Dear Editors,

We would like to thank the associate editor and the two anonymous referees for their insightful comments and for taking the time to read our paper carefully. Their feedback helped us improve the paper immensely and we are very gratefully for that. Below, we provide point-by-point response to these comments and state what we did accordingly to improve the paper. Reviewer's comments are presented in *italic* characters, while our responses are indicated in **bold**. Text modifications, as reflected in the revised manuscript, are highlighted in **blue**.

The authors.

Response to Referee #2

In this manuscript, a stochastic model is considered for cloud cover, and it has the form of an Ising model. The two particular aims of this manuscript are to calibrate the model and to compare the model against MODIS cloud area fraction distributions.

I think this is a substantial contribution in terms of the ability of the stochastic model to represent the MODIS observations in a statistical sense of the distribution of mean cloud area fraction, as shown in Figure 4. I also appreciated the discussion in section 5.1 on the energy landscape, as a way to connect the Ising model behavior with the bimodality of the MODIS cloud area fraction. I also think the scientific approach and applied methods are valid. Section 3.2 provided a good discussion of the challenges involved in connecting a simplified statistical mechanics model with the realistic environmental states from reanalysis and MODIS data.

I recommend revisions to improve the presentation.

I feel that the Introduction section could be improved. I spent a long time reading the Introduction section and waiting to hear (i) what is done in this manuscript and (ii) why it is done. Finally, in the second-to-last paragraph of the Introduction section, the two main aims are stated, briefly.

I recommend that greater emphasis be given to what is stated very briefly in Lines 56 and 418-420: This work is aimed at "improved parameterization of cloud cover in climate models" and "this is the first time satellite data were used" in the authors' line of work on statistical mechanics models of cloud cover. Can you expand on how you envision this manuscript being used for sub grid scale parameterizations in climate models?

We agree that the manuscript would benefit from a clearer discussion of its potential relevance for sub-grid-scale parameterizations. In fact, each climate model grid cell can be viewed as containing many smaller cloud elements that are not explicitly resolved by some climate model. The Ising model provides a simple statistical representation of how these unresolved cloud elements organize and interact, allowing us to reproduce observed patterns of cloud cover and transitions between cloudy and clear conditions. The relationships identified between the Ising model parameters (J_0, h_0) and large-scale atmospheric variables the boundary-layer temperature for instance, suggest a possible pathway for developing stochastic cloud-cover parameterizations. Rather than prescribing a single cloud-cover value from the large-scale environment, such an approach could represent the variability and spatial structure of cloud fields within a grid cell in a probabilistic way.

For clarity the following test have been added in the introduction of the revised manuscript.

Specifically, satellite observations of cloud cover and atmospheric boundary layer (ABL) temperature from ERA5 are used to calibrate a Ising-based stochastic model using hybrid optimization methods. To our knowledge, such an approach has not previously been attempted.

In the proposed framework, mesoscopic cells (corresponding to climate model grid boxes) are divided into microscopic Ising states representing unresolved cloud elements. The inferred relationships between the calibrated model parameters and large-scale environmental conditions then provide a physically based foundation for stochastic subgrid-scale cloud parameterizations capable of reproducing cloud intermittency, spatial organization, and transitions between cloudy and clear-sky regimes. Such calibrated statistical representations may ultimately contribute to improving the parameterization of cloud cover in numerical weather prediction and climate models. Namely, the framework, presented here can be seamlessly extended to the case of muliple cloud-types and adopted to the stochastic mulicloud model of Khouider et al. (2010) and specifically for the case with local interactions Khouider (2014), which is yet to be tested in a conprehensive climate model. The SMCM framework, without local interactions, has been successfully tested in both idealized and comprehensive global models at coarse resolution, especially in terms of improving the tropical mode of variability of rainfall statistics (Deng et al., 2015; Goswami et al., 2017; Khouider et al., 2023; Peters et al., 2017, e.g.). The inclusion of local interactions is expected to improve the representation of sub-grid self-organization of convection in kilometer-scale to moderate (grey-scale) resolution for better simulation of phenomena such as mesoscale convective systems, coastal weather, and localized storms in general.

I also recommend drawing some connections between the topics in the first six paragraphs of the Introduction section. These six paragraphs feel quite disjoint. What I see in the current Introduction section is:

A first paragraph on clouds and the deficient representations of clouds in climate and weather models.

A second paragraph on low clouds and fog and the deficiencies of numerical weather forecast models.

A third paragraph on simplified models of clouds with stochastic processes and phase transition regimes akin to statistical mechanics.

A fourth (and fifth) paragraph on power laws and self-similarity in clouds and other atmospheric observations.

A sixth paragraph on the objectives of this paper: to calibrate an Ising-like model of clouds and to compare with MODIS data.

To me, the connections between these paragraphs are largely missing. I can see some general similarities, and I can try to read between the lines, but it would be helpful to provide some explicit explanations and connections.

We agree that the original Introduction was structured in a way that presented several relevant topics without sufficiently explicit transitions between them. In the revised version, we have improved the narrative flow by explicitly linking these paragraphs into a more coherent progression. In particular, we introduced the following sentence in:

Line 24: . . . . Within this broader context of cloud-related uncertainties, low clouds and fog constitute a particularly important phenomenon to model and parameterize, owing to their strong dependence on fine-scale boundary layer processes and their high spatial and temporal variability.

Line 35: . . . .Consequently, improving physically based representations of cloud organization and variability remains an important challenge for next-generation cloud parameterizations and modeling.

Line 37: In this context, statistical mechanics coupled to a stochastic representation of the

cloud cover provides a promising theoretical framework for representing cloud organization and dynamical transitions between weather regimes. . . .

Line 45: . . . .Such a framework is particularly attractive for subgrid-scale parameterization because it directly represents cloud cover statistically rather than deterministically, while accounting for regime transitions and spatial organization.

I think the manuscript would be improved if the Introduction section could provide connections between the topics that are mentioned there. Along these lines, it would be helpful if the Introduction section would explain more clearly why the models in Paragraphs 3-6 are relevant to the topics in Paragraphs 1-2.

In the revised manuscript, we substantially reorganized and expanded the Introduction to clarify the logical progression from the challenges of representing clouds in weather and climate models (Paragraphs 1–2) to the motivation for using stochastic statistical mechanics approaches (Paragraphs 3–6).

If that could be clearly explained in the Introduction section, I feel that it would help to clarify the motivation for and relevance of the paper.

References

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