Supplementary information: Instability Threshold Assessment for Storage Tanks and Stacked Freight Containers under Tsunami Loading.

1. Procedure for Calculating Damage State Functions for Storage Tanks

The stability of above-ground cylindrical storage tanks subjected to tsunami inundation was assessed considering two primary failure mechanisms: flotation due to buoyancy forces and sliding caused by the combined action of hydrostatic and hydrodynamic loads. The methodology was applied to three storage conditions representing low, intermediate, and near-full crude oil inventories (5%, 50%, and 95% filling ratios, respectively), for an oil tank 10 meters high and 3 meters in diameter, assuming the density of water $\rho = 1025\ kg/m^3$ and diesel crude oil $\rho_{oil} = 850\ kg/m^3$.

1.1. Tank characteristics and material properties

The storage tank is represented as a vertical cylindrical steel structure characterized by its radius (R), height (H_{tank}), shell thickness (t_{shell}), and bottom plate thickness (t_{bottom}).

The following material properties were adopted:

- Water density: ($\rho = 1025\ kg/m^3$)
- Crude oil density: ($\rho_{oil} = 850\ kg/m^3$)
- Steel density: ($\rho_{steel} = 7850\ kg/m^3$)
- Gravitational acceleration: ($g = 9.81\ m/s^2$)
- Drag coefficient: ($C_d = 1.2$)
- Friction coefficient: ($\mu = 0.5$)

The structural mass of the tank is estimated from the volume of the shell and bottom plate (Eq. S1):

$$m_{shell} = V_{shell} + V_{bottom} \tag{S1}$$

Where: V_{shell} and V_{bottom} as (Eq. S2 and S3):

$$V_{shell} = 2\pi \cdot R \cdot H_{tank} \cdot t_{shell} \tag{S2}$$

$$V_{bottom} = \pi \cdot R^2 \cdot t_{bottom} \tag{S3}$$

The mass of the stored crude oil is calculated as (Eq. S4):

$$m_{content} = \rho_{oil} \cdot V_{content} \tag{S4}$$

With $V_{content}$ as (Eq. S5):

$$V_{content} = \pi \cdot R^2 \cdot H_{tank} \cdot f_{\%} \tag{S5}$$

Where $f_{\%}$ denotes the filling ratio. The total mass and corresponding weight are then obtained as (Eq. S6 and S7):

$$m_{tot} = m_{shell} + m_{content} \tag{S6}$$

$$W_{tot} = m_{tot} \cdot g \quad \text{(S7)}$$

1.2. Flotation instability

The uplift force generated by the inundating water is calculated according to Archimedes' principle. For a submerged water depth h_{sub}, the buoyancy force F_b is (Eq. S8):

$$F_b = \rho \cdot g \cdot V_{sub} \quad \text{(S8)}$$

where the displaced volume V_{sub} is (Eq. S9):

$$V_{sub} = \pi \cdot R^2 \cdot h_{sub} \quad \text{(S9)}$$

Flotation is assumed to occur when the buoyancy force equals the total downward weight of the tank system W_{tot} (Eq. S10):

$$F_b = W_{tot} \quad \text{(S10)}$$

Substituting the above expressions yields the critical flotation depth h_{flot} (Eq. S11):

$$h_{flot} = \frac{W_{tot}}{\rho \cdot g \cdot \pi \cdot R^2} \quad \text{(S11)}$$

This threshold depends exclusively on tank geometry and filling condition and is independent of flow velocity.

1.3. Sliding Instability

Sliding occurs when the horizontal tsunami-induced forces exceed the frictional resistance between the tank bottom and the foundation.

The available resisting force F_{res} is expressed as (Eq. S12):

$$F_{res} = \mu \cdot W_{tot} \quad \text{(S12)}$$

The horizontal loading is represented as the sum of hydrostatic and hydrodynamic drag forces. The hydrostatic component F_{hydro} is approximated as (Eq. S13):

$$F_{hydro} = \rho \cdot g \cdot R \cdot {h_{sub}}^2 \quad \text{(S13)}$$

while the drag force F_{drag} is calculated as (Eq. S14):

$$F_{drag} = \rho \cdot C_d \cdot R \cdot h_{sub} \cdot U^2 \quad \text{(S14)}$$

where (U) is the depth-averaged flow velocity. The sliding failure condition is therefore defined as (Eq. S15):

$$F_{hydro} + F_{drag} \geq F_{res} \quad \text{(S15)}$$

For a given flow velocity, the critical inundation depth associated with sliding is obtained by numerically solving the above equation using a bisection algorithm. The velocity-dependent sliding solutions are subsequently approximated using a power-law relationship of the form (Eq. S16):

$$(h_{sub} \cdot U^2) = k_{slide} \quad \text{(S16)}$$

where k_{slide} is a fitting parameter that depends on the tank geometry and filling condition. The coefficient k_{slide} is obtained through ordinary least-squares regression using the computed instability points. This formulation allows the sliding threshold to be expressed directly in terms of flow depth and velocity and can be readily incorporated into tsunami risk models based on momentum flux.

1.4. Numerical Solution Procedure

For each filling condition, the following procedure is applied:

a) Calculate the structural mass of the tank.

b) Calculate the mass of stored crude oil.

c) Determine the total weight of the tank system.

d) Compute the critical flotation depth (h_{flot})

e) Calculate the frictional resistance force.

f) Define a range of flow velocities between 0.1 and 10 m/s.

g) For each velocity, solve the sliding equilibrium equation using the bisection method.

h) Store the resulting pairs (h_{sub}, U^2) representing sliding instability conditions.

1.5. Damage State Definition

The maximum instability thresholds obtained from the flotation and sliding analyses are adopted as the reference values corresponding to complete failure. To represent intermediate damage conditions, scaled thresholds are defined as fractions of the maximum instability capacity. Three damage levels are considered:

- DS1: Less than 30% of the ultimate instability thresholds (h_{flot}, k_{slide}).
- DS2: Between the 30% and 100% of the ultimate instability thresholds (h_{flot}, k_{slide}).
- DS3: More than 100% of the ultimate instability thresholds (h_{flot}, k_{slide}).

For each filling condition, the methodology provides:

- Critical flotation depth h_{flot}.
- Critical sliding coefficient k_{slide}.
- Flotation damage thresholds.
- Sliding damage thresholds.

For flotation:

- $h_{DS1} = h < h_{flot} \cdot 0.3$
- $h_{DS2} = h_{flot} \cdot 0.3 \leq h < h_{flot}$
- $h_{DS3} = h \geq h_{flot}$

For sliding:

- $k_{DS1} = k < k_{slide} \cdot 0.3$
- $k_{DS2} = k_{slide} \cdot 0.3 \leq k < k_{slide}$
- $k_{DS3} = k \geq k_{slide}$

The resulting thresholds are calculated independently for each filling condition (5%, 50%, and 95% fill level).

1.6. Results for a Fill State: Empty (5% off fill level)

- $m_{tot} = 19465.70$ kg
- $W_{tot} = 190958.52$ N
- $F_{res} = 95479.26$ N
- $h_{flot} = 2.69$m
- $k_{slide} = 0.065\ m^3/s^2$

If the process is repeated for the other fill levels (50% and 95% of its capacity), the following Figure S1, Figure S2, Table S1 and Table S2 results are obtained.

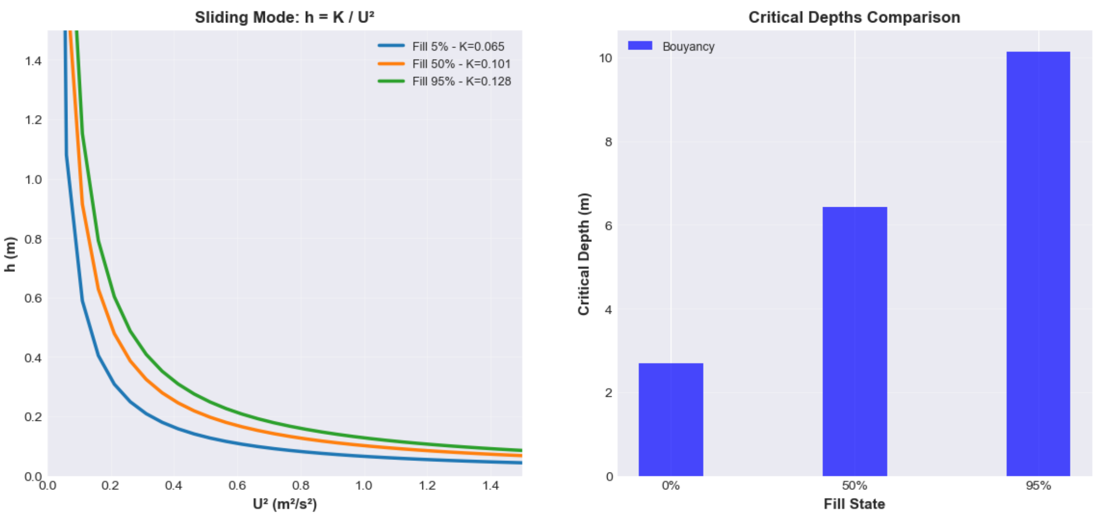


Figure S1. Results of the sliding threshold values k_{slide} and buoyancy values h_{flot} obtained for the example tank. For three fill levels: empty (5%), half-full (50%), and full (95%).

Table S1. instability-based damage-state functions used in the subsequent tsunami risk assessment for each tank.

	Flotation (m)			**Slide (m^3/s^2)**		
Damage state	**Empty**	**Half full**	**Full**	**Empty**	**Half-full**	**Full**
$\boldsymbol{DS1}$	$h \leq 1$	$h \leq 2$	$h \leq 3$	$k \leq 0.01$	$k \leq 0.03$	$k \leq 0.04$
$\boldsymbol{DS2}$	$1 < h \leq 2.7$	$2 < h \leq 6.4$	$3 < h \leq 10$	$0.01 < k \leq 0.07$	$0.03 < k \leq 0.10$	$0.04 < k \leq 0.13$
$\boldsymbol{DS3}$	$h > 2.7$	$h > 6.4$	$h > 10$	$k > 0.07$	$k > 0.10$	$k > 0.13$

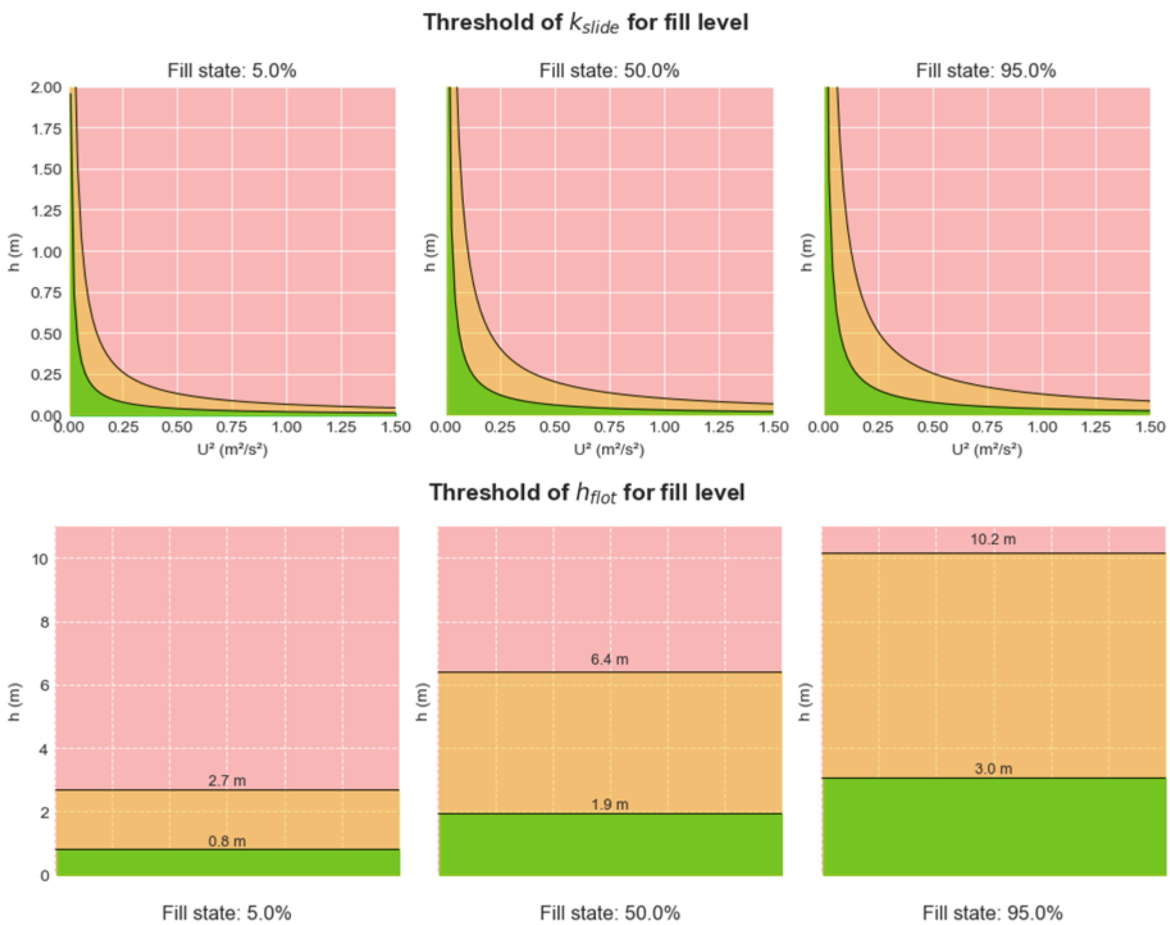
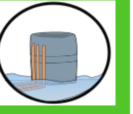


Figure S2. Graphical description of the proposed damage state function thresholds for a storage tank of 10 meters high and 3 meters in diameter, based on the stability equations for hydrodynamic loads.

Table S2 Description and definition of the proposed damage state function for a storage tank of 10 meters high and 3 meters in diameter.

DS	TIM	Fill level			Description	
		Empty	Half-full	Full		
DS1	Max. Inundation depth (m)	$h \leq 1.0$	$h \leq 2.0$	$h \leq 3$	Low probability of significant structural damage, no significant damage	
	Max. Momentum flux (m^3/seg^2)	$hU^2 \leq 0.01$	$hU^2 \leq 0.03$	$hU^2 \leq 0.04$		
DS2	Max. Inundation depth (m)	$1.0 < h \leq 2.7$	$2.0 < h \leq 6.4$	$3 < h \leq 10$	Considerable probability of damage to pipes or minor deformation of non-structural components, with limited risk of loss of containment.	
	Max. Momentum flux (m^3/seg^2)	$0.01 < hU^2 \leq 0.07$	$0.03 < hU^2 \leq 0.10$	$0.04 < hU^2 \leq 0.13$		
DS3	Max. Inundation depth (m)	$h > 2.7$	$h > 6.4$	$h > 10$	High probability of major failure modes such as floating, sliding, overturning, or buckling, with a high risk of loss of containment and secondary consequences.	
	Max. Momentum flux (m^3/seg^2)	$hU^2 > 0.07$	$hU^2 > 0.10$	$hU^2 > 0.13$		

2. Procedure for Calculating Damage State Functions for Containers

The vulnerability of stacked freight containers to tsunami-induced flooding was assessed through a mechanics-based approach considering two principal failure mechanisms: flotation caused by buoyancy forces and sliding induced by the combined action of hydrostatic and hydrodynamic loads. The methodology was developed to derive critical instability thresholds as a function of inundation depth and flow velocity (depth-averaged) and is applicable to different stacking configurations and loading conditions. Two loading states were considered: empty containers and fully loaded containers. Additionally, container stacks ranging from one to five levels were analyzed to represent common storage configurations in port terminals.

2.1. Container Characteristics

The analysis considers standard 40-foot ISO containers characterized by the dimensions:

- Container longitude: $b_l = 2.44\ (m)$
- Container length: $b_t = 12.19\ (m)$
- Container height: $h_c = 2.59\ (m)$

A clearance height (h_b) is introduced to represent the elevation of the container base above the ground surface, with $h_b = 0.14\ (m)$. As well as the calculation of storage tanks, the following physical parameters are adopted:

- Water density: $\rho = 1025\ kg/m3$
- Gravitational acceleration: ($g = 9.81\ m/s^2$)
- Drag coefficient: ($C_d = 1.2$)
- Friction coefficient: ($\mu = 0.5$)

2.2. Flotation instability

Flotation occurs when the buoyancy force generated by the inundating water exceeds the total weight of the container stack. The submerged volume V_{sub} is calculated as (Eq. S17):

$$V_{sub} = b_l * b_t * (h_{sub} - h_b) \tag{S17}$$

For water levels exceeding the base clearance. The corresponding buoyancy force F_b is (Eq. S18):

$$F_b = \rho * g * V_{sub} \tag{S18}$$

The flotation condition is reached when (Eq. S19):

$$F_b \geq W \tag{S19}$$

Which yields the critical inundation depth (Eq. S20):

$$h_{flot} = \frac{W}{\rho * g * b_l * b_t} + h_b \tag{S20}$$

This threshold defines the onset of complete loss of vertical stability. Because the total weight depends on the number of containers and their filling condition, flotation resistance increases with both stacking level and cargo mass.

2.3. Sliding instability

Sliding instability is evaluated through a force equilibrium between horizontal tsunami-induced loads and the available frictional resistance. Unlike conventional approaches that assume constant frictional resistance, the present methodology explicitly accounts for the reduction of effective weight caused by buoyancy. The effective normal force F_N acting on the stack is (Eq. A21):

$$F_N = W - F_b \quad \text{(S21)}$$

The available frictional resistance F_{fr} is therefore (Eq. S22):

$$F_{fr} = \mu * F_N \quad \text{(S22)}$$

As inundation depth increases, buoyancy progressively reduces the frictional resistance available to oppose sliding. The hydrostatic force acting on the container stack F_{hs} is represented as (Eq. S23):

$$F_{hs} = \frac{1}{2} * \rho * g * b_t * {h_{sub}}^2 \quad \text{(S23)}$$

Where h_{sub} is the inundation depth. The drag force generated by the flow F_d is computed as (Eq. S24):

$$F_d = \frac{1}{2} * \rho * C_d * A_p * U^2 \quad \text{(S24)}$$

Where A_p is the projected submerged area exposed to the flow and U is the depth-averaged flow velocity (Eq. S25):

$$A_p = b_t * (h - h_b) \quad \text{(S25)}$$

The total horizontal force F_H acting on the stack is (Eq. S26):

$$F_H = F_{hs} + F_d \quad \text{(S26)}$$

Sliding instability is assumed to occur when (Eq. S27):

$$F_{hs} + F_d - F_{fr} \geq 0 \quad \text{(S27)}$$

For a prescribed flow velocity, the critical inundation depth is obtained numerically by solving the equilibrium equation using a bisection algorithm. The numerical solutions indicate that the sliding threshold can be represented by a hyperbolic relationship between inundation depth and squared velocity. The velocity-dependent sliding solutions are subsequently approximated using a power-law relationship of the form (Eq. S28):

$$h * U^2 = k_{slide} \quad \text{(S28)}$$

Where critic k_{slide} is a fitting parameter that depends on the tank geometry and filling condition. The coefficient k_{slide} is obtained through ordinary least-squares regression using the computed instability points. This formulation allows the sliding threshold to be expressed directly in terms of flow depth and velocity and can be readily incorporated into tsunami risk models based on momentum flux.

Numerical Procedure

The instability assessment is performed according to the following workflow:

a) Define the stacking level (N).

b) Select the loading condition (empty, full).

c) Compute the total weight of the stack.

d) Determine the critical flotation depth (h_{crit}).

e) Generate a range of flow velocities between 0.05 and 10 m/s.

f) For each velocity, solve the sliding equilibrium equation.

g) Obtain the corresponding critical inundation depth.

h) Construct the instability envelope in the (h, U^2) domain (k_{slide}).

2.4. Damage State Definition

The maximum instability thresholds obtained from the flotation and sliding analyses are adopted as the reference values corresponding to complete failure. To represent intermediate damage conditions, scaled thresholds are defined as fractions of the maximum instability capacity. Three damage levels are considered:

- DS1: Less than 30% of the ultimate instability thresholds (h_{flot}, k_{slide}).
- DS2: Between the 30% and 100% of the ultimate instability thresholds (h_{flot}, k_{slide}).
- DS3: More than 100% of the ultimate instability thresholds (h_{flot}, k_{slide}).

For each filling condition, the methodology provides:

- Critical flotation depth h_{flot}.
- Critical sliding coefficient k_{slide}.
- Flotation damage thresholds.
- Sliding damage thresholds.

For flotation:

- $h_{DS1} = h < h_{flot} \cdot 0.3$
- $h_{DS2} = h_{flot} \cdot 0.3 \leq h < h_{flot}$
- $h_{DS3} = h \geq h_{flot}$

For sliding:

- $k_{DS1} = k < k_{slide} \cdot 0.3$
- $k_{DS2} = k_{slide} \cdot 0.3 \leq k < k_{slide}$
- $k_{DS3} = k \geq k_{slide}$

The resulting thresholds constitute the damage-state functions used to characterize the tsunami vulnerability of stacked freight containers within port environments. The final thresholds are reported separately for:

- Empty containers (Stacking levels from one to five containers).
- Fully loaded (Stacking levels from one to five containers).

These threshold values can subsequently be integrated into tsunami risk assessments and probabilistic damage analyses at the port scale.

2.5. Example: results for a one stacking level for empty and full containers

Container masses for one 40Ft container are defined as:

- Empty container mass:$m_e = 3800\ Kg$
- Fully loaded container mass $m_f = 30480\ Kg$

And the corresponding weight is $W = m * g$. Solving the equations, the critical thresholds for 1 stacking container are (Table S3):

Table S3. Instability-based damage-state functions used in the subsequent tsunami risk assessment for the 1 freight container

	Flotation $(\boldsymbol{m})$		Slide $(\boldsymbol{m^3/s^2})$	
Damage state	**Empty**	**Full**	**Empty**	**Full**
$\boldsymbol{DS1}$	$h < 0.15$	$h < 0.3$	$k < 1.4$	$k < 4.0$
$\boldsymbol{DS2}$	$0.15 \leq h < 0.26$	$0.3 \leq h < 1.1$	$1.4 \leq k < 4.8$	$4.0 \leq k < 13.5$
$\boldsymbol{DS3}$	$h \geq 0.26$	$h \geq 1.1$	$k \geq 4.8$	$k \geq 13.5$

By following this procedure for 2, 3, 4 and 5-tier container stacks, one can obtain the thresholds associated with the damage-state functions used in the subsequent tsunami risk assessment for each tank. The following Table S4, Figure S3, Table S5, Figure S4 and Table S6 shows the results for 1, 3 and 5 stacks conditions, for empty and full loaded containers.

Table S4. Proposed damage state function thresholds for a 1, 3 and 5 staking levels for empty containers.

DS	**TIM**	**Stacking for Empty Containers**		
		1	**3**	**5**
DS1	Max. Inundation depth (m)	$h \leq 0.15$	$h \leq 0.2$	$h \leq 0.25$
	Max. Momentum flux (m^3/seg^2)	$hU^2 \leq 1.4$	$hU^2 \leq 2$	$hU^2 \leq 3$
DS2	Max. Inundation depth (m)	$0.15 < h \leq 0.26$	$0.2 < h \leq 0.5$	$0.25 < h \leq 0.8$
	Max. Momentum flux (m^3/seg^2)	$1.4 < hU^2 \leq 4.8$	$2 < hU^2 \leq 7$	$3 < hU^2 \leq 10$
DS3	Max. Inundation depth (m)	$h > 0.26$	$h > 0.5$	$h > 0.8$

	Max. Momentum flux (m^3/seg^2)	$hU^2 > 4.8$	$hU^2 > 7$	$hU^2 > 10$

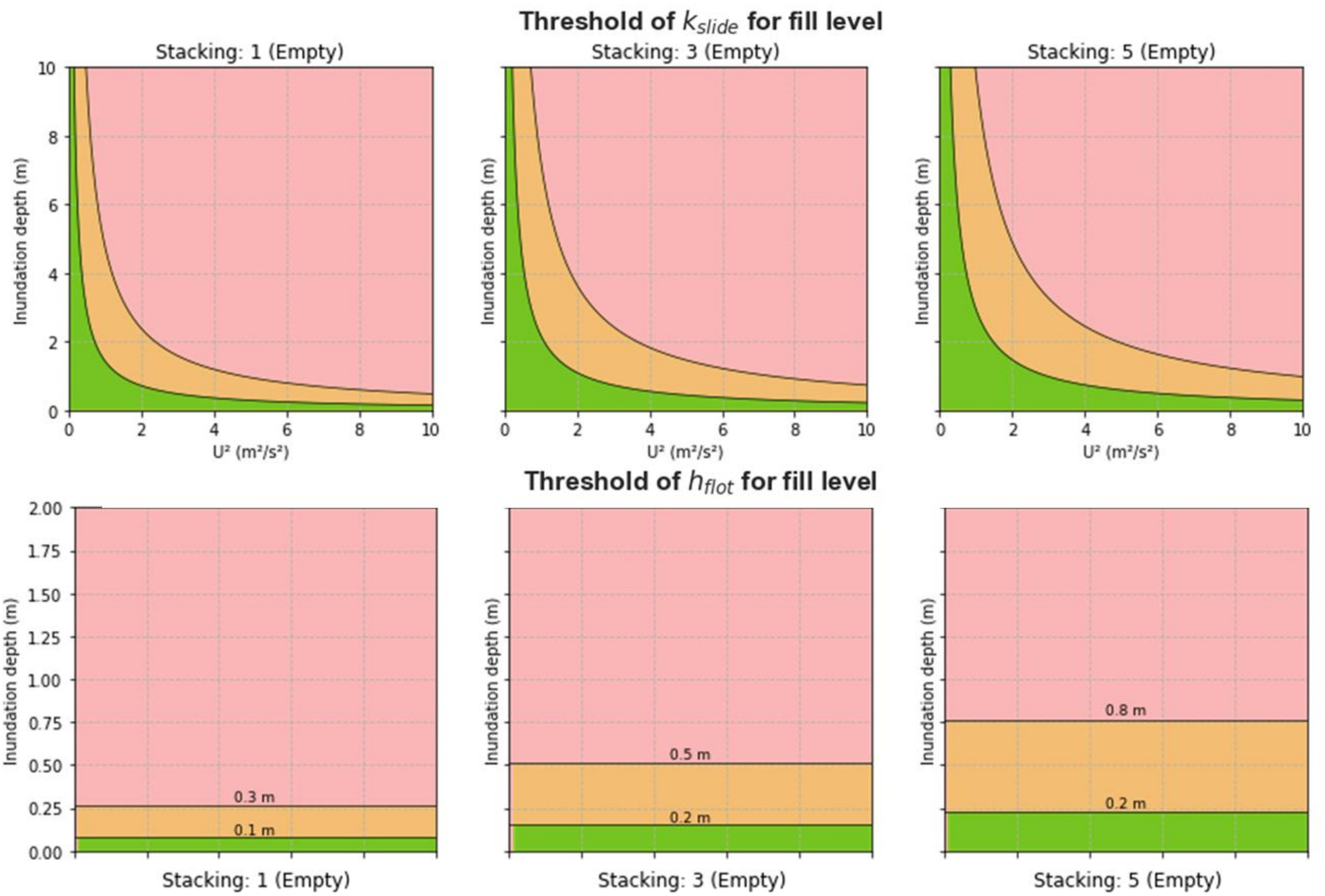


Figure S3. Graphical description of the proposed damage state function thresholds for a 1 (left), 3 (middle) and 5 (right) staking levels for empty containers, based on the stability equations for hydrodynamic loads.

Table S5. Proposed damage state function thresholds for a 1, 3 and 5 staking levels for fully loaded containers.

DS	TIM	Stacking for full Containers		
		1	**3**	**5**
DS1	Max. Inundation depth (m)	$h \leq 0.3$	$h \leq 1$	$h \leq 1.5$
	Max. Momentum flux	$hU^2 \leq 4$	$hU^2 \leq 8$	$hU^2 \leq 13$

	(m^3/seg^2)			
DS2	Max. Inundation depth (m)	$0.3 < h < 1.1$	$1 < h < 3$	$1.5 < h \leq 5$
	Max. Momentum flux (m^3/seg^2)	$4 < hU^2 \leq 13.5$	$8 < hU^2 \leq 30.0$	$13 < hU^2 \leq 40$
DS3	Max. Inundation depth (m)	$h > 1.1$	$h > 3$	$h > 5$
	Max. Momentum flux (m^3/seg^2)	$hU^2 > 13.5$	$hU^2 > 30$	$hU^2 > 40$

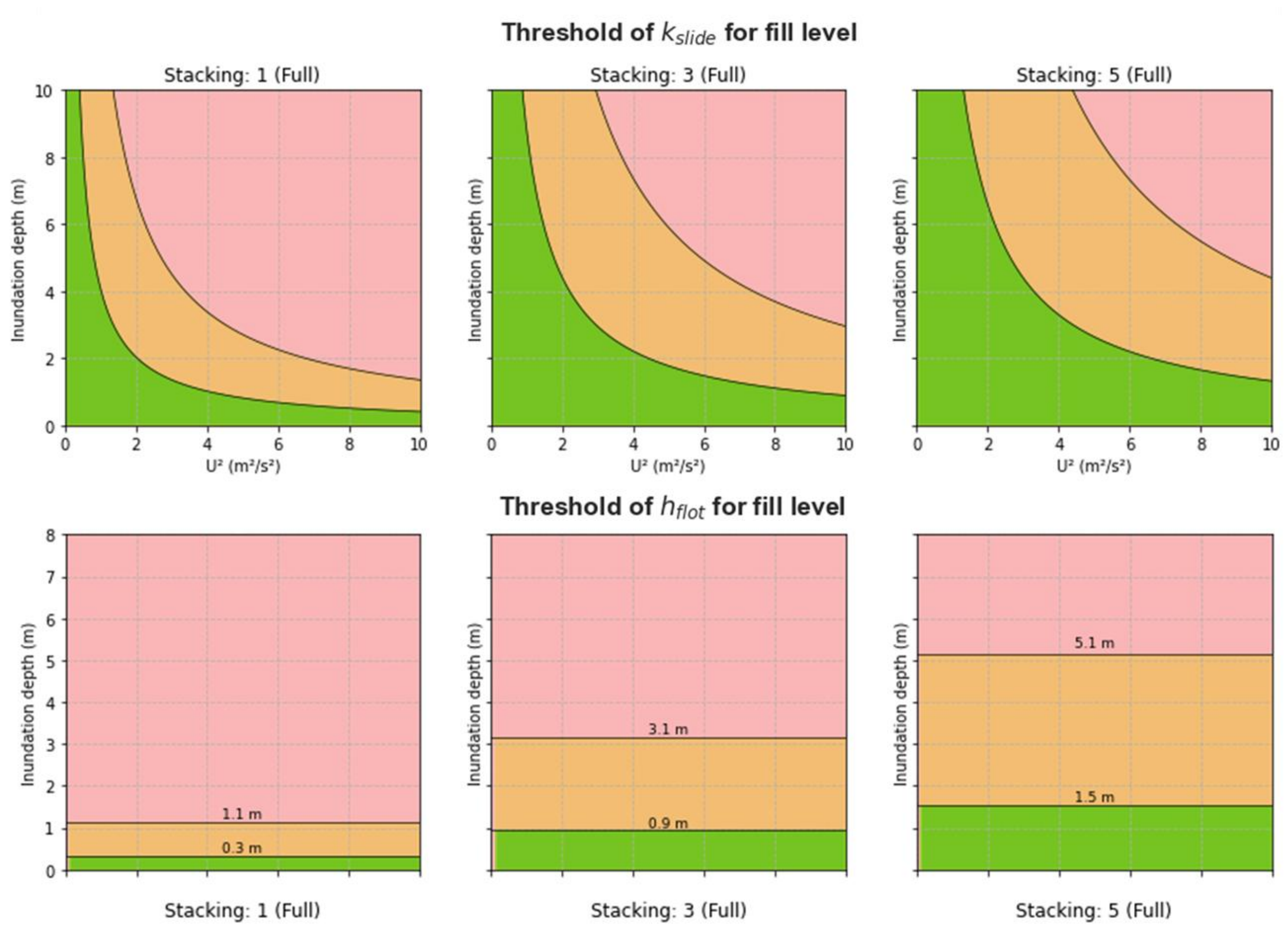


Figure S4. Graphical description of the proposed damage state function thresholds for a 1 (left), 3 (middle) and 5 (right) staking levels for fully loaded containers, based on the stability equations for hydrodynamic loads.

Table S6. Proposed damage state function thresholds for a 1, 3 and 5 staking levels for fully loaded containers.

DS	**TIM**	**Stacking for full Containers**		
		1	**3**	**5**
DS1	Max. Inundation depth (m)	$h \leq 0.3$	$h \leq 1$	$h \leq 1.5$
	Max. Momentum flux (m^3/seg^2)	$hU^2 \leq 4$	$hU^2 \leq 8$	$hU^2 \leq 13$
DS2	Max. Inundation depth (m)	$0.3 < h < 1.1$	$1 < h < 3$	$1.5 < h \leq 5$
	Max. Momentum flux (m^3/seg^2)	$4 < hU^2 \leq 13.5$	$8 < hU^2 \leq 30.0$	$13 < hU^2 \leq 40$
DS3	Max. Inundation depth (m)	$h > 1.1$	$h > 3$	$h > 5$
	Max. Momentum flux (m^3/seg^2)	$hU^2 > 13.5$	$hU^2 > 30$	$hU^2 > 40$