Figure 4: Figure 4 is not referenced in the text.

The figure was already referenced in the text in l. 267f: "The individual steps of the retrieval algorithm and the fit strategy are summarized in Fig. 4."

Figure 5b: There is artificial looking dark line at retrieved $v_{eff} \sim 0.025$. Can you address what aspects of the algorithm might be causing that?

To begin with, we are really grateful that the reviewer noted this and pointed this out. We found out that this was actually related to an issue with the $v_{eff} = 0.01$ phase functions (the lowermost in our table) for $r_{eff} > 200 \mu m$, which propagated some "NaN" values in single wavelengths when integrating over the spectral response function of the cameras. Now, the dark line shifted to the lowermost effective variance bin, which relates to the insensitivity of large r_{eff} to v_{eff} , as can be seen from Fig. 2, such that the algorithm ends at one or the other end of the LUT. We verified, that particularly the large r_{eff} (approx. $500 \mu m$ and larger) contribute to that stripe, which is also why the wrong phase functions made the stripe move up one bin. We corrected this and reproduced the Figs. 2, 5, 6, and 10-15 with the correct data.

Further, we added the following to the description of Fig. 5b in the text:

“Further, it can be seen that the retrieved effective variances accumulate both in the smallest ($v_{\text{eff}} = 0.01$) and the largest ($v_{\text{eff}} = 0.4$) effective variance bins, which is mainly due to insensitivities of the phase functions for large effective radii as seen in Fig. 2.”

Figure 7b: The RGB of DOLP is very dark and it's hard to see the cloudbow. The authors might consider additional image processing to make it more apparent.

Thanks for the comment. We processed the DOLP RGB such that the cloud/rainbow is better visible.

Figure 9a: Panel a in figure 9 is not mentioned in the caption.

We changed the second sentence in the caption to “Panel (a) shows the RGB image taken by the camera, while the cloud top height is projected to the image in panel (b).”

Figures 12-15: The yellow dashed line for single mode in panels c and d is very hard to see.

We changed the color to green.

569-570, 598,647: References that cite an entire textbook should be updated so they only cite the relevant sections/chapters.

We added the relevant sections in the citations within the text.

Technical corrections:

Please address the following grammar issues/ typos:

75: “...Atlantic before transferred to...”

Changed this to: “...Atlantic before being transferred to...”

227: “It exemplary shows the effect...”

Changed to “It illustrates the effect...”

294:” ...might be caused because of hitting...”

Corrected to “..might be caused by reaching...”

337:” ... but the sideward viewing geometry allows to observe rain...”

Changed this to “...but the sideward viewing geometry allows the observation of rain..”

573: Missing accent in "Bréon"

Applied the correction, thanks for noting!