Revisiting the thermodynamic derivation of the Penman-Monteith Equation

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The author provided a critical review of the derivation of the Penman-Monteith (PM) equation by pointing out the limited validity of the PM equation. Yet, the arguments leading to the correction conclusion are false. This research note highlights the prevailing misconceptions in evaporation physics starting long ago aggravated by Penman and Monteith through their popular evapotranspiration (ET) models.

"*The Penman-Monteith (PM) equation is the most commonly used method for determining latent heat fluxes (λE) from vegetated land surfaces and is regarded as a best practice for calculating evapotranspiration*" (line 2-3) is not true. The author appears to be unaware of the advancement in modeling ET over the past two decades. For example, Wang et al. (2024) critically evaluated three physics-based ET models, which overcome the weakness and limitation of the PM equation.

Wang, Y., Petrone, R. M., & Kompanizare, M. (2024). Toward a unified understanding of estimating evapotranspiration: The linkage between three effective parsimonious models. *Water Resources Research*, 60, e2023WR036910. https://doi.org/10.1029/2023WR036910

"*The resulting Penman-Monteith (PM) equation (Monteith, 1965) is today probably the most widely used method for determining latent heat fluxes from vegetated land surfaces and is regarded as the gold standard for calculating evapotranspiration*" (line 100-102) reflects the view of those who overlook the progress in the theoretical and modeling study of evaporation. Numerous improved ET models have been developed. The era of Penman and Monteith has passed. The author is encouraged to read (Wang et al., 2024).

This paper contains several misconceptions about ET process including:

- "… *higher water temperatures lead to a higher evaporation rate*" (line 39). This is not true in general. Temperature is only one of the factors in ET physics. Other factors include energy supply, water supply, and boundary layer turbulence.

- "… *the air is saturated with water vapor*" (line 43). It is well understood that saturation vapor (pressure) is only function of vapor temperature independent of whether water vapor is mixed with air or in vacuum. Dry air is not a porous medium where it becomes saturated when all pore space is filled up with water vapor molecules. The ideal gas law tells us that water vapor is part of the (moist) air if viewed as a "porous medium".

- "… *water concentration difference that triggers the diffusive flux drives the phase transition*" (line 62), "… *Evaporation can therefore be described as a transport of latent heat λE (in W m^{-2}) that is driven by the moisture-potential difference between the saturated (air close to the) surface e_s and the unsaturated air e_a*" (line 64-65) First, phase change is energy driven phenomenon. Water concentration difference does not necessarily cause phase change, at least not the dominant driver. In the Earth environment, it is evaporation, driven predominantly by

solar radiation, that drives moisture gradient near-surface instead of the opposite. Evaporation is the cause, not the effect, of the moisture-potential difference since solar radiation at land surface must be converted into latent, sensible and ground heat flux.

- "*The air supplies this energy in the form of sensible heat. The surface takes this sensible heat, which raises the kinetic-energy level of the water molecules, causing them to transition into the gas phase more frequently. As a result, the latent heat content of the air increases. In other words, the air transports energy in the form of sensible heat to the surface and, in return, receives an equal amount of energy as latent heat*" (line 188-191) This is one of the most nonsensical arguments of Monteith in deriving the PM equation. Downward sensible heat flux as the energy supply of evaporation rarely occurs, if at all, under natural conditions especially during daytime. Although downward sensible heat flux may occur over the oasis surface as a secondary energy supply to enhance evaporation, solar radiation remains the dominant driver of evaporation. I am not aware of any observations confirming downward sensible heat flux is the primary energy supply of evaporation.

- "… *air mass rises, … its capacity to hold water vapor*" (line 219), "*the capacity of the air mass to hold water vapor*" (line 319-320) This is a widespread misconception about water vapor repeated several times in this paper. As explained above, the air is not a porous medium like soil and does not have capacity to hold water vapor.

- "… *the air mass where evaporation occurs*" (line 335) Evaporation, i.e. the phase change of liquid water over soils and in plants, does not occur in the air as phase change always occurs at liquid-vapor interface. Certainly, this paper does not discuss rain drops evaporate as they travel toward surface as rainfall.

- "… *the spatial temperature difference between the surface and the near-surface air, which drives the sensible heat flux*" (line 408-409) Under natural conditions, temperature gradient is driven by sensible heat flux for the same reason that humidity (moisture-potential) gradient is driven by vapor flux since solar radiation must be converted into thermal energy fluxes at the land surface.

The two terms in the PM equation, commonly referred to as available energy (diabatic component) and atmospheric demand (adiabatic component), are mathematical artefact as the result of flux-gradient formula of vapor and heat flux and the approximate expression of surface temperature in terms of air temperature according to the Clausius-Clapeyron equation using the first order Taylor expansion. It is important to realize that Eq. (1) is empirical instead of based on physical principles. Decomposition of $e_s - e_a = (e_s - e_d) + (e_d - e_a)$ in Eq. (7) by introducing e_d, leading to the expressions of diabatic and adiabatic component of E_G in Eq. (12), is totally arbitrary, hence a mathematical artefact. In fact, E_1, E_2 are physically unreal and coupled mathematically. It is impossible to change one arbitrarily while fixing the other. In addition, G in Eq. (8) cannot be arbitrarily fixed while T_s (and T_a) changes. Just because the Penman equation has two components does not mean there are two separate evaporation processes.

Monteith's thermodynamic derivation of the PM based on the assumption of the existence of two evaporation processes is based on observational fact and the diabatic and adiabatic component

are not real physical processes. Monteith's interpretation of the diabatic and adiabatic component is based on imaginary processes, hence is misleading. Essentially, Monteith justified the mathematical artefact using false physical arguments to fit the Penman's formula. The thermodynamic derivation of the PM equation by Monteith is built on a series of misconceptions and unrealistic assumptions about ET process. Monteith justified the two terms in Penman's equation by associating them with unreal physical processes: diabatic and adiabatic evaporation. Monteith's questionable arguments appear to root in the misconception of evaporation occurring in the air. "*For the description of the adiabatic component of evaporation E_1, the air mass into which evaporation takes place and its contact with the surface are initially defined as a closed, adiabatic system that exchanges no energy with the surrounding environment*" (line 176-179) indicates E_1 is totally imaginary resulting from non-existent physical processes. "*The air supplies this energy in the form of sensible heat. The surface takes this sensible heat, which raises the kinetic-energy level of the water molecules, causing them to transition into the gas phase more frequently*" (line 188-189) is not what happens in a natural environment. During daytime, surface temperature increases due to solar radiation as the energy supply for evaporation, sensible heat flux into the air and ground heat flux into soil. Therefore, Eq. (13) is physically meaningless. Nighttime sensible het flux over land surface is indeed downward from air to surface, but nighttime evaporation is usually negligibly small if at all. Instead, condensation in the form of dew is a common occurrence that releases latent heat to partially balance longwave radiation loss. Since the decomposition of evaporation into diabatic (available energy) and adiabatic (atmospheric demand) component in Penman's equation is a mathematical artefact, it is not surprising that they are not associated with real physical processes. Monteith's thermodynamic derivation of the PM equation does not make physical sense. Even worse, Monteith's work has misled generations of hydrologists by spreading several misconceptions about evaporation physics.

The author's argument following those of Monteith to derive the PM equation for vegetated surface in Eq. (31) is also based on the same misconceptions including evaporation driven by moisture-potential difference. Without energy balance constraint, the claimed correct PM equation for vegetated surface goes back to pre-Penman models that Penman and Monteith proposed to improve. Therefore, the conclusion "*the Penman-Monteith equation is not applicable to vegetated surfaces*" (line 433) is partially correct but based on false premises. The claim "*The thermodynamic derivation of the PM equation is therefore internally consistent*" (line 396-398) is correct but should be understood as "consistently unphysical and misleading". This theoretical work would be useful if the author could provide an in-depth discussion of these common misconceptions about evaporation.

The paper contains too many equations (total 31). Some of them may be put in an appendix or "Variables and Symbols".