

Supplementary Information: Millennial-scale sensitivity of Pine Island and Thwaites Glaciers to effective pressure and ocean melt parameterisations

Matt Trevers¹, Stephen Cornford¹, and Anthony Payne¹

¹University of Bristol

Correspondence: Matt Trevers (matt.trevers@bristol.ac.uk)

Contents of this file

1. Text S1
2. Text S2
3. Text S3
- 5 4. Supplementary Figures

Additional Supporting Information

1. Information and Captions for Movies S1 to S8

S1. Model initialisation

We initialised a present-day configuration of the Amundsen Sea Embayment (ASE) to produce smooth and consistent fields of basal friction coefficient C , ice stiffening factor ϕ and a relaxed ice geometry. This was achieved through an iterative procedure following Bevan et al. (2023) which alternates model inversions to produce optimised fields of C and ϕ with relaxation cycles in which we allowed the surface geometry to evolve. We applied the most recently calculated fields of C and ϕ during each relaxation cycle, and the updated surface geometry was applied in the next iteration of the inverse problem. The initial ice thickness came from BedMachine v3 (Morlighem, 2022).

Fields of C and ϕ were estimated by solving an inverse problem. This method is described in detail in Cornford et al. (2015). In short, smooth fields of $C(x, y)$ and $\phi(x, y)$ are chosen that minimise the misfit between modelled and observed ice flow speeds. The observed flow speed came from (Mouginot et al., 2019). Where velocity observations are available we set as the initial guess at C ,

$$C_0 = \frac{\rho_i g h |\nabla s|}{|\mathbf{u}_o| + 1}, \quad (1)$$

elsewhere and outside of the ASE basin boundary we set $C_0 = 10^5$. We initially set $\phi_0 = 1$. A nonlinear conjugate gradient method was applied to find a minimum of an objective function composed of the velocity misfit function and penalty functions for $C(x, y)$ and $\phi(x, y)$. The penalty functions act to limit the magnitude of spatial gradients of $C(x, y)$ and $\phi(x, y)$, with the Tikhonov coefficients determining the relative weightings of spatial gradients of $C(x, y)$ and $\phi(x, y)$ within the objective function. The inclusion of penalty functions in the objective function serves two purposes. Firstly, without it, the inverse problem would be under-determined, that is we would be seeking values of two unknown and unconstrained fields with only one field of input data. Secondly, it limits overfitting to small changes and noise in the observed velocity. Without the penalty functions the problem would be ill-conditioned.

During the relaxation cycle, we prohibited thinning or thickening for floating ice by calculating an additional basal mass balance which opposed any thickness change. Over grounded ice we applied the mean surface mass balance from 1980 to 2018 from the MAR regional climate model (Agosta et al., 2019), from which the observed rate of thickness change from Smith et al. (2020) was subtracted. This follows a method introduced by van den Akker et al. (2025) which is intended to optimise agreement with modern observed thickening/thinning rates in the final relaxed state once the observed thickness change rate component is removed from the applied surface mass balance.

The initialisation procedure was run for 50 years. The inverse problem was solved before the first timestep of the relaxation, and then again at 10 year intervals. During both the inversion and the relaxation cycles, a linear viscous sliding law was applied:

$$\tau_{b,l} = -C_l \mathbf{u}_b. \quad (2)$$

The units and magnitude of C_l in Equation 2 differ from those of the Regularised law (Equation 1). Following the initialisation procedure, we therefore calculate C by equating $\tau_{b,r} = \tau_{b,l}$, taking the final modelled velocity at the end of the initialisation as u_b .

45 Figure S1 shows the state at the end of the initialisation and Figure S2 shows the model state following 1 year of a forwards model run initialised from the relaxed model state. Figures S3e and S3f show the basal shear stress τ_b (independent of sliding law) and ice stiffening factor ϕ produced by the initialisation procedure.

50 S2. Thermal spin-up

The three dimensional temperature field used in this study was derived from a thermal spin-up using the BISICLES ice sheet model. The spin-up was carried out for the whole Antarctic Ice Sheet.

Initially an inverse problem was solved to generate a realistic velocity structure for the ice sheet using the observed velocities
55 from (Mouginot et al., 2019). Note that unlike in Section S1, we simply performed a single model inversion rather than carrying out the full iterative procedure.

The spin-up was carried out for 100,000 years at a resolution of 8 km across the full ice sheet. The ice thickness was held constant throughout the spin-up. The ice column was divided into 24 vertical layers, with increasing vertical resolution towards
60 the bed. The mean of monthly surface temperatures from 1980 to 2018 from the MAR regional climate model (Agosta et al., 2019) provided the surface temperature boundary condition. The mean geothermal heat flow dataset from Burton-Johnson et al. (2020) provided the basal boundary condition.

Following the spin-up, the temperature field for the ASE domain was extracted directly from the full Antarctic field. Figure S3d
65 shows the depth averaged temperature field within the ASE.

S3. Basal melt parameterisation

The values of basal melt parameters a_0 , d_{tc} and p in Equation 3 were determined separately for the PIG and TG drainage
70 basins. While this would potentially produce a discontinuity in the melt rate across the basin boundary if applying melting to each basin simultaneously, the simulations in this study only apply melting to individual basins so this is not a consideration here. The melt parameterisations were calculated to produce melt volumes of 100 Gt yr⁻¹ for each ice shelf individually in the initial state (Rignot et al., 2013).

75 We take the balance melt rate produced during the model initialisation procedure (Section), which was the melt rate applied to

keep the ice shelf in a steady state. The observed thickness change rate from Smith et al. (2020) is subtracted from this balance melt rate to provide an estimate for the modern basal melt rate.

80 A series of test melt parameterisations were constructed across a range of values of p and d_{tc} , with corresponding a_0 values calculated to achieve 100 Gt yr^{-1} of melt volume. A least squares method was applied to select one parameterisation by comparison of the test parameterisations against the modern melt estimates and their associated ice shelf drafts. For PIIS the selected parameter values are $p = 2$, $d_{tc} = 0 \text{ m}$ and $a_0 = 9.07 \times 10^{-5}$ (Supplementary Figure S4). For TIS the parameter values are $p = 1$, $d_{tc} = 0 \text{ m}$ and $a_0 = 9.17 \times 10^{-2}$ (Supplementary Figure S5).

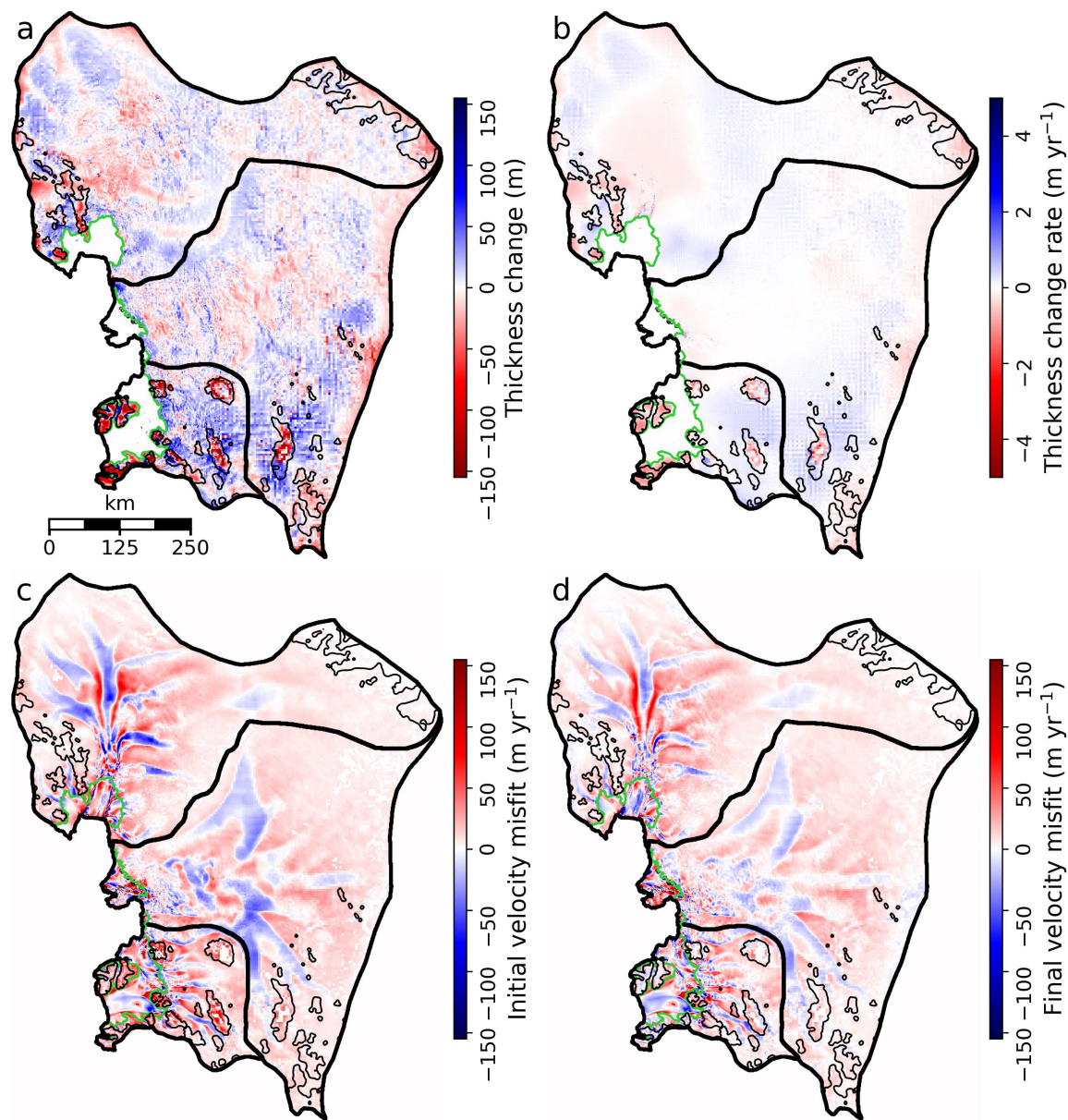


Figure S1. Initial state following the 50 year initialisation procedure. (a) Total thickness change from the original ice thickness. (b) Rate of thickness change after 50 years in the final relaxation iteration. (c) Initial ice velocity misfit. (d) Final ice velocity misfit. Thick black lines show the basin boundaries and ice extent while thin black lines show the 0 m contour of bed depth. Grounding lines are marked by green lines.

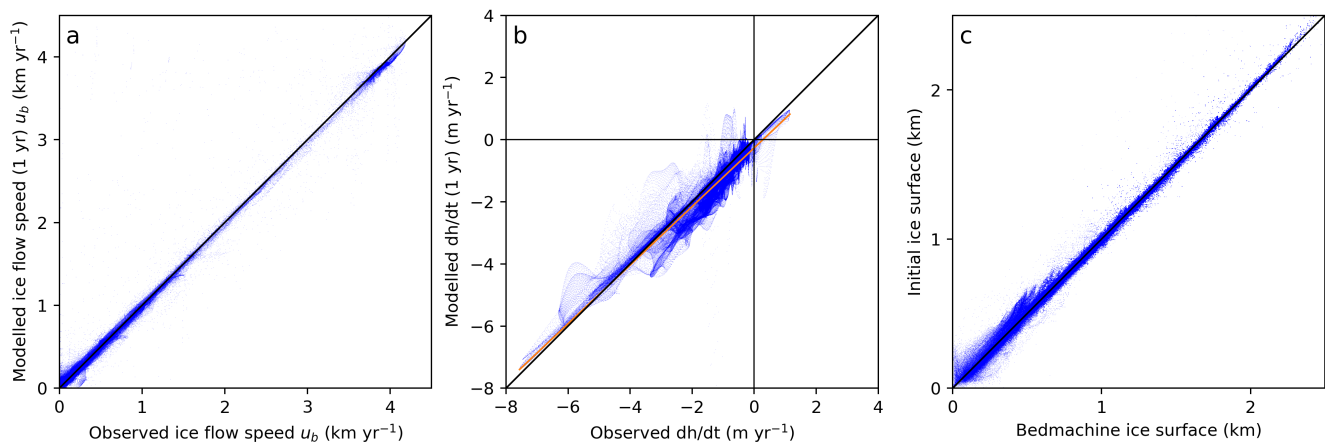


Figure S2. Model state following the first year of a forwards run started from the initial state shown in Figure S1. Scatterplots of: (a) Modelled versus observed ice flow speed. (b) Modelled versus observed rate of thickness change sampled where the flow speed exceeds 100 m/year, with orange line of best fit. (c) Initial ice surface elevation versus BedMachine v3 ice surface.

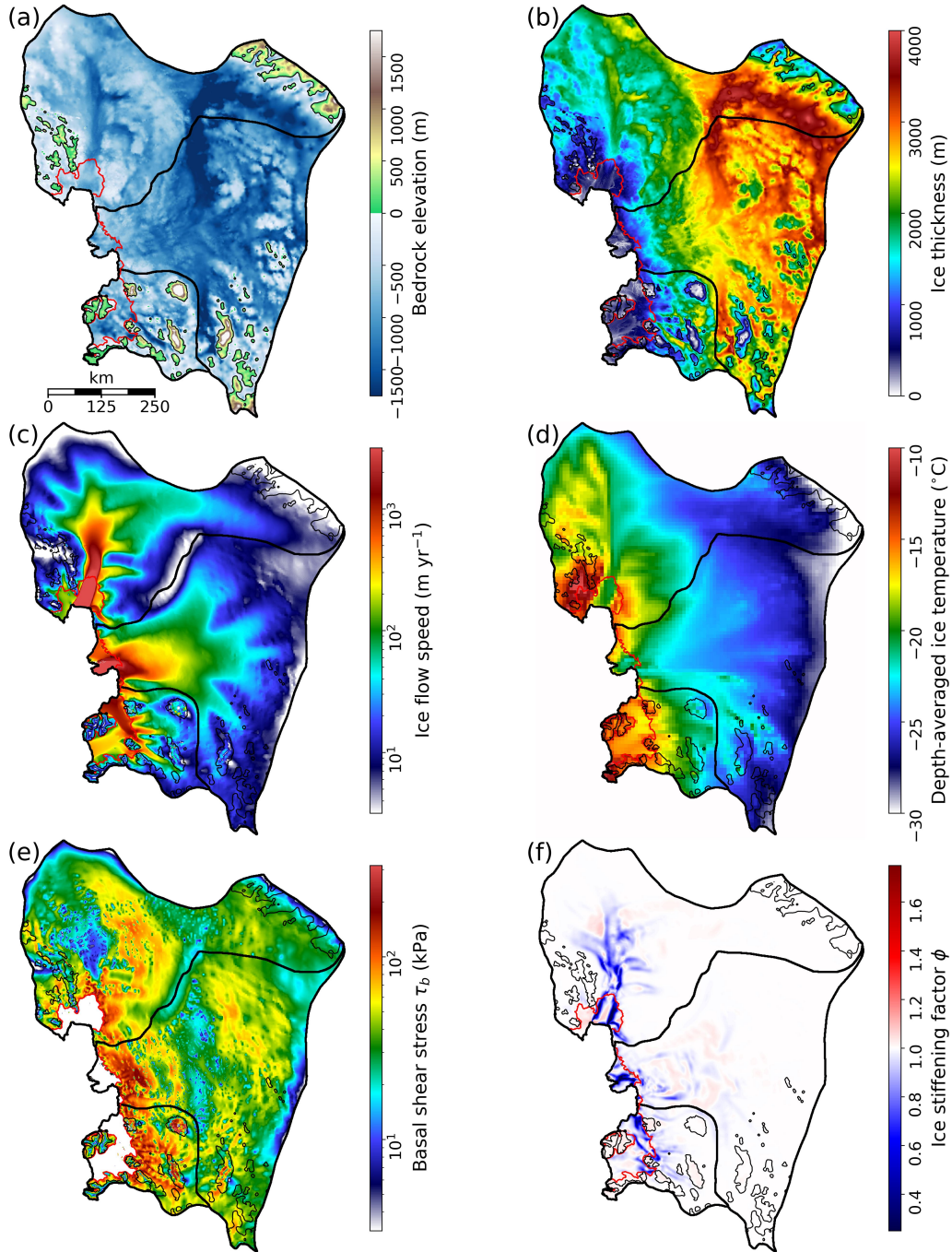


Figure S3. Model inputs following the initialization, shown only within the ASE. (a) Bed topography, (b) ice thickness, (c) modelled ice flow speed, (d) depth-averaged ice temperature, (e) basal shear stress and (f) ice stiffening factor. Thick black contours outline the drainage basins, thin black contours show the 0 m contour of bed depth and red lines highlight the grounding line.

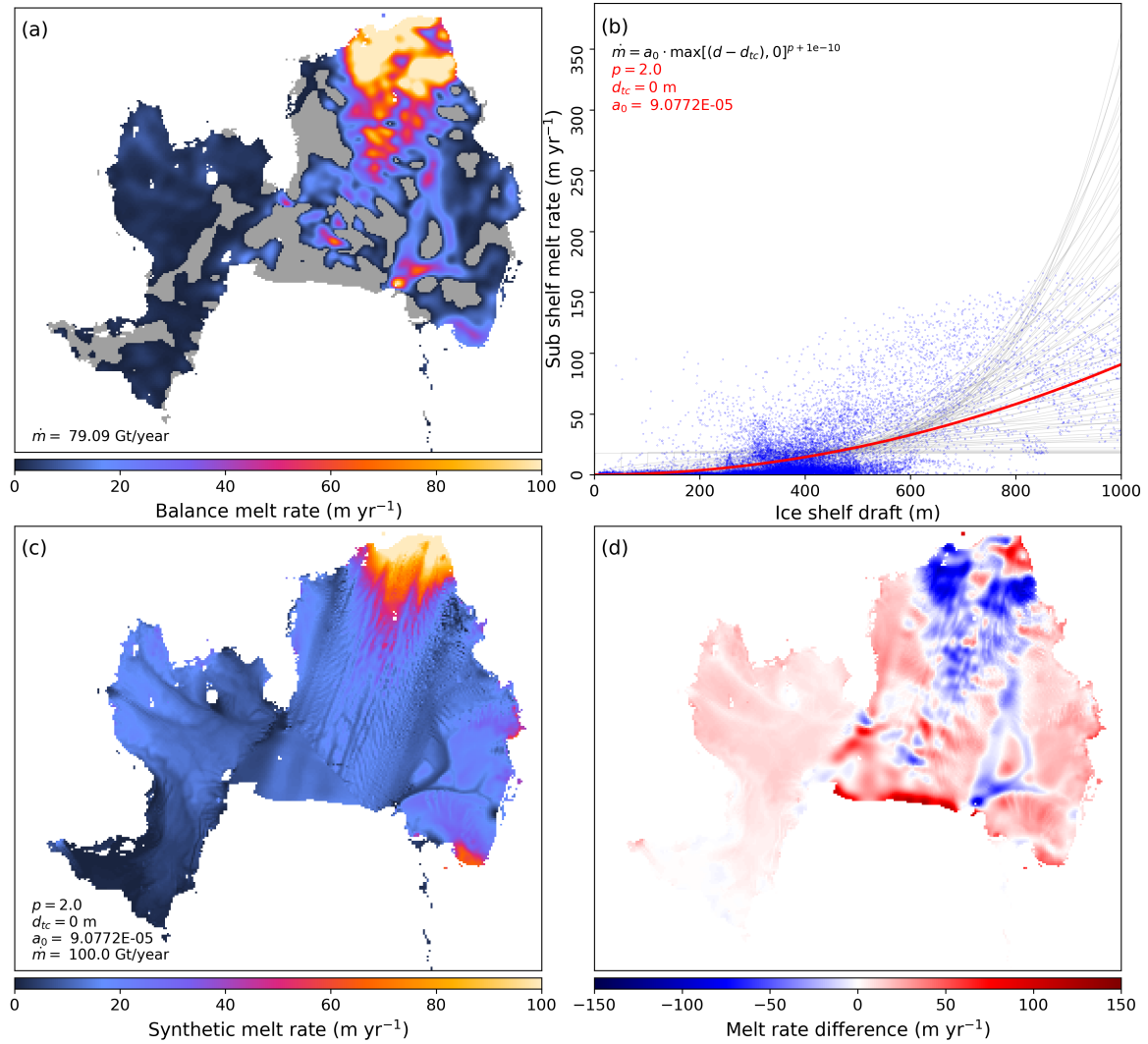


Figure S4. Detail of the basal melt rate parameterisation for PIIS. (a) The estimated present-day basal melt rate, constructed by subtracting the thickness change rate from Smith et al. (2020) from the balance melt rate produced by the model initialisation (Figure S2). Grey areas denote regions where the melt rate was negative (i.e. freezing-on occurred). (b) Estimated melt rate plotted against ice shelf draft for individual cells (blue scatter points). Grey lines denote trial melt parameterisations across a range of values of index p and thermocline depth d_{tc} . The red line denotes the selected parameterisation with values shown in red. a_0 is calculated to produce a total melt rate of 100 Gt/year across the ice shelf. (c) The parameterised synthetic melt rate applied to the PIIS initial geometry. (d) The difference between the synthetic melt rate (panel c) and the balance melt rate (panel a).

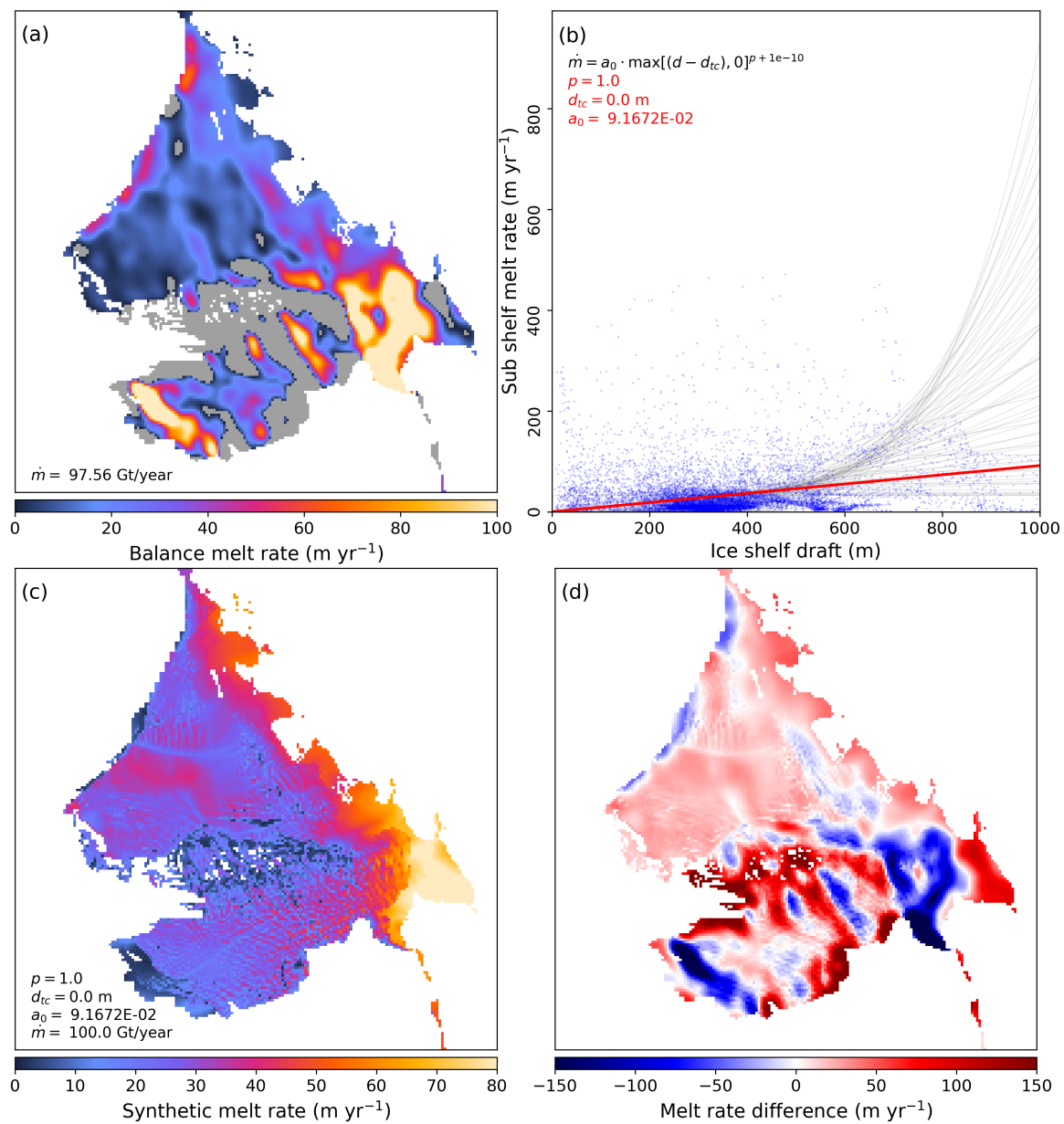


Figure S5. Detail of the basal melt rate parameterisation for the TIS. Panels (a) to (d) as in Figure S4.

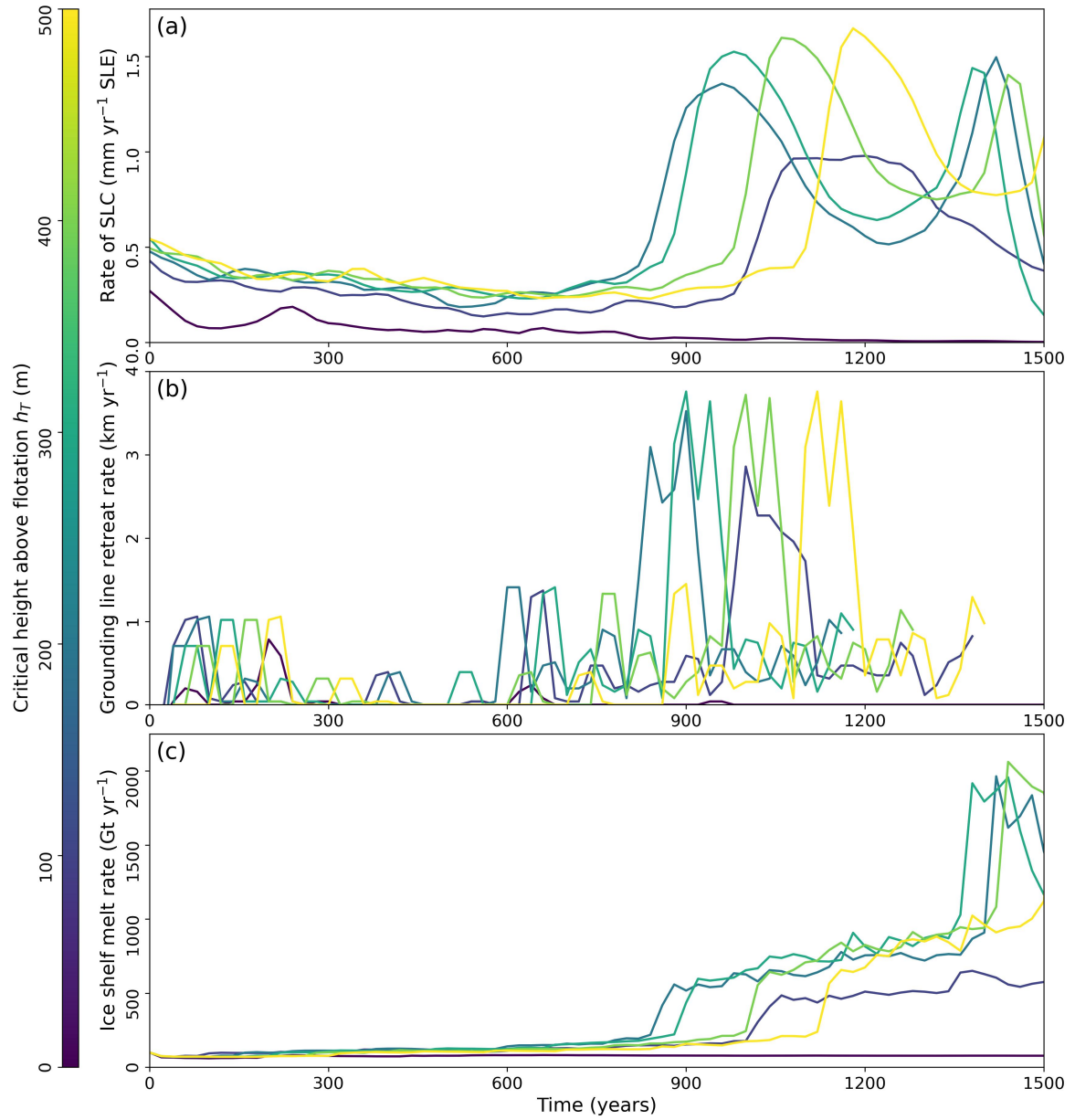


Figure S6. Additional results for PIG1 simulations across the range of h_T . (a) Rate of SLC. (b) Rate of grounding line retreat. (c) Total ice shelf melt rate.

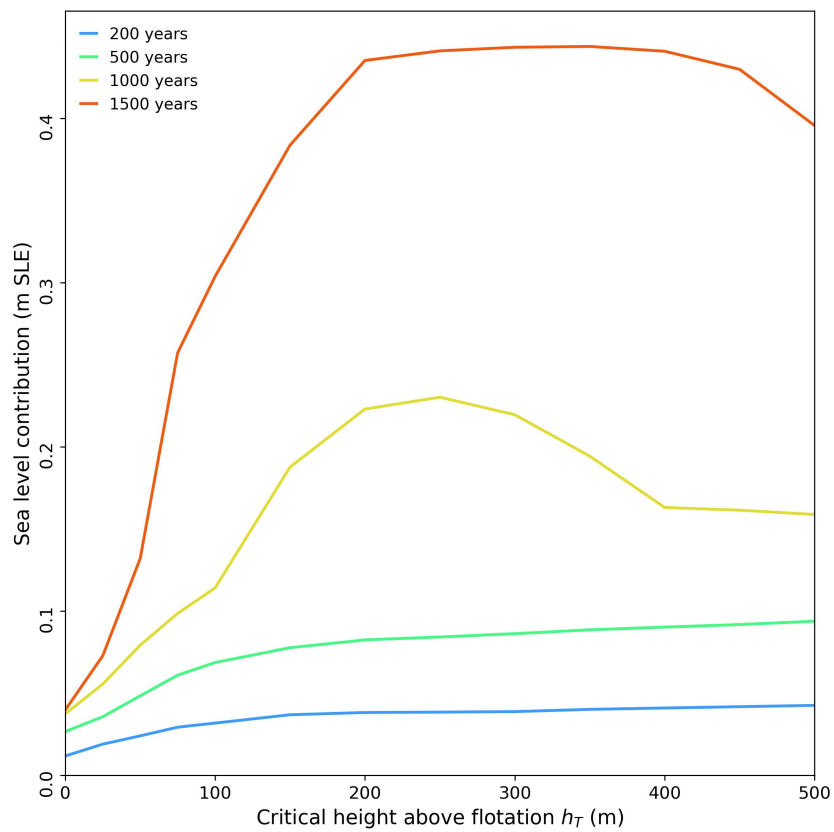


Figure S7. SLC measured at set times throughout the PIG1 simulations across the range of h_T .

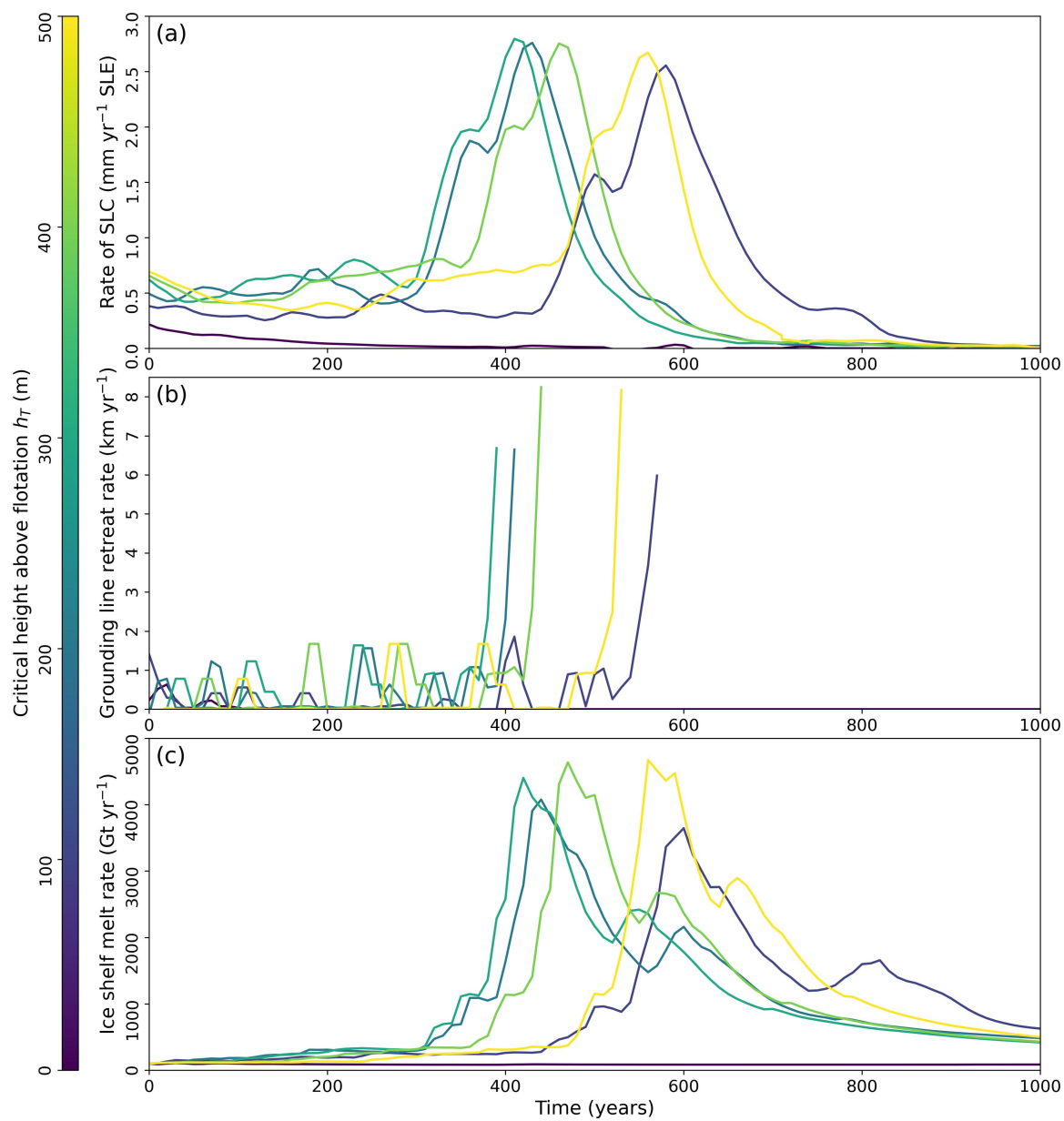


Figure S8. Additional results for TG1 simulations across the range of h_T . (a) to (c) as in Figure S6 but for the TG drainage basin.

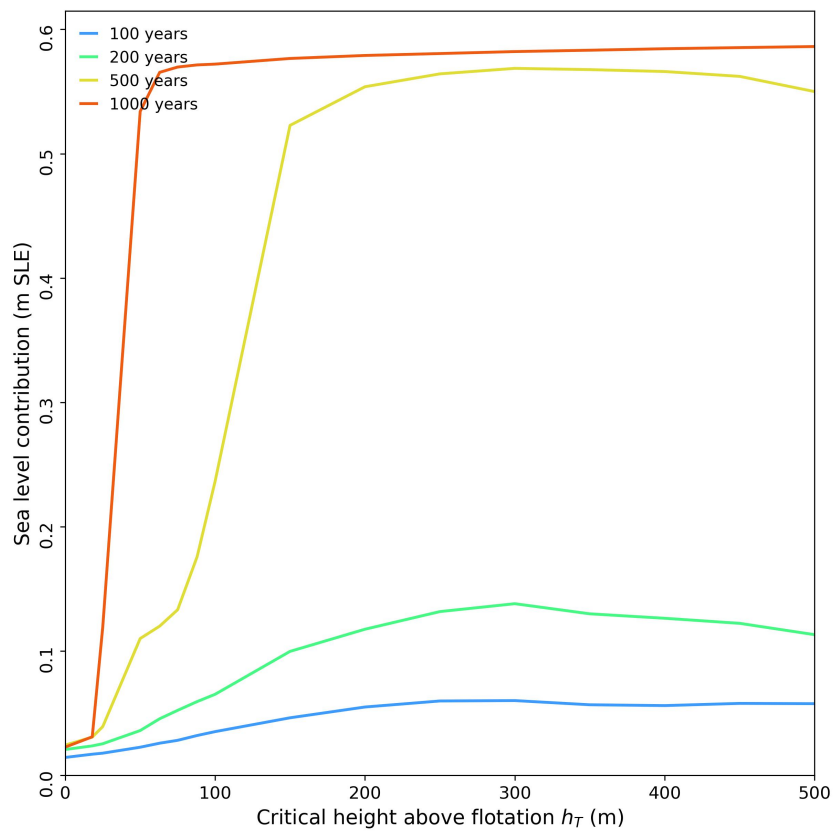


Figure S9. SLC measured at set times throughout the TG1 simulations across the range of h_T .

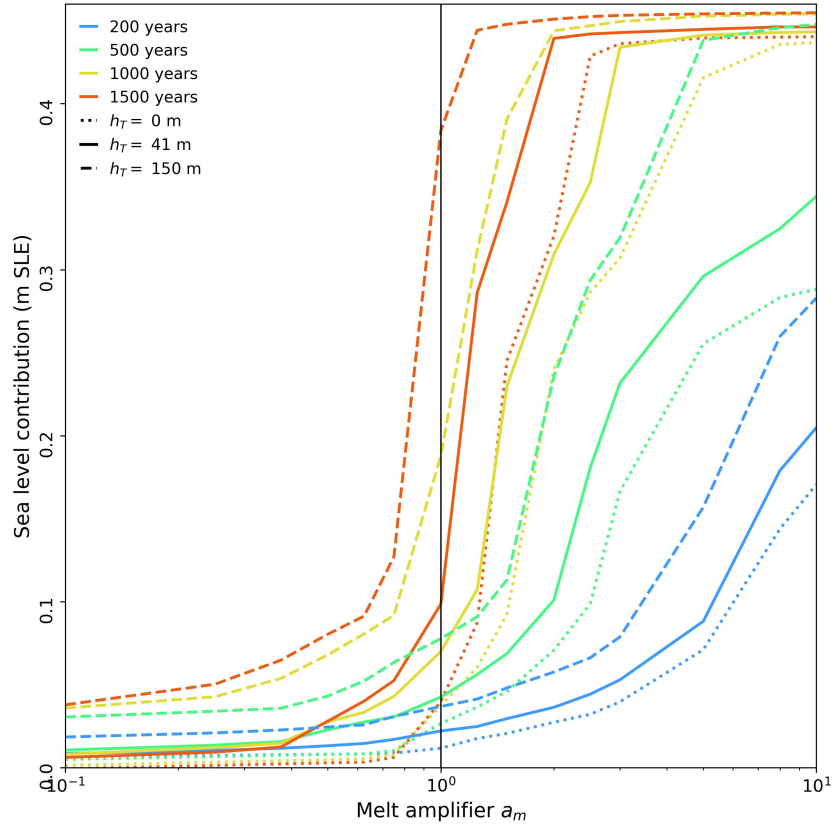


Figure S10. SLC measured at set times throughout the FIG2 simulations across the range of a_m and for all values of h_T .

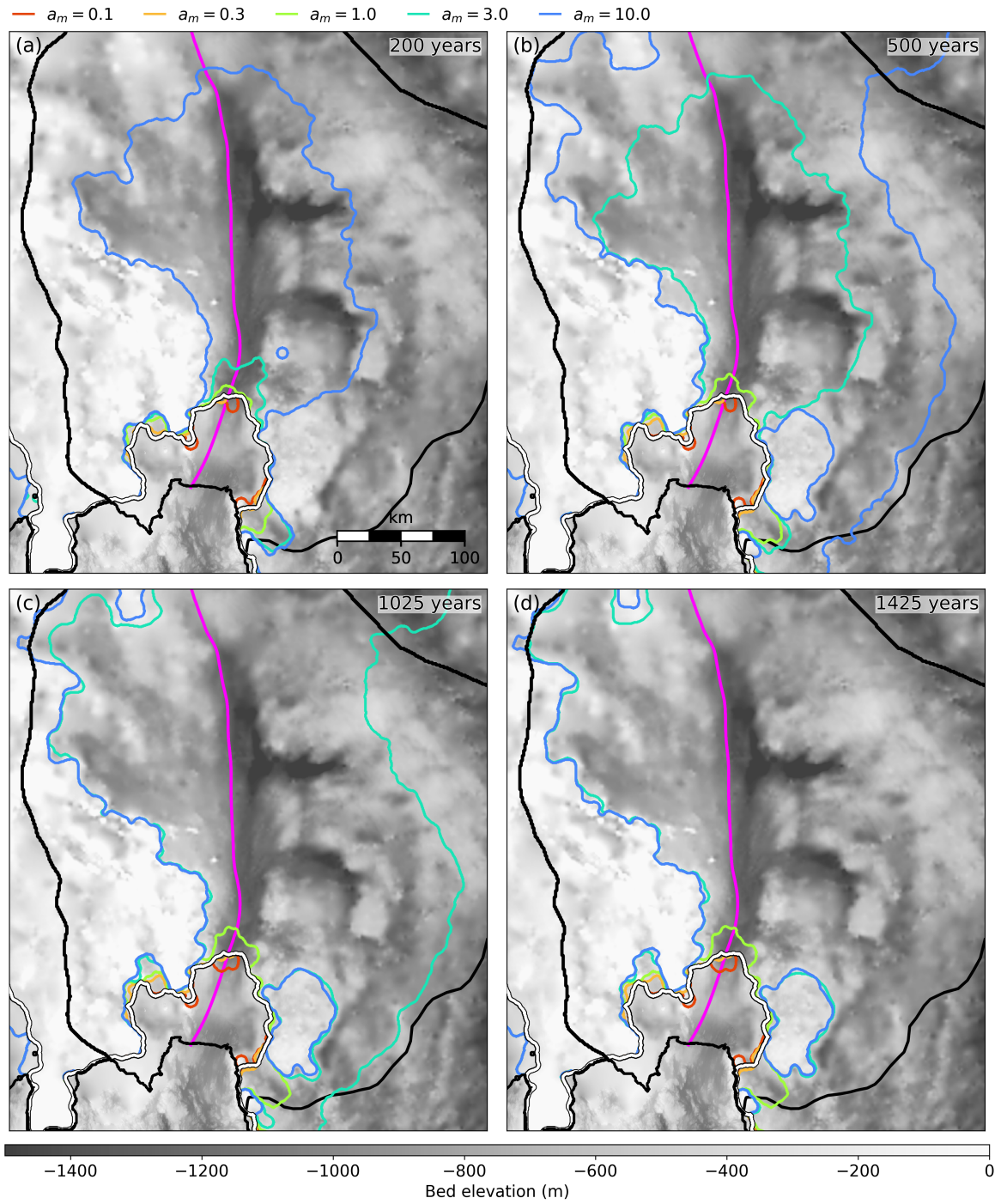


Figure S11. Maps of grounding line retreat for PIG2 simulations (Figure 7) for $h_T = 0$ m, for values of a_m between 0.1 and 10. Snapshots at 200 years (a), 500 years (b), 1025 years (c) and 1425 years (d).

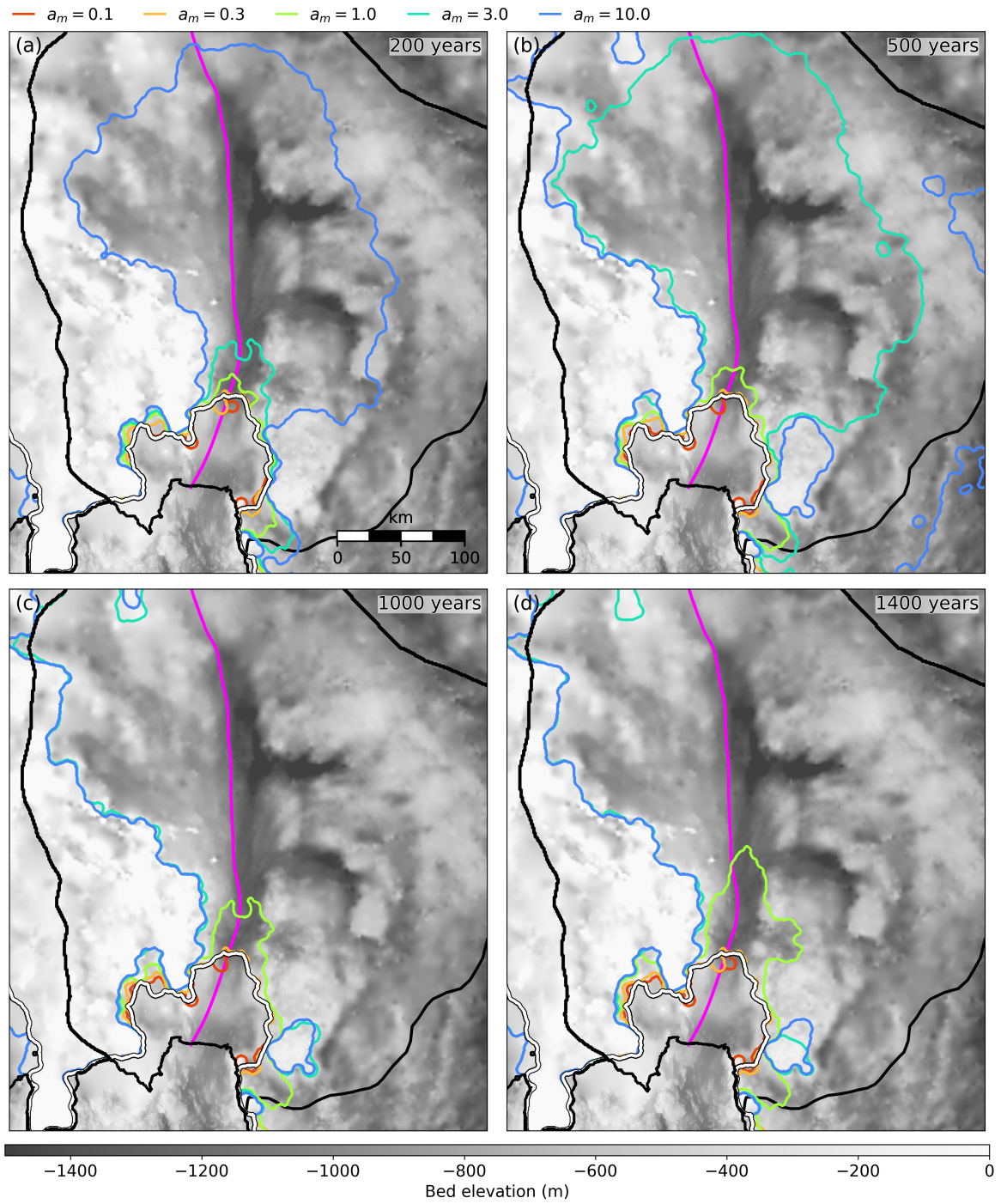


Figure S12. Maps of grounding line retreat for PIG2 simulations (Figure 7) for $h_T = 41$ m, for values of a_m between 0.1 and 10. Snapshots at 200 years (a), 500 years (b), 1000 years (c) and 1400 years (d).

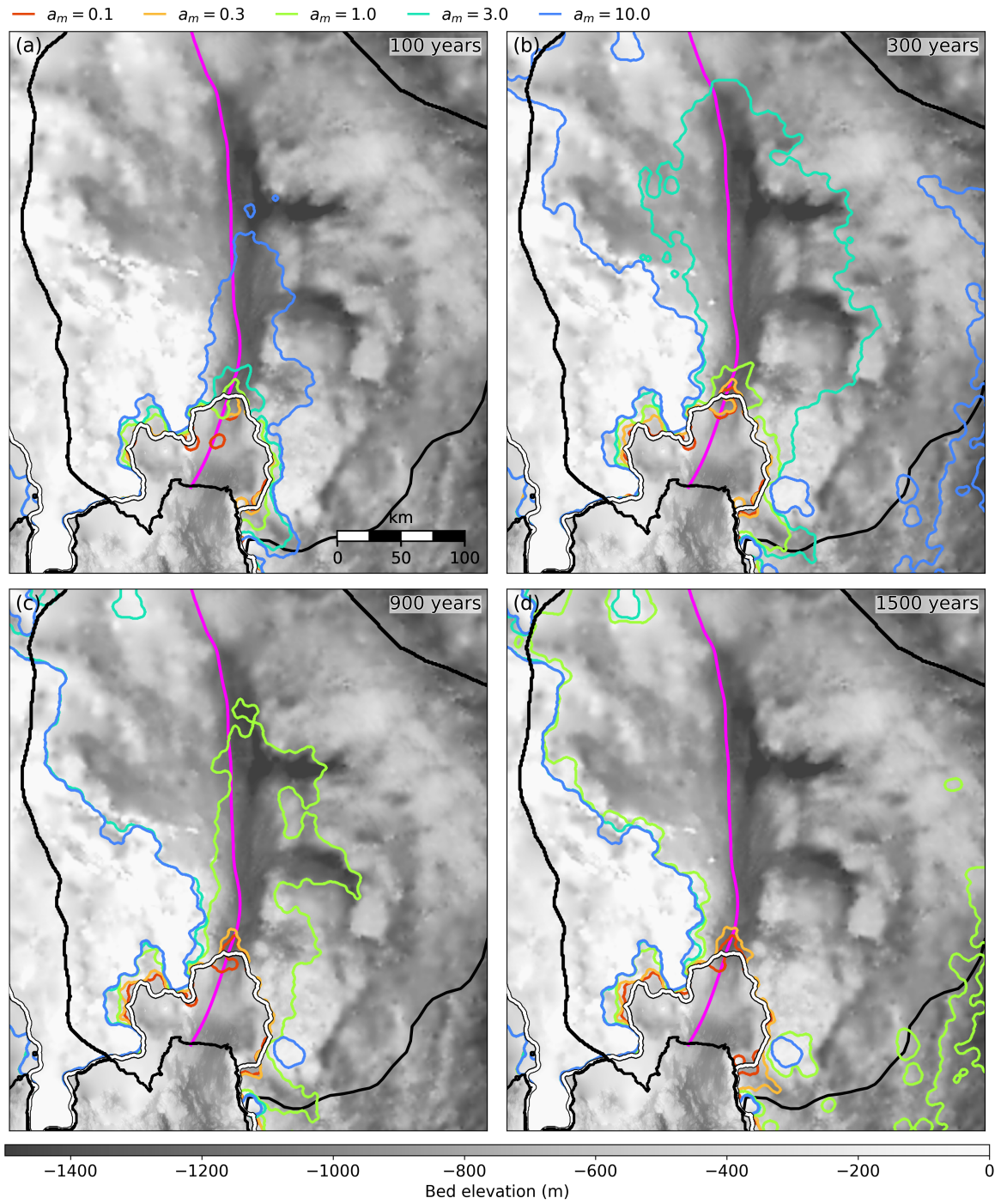


Figure S13. Maps of grounding line retreat for PIG2 simulations (Figure 7) for $h_T = 150$ m, for values of a_m between 0.1 and 10. Snapshots at 100 years (a), 300 years (b), 900 years (c) and 1500 years (d).

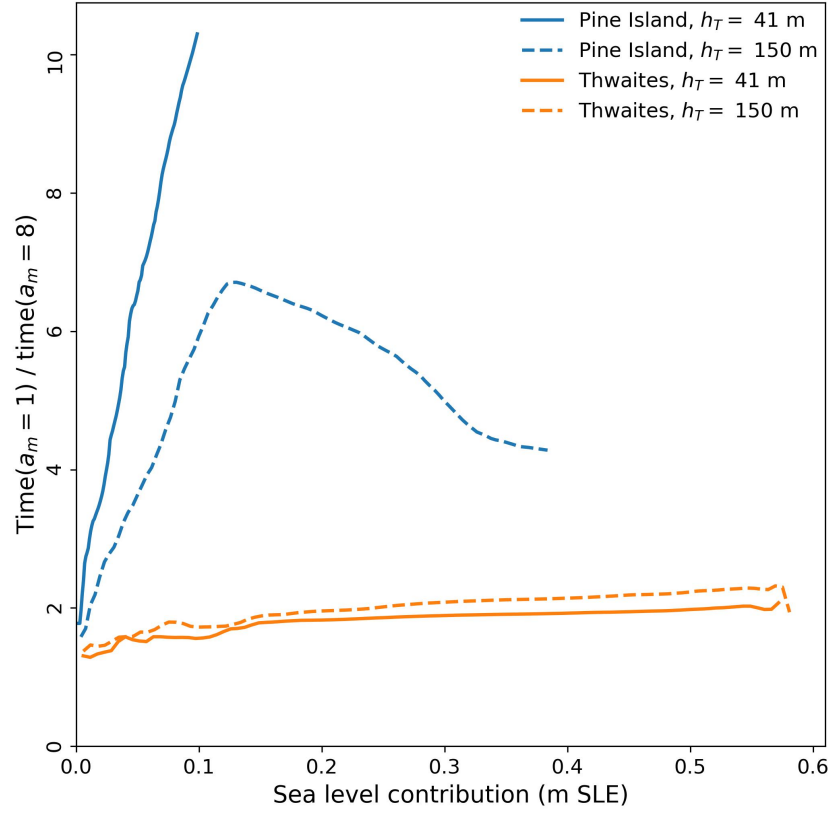


Figure S14. The ratio between the time taken to reach values of SLC, between $a_m = 1$ and $a_m = 8$, for PIG2 and TG2 simulations and with $h_T = 41$ m and $h_T = 150$ m.

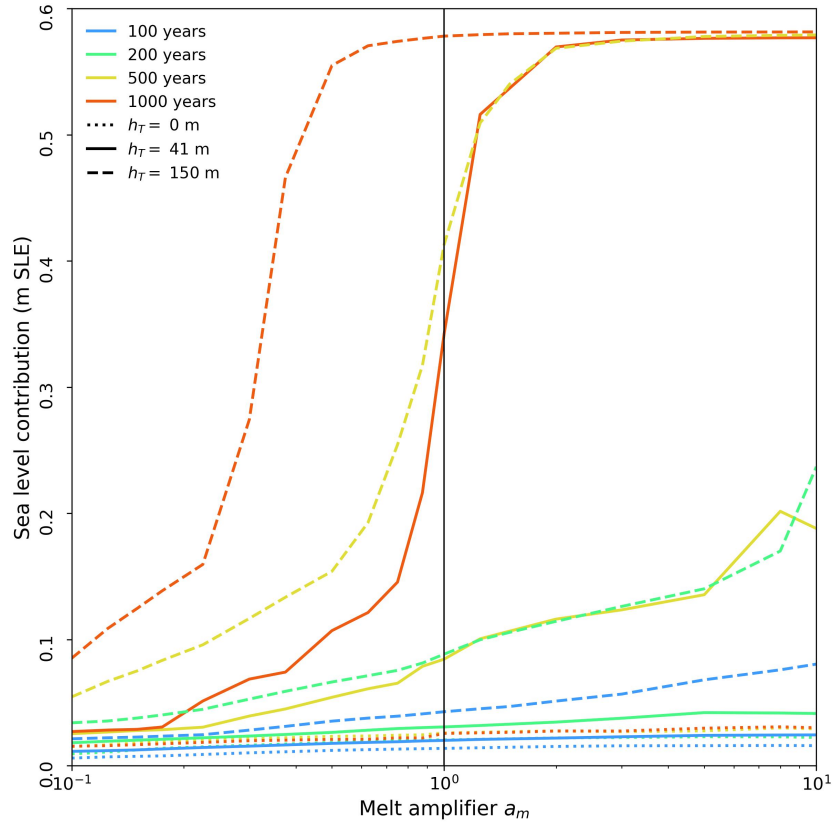


Figure S15. SLC measured at set times throughout the TG2 simulations across the range of a_m and for all values of h_T .

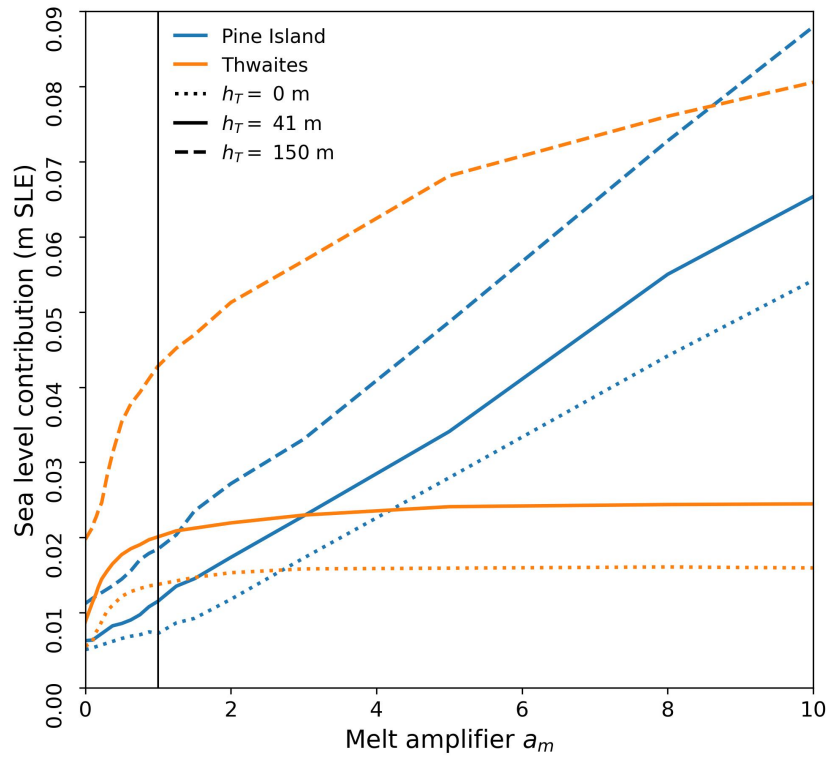


Figure S16. 100 year SLC from PIG2 and TG2 simulations across the range of a_m , and for the three values of h_T . This figure reproduces the details from in Figures S10 and S11 in increased detail, and with a linear horizontal axis.

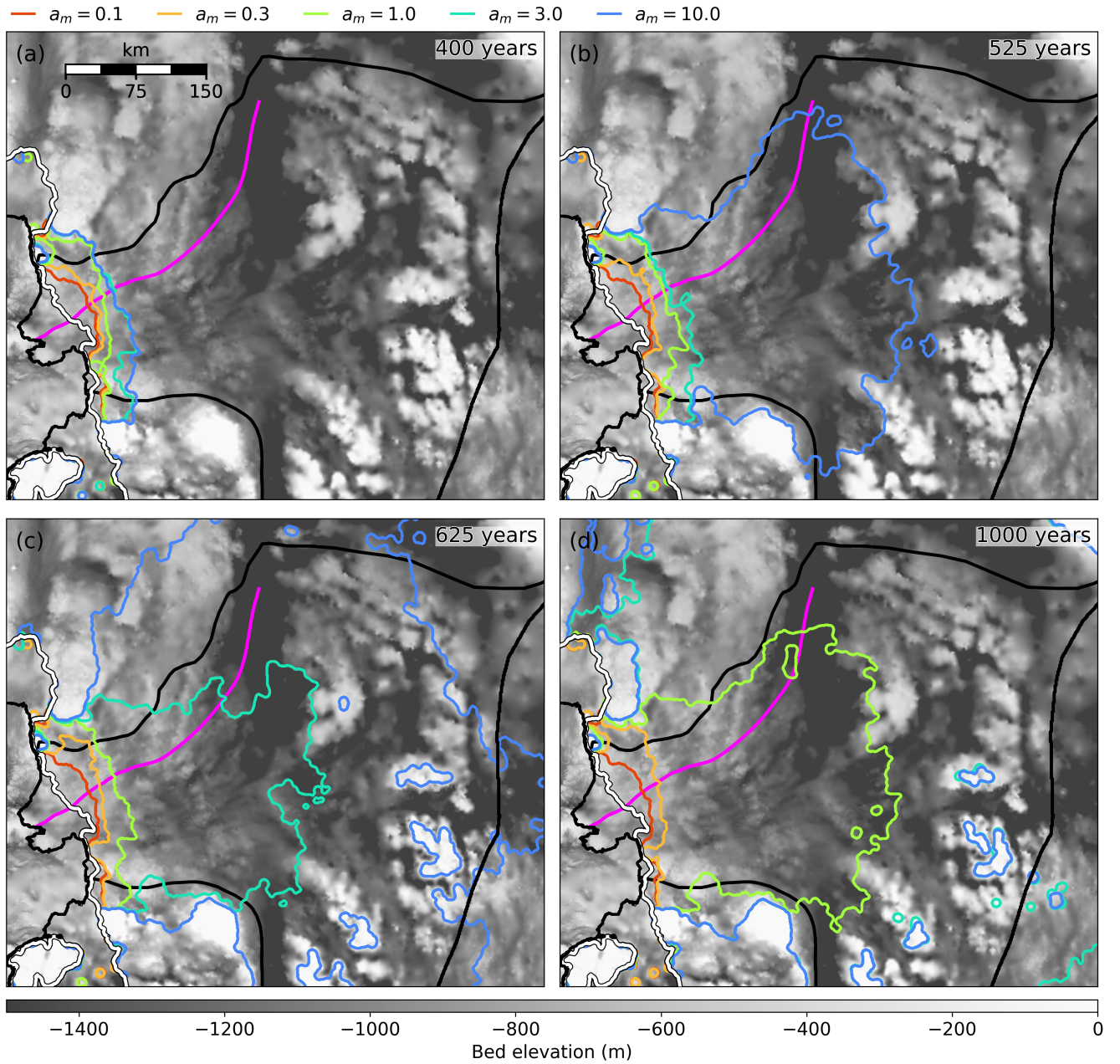


Figure S17. Maps of grounding line retreat for TG2 simulations (Figure 8) for $h_T = 41$ m, for values of a_m between 0.1 and 10. Snapshots at 400 years (a), 525 years (b), 625 years (c) and 1000 years (d).

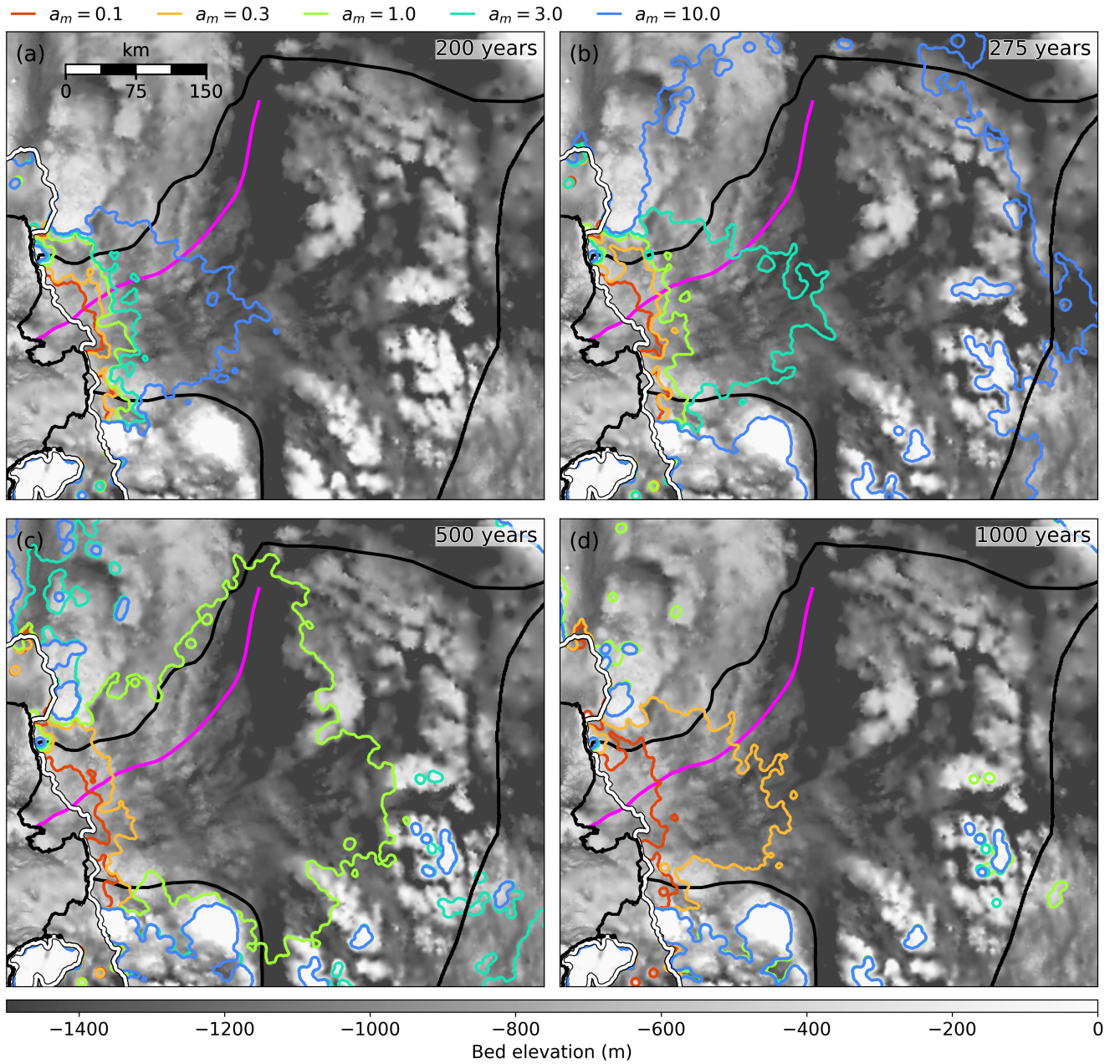


Figure S18. Maps of grounding line retreat for TG2 simulations (Figure 8) for $h_T = 150$ m, for values of a_m between 0.1 and 10. Snapshots at 200 years (a), 275 years (b), 500 years (c) and 1000 years (d).

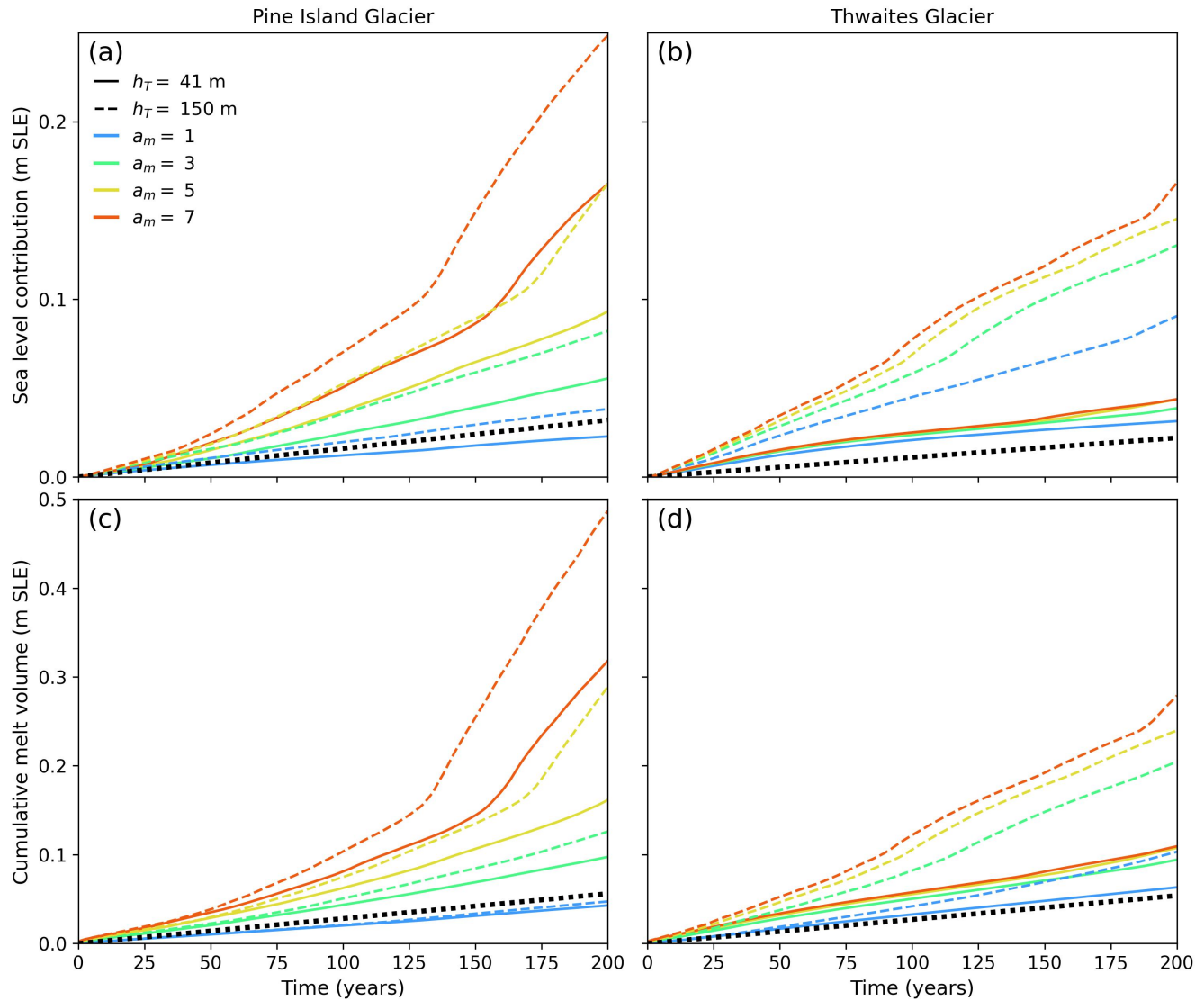


Figure S19. Time-series of SLC (top row) and cumulative melt volume (bottom row) for PIG (left) and TG (right) over 200 years. Results were plotted out for $h_T = 41$ m (solid lines) and 150 m (dashed lines) and for $a_m = 1, 3, 5$ and 7. Black dotted lines refer to totals extrapolated from modern observed rates for the corresponding basin from Smith et al. (2020) (top row) and Rignot et al. (2013) (bottom row).

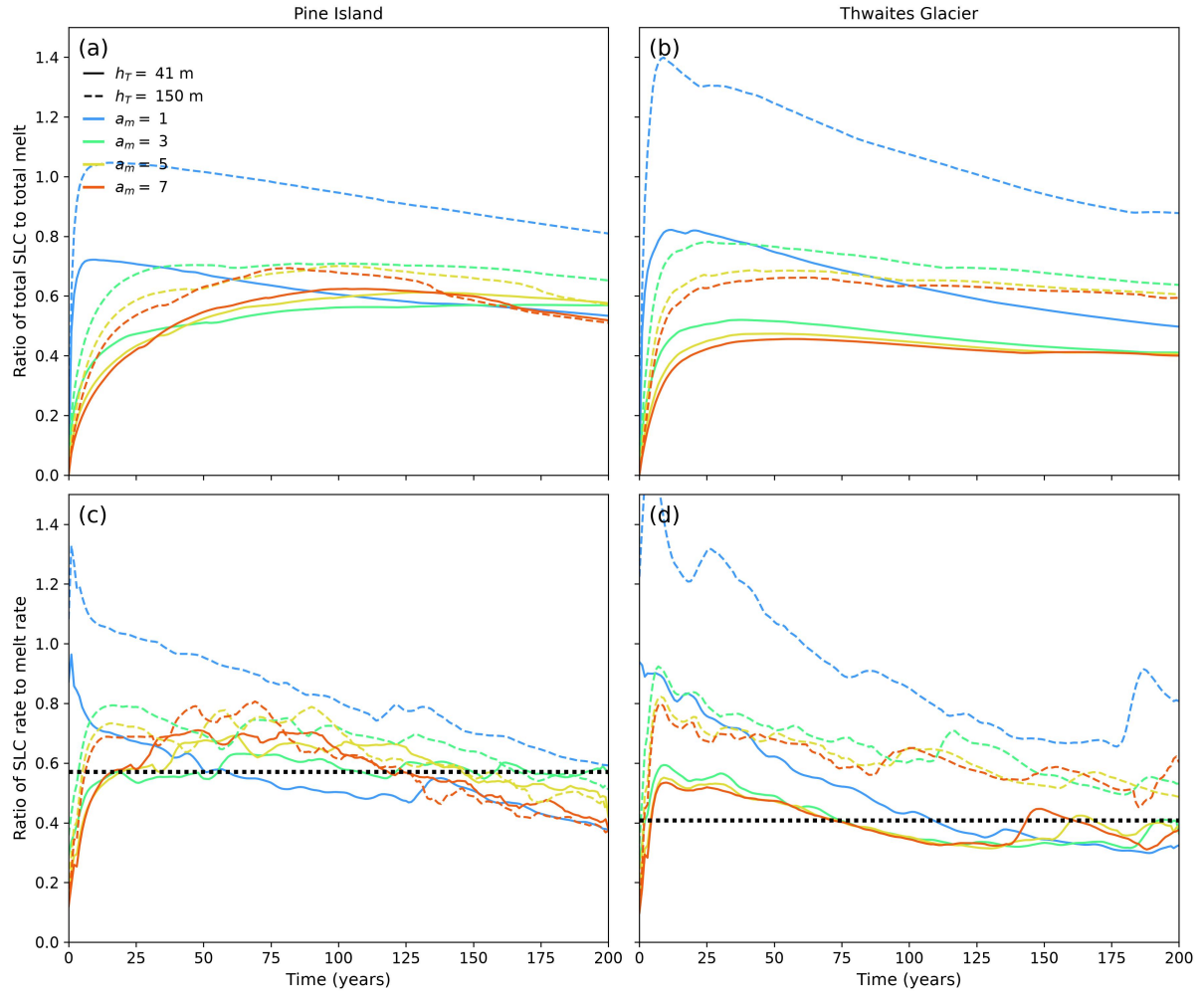


Figure S20. Ratio of total sea level contribution to total melt (top row), and ration of current SLC rate to melt rate (bottom row) for PIG (left) and TG (right). Black dotted lines in (c) and (d) show the ratio of present day observed SLC rate (Smith et al., 2020) and melt rate (Rignot et al., 2013).

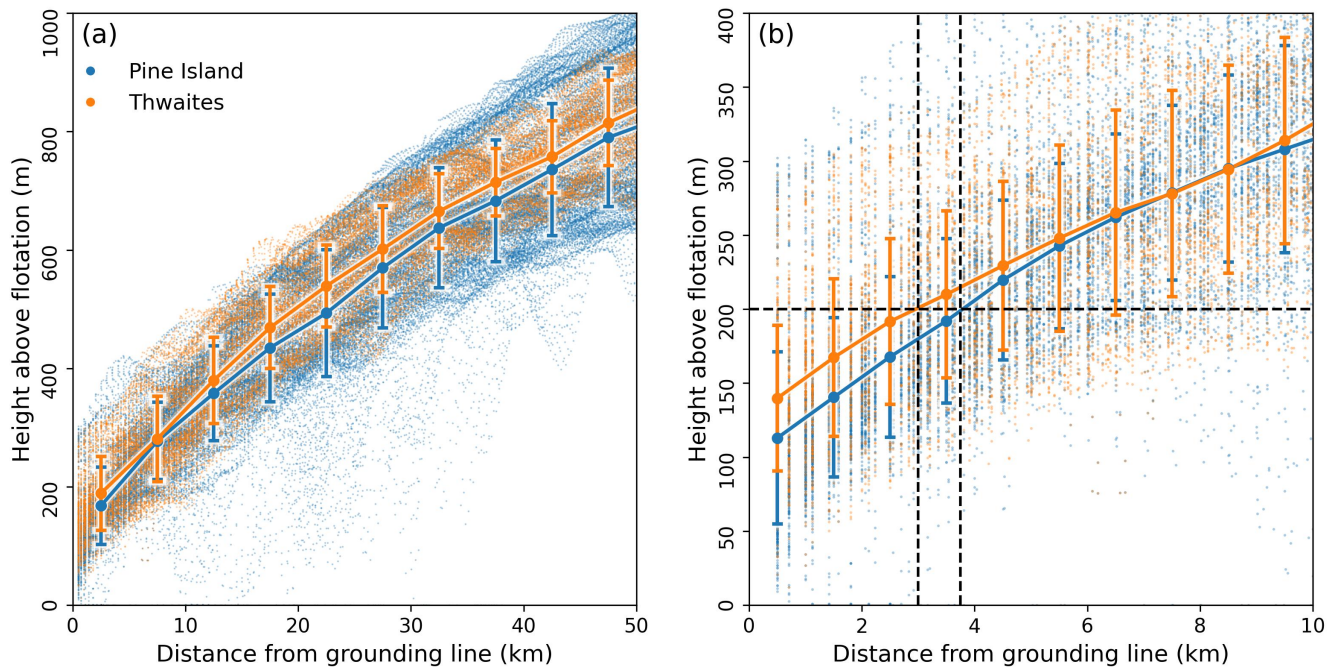


Figure S21. Height-above-flotation plotted as a function of distance from the grounding line for PIG and TG. (b) shows zoomed in detail of (a). Small points denote individual cell values while larger connected points are the mean values calculated within bins (5 km bins for (a) and 1 km bins for (b)). The horizontal dashed line in (b) is drawn at 200 m while vertical dashed lines are drawn at 3 km and 3.75 km where the horizontal line intersects the line plots.

Captions for Movies S1 to S8

Supplementary movies are available for download at <https://doi.org/10.5281/zenodo.21477193>.

90 **Movie S1.** Ice geometry throughout PIG1 simulations for $h_T = 0$ m, 200 m and 500 m, measured along the PIG flowline shown in Figure 1.

Movie S2. Grounding line retreat in PIG1 simulations across the range of h_T . This movie corresponds to Figure 4.

95 **Movie S3.** Grounding line retreat in TG1 simulations across the range of h_T . This movie corresponds to Figure 6.

Movie S4. Grounding line retreat in PIG2 simulations across the range of a_m , with $h_T = 0$ m. This movie corresponds to Figure S12.

100 **Movie S5.** Grounding line retreat in PIG2 simulations across the range of a_m , with $h_T = 41$ m. This movie corresponds to Figure S13.

Movie S6. Grounding line retreat in PIG2 simulations across the range of a_m , with $h_T = 150$ m. This movie corresponds to Figure S14.

105 **Movie S7.** Grounding line retreat in TG2 simulations across the range of a_m , with $h_T = 41$ m. This movie corresponds to Figure S17.

Movie S8. Grounding line retreat in TG2 simulations across the range of a_m , with $h_T = 150$ m. This movie corresponds to Figure S18.

- Agosta, C., Amory, C., Kittel, C., Orsi, A., Favier, V., Gallée, H., van den Broeke, M. R., Lenaerts, J. T. M., van Wessem, J. M., van de Berg, W. J., and Fettweis, X.: Estimation of the Antarctic surface mass balance using the regional climate model MAR (1979–2015) and identification of dominant processes, *The Cryosphere*, 13, 281–296, <https://doi.org/10.5194/tc-13-281-2019>, 2019.
- Bevan, S., Cornford, S. L., Gilbert, L., Otsuka, I., Martin, D. F., and Surawy-Stepney, T.: Amundsen Sea Embayment ice-sheet mass-loss predictions to 2050 calibrated using observations of velocity and elevation change, *Journal of Glaciology*, pp. 1–11, <https://doi.org/10.1017/jog.2023.57>, 2023.
- Burton-Johnson, A., Dziadek, R., and Martin, C.: Review article: Geothermal heat flow in Antarctica: current and future directions, *The Cryosphere*, 14, 3843–3873, <https://doi.org/10.5194/tc-14-3843-2020>, 2020.
- Cornford, S. L., Martin, D. F., Payne, A. J., Ng, E. G., Le Brocq, A. M., Gladstone, R. M., Edwards, T. L., Shannon, S. R., Agosta, C., Van Den Broeke, M. R., Hellmer, H. H., Krinner, G., Ligtenberg, S. R. M., Timmermann, R., and Vaughan, D. G.: Century-scale simulations of the response of the West Antarctic Ice Sheet to a warming climate, *Cryosphere*, 9, 1579–1600, <https://doi.org/10.5194/tc-9-1579-2015>, 2015.
- Morlighem, M.: MEaSUREs BedMachine Antarctica, Version 3 [Data Set], <https://doi.org/10.5067/FPSU0V1MWUB6>, 2022.
- Mouginot, J., Rignot, E., and Scheuchl, B.: MEaSUREs Phase-Based Antarctica Ice Velocity Map, Version 1 [Data Set], <https://doi.org/https://doi.org/10.5067/PZ3NJ5RXRH10>, 2019.
- Rignot, E., Jacobs, S., Mouginot, J., and Scheuchl, B.: Ice-Shelf Melting Around Antarctica, *Science*, 341, 266–270, <https://doi.org/10.1126/science.1235798>, 2013.
- Smith, B. E., Fricker, H. A., Gardner, A. S., Medley, B., Nilsson, J., Paolo, F. S., Holschuh, N., Adusumilli, S., Brunt, K., Csatho, B., Harbeck, K., Markus, T., Neumann, T., Siegfried, M. R., and Zwally, H. J.: Pervasive ice sheet mass loss reflects competing ocean and atmosphere processes, *Science*, 368, 1239–1242, <https://doi.org/10.1126/science.aaz5845>, 2020.
- van den Akker, T., Lipscomb, W. H., Leguy, G. R., Bernalles, J., and Berends, C. J.: Present-day mass loss rates are a precursor for West Antarctic Ice Sheet collapse, *The Cryosphere*, 19, 283–301, <https://doi.org/10.5194/tc-19-283-2025>, 2025.