**Table S1.** Sensitivity of MLS $H_2O$ tracer propagation diagnostics using monthly averaged anomaly lag correlations during the pre-eruption period (2004–2021). Monthly aggregation reproduces the same overall vertical transport hierarchy identified in the daily anomaly diagnostics, with faster transport within the 400–500 K layer and progressively slower propagation above 500 K. Final lag times were selected using physically constrained minimum thresholds of 45 days for tropical-to-midlatitude transport and 90 days for tropical-to-polar transport. Effective propagation speed represents great-circle meridional distance divided by the selected lag timescale.

| **Layer** | **Source → Target** | **Final lag (days)** | **rmax** | **p-value** | **Effective speed (km day$^{-1}$)** |
|---|---|---|---|---|---|
| 400–500 K | SH tropics → SH midlatitudes | 62 | 0.83 | <0.001 | 53.8 |
| 400–500 K | SH tropics → SH polar | 124 | 0.40 | <0.001 | 53.8 |
| 400–500 K | NH tropics → NH midlatitudes | 62 | 0.84 | <0.001 | 53.8 |
| 400–500 K | NH tropics → NH polar | 93 | 0.75 | <0.001 | 71.7 |
| 500–600 K | SH tropics → SH midlatitudes | 124 | 0.33 | <0.001 | 26.9 |
| 500–600 K | SH tropics → SH polar | 279 | 0.15 | 0.030 | 23.9 |
| 500–600 K | NH tropics → NH midlatitudes | 62 | 0.60 | <0.001 | 53.8 |
| 500–600 K | NH tropics → NH polar | 341 | 0.19 | 0.007 | 19.6 |
| 600–800 K | SH tropics → SH midlatitudes | 155 | 0.53 | <0.001 | 21.5 |
| 600–800 K | SH tropics → SH polar | 217 | 0.54 | <0.001 | 30.7 |
| 600–800 K | NH tropics → NH midlatitudes | 217 | 0.60 | <0.001 | 15.4 |
| 600–800 K | NH tropics → NH polar | 248 | 0.49 | <0.001 | 26.9 |