



1 Efficient computation of global high-resolution Jacobian matrices for analytical
2 inversions of satellite observations of greenhouse gases: Application to the
3 Integrated Methane Inversion (IMI) version 2.3

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14



15 Abstract

16 Dense satellite observations of atmospheric composition call for global high-resolution
17 inversions to infer surface fluxes from the observations, using a chemical transport model (CTM)
18 as forward model to relate the fluxes to the observed concentrations. The inversion can be done
19 analytically when the relationship of surface fluxes to the observable atmospheric concentrations
20 is linear. The main cost of analytical inversions is the explicit construction of the Jacobian matrix
21 describing the CTM sensitivity of atmospheric concentrations to the gridded surface fluxes (state
22 vector elements) and requiring perturbation simulations for all individual state vector elements.
23 This cost has so far limited global analytical inversions to coarse resolution (~200 km) despite
24 the availability of fine-resolution (~10 km) satellite observations. Here we show how the
25 stretched-grid capability of the high-performance GEOS-Chem CTM (GCHP) operating on a
26 cubed-sphere grid can considerably reduce the computational and storage costs of global
27 Jacobian construction by achieving high resolution in the near-field of the state vector elements
28 (target face) while gradually relaxing to coarse resolution at the antipode. Application to a stretch
29 factor of 10 on the C36 cubed-sphere GCHP grid (C36S10) enables generation of a compact
30 global Jacobian archive at 25-km resolution with 100 times lower computational and storage
31 costs than a regular-grid C360 GCHP with global quasi-uniform 25-km resolution. We have
32 implemented this capability in the Integrated Methane Inversion (IMI) version 2.3. Beyond its
33 use for global inversions, the precomputed C36S10 Jacobian matrix archive can be used for
34 regional inversions, which in the IMI rely on a nested rectilinear grid with boundary conditions
35 at the edges. Conservative remapping of the stretched-grid Jacobian archive to the rectilinear grid
36 reduces the cost of these regional inversions by an order of magnitude by skipping the Jacobian
37 calculation. We demonstrate this capability with a 1-month regional inversion of TROPOMI
38 methane satellite data over the South-Central US where the remapped C36S10 Jacobian
39 reproduces closely the standard IMI results (posterior fluxes) at $0.25^\circ \times 0.3125^\circ$ resolution.

40 1 Introduction

41 The rapidly growing satellite observations of atmospheric methane and CO₂ provide top-down
42 information on surface fluxes of these gases to improve bottom-up activity-based estimates
43 (Palmer et al., 2021; Pandey et al., 2024). Exploiting this information involves inversion of the



44 satellite data using a chemical transport model (CTM) as forward model to relate the surface
45 fluxes to atmospheric concentrations, and Bayesian inference with prior bottom-up information
46 to obtain optimized (posterior) estimates of the surface fluxes (Houweling et al., 2017). If the
47 relationship between surface fluxes and atmospheric concentrations is linear, as is the case with
48 methane and CO₂, then an optimal estimate can be obtained analytically (analytical inversion) by
49 constructing the CTM Jacobian matrix that expresses this linear relationship (Brasseur and
50 Jacob, 2017). The analytical inversion is powerful because it provides the optimal estimate
51 together with closed-form error statistics, and allows immediate generation of inversion
52 ensembles by varying inversion parameters (Jacob et al., 2022). But construction of the Jacobian
53 matrix column-by-column is expensive, requiring CTM perturbation simulations for individual
54 surface flux elements (state vector of the inversion). Here we present a new method to construct
55 high-resolution Jacobian matrices for global inversions that improves cost and storage by two
56 orders of magnitude over standard methods and provides a Jacobian archive for use in regional
57 inversions and for training machine learning algorithms.

58 Our work is motivated by the inversion of methane column concentration data from the
59 TROPOMI satellite instrument (Lorente et al., 2021) using the analytical Integrated Methane
60 Inversion (IMI) open-source software tool (Estrada et al., 2025; Varon et al., 2022). TROPOMI
61 provides global daily observations of methane columns with $5.5 \times 7 \text{ km}^2$ nadir pixel resolution
62 and a ~3% retrieval success rate. The IMI in its standard configuration uses the GEOS-Chem
63 Classic (GCC) CTM as forward model with rectilinear grid resolution of $0.25^\circ \times 0.3125^\circ$ ($\approx$
64 $25 \times 25 \text{ km}^2$) or $0.125^\circ \times 0.15625^\circ$ ($\approx 12 \times 12 \text{ km}^2$) (Wang et al., 2026a). Methane surface
65 fluxes can be optimized at those resolutions in small-scale regional inversions where the task of
66 Jacobian construction is eased by the limited domain (Hancock et al., 2026; Wang et al., 2026b).
67 However, global inversions have been limited to $2^\circ \times 2.5^\circ$ ($\approx 200 \times 200 \text{ km}^2$) resolution because
68 of the cost of Jacobian construction (He et al., 2026; Qu et al., 2024). Such coarse resolution
69 does not adequately exploit the spatial information available from the satellite observations.

70 Here we show how the stretched-grid GEOS-Chem CTM in its cubed-sphere high-performance
71 configuration (GCHP) enables construction of high-resolution global Jacobian matrices. In
72 stretched-grid GCHP, the Schmidt transform (Schmidt, 1977) is applied to the global six-faced
73 cubed-sphere grid to achieve high resolution over a selected target face while proportionally



74 coarsening resolution at the antipode (Bindle et al., 2021; Harris et al., 2016). Although not
75 previously recognized, this has useful application for Jacobian matrix construction where fine
76 spatial resolution is needed in the near field but not in the far field. A stretch factor (a parameter
77 for the resolution refining power) of 10 increases the resolution over the target face by two
78 orders of magnitude relative to the uniform GCHP grid, enabling fine-scale Jacobian
79 construction at only one percent of the computational and storage cost of equivalent uniform
80 global simulations. Global construction of the high-resolution Jacobian matrix is achieved by
81 tiling the world with target faces. The resulting compact global Jacobian matrix can then be
82 unpacked for high-resolution global inversions. We further show that the compact global
83 Jacobian matrix can be conservatively remapped for use in regional inversions anywhere in the
84 world with the standard rectilinear configuration of the IMI, avoiding costly Jacobian
85 construction for these regional inversions.

86 2 Analytical inversion with the IMI

87 The IMI computes an optimized posterior estimate of 2-D gridded surface fluxes, assembled in a
88 state vector $\mathbf{x}$ of dimension n , by analytical minimization of the Bayesian cost function $J(\mathbf{x})$
89 assuming normal error statistics:

$$90 \quad J(\mathbf{x}) = (\mathbf{x} - \mathbf{x}_a)^T \mathbf{S}_a^{-1} (\mathbf{x} - \mathbf{x}_a) + \gamma (\mathbf{y} - \mathbf{K}\mathbf{x})^T \mathbf{S}_0^{-1} (\mathbf{y} - \mathbf{K}\mathbf{x}) \quad (1)$$

91 Here $\mathbf{x}_a$ is the prior (typically bottom-up) estimate, $\mathbf{S}_a$ ($n \times n$) is the prior error covariance
92 matrix, γ is a regularization factor to prevent overfitting to observations, $\mathbf{y}$ is an observation
93 vector of dimension m assembling the satellite data, $\mathbf{S}_0$ ($m \times m$) is the observational error
94 covariance matrix, and $\mathbf{K} = \partial \mathbf{y} / \partial \mathbf{x}$ ($m \times n$) is the Jacobian matrix quantifying the sensitivity of
95 observations to emissions as described by the GEOS-Chem CTM. The analytical solution to
96 $\partial J / \partial \mathbf{x} = \mathbf{0}$ yields the optimal (posterior) estimate $\hat{\mathbf{x}}$ and associated posterior error covariance
97 matrix $\hat{\mathbf{S}}$ (Rodgers, 2000):

$$98 \quad \hat{\mathbf{x}} = \mathbf{x}_a + (\gamma \mathbf{K}^T \mathbf{S}_0^{-1} \mathbf{K} + \mathbf{S}_a^{-1})^{-1} \gamma \mathbf{K}^T \mathbf{S}_0^{-1} (\mathbf{y} - \mathbf{K}\mathbf{x}_a) \quad (2)$$

$$99 \quad \hat{\mathbf{S}} = (\gamma \mathbf{K}^T \mathbf{S}_0^{-1} \mathbf{K} + \mathbf{S}_a^{-1})^{-1} \quad (3)$$

100 The averaging kernel sensitivity matrix $\mathbf{A} = \partial \hat{\mathbf{x}} / \partial \mathbf{x}$ quantifying the sensitivity of the posterior



101 state estimate to the true state is given by:

$$102 \quad \mathbf{A} = \mathbf{I}_n - \hat{\mathbf{S}}\mathbf{S}_a^{-1} \quad (4)$$

103 The trace of $\mathbf{A}$ gives the degrees of freedom for signal (DOFS), which estimates the number of
104 independent pieces of information from the inversion.

105 The Jacobian matrix is constructed column by column in GEOS-Chem by perturbing the surface
106 flux x_i for an individual state vector element (grid cell) i and computing the resulting linear
107 perturbation to concentrations $\Delta\mathbf{y}/\Delta x_i = \partial\mathbf{y}/\partial x_i$ over the inversion time window (Estrada et al.,
108 2025). Here Δ denotes the difference between the perturbed simulation and a base simulation
109 with no emissions. The state vector includes all emitting grid cells in the prior bottom-up
110 estimate, which in the case of methane excludes oceans (except for offshore oil/gas) and barren
111 land, and may be clustered for faster Jacobian construction at the cost of degraded resolution
112 (Nesser et al., 2021). The GEOS-Chem observations ($\mathbf{K}\mathbf{x}$) replicate the quantity observed by the
113 satellite instrument, which in the case of TROPOMI is the dry column mixing ratio X_{CH_4} (ppb)
114 with averaging kernels applied to the vertical distribution (Lorente et al., 2021). Construction of
115 the Jacobian matrix thus requires $1 + n/p$ GEOS-Chem simulations, including the base
116 simulation and n/p perturbed simulations where p is the number of independent state vector
117 elements included in a GEOS-Chem perturbation simulation as separate transported species
118 (tracers). The choice of p balances compute time and wall time requirements (Estrada et al.,
119 2025).

120 The standard IMI as described by Estrada et al. (2025) uses the GEOS-Chem Classic CTM
121 driven by an archive of assimilated Goddard Earth Observing System Forward Processing
122 (GEOS-FP) meteorological data from the NASA Global Modeling and Assimilation Office
123 (GMAO). That archive is provided operationally by GMAO on a $0.25^\circ \times 0.3125^\circ$ rectilinear
124 grid with 72 vertical levels, remapped from the native-resolution cubed-sphere C720 GEOS-FP
125 output, where the C720 notation refers to 720×720 grid cells on each cubed-sphere face (≈ 12 -
126 km resolution). GEOS-Chem Classic uses the GEOS-FP operational archive on the
127 $0.25^\circ \times 0.3125^\circ$ rectilinear grid or on coarser rectilinear grids ($2^\circ \times 2.5^\circ$ for global inversions).
128 More recently, GMAO has started to produce a native-resolution C720 advection archive (mass
129 fluxes, surface pressure, and water vapor) which is remapped for GEOS-Chem Classic on a



130 $0.125^\circ \times 0.15625^\circ$ rectilinear grid and enables inversions at ≈ 12 -km resolution (Hancock et al.,
131 2026; Wang et al., 2026a, b).

132 **3 Construction of global high-resolution Jacobians**

133 Our first task in enabling the construction of global high-resolution Jacobians was to implement
134 GCHP as an alternative forward model into the IMI version 2.3 (Estrada et al., 2026). GCHP
135 operates on the same native cubed-sphere grid as GEOS and uses the FV3 advection algorithm
136 native to that grid (Putman and Lin, 2007), with improved accuracy over the TPCORE advection
137 algorithm used in the rectilinear GCC (Lin and Rood, 1996). The cubed-sphere grid enables
138 GCHP to achieve global high-resolution simulations with high scalability on thousands of cores
139 in massively parallel computing environments (Eastham et al., 2018; Martin et al., 2022; Zhang
140 et al., 2023).

141 The massively parallel capability of GCHP is not of intrinsic value for the Jacobian matrix
142 construction, which is embarrassingly parallel in any case (the $1 + n/p$ simulations are conducted
143 independently). The high-resolution capability of GCHP is also not of intrinsic value because it
144 still involves prohibitive compute time. The critical feature is the stretched-grid capability of
145 GCHP (Bindle et al., 2021), which increases the grid resolution over a target cubed-sphere face
146 while coarsening the resolution gradually away from that face. Each edge of the target face is
147 reduced by a stretch factor S , reducing the spatial domain of the target face by a factor S^2 , with
148 the opposite change for the antipode face. By using the stretched-grid GCHP for global Jacobian
149 construction, we achieve high resolution for the state vector elements in the near field (target
150 face) where it matters, while maintaining continuity and mass conservation in the far field, for a
151 $\sim S^2$ decrease in computational and Jacobian storage costs relative to a global high-resolution
152 simulation.

153 Here we use the C36S10 stretched-grid GCHP, representing a C36 global grid with stretch factor
154 $S = 10$, for Jacobian construction over a full-year (2024) inversion time window. The regular
155 C36 global grid features the ≈ 250 -km resolution typically used for global inversions, and the $S =$
156 10 stretch factor increases the resolution to 25-km for the target face while decreasing it to 2500-
157 km for the antipode (Figure 1). The C36S10 simulation reproduces the results of a regular C360
158 simulation for the target face (Bindle et al., 2021) and does so at 1% of the computational cost of



the C360 simulation. In this manner, we compute global Jacobian columns with 25-km resolution for the target face ($\approx 900 \times 900 \text{ km}^2$ domain), gradually decreasing in resolution with distance away from the target face, for the cost of a C36 simulation. We do so for all state vector elements in the target face (up to $36 \times 36 = 1296$ elements, depending on the number of methane-emitting grid cells: Figure 1). Each GEOS-Chem perturbation simulation carries up to 200 state vector elements for computational efficiency (Figure S2), so that up to 7 perturbation simulations are needed to cover a target face. We then successively shift the location of the target face to achieve global coverage for methane-emitting grid cells, allowing for overlap to avoid spatial gaps. We tile the world in this manner with 220 target faces, resulting in 168,862 state vector elements at 25-km resolution and a total of 957 GEOS-Chem perturbation simulations to construct the full Jacobian matrix. The C36S10 Jacobians regridded to C360 will be applied to global 25-km inversions for 2024 with the results to be presented in a future paper. Here we demonstrate the capability with a 1-month test for May 2024.

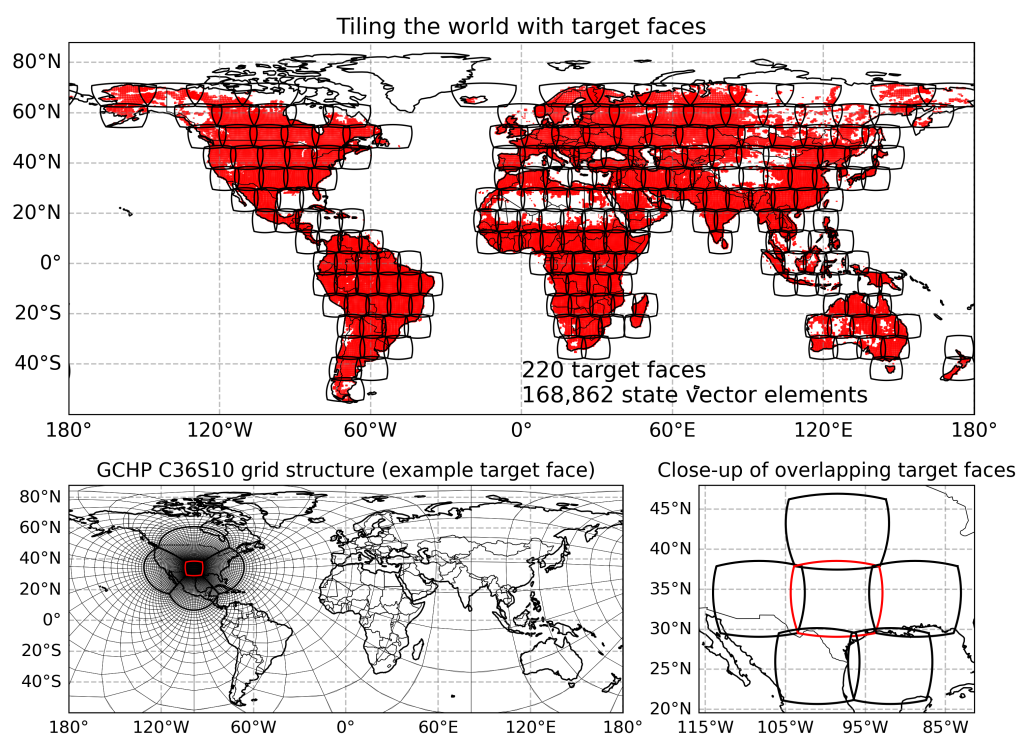
An example C36S10 Jacobian column is shown in Figure 2 and compared to the corresponding C360 Jacobian column. C36S10 and C360 are on different grids, even for the target face, so that quantitative comparison would require regridding. Figure 2 illustrates that C36S10 closely replicates C360 for the target face while smoothing the sensitivity in the far field. Sensitivities decrease rapidly away from the target face in both the C36S10 and C360 cases, supporting our premise that high resolution is necessary only in the near field. The C36S10 has higher sensitivity at the antipodes and in the opposite hemisphere than C360, reflecting the coarse resolution, but these sensitivities are very low. Notably, the C360 Jacobian column does not show any significant far-field structure that would be missed in the C36S10 Jacobian column.

The complete C36S10 Jacobian matrix compiles the sensitivity of all TROPOMI observations over the 1-year inversion time window, at 1% of the storage cost of a C360 Jacobian matrix. It only requires 0.5 TB storage while the C360 Jacobian matrix would require 50 TB. The global C360 inversion still requires the C360 Jacobian matrix, and this is done by conservative remapping of C36S10 Jacobian columns to C360 at runtime. There is no need to store the global C360 Jacobians.

Tiling the world with the ensemble of C36S10 target faces allows for effective construction of a



188 global Jacobian matrix at C360 resolution. This can be compared to the alternative tiling-the-
189 world approach using regional inversions in the rectilinear GCC to cover all methane-emitting
190 areas worldwide at high resolution (East et al., 2025; Shen et al., 2023). The fundamental
191 difference between the two approaches is that no consistency can be enforced in the boundary
192 conditions for the different regional inversions, so that results may be inconsistent with the
193 global methane budget, and the global methane sink from oxidation by OH cannot be optimized,
194 whereas a global inversion enables optimization of the methane sink (Penn et al., 2025). In
195 addition, there is some arbitrariness in the choices of domains for the regional inversions, so that
196 results for different domains or varying influences from the domain boundaries (Nesser et al.,
197 2025) may not be fully consistent.

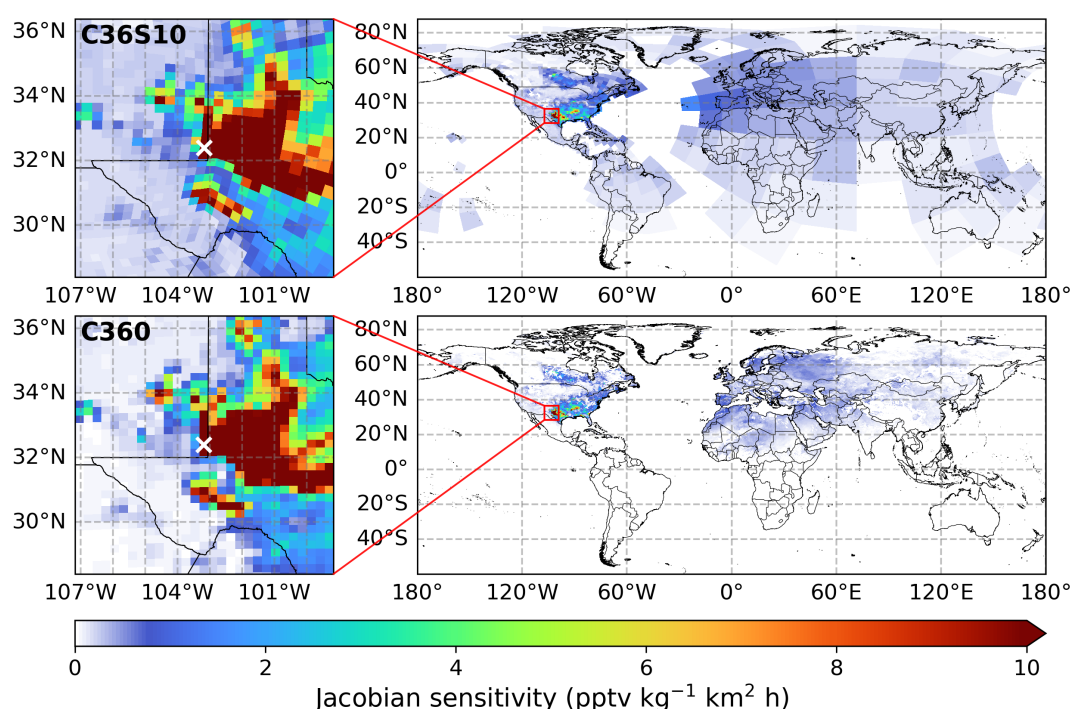


198

199 Figure 1. Tiling the world with target faces using the stretched-grid GCHP at C36S10 resolution
200 for global C360 ($\approx 25 \times 25 \text{ km}^2$) state vector construction. Top panel shows the global ensemble
201 of target faces covering methane-emitting land (in red). Non-emitting land cells are defined here
202 as having annual emissions smaller than $1.7 \times 10^{-11} \text{ kg m}^{-2} \text{ s}^{-1}$, for a total corresponding to less



than 2% of global emissions (Figure S3) in the IMI default prior estimate (Estrada et al., 2025). The black quasi-square lines show the outline of each target face. Total numbers of state vector elements and target faces are given inset. The global C36S10 grid structure for a sample target face in the South-Central US is shown in the bottom left panel. The target face is outlined in red and the five other faces are outlined in black. The bottom right panel shows the spatial overlap of that target face with neighboring target faces to avoid spatial gaps in state vector elements.



209

Figure 2. Spatial distribution of an example Jacobian column describing the GEOS-Chem sensitivity of TROPOMI X_{CH_4} observations in May 2024 to methane emissions in a C360 grid cell centered over the South-Central US at (32.5°N, 103.125°W; white cross). Results from the Jacobian construction at C36S10 (top) and C360 (bottom) resolutions are compared.

214 4 Use of global high-resolution Jacobian archive for regional inversions

A global analytical inversion at 25-km resolution with $n=166,862$ state vector elements is not computationally tractable, even with a precomputed Jacobian matrix, due to the need to invert $n \times n$ matrices. A reduced-rank or reduced-dimension approach is needed (Nesser et al., 2021).



218 In a follow-up publication we will apply smart state vector clustering (Estrada et al., 2025) with
219 corresponding addition of Jacobian columns to retain 25-km resolution in high-emission areas
220 while clustering weakly emitting areas.

221 But another application of our global C36S10 Jacobian matrix is to enable regional inversions at
222 25-km resolution for any domain worldwide within the IMI framework by selecting precomputed
223 Jacobian columns for the inversion domain and adding boundary conditions. Jacobian
224 construction represents about 90% of the computational cost of regional inversions in the IMI
225 (Varon et al., 2022), so that use of our precomputed C36S10 archive as an available resource can
226 reduce cost by a factor of 10. A complication is that regional inversions in the IMI rely on the
227 nested rectilinear grid of GCC for forward model simulations. We therefore need to remap our
228 precomputed Jacobians from the stretched cubed-sphere C36S10 GCHP grid over the target
229 faces to the corresponding rectilinear $0.25^\circ \times 0.3125^\circ$ GCC grid. Figure 3 illustrates this task
230 with the $0.25^\circ \times 0.3125^\circ$ state vector elements for a regional inversion over the South-Central
231 US including the Permian oil basin ($109.53125^\circ - 98.28125^\circ\text{W}$, $27.375^\circ - 36.625^\circ\text{N}$). In this
232 case, the regional domain is covered by four corresponding C36S10 target faces with spatial
233 overlap.

234 Converting the Jacobian matrix from the stretched-grid cubed-sphere to the rectilinear grid
235 requires conservative remapping with multiple C36S10 grid cells contributing to a single
236 $0.25^\circ \times 0.3125^\circ$ destination cell. Since $\mathbf{x}$ in the Jacobian matrix definition $\mathbf{K} = \partial\mathbf{y}/\partial\mathbf{x}$ is a
237 vector of surface fluxes ($\text{kg m}^{-2} \text{s}^{-1}$), it follows that Jacobian vector elements are additive on an
238 area basis. This additivity means that when multiple C36S10 source grid cells overlap a single
239 $0.25^\circ \times 0.3125^\circ$ destination cell, the Jacobian column for the destination cell can be calculated
240 as the area-weighted average of the Jacobian columns from the contributing cubed-sphere cells.

241 To demonstrate the use of the precomputed C36S10 Jacobian archive for regional inversions,
242 here we apply it over the South-Central US (domain of Figure 3) for a 1-month inversion of
243 TROPOMI methane observations in May 2024. Figure 4 compares the reconstructed Jacobian
244 column for the destination state vector element of Figure 3 to the native Jacobian column directly
245 computed on the $0.25^\circ \times 0.3125^\circ$ grid with the nested GCC. The spatial distribution of the
246 reconstructed Jacobian column matches closely ($r = 0.95$) that calculated explicitly from the



247 nested GCC. That particular state vector element resides within the intersection of two target
248 faces, so that we have a total of 8 overlapping source C36S10 grid cells to reconstruct the
249 Jacobian column for that element. The reconstructed Jacobian column features a reduced peak
250 value at the center of the emission plume from the state vector element, with higher values over
251 adjacent downwind cells due to the smearing effects from the fractional overlap. Different
252 advection schemes in GCHP and GCC may also play a role. We find that the similarity of
253 reconstructed and native Jacobian columns across all the state vector elements in this regional
254 application is uniformly high, as measured by the correlation coefficient for the spatial
255 distribution ($r = 0.95 \pm 0.03$).

256 The near-field smearing of the reconstructed Jacobians would be a concern if it degraded
257 significantly the inversion results but we find that it does not. Figure 5 shows that the posterior
258 emissions obtained from a 1-month South-Central US regional inversion using the standard IMI
259 version 2.3 are virtually indistinguishable between using the precomputed C36S10 Jacobians or
260 using the native-resolution Jacobians on the $0.25^\circ \times 0.3125^\circ$ grid. The overall normalized mean
261 bias (NMB) of posterior/prior emission scale factors differs by less than 2%. The largest errors
262 are for isolated point sources, as expected from spatial smearing, but even there they are only
263 around 20%. The smearing manifests itself in the computed DOFS which decreases from 4.3
264 with the native-resolution Jacobians to 3.5 with the precomputed Jacobians. The DOFS is usually
265 taken as indicator of information content in an inversion, but in the inversion with the
266 precomputed Jacobians we find that the smearing moves some of the information to the
267 immediate off-diagonals of the averaging kernel matrix so that it is not included in the DOFS.
268 The lack of impact on the posterior emissions is because the spatial resolution of the IMI is
269 effectively coarser than $0.25^\circ \times 0.3125^\circ$ due to numerical diffusion in the advection scheme and
270 limited prior knowledge of the location of emissions (Wang et al., 2025). The added smearing
271 effects from Jacobian remapping appear to be marginal.

272 Regridding of the precomputed C36S10 Jacobian for regional inversion applications can also be
273 done within the stretched-grid framework using the same regridding strategy as for the rectilinear
274 grid but without the need for boundary conditions, whose uncertainties would propagate into the
275 posterior uncertainties (Nesser et al., 2025). For example, the domain of Figure 3 can be
276 represented by a C36S10 grid with target face centered on the domain. A C120S3 grid would



allow 25-km resolution over the contiguous US.

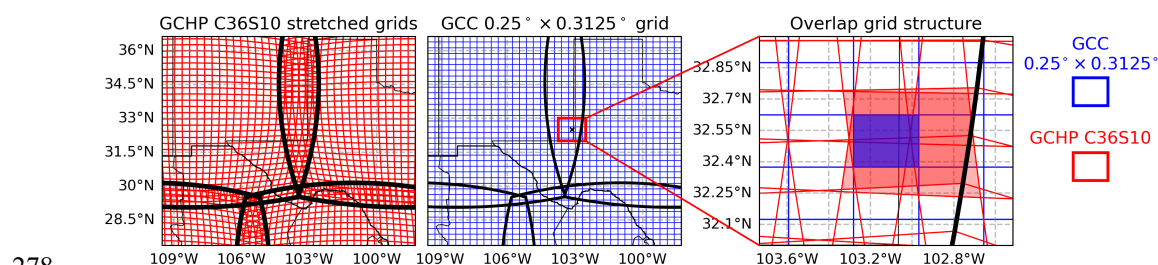
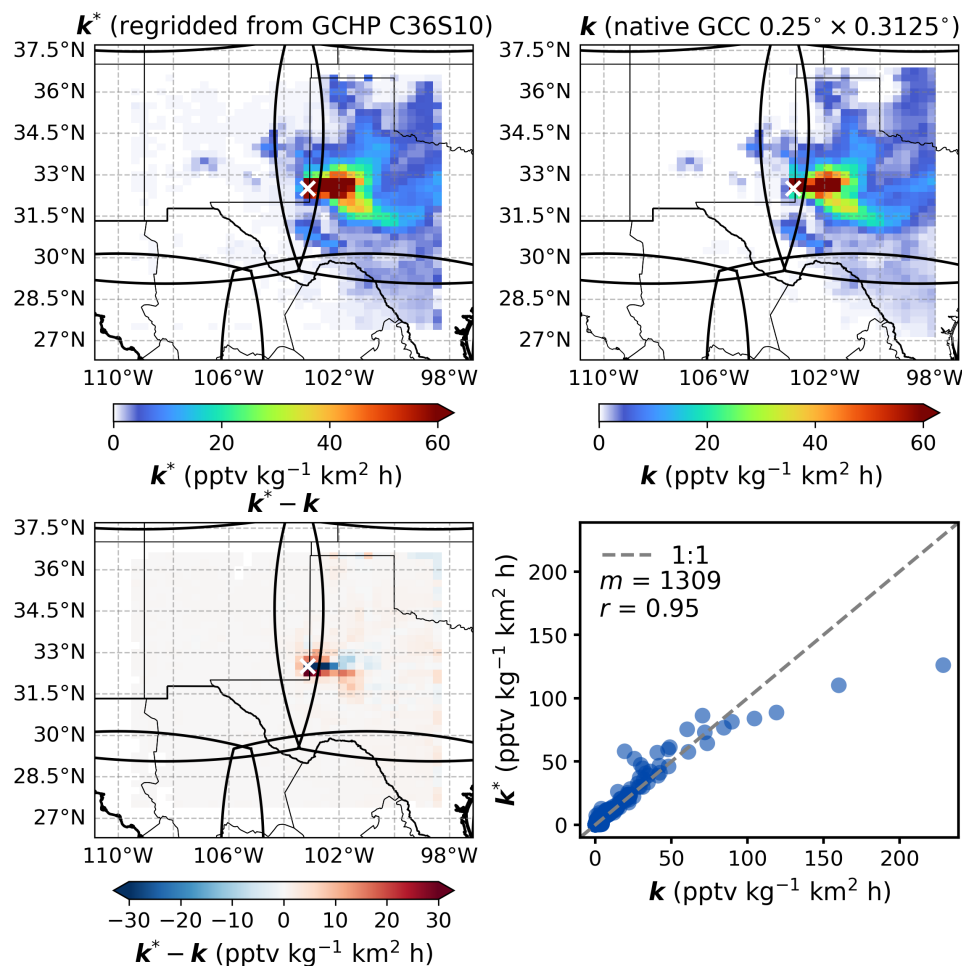
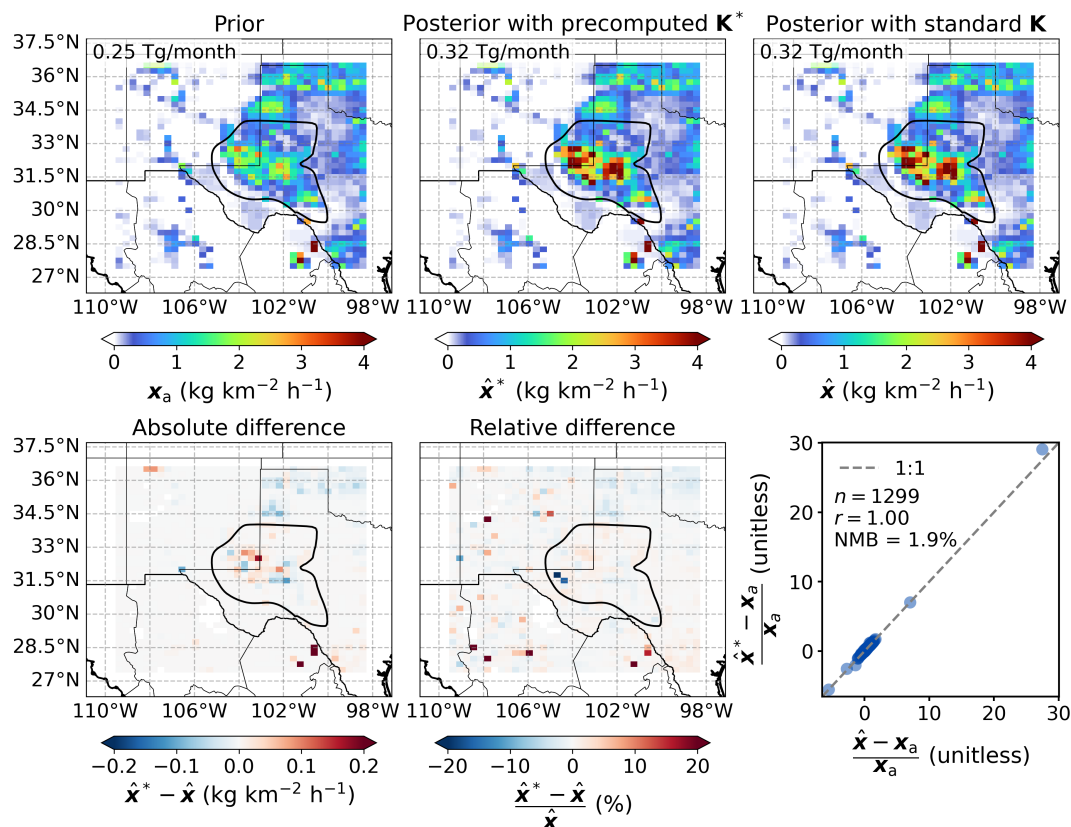


Figure 3. Grid structures at C36S10 (left) and $0.25^{\circ} \times 0.3125^{\circ}$ (middle) resolutions for a regional inversion over the South-Central US covering a rectilinear domain ($109.53125^{\circ} - 98.28125^{\circ}\text{W}$, $27.375^{\circ} - 36.625^{\circ}\text{N}$). The right panel shows a zoomed-in region around a specific destination GCC grid cell filled in blue, with the GCC $0.25^{\circ} \times 0.3125^{\circ}$ grid and the GCHP C36S10 grid. That specific GCC grid cell is in an area of overlap between two C36S10 target faces and the corresponding overlapping grid cells are filled in red. The thick black lines denote the edges of target faces.



287

288 Figure 4. Comparison of regridded and native Jacobian column (denoted k) for a regional
 289 inversion at rectilinear $0.25^\circ \times 0.3125^\circ$ resolution over the South-Central US in May 2024. The
 290 state vector element corresponding to this particular Jacobian column is indicated by the white
 291 cross. The Figure shows the Jacobian column regridded from our precomputed C36S10 global
 292 archive to the $0.25^\circ \times 0.3125^\circ$ grid (k^* ; top left), its counterpart computed natively at
 293 $0.25^\circ \times 0.3125^\circ$ resolution (k ; top right), and their differences (bottom left). The bottom right
 294 panel shows a scatterplot between the standard and remapped Jacobian columns where each
 295 point represents a $0.25^\circ \times 0.3125^\circ$ grid cell. The total number of comparison pairs (m) and the
 296 correlation coefficient (r) are listed inset. The black lines show the outline of the target faces.



297

298 Figure 5. Posterior methane emissions over the South-Central US in May 2024 computed by 1-
 299 month inversion of TROPOMI data with IMI version 2.3 at $0.25^\circ \times 0.3125^\circ$ resolution and
 300 default settings (Estrada et al., 2025) including the US EPA GHGI prior inventory (Maasakkers
 301 et al., 2023). The Figure compares the inversion with the standard IMI, using a Jacobian matrix
 302 $\mathbf{K}$ computed on the $0.25^\circ \times 0.3125^\circ$ grid, to an inversion using the precomputed C36S10
 303 Jacobian matrix $\mathbf{K}^*$ remapped to the $0.25^\circ \times 0.3125^\circ$ grid. Prior and posterior emissions are
 304 shown in the top panels with the total emissions shown in the inset at the top left. The spatial
 305 distributions of their posterior differences are shown in the bottom left and middle panels.
 306 Comparison statistics in the scatterplot include the number of pairs (n), the correlation coefficient
 307 (r), and the normalized mean bias (NMB), where each point represents a $0.25^\circ \times 0.3125^\circ$ grid
 308 cell (state vector element). State boundaries and the Permian oil production basin are outlined.



309 5 Conclusions

310 We developed a stretched-grid capability to construct high-resolution Jacobian matrices for
311 global analytical inversions of atmospheric composition data at 100-fold lower computational
312 and storage costs than regular-grid forward model perturbation methods. Jacobian matrix
313 construction is the computational bottleneck of analytical inversions, and our method enables
314 global high-resolution inversions that can exploit the observation density and worldwide
315 coverage of satellite observations. This capability is now in place in the Integrated Methane
316 Inversion (IMI) version 2.3 (<https://carboninversion.com>).

317 Our approach is to use the stretched-grid mode of the cubed-sphere high-performance GEOS-
318 Chem chemical transport model (GCHP) as forward model for constructing the Jacobian matrix
319 that expresses the sensitivity of atmospheric concentrations to surface fluxes. The stretched grid
320 enables GCHP to achieve fine resolution in the region of interest while gradually degrading to
321 coarse resolution at the antipode and enforcing mass conservation. We demonstrated this
322 capability in the IMI with a C36S10 stretched-grid cubed-sphere resolution. The C36 global grid
323 has about 250-km resolution, typical of global inversions, but the 10-fold stretching of the target
324 face achieves ~25-km resolution in the near field for the Jacobian matrix construction. By tiling
325 the world with target faces covering the domain to be optimized, a compact global Jacobian
326 matrix at ~25-km resolution is constructed. This compact C36S10 Jacobian matrix can then be
327 regridded on the fly to the full C360 Jacobian matrix, which does not need to be stored. The
328 C36S10 Jacobian matrix for a 1-year inversion of TROPOMI methane satellite observations is
329 only 0.5 TB in size, while the full C360 matrix would be 50 TB.

330 The global C36S10 Jacobian archive generated with stretched-grid GCHP can in addition be
331 applied to limited-domain regional inversions anywhere in the world, avoiding the need to
332 construct regional Jacobians for these applications and achieving a 10-fold reduction in
333 computational cost. These regional applications in the IMI use the rectilinear (latitude $\times$
334 longitude) grid of GEOS-Chem Classic (GCC), which allows nested-domain simulations with
335 boundary conditions that are optimized as part of the inversion. We showed how the C36S10
336 stretched-grid Jacobian matrix can be remapped conservatively to the $0.25^\circ \times 0.3125^\circ$ GCC grid
337 to drive regional IMI inversions, and demonstrated with a 1-month inversion for the South-



338 Central US that the inversion using the precomputed Jacobian matrix reproduces the posterior
339 fluxes from the standard $0.25^\circ \times 0.3125^\circ$ inversion with a normalized mean bias of less than 2%.
340 The remapping of the Jacobian matrix introduces some marginal loss of information due to
341 fractional overlap between grid cells but the effect is negligibly small.

342 **Code availability.** The source code used in this study was built upon standard IMI version 2.3
343 and can be accessed at <https://doi.org/10.5281/zenodo.21515056> (Zhang et al., 2026). The source
344 code for GCHP v14.7.0 can be accessed at <https://doi.org/10.5281/zenodo.18554732> (The
345 International GEOS-Chem User Community, 2026)

346 **Data availability.** The blended TROPOMI+GOSAT data for atmospheric methane
347 concentrations in May 2024 used in this study can be accessed at
348 <https://doi.org/10.5281/zenodo.22962933> (Balasus et al., 2023; Zhang, 2026).

349 **Author contributions.** The manuscript was written by DZ and DJJ with contributions from all
350 authors. DZ, DJJ and DJV designed the study. DZ developed the model code and conducted
351 inverse analyses. EWL contributed to the model development of GCHP carbon simulation for
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