Supplementary material

This document provides supplementary figures supporting the methodology and spatial analyses presented in the main manuscript. The contents are structured to provide full transparency of the models' behaviour across all evaluated temporal windows.

- **Figure S1–Figure S3** outline the SHAP dependence plots, local feature importance maps (FI) and spatial SHAP maps for the full 13 predictors used in the optimal October–December (Short Rains) window.
- **Figure S4–Figure S12** present the SHAP dependence plots, local FI maps and spatial SHAP maps for the suboptimal temporal windows: June–August (dry season), March–May (Long Rains), and an Annual climatological baseline.

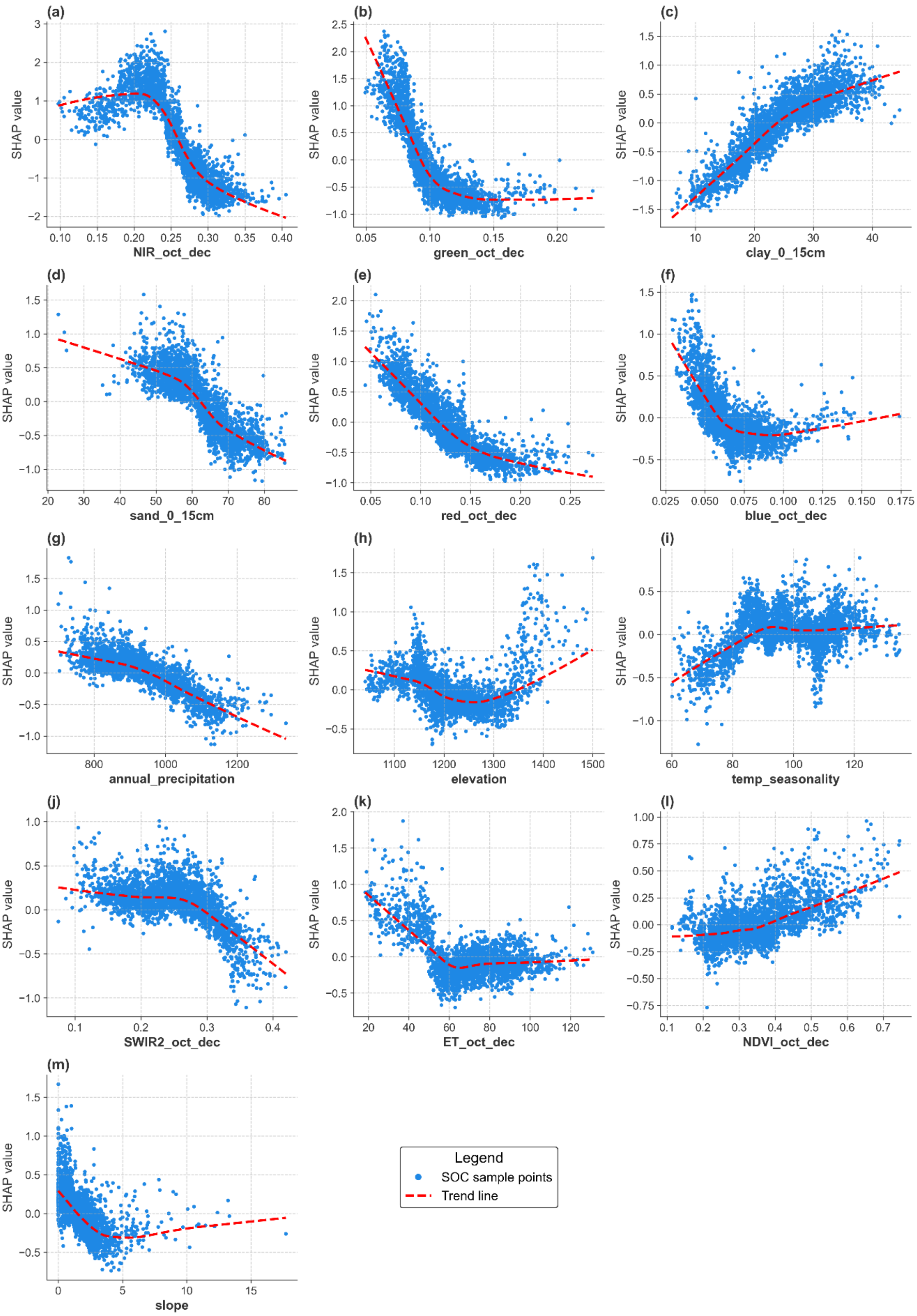

Figure S1 SHAP dependence plots illustrating the functional forms of predictor–SOC relationships for the Oct-Dec (Short Rains) model. **Derived exclusively from the global Random Forest model, these plots reveal the mechanistic influence of each predictor across its sampled range.** Predictors: **seasonal optical satellite bands (near-infrared [NIR], green, red, blue, shortwave infrared 2 [SWIR2]), physical soil properties (clay and sand content at 0–15 cm depth), bioclimatic variables (annual precipitation, temperature seasonality), topographic features (elevation, slope), Normalised Difference Vegetation Index [NDVI] and evapotranspiration [ET].**

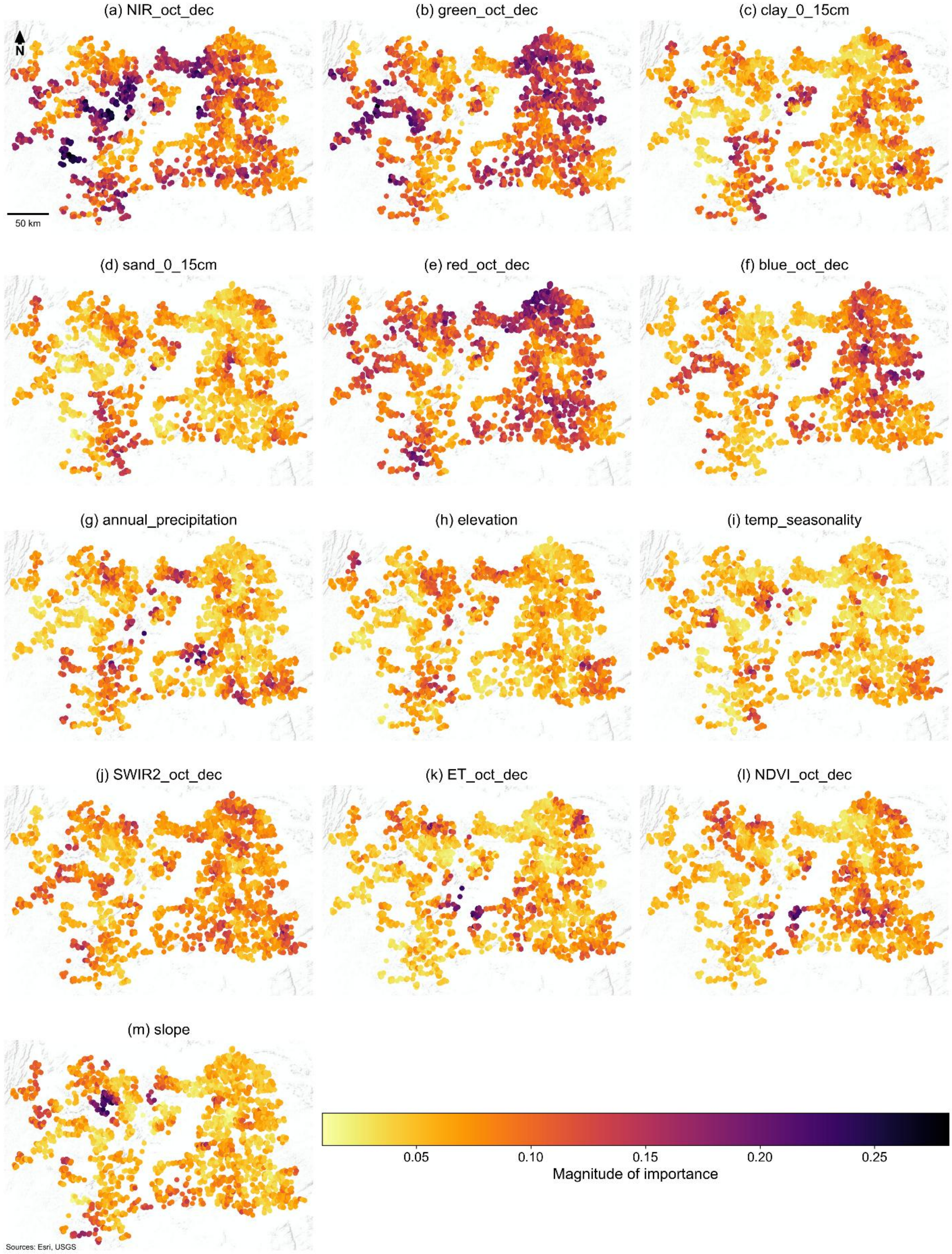

Figure S2 Local feature importance (FI) maps for all 13 predictors used in the optimal Oct–Dec (Short Rains) model. **These maps illustrate the spatial non-stationarity of the variables, demonstrating how the PyGRF model adapts to localised eco-pedological relationships to predict SOC across northern Tanzania during the Short Rains window.**

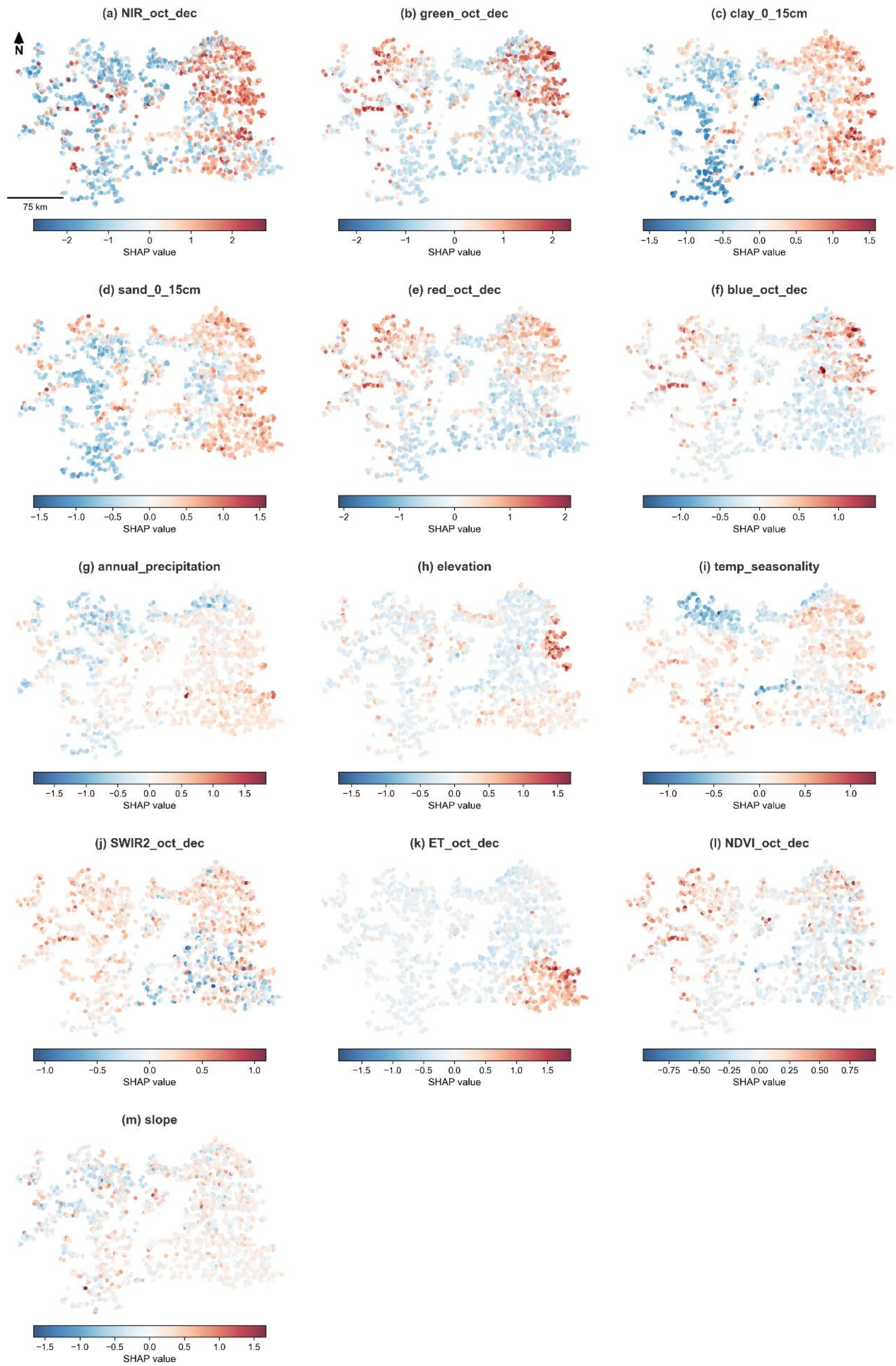

Figure S3 Spatial distribution of SHAP (SHapley Additive exPlanations) values for all 13 predictors in the optimal Oct–Dec (Short Rains) model. **The maps illustrate the localised impact of each variable on global SOC predictions across the study area. The colour gradient indicates the strength and direction of a feature's effect on the model output: red dots represent positive SHAP values (the variable drives SOC prediction higher at this location), while blue dots denote negative SHAP values (the variable drives SOC prediction lower at this location).**

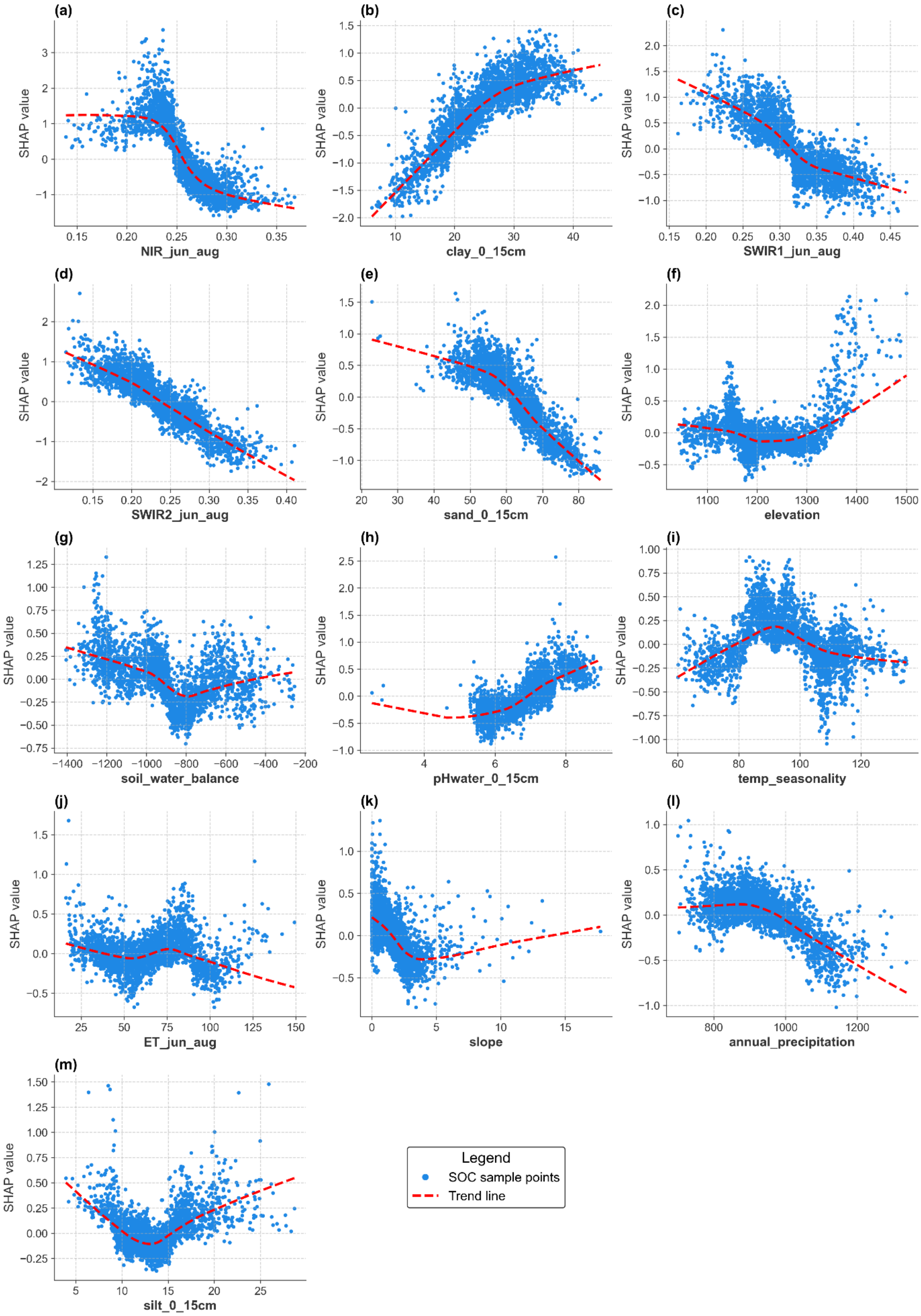

Figure S4 SHAP dependence plots illustrating the functional forms of predictor–SOC relationships for the Jun–Aug (dry season) model. **Predictors: seasonal optical satellite bands (NIR, SWIR1, SWIR2), physical and chemical soil properties (clay, sand & silt content, and pHwater at 0–15 cm depth), bioclimatic variables (annual precipitation, temperature seasonality), topographic features (elevation, slope), NDVI and evapotranspiration [ET].**

Figure S5 Local feature importance maps for all 13 predictors used in the Jun–Aug (dry season) model. **These maps illustrate the spatial non-stationarity of the variables, demonstrating how the PyGRF model adapts to localised eco-pedological relationships to predict SOC across northern Tanzania during the dry season window.**

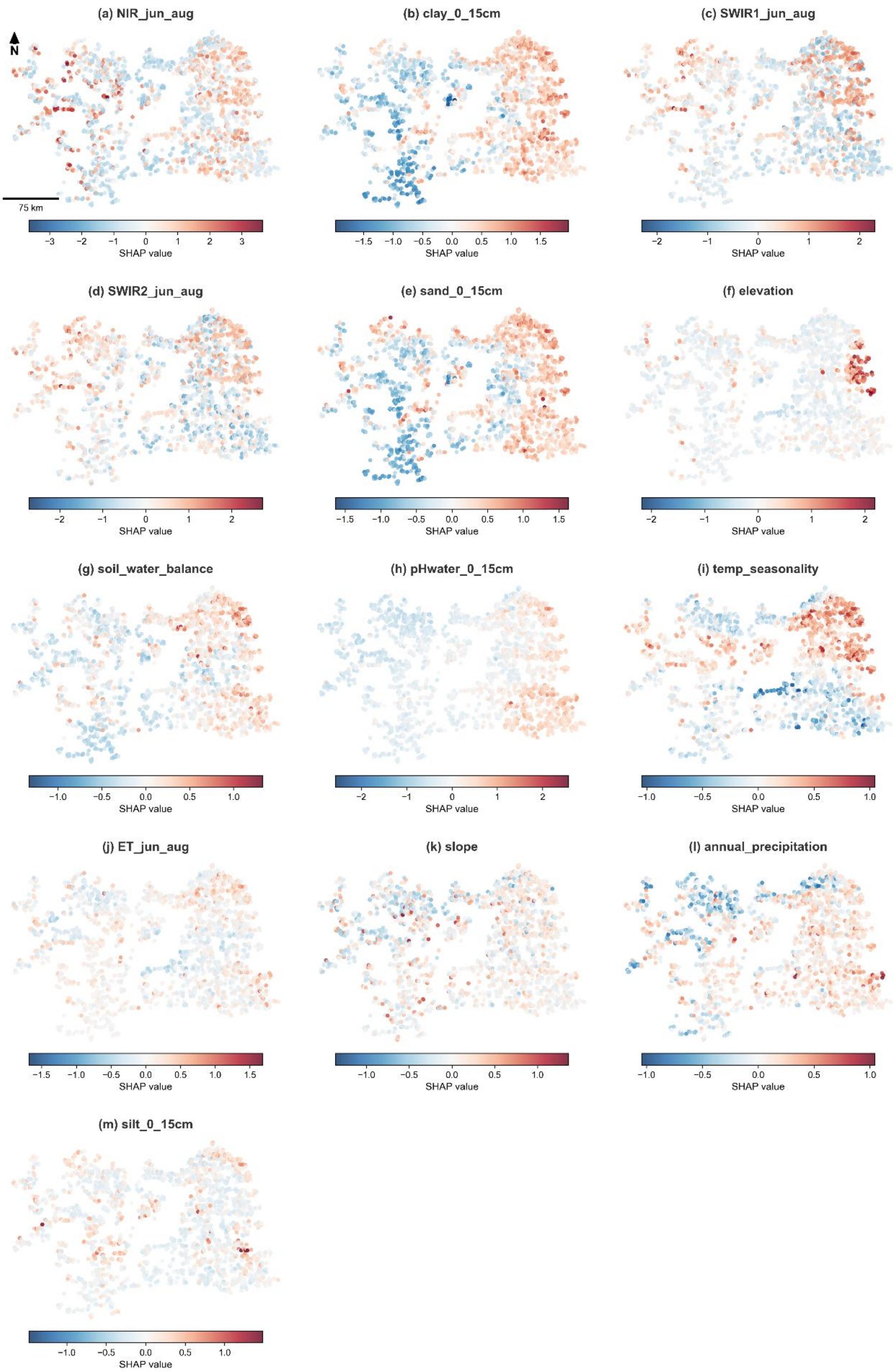

Figure S6 Spatial distribution of SHAP for all 13 predictors in the Jun–Aug (dry season) model. **The maps illustrate the localised impact of each variable on global SOC predictions across the study area. The colour gradient indicates the strength and direction of a feature's effect on the model output: red dots represent positive SHAP values (the variable drives SOC prediction higher at this location), while blue dots denote negative SHAP values (the variable drives SOC prediction lower at this location).**

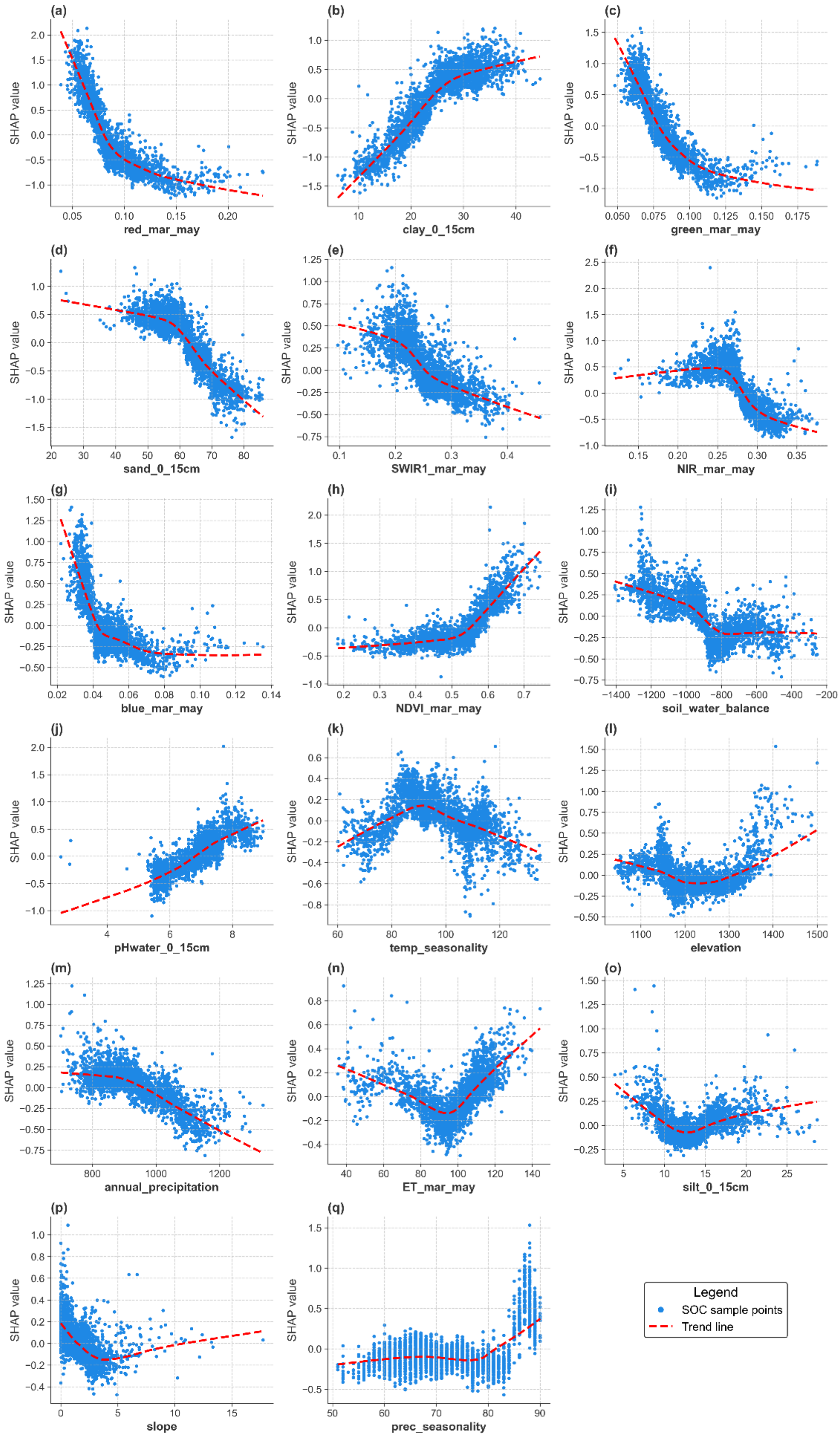

Figure S7 SHAP dependence plots illustrating the functional forms of predictor–SOC relationships for the Mar–May (Long Rains) model. **Predictors: seasonal optical satellite bands (NIR, green, red, blue, SWIR1), physical and chemical soil properties (clay, sand & silt content, and pHwater at 0–15 cm depth), bioclimatic variables (annual precipitation, temperature seasonality, precipitation seasonality and soil water balance), topographic features (elevation, slope), NDVI and evapotranspiration [ET].**

Figure S8 Local feature importance maps for all 17 predictors used in the Mar–May (Long Rains) model. **These maps illustrate the spatial non-stationarity of the variables, demonstrating how the PyGRF model adapts to localised eco-pedological relationships to predict SOC across northern Tanzania during the Long Rains window.**

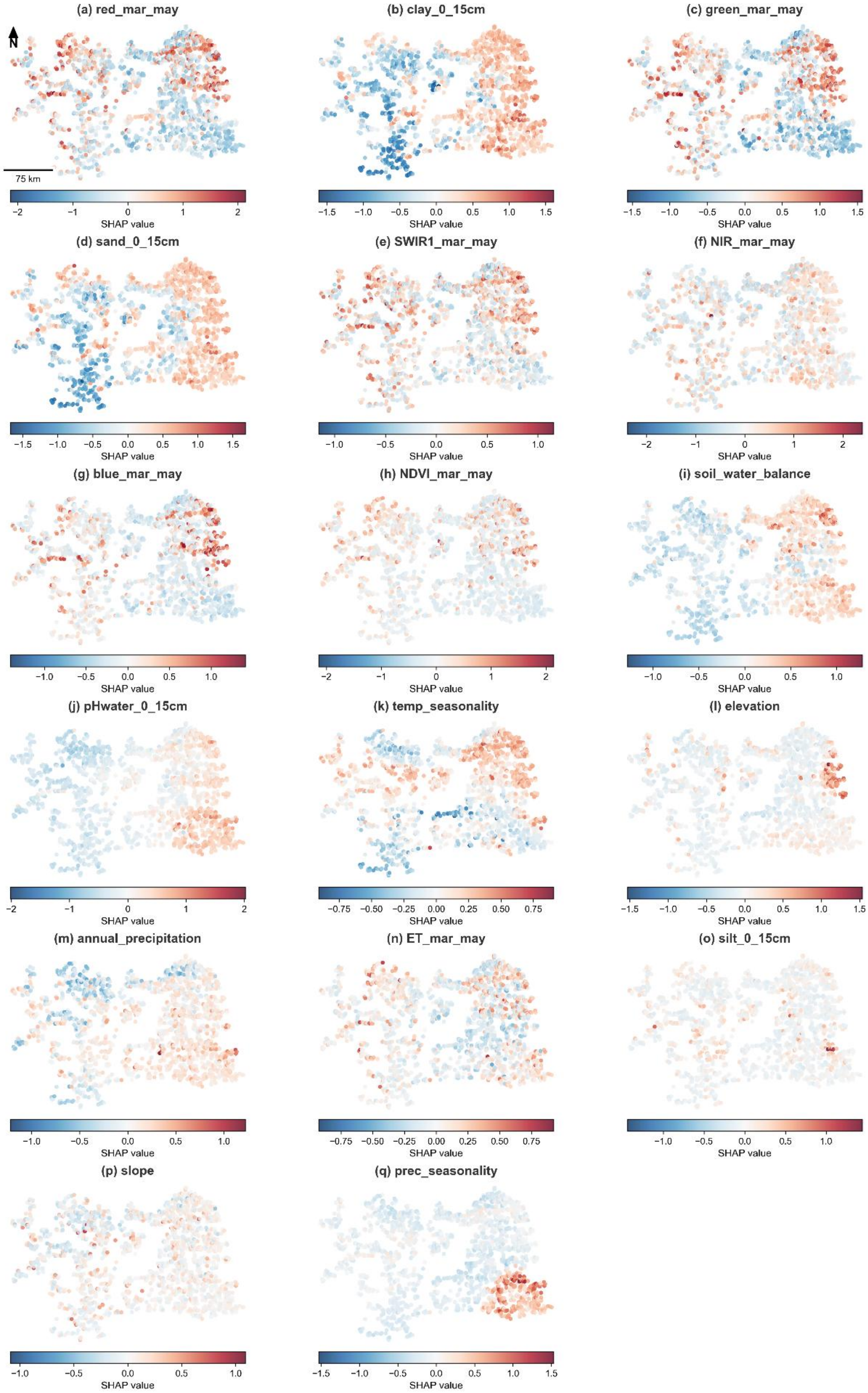

Figure S9 Spatial distribution of SHAP for all 17 predictors in the Mar–May (Long Rains) model. **The maps illustrate the localised impact of each variable on global SOC predictions across the study area. The colour gradient indicates the strength and direction of a feature's effect on the model output: red dots represent positive SHAP values (the variable drives SOC prediction higher at this location), while blue dots denote negative SHAP values (the variable drives SOC prediction lower at this location).**

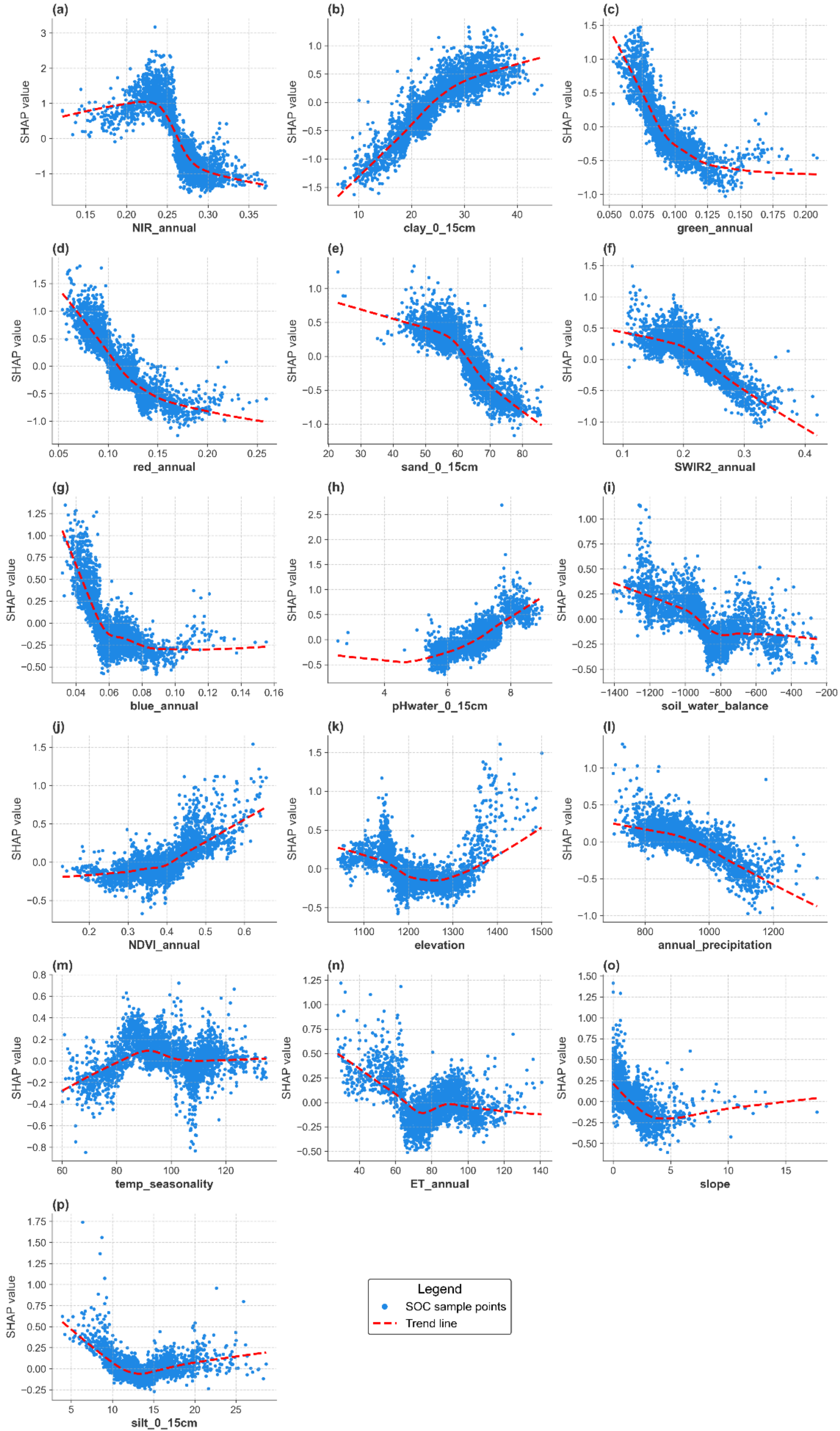

Figure S10 SHAP dependence plots illustrating the functional forms of predictor–SOC relationships for the Annual model. **Predictors: optical satellite bands (NIR, green, red, blue, SWIR2), physical and chemical soil properties (clay, sand & silt content, and pHwater at 0–15 cm depth), bioclimatic variables (annual precipitation, temperature seasonality and soil water balance), topographic features (elevation, slope), NDVI and evapotranspiration [ET].**

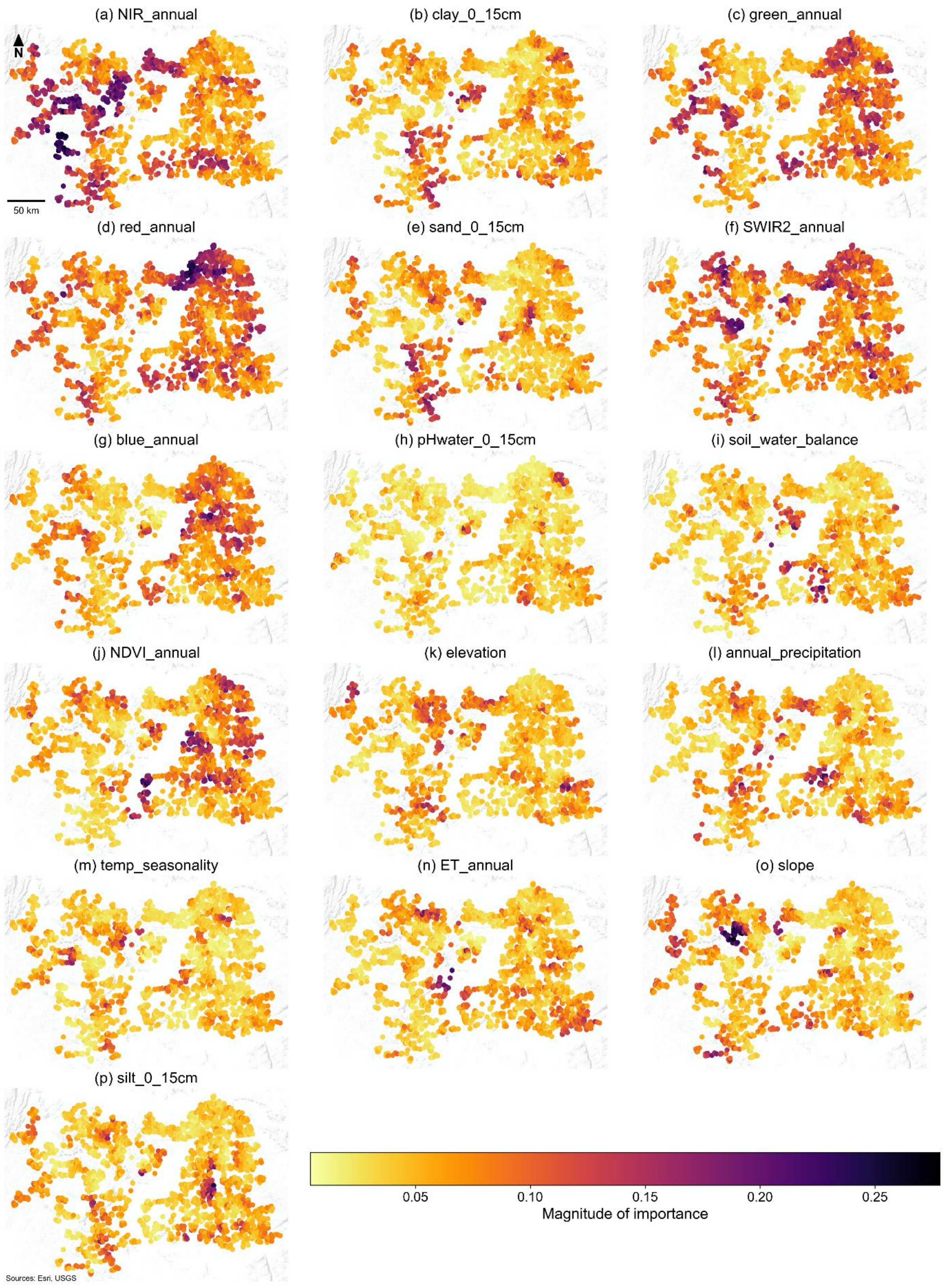

Figure S11 Local feature importance maps for all 16 predictors used in the Annual model. **These maps illustrate the spatial non-stationarity of the variables, demonstrating how the PyGRF model adapts to localised eco-pedological relationships to predict SOC across northern Tanzania when phenological variance was collapsed into an annual composite.**

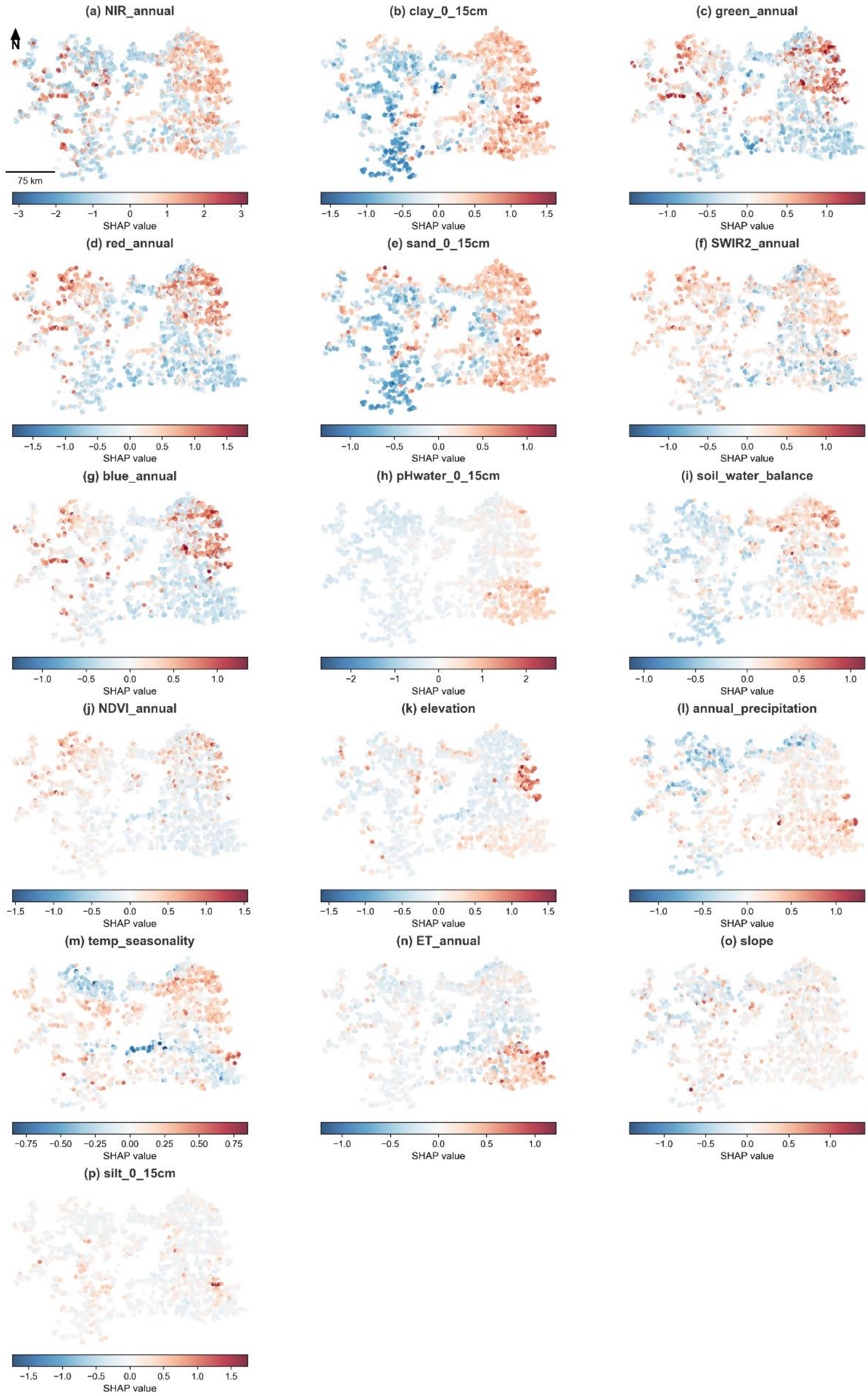

Figure S12 Spatial distribution of SHAP for all 16 predictors in the Annual model. **The maps illustrate the localised impact of each variable on global SOC predictions across the study area. The colour gradient indicates the strength and direction of a feature's effect on the model output: red dots represent positive SHAP values (the variable drives SOC prediction higher at this location), while blue dots denote negative SHAP values (the variable drives SOC prediction lower at this location).**