



GeoEnthalpy-Delta v1.0: an enthalpy-based model for coupled subaerial and subaqueous delta evolution with diagnostic moving boundaries

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Abstract

Deltaic systems are vulnerable landscapes and stratigraphic archives, but modeling their evolution over long timescales requires methods that conserve sediment while allowing geomorphic boundaries to move. We present GeoEnthalpy-Delta v1.0, a fixed-grid enthalpy model for idealized fluvial-deltaic evolution in one and two spatial dimensions. The model evolves sediment thickness over a prescribed non-erodible basement, reconstructs the sediment surface, and diagnoses the alluvial–bedrock transition, shoreline, and delta toe from the evolving thickness and surface-elevation fields. Sediment redistribution is represented by a thresholded, slope-driven transport law with separate topset, foreset, down-system, and lateral mobility parameters, allowing finite foreset storage and fan-delta planform spreading without remeshing or explicit front tracking.

We verify the one-dimensional implementation against analytical benchmarks for fixed sea level, square-root sea-level histories, vertical-foreset and graded-river limits, and post-autobreak retreat under constant sea-level rise. Two-dimensional experiments are evaluated using mass conservation, grid consistency, and moving-boundary diagnostics. Model demonstrations show that shoreline retreat can occur while the toe continues to prograde during cyclic sea-level forcing, and that lateral thresholds, foreset thresholds, sediment input, and feeder width control fan width, shoreline-to-toe separation, and sediment partitioning. GeoEnthalpy-Delta v1.0 provides a compact and reproducible baseline for studying coupled delta morphology and stratigraphy and for future incorporation of hydrodynamic, coastal, and subsidence processes.

Keywords: GeoEnthalpy-Delta v1.0; enthalpy method; Stefan problem; moving-boundary problem; fluvial delta; finite foreset; shoreline reversal; fan-delta planform; model verification.

Non-technical summary (500-character)

GeoEnthalpy-Delta v1.0 models river-delta growth and retreat by conserving sediment on a fixed grid. The shoreline, upstream sediment limit, and offshore delta toe emerge from the evolving deposit rather than being prescribed. The model matches analytical tests and shows how sea level, sediment supply, and transport thresholds shape delta profiles, fan geometry, and preserved strata.



40 1. Introduction

41 Deltaic systems are among the most dynamic and societally important coastal landscapes. They
42 support large populations, ecosystems, agriculture, navigation, and infrastructure, while also
43 preserving stratigraphic records of sediment supply, relative sea-level change, subsidence, basin
44 geometry, and environmental forcing over a wide range of timescales (Galloway, 1975; Orton
45 and Reading, 1993; Blum and Törnqvist, 2000; Syvitski and Saito, 2007; Paola et al., 2011;
46 Giosan et al., 2014). Interpreting these records, and anticipating future delta response to
47 changing sediment delivery and sea level, requires models that conserve sediment while coupling
48 subaerial fluvial transport, shoreline migration, subaqueous deposition, and evolving
49 accommodation.

50 A wide range of morphodynamic models can be used to study delta evolution, but different
51 model classes serve different purposes. Process-resolving models, including Delft3D-style
52 hydrodynamic and sediment-transport models (Lesser et al., 2004), are valuable for studying
53 event- to decadal-scale dynamics, flow routing, waves, tides, channel evolution, and river-mouth
54 processes. However, their computational cost and parameter requirements can make systematic
55 exploration difficult over centennial to millennial timescales or across large parameter spaces.
56 Idealized or reduced-order morphodynamic models take a complementary approach: they
57 simplify unresolved process detail in order to isolate long-term conservation constraints,
58 dominant transport feedbacks, and the motion of geomorphic boundaries (Swenson et al. 2000;
59 Paola, 2000; Paola and Voller 2005; Lorenzo-Trueba et al. 2009; Paola et al., 2011; Edmonds et
60 al., 2022). GeoEnthalpy-Delta v1.0 belongs to this second class. It is not intended as a substitute
61 for process-resolving morphodynamic models; instead, it is designed as a transparent framework
62 for quantifying how sediment conservation, transport thresholds, sea-level forcing, and finite
63 foreset storage control the motion of delta boundaries.

64 A useful way to frame this problem is as a moving-boundary problem. In the idealized setting
65 considered here, the primary geomorphic boundaries are the alluvial–bedrock transition (ABT),
66 which marks the upstream limit of finite sediment accumulation; the shoreline (SH), which
67 separates subaerial and subaqueous environments; and the delta toe (TOE), which marks the
68 offshore limit of finite sediment thickness (Fig. 1). These boundaries are not known in advance.
69 They migrate in response to sediment fluxes, transport efficiency, basement geometry, and
70 relative sea-level forcing. This is directly analogous to classical Stefan problems in heat transfer,
71 where an interface such as a melting front is determined as part of the solution rather than
72 imposed externally (Crank, 1984; Swenson et al., 2000; Voller et al., 2004). In the deltaic
73 analogue, sediment flux and accommodation determine the position of geomorphic boundaries,
74 and the sediment volume needed to fill accommodation plays a role analogous to latent heat in
75 the thermal problem.

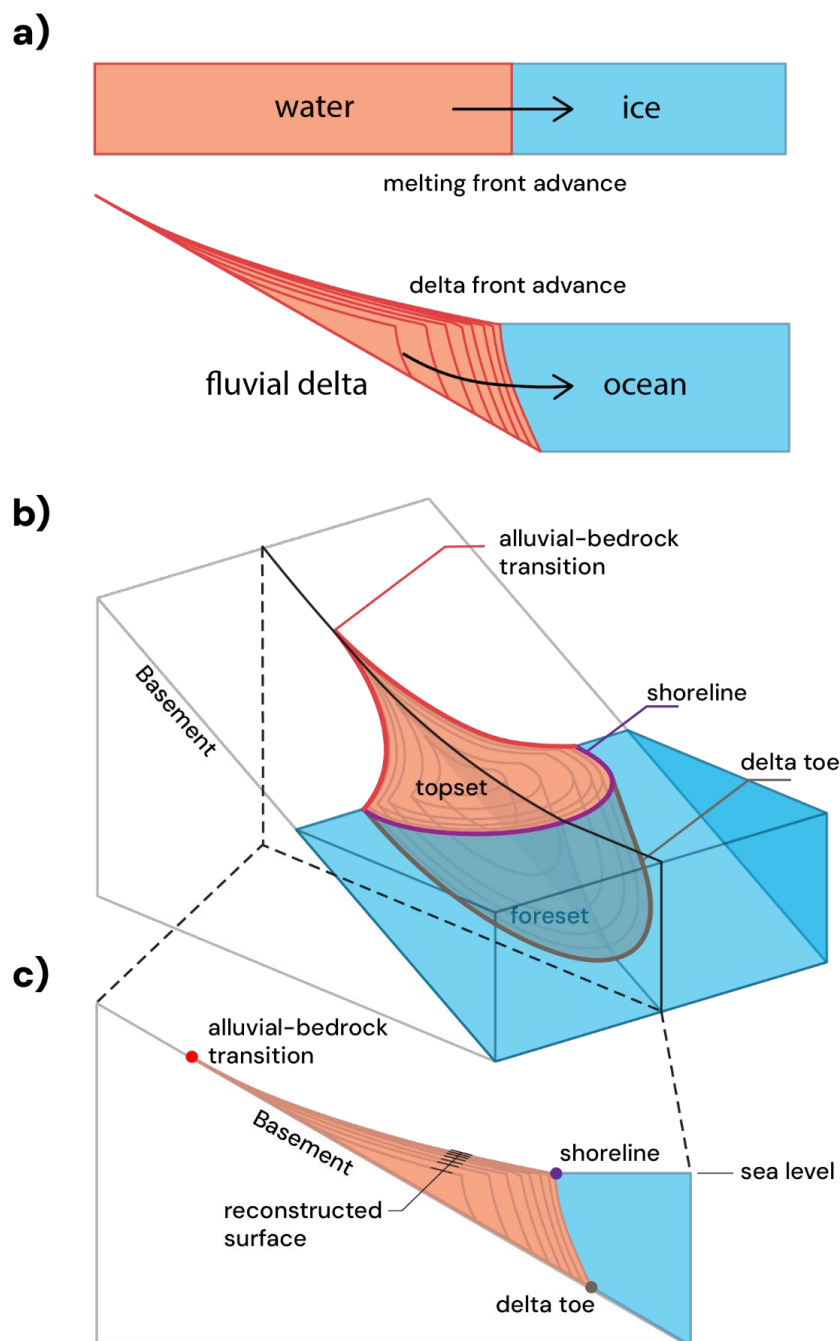
76 Despite substantial progress in analytical, numerical, and experimental models of delta evolution,
77 important limitations remain for the class of problems addressed here. Many existing reduced-
78 complexity formulations treat only one or two moving boundaries, assume a vertical or
79 prescribed foreset geometry, or rely on explicit front tracking that becomes difficult to generalize
80 when boundaries reverse direction or evolve in two horizontal dimensions. These simplifications



81 have been extremely useful for theory and benchmarking, but they limit the ability to represent
82 finite foreset storage, independent shoreline and TOE motion, shoreline reversal, TOE
83 abandonment, and fan-delta planform spreading within a single conservative framework.

84 Here we present GeoEnthalpy-Delta v1.0, an enthalpy-based framework for coupled subaerial
85 and subaqueous delta evolution in one and two spatial dimensions. The model evolves sediment
86 thickness on a fixed Eulerian grid, reconstructs the sediment surface relative to a prescribed
87 basement and sea-level history, and diagnoses the ABT, SH, and TOE from the evolving fields.
88 The objectives of this paper are to (i) develop a conservative enthalpy-based delta model in
89 which the ABT, SH, and TOE emerge diagnostically from an evolving sediment-thickness field;
90 (ii) implement this framework in one-dimensional (1D) and two-dimensional (2D) configurations
91 using a thresholded, slope-driven transport closure; (iii) verify the 1D model against analytical
92 and similarity solutions for finite-foreset and constant-sea-level-rise cases; and (iv) evaluate and
93 demonstrate the 2D model using conservation tests, benchmark diagnostics, sea-level forcing
94 experiments, and planform sensitivity analyses.

95



96

97 **Figure 1. Moving-boundary interpretation of delta evolution and the GeoEnthalpy-Delta**
 98 **v1.0 extension.** (a) Analogy between a classical Stefan problem, in which a melting front
 99 advances as part of the solution, and delta progradation, in which shoreline motion is determined



by sediment conservation. The phase-change analogy motivates the fixed-grid boundary-diagnostic strategy; it is less literal in the present finite-foreset formulation because subaqueous storage and transport are resolved explicitly. (b) 3D schematic of the 2D delta problem considered here. The alluvial–bedrock transition, shoreline, and TOE are evolving curves rather than prescribed points. (c) Cross-sectional schematic of the finite-foreset formulation. The reconstructed sediment surface is continuous across the shoreline. The SH is diagnosed from the contour $\eta = Z(t)$, whereas the TOE is diagnosed from the offshore limit of finite sediment thickness. This differs from vertical-foreset or shoreline-shock formulations in which the offshore geometry is imposed or collapsed into a boundary condition.

2. Background: analytical and numerical moving-boundary models for delta evolution

A large body of analytical, similarity, experimental, and numerical work has used the moving-boundary perspective to understand delta and alluvial-plain evolution. Early and related formulations examined shoreline motion, alluvial-channel aggradation, autogenic shoreline retreat, and the coupling between sediment supply, water discharge, basement slope, and sea-level change (Muto, 2001; Parker and Muto, 2003; Muto and Swenson, 2005; Swenson and Muto, 2007; Parker et al., 2008). Other studies developed solutions for idealized 1D systems involving one or two moving boundaries, including shoreline-only problems, coupled alluvial–bedrock transition and shoreline migration, and related morphodynamic free-boundary problems (Capart et al., 2007; Lai and Capart, 2007, 2009; Kim and Muto, 2007; Lorenzo-Trueba et al., 2009; Lorenzo-Trueba and Voller, 2010; Lorenzo-Trueba et al., 2013). These studies clarified how sediment supply, accommodation creation, and transport efficiency control shoreline advance or retreat, topset length, stratigraphic architecture, and the onset of autoretreat or autobreak behavior.

Analytical and similarity solutions remain valuable because they reveal the parameter combinations that control delta response and provide benchmarks for numerical models. However, they typically require simplified geometries, steady or self-similar forcing, prescribed boundary relationships, or idealized foreset treatments. Many formulations either collapse the subaqueous domain into a geometric rule or assume a vertical foreset, so the offshore termination of finite sediment thickness is not treated as an independent moving boundary (Swenson et al., 2000; Voller et al., 2004; Lorenzo-Trueba et al., 2009; Lorenzo-Trueba and Voller, 2010; Anderson et al., 2019). This simplification is useful for theory, but it limits the ability to represent finite foresets, subaqueous redistribution, and the stratigraphic consequences of SH and TOE migration.

A particularly relevant contribution is Swenson et al. (2005), who modeled coupled subaerial and subaqueous delta progradation and compound-clinoform development. Their model linked fluvial and shallow-marine sediment dynamics through the surf zone, which was collapsed to a shock condition and treated as a moving boundary. This approach demonstrated that shoreline motion, clinoform-rollover motion, and sediment partitioning between subaerial and subaqueous environments can differ substantially, and that shoreline and rollover positions need not coincide. The present model builds on this insight but adopts a different closure: rather than imposing a surf-zone shock or prescribing the offshore geometry, GeoEnthalpy-Delta v1.0 diagnoses the shoreline from the reconstructed sediment surface and the TOE from the finite-thickness



boundary of the deposit. In this way, shoreline migration, finite foreset storage, and TOE migration can be represented as distinct outcomes of the same sediment-conservation update.

Numerical approaches have also been developed to handle more general forcing histories and boundary behavior. Deforming-grid and front-tracking methods can represent moving boundaries explicitly in one dimension, and they have been used effectively to study shoreline migration, base-level change, and autogenic delta response (Parker and Muto, 2003; Muto and Swenson, 2005; Swenson and Muto, 2007; Parker et al., 2008). However, explicit tracking becomes cumbersome when multiple boundaries interact, reverse direction, merge, or split. The difficulty increases substantially in two horizontal dimensions, where the ABT, SH, TOE, and fan margins become evolving curves rather than single points.

Fixed-grid enthalpy methods provide an alternative. In phase-change modeling, enthalpy methods avoid explicitly tracking the phase boundary by solving for a conserved state variable on a fixed grid; the boundary is then diagnosed from the state variable after each update (Voller et al., 2004; Voller et al., 2006; Voller, 2010). Applied to delta evolution, this approach represents the deposit using a thickness-like variable and reconstructs the surface elevation relative to sea level and basement geometry. Lorenzo-Trueba and Voller (2010) and Anderson et al. (2019) showed that enthalpy formulations can reproduce moving-boundary delta behavior while handling sea-level histories, shoreline reversals, and stratigraphic development without explicit front tracking. In particular, Anderson et al. (2019) demonstrated the usefulness of the enthalpy approach for 1D delta response to sea-level cycles, but the formulation retained a simplified offshore geometry.

The present manuscript extends this line of work in two main ways. First, we treat the ABT, SH, and TOE as three coupled geomorphic boundaries, allowing the offshore TOE of the deposit to emerge from the solution rather than being prescribed through a vertical-foreset, shock, or fixed-geometry approximation. Second, we formulate the model on a fixed grid in both 1D and 2D, so that the same conservation logic applies when the moving boundaries become curves in plan view.

3. Conceptual model and scope

This section defines the idealized geometry, state variables, and diagnostic moving interfaces used throughout the manuscript. Unless otherwise stated, variables introduced in this section are dimensional physical quantities. Slopes such as the basement slope β are dimensionless ratios of vertical to horizontal length. Nondimensional variables and parameter groups are introduced explicitly in Sect. 4.3; after nondimensionalization, the same symbols are used for simplicity.

GeoEnthalpy-Delta v1.0 follows the moving-boundary interpretation introduced in Sects. 1 and 2, but implements the moving interfaces differently from explicit front-tracking approaches. Instead of advancing the ABT, SH, and TOE as computational boundaries, the model evolves sediment thickness on a fixed grid and diagnoses these interfaces from the updated sediment-thickness field, reconstructed surface, and prescribed sea level. The purpose of this section is to define those fields and interfaces before presenting the conservation law and numerical implementation in Sect. 4.



187 **3.1 Idealized geometry and forcing**

188 As shown in Fig. 2, we consider a delta deposit evolving over a prescribed, non-erodible
 189 basement surface $E(x, y)$. The coordinate x increases seaward in the primary transport direction,
 190 and y denotes the lateral direction. Unless otherwise stated, the basement is planar in the along-
 191 profile direction,

$$192 \quad E(x, y) = -\beta x, \quad (1)$$

193

194 where β is the basement slope. This choice provides a minimal geometry for isolating the effects
 195 of sediment supply, sea-level forcing, and moving-boundary coupling. More complex basement
 196 geometries can be represented by prescribing a different function $E(x, y)$, but spatially variable
 197 basement topography is not explored in this version of the model.

198 The evolving deposit is a sedimentary body superimposed on this basement. It includes a
 199 subaerial topset and a subaqueous foreset, separated by a shoreline (Figure 2). In 1D, the model
 200 represents the longitudinal profile of this deposit. In 2D, the same framework represents
 201 planform spreading from a finite-width sediment source, allowing the shoreline and offshore
 202 sediment body to evolve as curves rather than as single points.

203 The model is forced by a prescribed sea-level history $Z(t)$ and an imposed upstream sediment
 204 input. In 1D, the sediment input is represented as a flux per unit width, q_0 . In 2D, the total input,
 205 Q_{in} , is distributed across an upstream feeder of prescribed width. A laterally uniform feeder and
 206 laterally uniform initial conditions recover the 1D limit, whereas a finite-width feeder allows
 207 planform spreading from a localized sediment source.

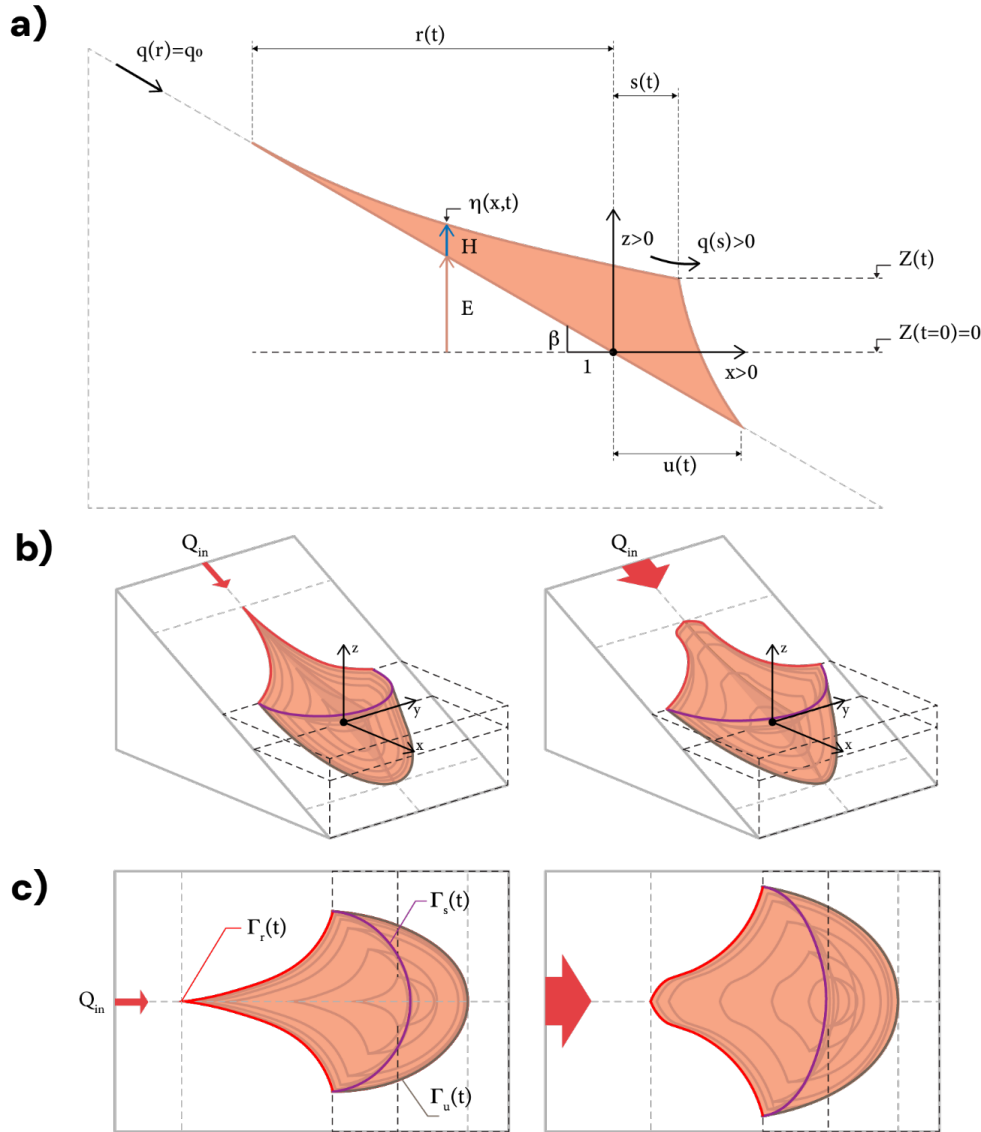


Figure 2. Conceptual geometry and diagnostic moving boundaries for GeoEnthalpy-Delta v1.0.
 (a) 1D cross-section showing the prescribed basement elevation $E(x)$, sediment thickness $H(x, t)$, reconstructed sediment surface $\eta(x, t) = E(x) + H(x, t)$, and prescribed sea level $Z(t)$. The alluvial-bedrock transition $r(t)$, shoreline $s(t)$, and toe $u(t)$ define the upstream limit of finite sediment thickness, the subaerial-subaqueous transition $\eta = Z(t)$, and the offshore limit of finite sediment thickness, respectively. (b) 3D schematic of the 2D model geometry, showing a finite-width upstream feeder, the evolving deposit, sea level, and diagnostic moving-boundary curves. (c) Plan-view representation of the same 2D geometry, showing the alluvial-bedrock transition $\Gamma_r(t)$, shoreline $\Gamma_s(t)$, toe $\Gamma_u(t)$, and lateral fan margins.



218 3.2 Sediment thickness and reconstructed surface

219 The primary state variable is the sediment thickness above basement, $H(x, y, t)$. Given the
220 prescribed basement elevation $E(x, y)$, the sediment surface elevation relative to the fixed
221 vertical datum is

$$222 \quad \eta(x, y, t) = E(x, y) + H(x, y, t). \quad (2)$$

223 Sea level is prescribed externally as $Z(t)$. Within the feeder-associated active deposit, the
224 reconstructed surface η determines the local subaerial or subaqueous transport regime. Cells
225 satisfying

$$226 \quad \eta(x, y, t) \geq Z(t) \quad (3)$$

227 are treated as topset cells, whereas cells satisfying

$$228 \quad \eta(x, y, t) < Z(t) \quad (4)$$

229 are treated as foreset cells. Thus, H is the prognostic variable evolved by the model, η is the
230 reconstructed sediment surface used to compute transport slopes, and $Z(t)$ determines the local
231 topset–foreset partition.

232 This distinction is important for the formulation in Sect. 4. The model does not evolve only
233 exposed height above sea level, nor does it clip the deposit at the shoreline. Instead, it evolves
234 total sediment thickness above the basement. Transport slopes are computed from the continuous
235 reconstructed surface η , allowing the subaerial and subaqueous parts of the deposit to be
236 represented within a single conserved thickness field.

237 3.3 Diagnostic moving geomorphic interfaces

238 The three moving interfaces introduced above are treated here as diagnostic features of H , η , and
239 $Z(t)$. In classical moving-boundary formulations, these interfaces are unknown functions of time
240 that must be advanced explicitly or closed using Stefan-type boundary conditions. In the
241 enthalpy formulation used here, they are recovered after each update from the evolving fields.

242 The ABT is the upstream limit of finite sediment thickness. Landward of the ABT, sediment may
243 bypass the non-erodible basement without forming a finite alluvial cover; seaward of the ABT,
244 sediment thickness becomes positive and the depositional domain expands. In 1D, the ABT is
245 denoted by $x = r(t)$. In 2D, it forms a curve $\Gamma_r(t)$.

246 The SH separates subaerial and subaqueous portions of the system. It is defined as the locus
247 where the sediment surface intersects sea level, $\eta(x, y, t) = Z(t)$. In one dimension, the
248 shoreline is denoted by $x = s(t)$. In two dimensions, it forms a curve $\Gamma_s(t)$ that can migrate and
249 deform in plan view.

250 The TOE marks the offshore termination of the feeder-associated sediment body. In 1D, it is
251 denoted by $x = u(t)$. In 2D, it forms the seaward portion of the finite-thickness boundary,
252 denoted here by $\Gamma_u(t)$. Conceptually, the TOE is the seaward limit where $H \rightarrow 0$; numerically, it
253 is extracted from a small finite-thickness tolerance, $H = \epsilon_H$, as described in Sect. 4.7. This



254 definition distinguishes the TOE from the shoreline and avoids prescribing the offshore
 255 termination of the deposit geometrically.

256 The same thickness field also defines the lateral fan margins in 2D. These margins are part of the
 257 finite-thickness boundary of the feeder-associated active deposit, but they are not counted as
 258 longitudinal moving boundaries. The ABT and TOE correspond to the landward and seaward
 259 portions of the finite-thickness boundary, respectively, whereas the shoreline is an internal
 260 contour of the active deposit defined by $\eta = Z(t)$.

261 A useful feature of this formulation is that the number of active geomorphic interfaces can vary
 262 without changing the underlying conservation law. Shoreline-only configurations recover one-
 263 boundary Stefan-type delta problems (Swenson et al., 2000; Voller et al., 2004). ABT–shoreline
 264 configurations recover two-boundary delta-growth problems in which the offshore geometry is
 265 prescribed, collapsed into a shoreline condition, or otherwise not treated as an independent finite-
 266 thickness boundary (Lorenzo-Trueba et al., 2009; Lorenzo-Trueba and Voller, 2010; Anderson et
 267 al., 2019). Coupled subaerial–subaqueous formulations with shock or prescribed shoreface
 268 conditions demonstrate the importance of separating shoreline motion from subaqueous
 269 clinoform development (Swenson et al., 2005), but still impose part of the offshore coupling
 270 geometrically. The full ABT–SH–TOE configuration used here resolves finite foreset storage
 271 and TOE migration explicitly, providing a hierarchy of moving-boundary problems within a
 272 single fixed-grid enthalpy framework.

273

274 **4. Model formulation and numerical implementation**

275 Here, “enthalpy” is used in the fixed-grid numerical sense: the model advances a conserved
 276 sediment-thickness field on a stationary mesh and diagnoses moving geomorphic boundaries
 277 from that field. Because subaqueous sediment storage and transport are resolved explicitly, H is
 278 not a literal thermodynamic enthalpy formed by combining separate sensible- and latent-heat
 279 contributions; this distinction is discussed further in Sect. 7.

280 This section presents the governing formulation and numerical implementation of GeoEnthalpy-
 281 Delta v1.0. The aim is to isolate the minimal ingredients needed to test the moving-boundary
 282 framework: sediment conservation, a fixed non-erodible basement, externally prescribed relative
 283 sea level, and thresholded slope-driven sediment transport. Processes such as backwater
 284 hydrodynamics, waves, tides, sediment sorting, compaction, and evolving basement motion are
 285 not included in this version of the model and are discussed later as limitations and possible
 286 extensions.

287 The geometry, state variables, and diagnostic moving boundaries were introduced in Sect. 3.
 288 Here we describe the sediment-thickness conservation law, the thresholded transport closure, the
 289 nondimensional parameterization, the fixed-grid finite-volume update, and the diagnostic
 290 procedure used to extract the ABT, SH, TOE from the numerical solution. The formulation is
 291 presented first in two dimensions because this is the most general implementation. The 1D model
 292 used for analytical verification is then obtained as a reduction of the same numerical structure.

293 **4.1 Conservation law and enthalpy-style fixed-grid reformulation**



294 Using the notation introduced in Sect. 3, the prognostic variable is the sediment thickness
295 $H(x, y, t)$ and the sediment mass conservation is written in flux form as

$$296 \quad \frac{\partial H}{\partial t} = -\nabla \cdot \mathbf{q}, \quad (5)$$

297 where $\mathbf{q} = (q_x, q_y)$ is the depth-integrated sediment flux vector. Because the basement is fixed,
298 $\partial \eta / \partial t = \partial H / \partial t$ wherever E does not vary in time. Thus, Eq. (5) is equivalent to a standard
299 bed-elevation conservation equation within the active deposit, but it is written in terms of H ,
300 which is naturally defined on the entire computational grid.

301 This is the enthalpy-style fixed-grid reformulation used in the model. Rather than solving on a
302 moving depositional domain bounded by the ABT, SH, and TOE, the model evolves H on a
303 fixed grid. Cells with $H = 0$ represent exposed basement or regions not yet reached by the
304 deposit, whereas cells with $H > 0$ belong to the sedimentary body. The ABT and TOE are
305 therefore diagnosed from the limits of finite sediment thickness, while the shoreline is diagnosed
306 from $\eta = Z(t)$. The geomorphic interfaces are not computational boundaries; the fixed upstream,
307 lateral, and downstream edges of the rectangular grid are the computational boundaries (Figure
308 3).

309 **4.2 Threshold transport closure and regime switching**

310 Sediment transport is represented using a reduced morphodynamic closure rather than an explicit
311 hydrodynamic calculation. The model does not solve for water depth, velocity, turbulence, or
312 channel-scale routing. Instead, sediment redistribution is related locally to the gradient of the
313 reconstructed sediment surface η , following earlier reduced-complexity moving-boundary and
314 enthalpy models for delta evolution (Swenson et al., 2000; Voller et al., 2004, 2006; Lorenzo-
315 Trueba et al., 2009; Lorenzo-Trueba and Voller, 2010; Anderson et al., 2019; Lai and Capart,
316 2007, 2009; Lai et al., 2017).

317 The local transport regime is determined from the reconstructed surface relative to sea level:

$$k(x, y, t) = \begin{cases} \text{top}, & \eta(x, y, t) \geq Z(t), \\ \text{fore}, & \eta(x, y, t) < Z(t). \end{cases} \quad (6)$$

318 Here $k = \text{top}$ denotes topset transport and $k = \text{fore}$ denotes foreset transport. This local
319 switching allows transport parameters to change across the shoreline without treating the
320 shoreline as a computational boundary. The shoreline remains the diagnostic contour $\eta = Z(t)$.

321 In the down-system direction, sediment transport is assumed to be seaward directed. With x
322 increasing seaward, the dimensional down-system flux is

$$q_x = v_k^x \max\left(-\frac{\partial \eta}{\partial x} - C_k^x, 0\right), \quad (7)$$

323 where v_k^x is the dimensional transport coefficient and C_k^x is the critical slope for transport in the
324 down-system direction. The critical slopes are dimensionless slopes, but they appear in the
325 dimensional flux law because they are compared directly with surface gradients.



In the lateral direction, sediment can move in either direction depending on the sign of the cross-slope gradient (Figure 3). If q_y is positive in the $+y$ direction, the dimensional lateral flux is

$$q_y = -v_k^y \operatorname{sgn}\left(\frac{\partial \eta}{\partial y}\right) \max\left(\left|\frac{\partial \eta}{\partial y}\right| - C_k^y, 0\right). \quad (8)$$

Thus, lateral transport is directed down the local gradient of η and ceases when the magnitude of the lateral slope is below the corresponding critical threshold. The parameters C_{top}^x , C_{top}^y , C_{fore}^x , and C_{fore}^y allow down-system progradation, offshore redistribution, and lateral spreading to respond differently to local slope.

4.3 Nondimensional form and diagnostic parameter groups

The numerical codes solve the nondimensional equations. Let ℓ be a characteristic horizontal length scale, $\beta\ell$ the corresponding vertical length scale set by the basement slope, and let v_{top}^x be the characteristic topset transport coefficient in the down-system direction. We introduce nondimensional variables, denoted temporarily with hats, as

$$\hat{x} = \frac{x}{\ell}, \quad \hat{y} = \frac{y}{\ell}, \quad \hat{H} = \frac{H}{\beta\ell}, \quad \hat{\eta} = \frac{\eta}{\beta\ell}, \quad \hat{E} = \frac{E}{\beta\ell}, \quad \hat{Z} = \frac{Z}{\beta\ell} \quad (9)$$

and

$$\hat{t} = \frac{v_{\text{top}}^x t}{\ell^2}, \quad \hat{\mathbf{q}} = \frac{\mathbf{q}}{\beta v_{\text{top}}^x}. \quad (10)$$

With this scaling, the reconstructed surface remains

$$\hat{\eta}(\hat{x}, \hat{y}, \hat{t}) = \hat{E}(\hat{x}, \hat{y}) + \hat{H}(\hat{x}, \hat{y}, \hat{t}), \quad (11)$$

and a planar dimensional basement $E = -\beta x$ becomes $\hat{E} = -\hat{x}$. The nondimensional conservation law is

$$\frac{\partial \hat{H}}{\partial \hat{t}} = -\hat{\nabla} \cdot \hat{\mathbf{q}}. \quad (12)$$

The nondimensional fluxes are

$$\hat{q}_x = D_k^x \max\left(-\frac{\partial \hat{\eta}}{\partial \hat{x}} - \hat{C}_k^x, 0\right), \quad (13a)$$

$$\hat{q}_y = -D_k^y \operatorname{sgn}\left(\frac{\partial \hat{\eta}}{\partial \hat{y}}\right) \max\left(\left|\frac{\partial \hat{\eta}}{\partial \hat{y}}\right| - \hat{C}_k^y, 0\right) \quad (13b)$$

where

$$D_k^x = \frac{v_k^x}{v_{\text{top}}^x}, \quad D_k^y = \frac{v_k^y}{v_{\text{top}}^x}, \quad (14)$$

And in Eq. (15) only, the symbols on the right-hand side denote the corresponding dimensional critical slopes before beta-normalization.



$$\hat{C}_k^x = \frac{C_k^x}{\beta}, \hat{C}_k^y = \frac{C_k^y}{\beta}, \quad (15)$$

Thus, after nondimensionalization, critical slopes are interpreted as threshold slopes normalized by the dimensional basement slope. The dimensional upstream sediment supply per unit width is nondimensionalized as

$$R_{ab} = \frac{q_0}{\beta v_{top}^x}. \quad (16)$$

Thus, after nondimensionalization, the imposed 1D upstream flux is R_{ab} . In 2D, the total imposed input is the integral of the upstream flux density across the feeder. Its nondimensional form is

$$\hat{Q}_{in} = \frac{Q_{in}}{\beta v_{top}^x \ell}. \quad (17)$$

With thresholded transport, R_{ab} alone does not determine the active topset slope. The beta-normalized surface slope required to transmit the imposed upstream flux is

$$R_{slope} = C_{top}^x + \frac{R_{ab}}{D_{top}^x}, \quad (18)$$

which reduces to $R_{slope} = C_{top}^x + R_{ab}$ when $D_{top}^x = 1$ and $q_0 = R_{ab}$. For the planar single-slope basement used here, the beta-normalized basement slope is one. Therefore $R_{slope} < 1$ indicates that the active topset surface slope remains lower than the basement-normalized reference slope. Similarly, a finite down-system foreset TOE requires $C_{fore}^x > 1$ in the beta-normalized variables.

After nondimensionalization, hats are dropped for simplicity. Unless otherwise stated, all variables and parameters in the numerical implementation and in the parameter tables are beta-normalized nondimensional quantities.

4.4 Computational domain, forcing, and boundary conditions

The 2D model is solved on a fixed rectangular grid with n_x cells in the down-system direction and n_y cells in the lateral direction (Figure 3). Cell centers are denoted by (x_i, y_j) , where $i = 1, \dots, n_x$ increases in the positive x direction and $j = 1, \dots, n_y$ increases in the positive y direction. Grid spacings are uniform and are denoted by Δx and Δy ; the simulations presented here use $\Delta x = \Delta y$ unless otherwise stated. The geomorphic interfaces are not imposed as computational boundaries; they are diagnosed after each saved update from H , η , and $Z(t)$.

Numerical fluxes across the four faces of cell (i, j) are denoted by $F_{i,j}^N$, $F_{i,j}^S$, $F_{i,j}^E$, and $F_{i,j}^W$. The north and south faces are normal to the x direction, with the south face lying in the positive x direction. The west and east faces are normal to the y direction, with the east face lying in the positive y direction. The south-face flux $F_{i,j}^S$ is non-negative and represents transport from cell (i, j) to cell $(i + 1, j)$. The east-face flux $F_{i,j}^E$ is signed: $F_{i,j}^E > 0$ represents transport from cell (i, j) to cell $(i, j + 1)$, whereas $F_{i,j}^E < 0$ represents transport from cell $(i, j + 1)$ to cell (i, j) . Interior north and west fluxes are shared with neighboring cells,

$$F_{i,j}^N = F_{i-1,j}^S \quad (i > 1), \quad F_{i,j}^W = F_{i,j-1}^E \quad (j > 1). \quad (19)$$



384 Sediment enters through a prescribed upstream source. In 1D, the upstream boundary flux is
 385 $q_{1/2} = q_0$. In 2D, the imposed upstream source can be specified either as a feeder flux density or
 386 as a total sediment input integrated across the feeder. Let $\mathcal{F}$ denote the set of active feeder cells
 387 along the upstream boundary and let $N_f = |\mathcal{F}|$. If the nondimensional feeder flux density is q_f ,
 388 the imposed north-face flux is

$$389 \quad F_{1,j}^N = \begin{cases} q_f, & j \in \mathcal{F}, \\ 0, & j \notin \mathcal{F}. \end{cases} \quad (20)$$

390 The corresponding total nondimensional sediment input is

$$391 \quad Q_{\text{in}} = \sum_{j \in \mathcal{F}} F_{1,j}^N \Delta y = q_f N_f \Delta y. \quad (21)$$

392 In the continuum limit, $N_f \Delta y$ is the nondimensional feeder width w_f , so $Q_{\text{in}} = q_f w_f$. The 1D
 393 input parameter R_{ab} is a flux per unit width. Therefore, if the 2D feeder flux density is chosen to
 394 match the 1D upstream supply, $q_f = R_{ab}$, then

$$395 \quad Q_{\text{in}} = R_{ab} w_f \quad (22)$$

396 in the continuum limit, or $Q_{\text{in}} = R_{ab} N_f \Delta y$ on the discrete grid. Conversely, when Q_{in} is held fixed
 397 across simulations with different feeder widths, the feeder flux density is adjusted as

$$398 \quad q_f = \frac{Q_{\text{in}}}{N_f \Delta y}. \quad (23)$$

399 Thus, R_{ab} and Q_{in} describe different quantities: R_{ab} is the nondimensional 1D flux per unit
 400 width, whereas Q_{in} is the nondimensional total 2D sediment input integrated across the feeder.

401 The lateral computational boundaries are no-flux boundaries,

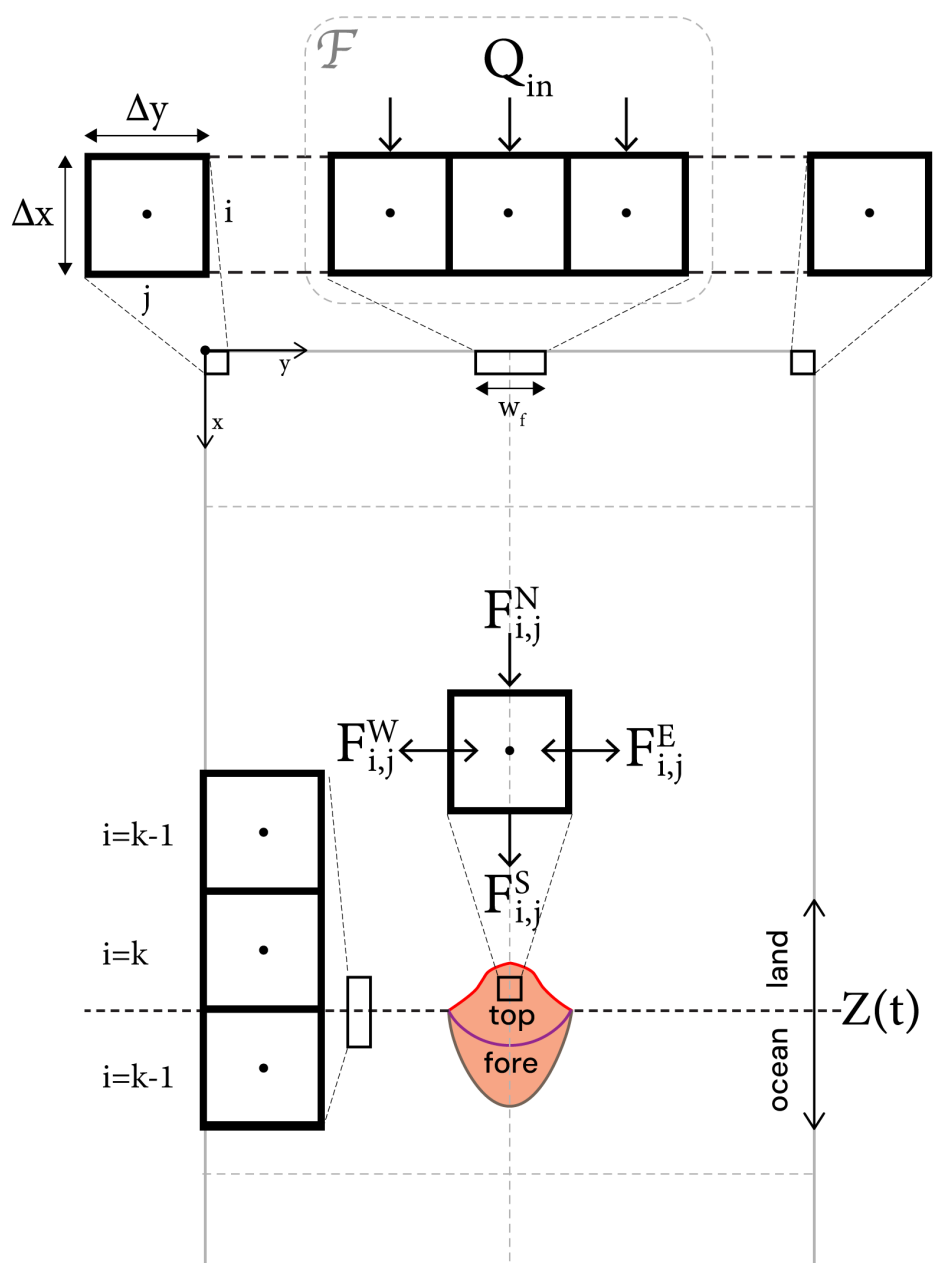
$$402 \quad F_{i,1}^W = 0, \quad F_{i,n_y}^E = 0, \quad (24)$$

403 and the downstream computational boundary is also treated as no flux in the present
 404 implementation,

$$405 \quad F_{n_x,j}^S = 0. \quad (25)$$

406 This downstream condition is appropriate only when the computational domain is large enough
 407 that the TOE remains well inside the domain over the interval analyzed. Simulations in which the
 408 feeder-associated active deposit approaches a lateral or downstream boundary are repeated on a
 409 larger domain or excluded from interpretation.

410 Sea level is prescribed externally as $Z(t)$. The finite-volume method does not require a specific
 411 sea-level history. Fixed sea level, square-root sea-level histories, constant sea-level rise, and
 412 cyclic sea-level forcing are defined in the corresponding model-evaluation and model-
 413 exploration sections.



415 **Figure 3.** Discrete computational domain, finite-volume notation, and thresholded transport closure used
416 in GeoEnthalpy-Delta v1.0. The 2D model is solved on a fixed rectangular grid with x positive seaward



and y lateral. Sediment input Q_{in} is distributed over the feeder-cell set $\mathcal{F}$, with nondimensional feeder width w_f . Numerical face fluxes through a representative cell (i, j) are denoted $F_{i,j}^N$, $F_{i,j}^S$, $F_{i,j}^E$, and $F_{i,j}^W$, following the sign convention in Sect. 4.4. The local transport regime switches between topset and foreset according to the diagnostic shoreline condition $\eta = Z(t)$. Candidate fluxes are computed from reconstructed-surface slopes and thresholded transport laws and are then limited by the positivity-preserving availability limiter in Sect. 4.6. The ABT, SH, and TOE are diagnostic contours rather than computational boundaries.

4.5 Candidate face fluxes on the two-dimensional finite-volume grid

Cell-centered values at time level n are denoted by $H_{i,j}^n$ and $\eta_{i,j}^n$. At each time step, sea level is updated from the prescribed forcing, $Z^n = Z(t^n)$, and the reconstructed surface is evaluated as the discrete form of Eq. (2),

$$\eta_{i,j}^n = E_{i,j} + H_{i,j}^n. \quad (26)$$

The local transport regime is the discrete form of Eq. (6),

$$k_{i,j}^n = \begin{cases} \text{top}, & \eta_{i,j}^n \geq Z^n, \\ \text{fore}, & \eta_{i,j}^n < Z^n. \end{cases} \quad (27)$$

Candidate face fluxes are first evaluated from the thresholded transport closure. These candidate fluxes represent the local transport capacities implied by the reconstructed-surface slope and the critical threshold. They are not yet the fluxes used in the conservative update, because a cell may not contain enough sediment to satisfy all candidate outgoing fluxes during one explicit time step. The actual face fluxes are obtained after applying the positivity-preserving availability limiter in Sect. 4.6.

The down-system slope across the south face of cell (i, j) is

$$S_{i,j}^S = \frac{\eta_{i,j}^n - \eta_{i+1,j}^n}{\Delta x}. \quad (28)$$

The candidate south-face flux is

$$F_{i,j}^{S,\text{cand}} = D_{k_{i,j}^n}^x \max(S_{i,j}^S - C_{k_{i,j}^n}^x, 0). \quad (29)$$

This candidate flux is non-negative and represents transport from cell (i, j) toward cell $(i + 1, j)$. The transport coefficient and critical slope are selected from the donor cell (i, j) , because sediment is exported through the south face of that cell.

The lateral slope across the east face of cell (i, j) is defined as

$$S_{i,j}^E = \frac{\eta_{i,j}^n - \eta_{i,j+1}^n}{\Delta y}. \quad (30)$$

With this sign convention, $S_{i,j}^E > 0$ means that cell (i, j) is higher than its eastern neighbor, so candidate transport is directed eastward out of cell (i, j) . Conversely, $S_{i,j}^E < 0$ means that the eastern neighbor is higher, so candidate transport is directed westward into cell (i, j) . For $S_{i,j}^E > 0$, the donor cell is (i, j) and the candidate east-face flux is



$$F_{i,j}^{E,\text{cand}} = D_{k_{i,j}}^y \max(S_{i,j}^E - C_{k_{i,j}}^y, 0). \quad (31a)$$

For $S_{i,j}^E < 0$, the donor cell is $(i, j + 1)$ and the candidate east-face flux is

$$F_{i,j}^{E,\text{cand}} = -D_{k_{i,j+1}}^y \max(-S_{i,j}^E - C_{k_{i,j+1}}^y, 0). \quad (31b)$$

For $S_{i,j}^E = 0$, $F_{i,j}^{E,\text{cand}} = 0$. Positive $F_{i,j}^{E,\text{cand}}$ represents candidate transport from cell (i, j) into cell $(i, j + 1)$, whereas negative $F_{i,j}^{E,\text{cand}}$ represents candidate transport from cell $(i, j + 1)$ into cell (i, j) . In both cases, transport parameters are selected from the donor cell. Only south- and east-face candidate fluxes need to be computed independently; north- and west-face fluxes are shared with neighboring cells according to Eq. (19).

4.6 Positivity limiter, conservative update, and stability

Because the basement is non-erodible, sediment thickness must remain non-negative. In two dimensions, sediment can leave a cell laterally and down-system during the same time step. The implementation therefore uses a positivity-preserving availability limiter. The limiter is applied in two stages: first to the signed lateral fluxes and then to the down-system fluxes during an upstream-to-downstream sweep. This ordering is useful because down-system transport is one-directional in the present model, whereas lateral transport can occur in either direction.

The candidate lateral flux across the east face of cell (i, j) , $F_{i,j}^{E,\text{cand}}$, is signed. Positive values represent transport from cell (i, j) to cell $(i, j + 1)$, and negative values represent transport from cell $(i, j + 1)$ to cell (i, j) . The candidate lateral outflow rate from cell (i, j) is

$$O_{i,j}^{y,\text{cand}} = \frac{\max(F_{i,j}^{E,\text{cand}}, 0) + \max(-F_{i,j-1}^{E,\text{cand}}, 0)}{\Delta y}. \quad (32)$$

The sediment available for lateral export from cell (i, j) during one explicit time step is $H_{i,j}^n / \Delta t$. We define the lateral limiting factor

$$\alpha_{i,j}^y = \begin{cases} \min\left(1, \frac{H_{i,j}^n / \Delta t}{O_{i,j}^{y,\text{cand}}}\right), & O_{i,j}^{y,\text{cand}} > 0, \\ 1, & O_{i,j}^{y,\text{cand}} = 0. \end{cases} \quad (33)$$

The limited east-face flux is then assigned using the limiting factor of the donor cell:

$$F_{i,j}^E = \begin{cases} \alpha_{i,j}^y F_{i,j}^{E,\text{cand}}, & F_{i,j}^{E,\text{cand}} > 0, \\ \alpha_{i,j+1}^y F_{i,j}^{E,\text{cand}}, & F_{i,j}^{E,\text{cand}} < 0, \\ 0, & F_{i,j}^{E,\text{cand}} = 0. \end{cases} \quad (34)$$



After the lateral fluxes are limited, the down-system fluxes are computed by sweeping from the upstream boundary toward the downstream boundary. For cell (i, j) , the sediment available for south-face export includes the sediment already stored in the cell, the flux entering through the north face, and the net contribution of the limited lateral fluxes:

$$A_{i,j}^x = F_{i,j}^N + \Delta x \left[\frac{H_{i,j}^n}{\Delta t} + \frac{F_{i,j}^W - F_{i,j}^E}{\Delta y} \right]. \quad (35)$$

The limited south-face flux is

$$F_{i,j}^S = \min \left(F_{i,j}^{S,\text{cand}}, \max(A_{i,j}^x, 0) \right). \quad (36)$$

Interior north and west fluxes are shared with neighboring cells,

$$F_{i,j}^N = F_{i-1,j}^S, F_{i,j}^W = F_{i,j-1}^E, \quad (37)$$

with the upstream, lateral, and downstream boundary fluxes defined in Sect. 4.4.

The conservative finite-volume update is

$$H_{i,j}^{n+1} = H_{i,j}^n + \Delta t \left[\frac{F_{i,j}^N - F_{i,j}^S}{\Delta x} + \frac{F_{i,j}^W - F_{i,j}^E}{\Delta y} \right]. \quad (38)$$

The reconstructed surface is then updated as

$$\eta_{i,j}^{n+1} = E_{i,j} + H_{i,j}^{n+1}. \quad (39)$$

The same limited face flux is used by both cells sharing an internal face, so the update is conservative. The limiter prevents lateral export from exceeding local storage and prevents down-system export from exceeding the sediment available after upstream and lateral contributions have been accounted for. Incoming sediment from neighboring cells and from the upstream source is added through the conservative update.

The scheme is explicit and is therefore subject to a diffusion-type stability constraint. For isotropic unit diffusivity on a uniform grid, the standard explicit stability limits are $\Delta t / \Delta x^2 \leq 1/2$ in 1D and $\Delta t / \Delta x^2 \leq 1/4$ in 2D. More generally, for anisotropic transport coefficients on a rectangular grid, we use

$$\Delta t \left(\frac{D_{\max}^x}{\Delta x^2} + \frac{D_{\max}^y}{\Delta y^2} \right) \leq \frac{1}{2}, \quad (40)$$

where $D_{\max}^x$ and $D_{\max}^y$ are the maximum nondimensional transport coefficients in the two coordinate directions. The thresholded flux law reduces the spatial extent of active transport, but it does not remove the need for conservative time-step selection. Grid and time-step consistency are evaluated in Sect. 5.1.



505 4.7 Diagnostic extraction of moving boundaries in two dimensions

506 The diagnostic boundary extraction follows the fixed-grid enthalpy approach used in previous
 507 Stefan-type geomorphic models, in which the governing equation is solved for a conserved state
 508 variable on a fixed grid and the moving boundary is recovered after each update rather than
 509 imposed as a computational boundary (Crank, 1984; Swenson et al., 2000; Voller et al., 2004,
 510 2006; Lorenzo-Trueba and Voller, 2010; Anderson et al., 2019). Anderson et al. (2019), for
 511 example, diagnosed the ABT and SH from the updated enthalpy/thickness field in 1D. The
 512 present implementation extends that diagnostic idea to the 2D, three-boundary problem by
 513 identifying the ABT, SH, and TOE from the updated sediment thickness H , reconstructed surface
 514 η , and prescribed sea level $Z(t)$.

515 After each saved time step, the diagnostic fields are

$$516 \phi_H^n(x, y) = H^n(x, y) - \epsilon_H, \quad (41a)$$

$$517 \phi_S^n(x, y) = \eta^n(x, y) - Z^n. \quad (41b)$$

518 where ϵ_H is a small positive sediment-thickness tolerance. The tolerance avoids sensitivity to
 519 roundoff error and isolated cells with numerically negligible sediment thickness. Unless
 520 otherwise stated, ϵ_H is chosen to be several orders of magnitude smaller than the characteristic
 521 deposit thickness and is checked in the grid-consistency tests.

522 At each saved time, the finite-thickness set $\mathcal{M}_\epsilon^n = \{(x_i, y_j): H_{i,j}^n > \epsilon_H\}$ is partitioned into
 523 components using four-neighbour connectivity, consistent with the face-based finite-volume
 524 stencil. Because sediment may bypass over bare basement landward of the ABT, the retained
 525 component is not required to intersect the upstream computational boundary. Instead, we define
 526 the feeder-associated active deposit, Ω_ϵ^n , as the connected component containing the most
 527 landward active cell within the lateral footprint of the feeder. If no active cell lies within that
 528 footprint, the component nearest the feeder is selected, with ties resolved using component
 529 sediment volume. All ABT, SH, TOE, and lateral-margin diagnostics are restricted to Ω_ϵ^n . Other
 530 finite-thickness components remain part of the modeled sediment-thickness field and sediment
 531 budget but are excluded from the active-boundary diagnostics. This procedure prevents detached
 532 sediment patches from producing artificial TOE progradation, spurious shoreline segments, or
 533 inflated planform dimensions.

534 The SH is extracted as the zero contour of ϕ_S^n within the feeder-associated active deposit,

$$535 \Gamma_S(t^n) = \{(x, y) \in \Omega_\epsilon^n: \eta^n(x, y) = Z^n\}, \quad (42)$$

536 For the approximately single-valued planform boundaries considered here, the implementation
 537 stores row-wise SH positions obtained by linear interpolation between neighboring cells that
 538 straddle $\phi_S^n = 0$. Restricting this interpolation to Ω_ϵ^n prevents intersections between sea level and
 539 bare basement outside the deposit from being identified as shoreline.

540 The ABT and TOE are extracted from the finite-thickness boundary $\phi_H^n = 0$, but they
 541 correspond to different portions of that boundary. The ABT is the landward portion of the feeder-
 542 associated active deposit boundary, and the TOE is the seaward portion. The remaining portions



of the $H = \epsilon_H$ contour are lateral fan margins; they are useful for measuring fan width and opening angle but are not counted as longitudinal moving boundaries.

For simulations in which the contours can be represented approximately as single-valued functions of the lateral coordinate, we also store row-wise sub-grid positions. Let

$$I_j^n = \{i: (x_i, y_j) \in \Omega_\epsilon^n\}, \quad (43)$$

for a row connected to the upstream sediment source. If $i_r = \min I_j^n$ and $i_u = \max I_j^n$, the raw ABT and TOE positions along row j are estimated by linear interpolation as

$$r(y_j, t^n) = x_{i_r-1} + \frac{\epsilon_H - H_{i_r-1,j}^n}{H_{i_r,j}^n - H_{i_r-1,j}^n} (x_{i_r} - x_{i_r-1}), \quad (44a)$$

and

$$u(y_j, t^n) = x_{i_u} + \frac{\epsilon_H - H_{i_u,j}^n}{H_{i_u+1,j}^n - H_{i_u,j}^n} (x_{i_u+1} - x_{i_u}). \quad (44b)$$

The row-wise SH position is extracted from $\phi_s^n = \eta^n - Z^n$. For the cell pair within the feeder-associated active deposit satisfying $\phi_{s,i,j}^n \phi_{s,i+1,j}^n \leq 0$, the raw SH position is

$$s(y_j, t^n) = x_i - \phi_{s,i,j}^n \frac{x_{i+1} - x_i}{\phi_{s,i+1,j}^n - \phi_{s,i,j}^n}. \quad (45)$$

For centerline comparisons, we use $r_c(t^n) = r(y_c, t^n)$, $s_c(t^n) = s(y_c, t^n)$, and $u_c(t^n) = u(y_c, t^n)$, where y_c is the feeder centerline.

All analytical comparisons and grid-sensitivity tests use the raw sub-grid estimates obtained from Eqs. (44a), (44b), and (45). Smoothing is not used to improve agreement with analytical solutions and does not affect the finite-volume update or sediment conservation. When diagnostic quantities require time derivatives, such as boundary speeds or intervals of simultaneous SH retreat and TOE progradation, derivatives may be estimated from the raw trajectories using a short moving-window average. When such smoothing is used, the window length is reported with the corresponding diagnostic or figure caption.

This diagnostic procedure is central to the model formulation. The ABT and TOE are defined by finite sediment thickness, while the SH is defined by the level set $\eta = Z(t)$. The SH and TOE are therefore not forced to move together. SH retreat, continued TOE progradation, TOE abandonment, and changing SH–TOE separation are all diagnosed directly from the evolving sediment-thickness and surface-elevation fields.

4.8 One-dimensional reduction and boundary extraction for analytical verification

The 1D model is obtained by suppressing lateral variation and setting lateral fluxes to zero. It can also be viewed as the laterally uniform limit of the 2D implementation: if all upstream cells receive the same sediment input, all parameters are uniform in y , and the initial condition is laterally uniform, then lateral gradients vanish and the 2D equations reduce to the 1D conservation law and thresholded down-system flux. This reduction is used for analytical verification because several exact or similarity solutions are available in 1D.



577 In 1D, Eq. (5) becomes

$$578 \quad \frac{\partial H}{\partial t} = -\frac{\partial q}{\partial x}. \quad (46)$$

579 The reconstructed surface is $\eta_i^n = E_i + H_i^n$, with beta-normalized planar basement $E_i = -x_i$.

580 The face slope between cells i and $i + 1$ is

$$581 \quad S_{i+1/2}^n = \frac{\eta_i^n - \eta_{i+1}^n}{\Delta x}. \quad (47)$$

582 The candidate flux is

$$q_{i+1/2}^{\text{cand}} = D_k^x \max(S_{i+1/2}^n - C_k^x, 0), \quad (48)$$

583 where $k = \text{top}$ if $\eta_i^n \geq Z^n$ and $k = \text{fore}$ otherwise. The 1D implementation used for analytical
584 verification follows the upstream-to-downstream availability limiter used in previous enthalpy
585 calculations (Anderson et al. 2019). The flux exported from cell i is limited by the candidate
586 transport capacity and by the sediment already stored in the cell plus the flux entering from
587 upstream during the same time step:

$$588 \quad q_{i+1/2} = \min\left(q_{i+1/2}^{\text{cand}}, q_{i-1/2} + \frac{H_i^n \Delta x}{\Delta t}\right). \quad (49)$$

589 The conservative update is

$$H_i^{n+1} = H_i^n + \frac{\Delta t}{\Delta x} (q_{i-1/2} - q_{i+1/2}). \quad (50)$$

590 The imposed upstream sediment supply is applied as $q_{1/2} = q_0$, where $q_0 = R_{ab}$ after beta-
591 normalized nondimensionalization. Internal fluxes are determined by the thresholded transport
592 law and the sediment-availability limiter; q_0 controls the sediment entering the domain but is not
593 used as a cap on all internal fluxes.

594 The 1D moving boundaries are extracted after each saved time step using the same diagnostic
595 criteria as in 2D. The active interval is

$$596 \quad I_\epsilon^n = \{i: H_i^n > \epsilon_H\}, \quad i_r^n = \min I_\epsilon^n, \quad i_u^n = \max I_\epsilon^n. \quad (51)$$

597 If more than one active interval is present because of roundoff or local reworking, the interval
598 connected to the sediment source is retained. The raw ABT and TOE positions are estimated by
599 linear interpolation across the landward and seaward finite-thickness crossings:

$$600 \quad \tilde{r}^n = x_{i_r-1} + \frac{\epsilon_H - H_{i_r-1}^n}{H_{i_r}^n - H_{i_r-1}^n} (x_{i_r} - x_{i_r-1}), \quad (52a)$$

$$\tilde{u}^n = x_{i_u} + \frac{\epsilon_H - H_{i_u}^n}{H_{i_u+1}^n - H_{i_u}^n} (x_{i_u+1} - x_{i_u}). \quad (52b)$$

601 The SH is extracted from the reconstructed surface. Let $\phi_i^n = \eta_i^n - Z^n$. For the cell pair
602 satisfying $\phi_i^n \phi_{i+1}^n \leq 0$ within the feeder-associated active deposit, the raw SH position is



$$\tilde{s}^n = x_i - \phi_i^n \frac{x_{i+1} - x_i}{\phi_{i+1}^n - \phi_i^n}. \quad (53)$$

603 These raw sub-grid estimates are compared directly with analytical boundary positions in
 604 verification tests. For plotting or velocity-based diagnostics, stored raw trajectories may be
 605 smoothed only as post-processing and only when stated explicitly. In GeoEnthalpy-Delta v1.0,
 606 the ABT and TOE are both diagnosed from H , while the SH is diagnosed from $\eta = Z(t)$,
 607 allowing the three moving boundaries to evolve independently.

608

609 **Table 1. State variables and coordinates used in the model formulation.**

Symbol	Meaning	Dimension before scaling
x	Down-system coordinate	L
y	Lateral coordinate	L
t	Time	T
$E(x, y)$	Prescribed basement elevation	L
$H(x, y, t)$	Sediment thickness above basement	L
$\eta(x, y, t)$	Reconstructed sediment surface, $\eta = E + H$	L
$Z(t)$	Prescribed sea level	L
$\mathbf{q}$	Depth-integrated sediment flux vector	$L^2 T^{-1}$

610

611 **Table 2.** Dimensionless transport, forcing, and geometric parameter groups used in the model
 612 formulation. The subscript “dim” is used only in this table to distinguish dimensional quantities
 613 before scaling from the nondimensional symbols used after hats are dropped. The horizontal
 614 length scale is ℓ , the vertical length scale is $\beta\ell$, and the reference transport coefficient is v_{top}^x .
 615 Unless otherwise stated, symbols used after Sect. 4.3 refer to the beta-normalized
 616 nondimensional quantities.

Symbol Definition

Interpretation / role

$$D_k^x \quad D_k^x = v_k^x / v_{\text{top}}^x$$

Down-system transport coefficient normalized by the topset reference value; $k = \text{top, fore}$.



Symbol	Definition	Interpretation / role
D_k^y	$D_k^y = v_k^y / v_{\text{top}}^x$	Lateral transport coefficient normalized by the topset down-system reference value; $k = \text{top, fore}$.
C_k^x	$C_k^x = C_{k,\text{dim}}^x / \beta$	Down-system critical slope normalized by the dimensional basement slope; $k = \text{top, fore}$.
C_k^y	$C_k^y = C_{k,\text{dim}}^y / \beta$	Lateral critical slope normalized by the dimensional basement slope; $k = \text{top, fore}$.
R_{ab}	$R_{ab} = q_{0,\text{dim}} / (\beta v_{\text{top}}^x)$	Nondimensional 1D upstream sediment input per unit width; equivalent to the imposed nondimensional 1D upstream flux.
q_f	$q_f = q_{f,\text{dim}} / (\beta v_{\text{top}}^x)$	Nondimensional feeder flux density in 2D.
Q_{in}	$Q_{\text{in}} = Q_{\text{in},\text{dim}} / (\beta v_{\text{top}}^x \ell)$	Nondimensional total sediment input integrated across the 2D feeder.
w_f	$w_f = w_{f,\text{dim}} / \ell$	Nondimensional feeder width.
R_v	$R_v = D_{\text{fore}}^x / D_{\text{top}}^x = v_{\text{fore}}^x / v_{\text{top}}^x$	Foreset-to-topset down-system transport-coefficient ratio used in the 1D analytical solutions.
R_{slope}	$R_{\text{slope}} = C_{\text{top}}^x + R_{ab} / D_{\text{top}}^x$	Beta-normalized topset surface slope required to transmit the imposed 1D upstream flux. For $D_{\text{top}}^x = 1$, $R_{\text{slope}} = C_{\text{top}}^x + R_{ab}$.
a	$a = a_{\text{dim}} \ell / (\beta v_{\text{top}}^x)$	Nondimensional constant sea-level-rise or sea-level-fall rate when $Z(t) = at$.
λ_z	$\lambda_z = \lambda_{z,\text{dim}} / (\beta \sqrt{v_{\text{top}}^x})$	Nondimensional square-root sea-level-forcing amplitude when $Z(t) = 2\lambda_z \sqrt{t}$.

617

618 5. Verification, analytical benchmarks, and two-dimensional demonstrations

619 This section evaluates GeoEnthalpy-Delta v1.0 using a hierarchy of analytical comparisons,
 620 mass-conservation checks, numerical-consistency tests, and 2D benchmark demonstrations. We
 621 avoid the term validation because comparison with independent laboratory or field observations
 622 is outside the scope of this manuscript. Instead, the goal is to verify the numerical
 623 implementation against exact analytical solutions where the governing equations match, to test
 624 limiting cases where simpler moving-boundary behavior should be recovered, and to
 625 demonstrate the behavior of the 2D implementation under controlled forcing scenarios. This



usage follows the common distinction between verification as testing whether equations are solved correctly and validation as comparison with independent observations (Oreskes et al., 1994; Roache, 1998).

The full model introduced in Sect. 3 includes thresholded sediment transport, finite foreset evolution, diagnostic ABT, shoreline, and TOE positions, prescribed sea-level change, and 2D planform spreading. In one dimension, closed-form solutions are available for selected forcing histories and transport regimes. We use these solutions as exact verification tests where their assumptions match the numerical model. In 2D, the corresponding free fan-delta problem has unknown SH, TOE, and lateral-margin curves. A closed-form solution is not available without imposing additional geometric constraints. We therefore evaluate the 2D model using sediment-mass conservation, grid and time-step consistency, physically ordered diagnostic boundaries, and recovery of the expected post-autobreak translation in physical and moving coordinates.

We proceed in stages. First, we test sediment mass conservation and numerical consistency. Second, we verify the 1D model against a closed-form three-moving-boundary solution for fixed sea level and square-root sea-level histories. Third, we examine limiting cases of that solution, including the vertical-foreset and fixed-ABT graded-river limits. Fourth, we verify the 1D model against an independent steady migrating analytical solution for constant sea-level rise. Finally, we use 2D constant-SLR experiments to demonstrate analogous steady behavior and planform sensitivity in the full 2D model. The input parameter values for the 1D and 2D model-evaluation and model-exploration experiments are summarized in Tables 3 and 4, respectively.

5.1 Mass conservation and numerical consistency

For all 1D and 2D simulations presented in this section, we evaluate sediment mass conservation by comparing the total sediment stored in the computational domain with the cumulative imposed sediment input. This check is independent of the analytical benchmark used in a given experiment and is applied to all model runs. Because the numerical scheme is formulated as a conservative flux update for sediment thickness H , this comparison provides a direct check on the finite-volume implementation described in Sect. 4.

In one dimension, the stored sediment volume per unit width is

$$V(t) = \sum_i H_i(t) \Delta x, \quad (54)$$

and the cumulative sediment input is

$$V_{\text{in}}(t) = \int_0^t q_0(t') dt'. \quad (55)$$

For constant sediment supply, this reduces to $V_{\text{in}} = q_0 t$. In two dimensions, the stored sediment volume is

$$V(t) = \sum_{i,j} H_{i,j}(t) \Delta x \Delta y, \quad (56)$$

and the cumulative input is



$$V_{\text{in}}(t) = \int_0^t \left(\sum_j F_{1,j}^N(t') \Delta y \right) dt', \quad (57)$$

where $F_{1,j}^N$ is the imposed upstream flux through the feeder cells. We define the relative mass-balance error as

$$\epsilon_V(t) = \frac{|V(t) - V_{\text{in}}(t)|}{V_{\text{in}}(t)}. \quad (58)$$

For simulations in which the deposit remains away from the lateral and downstream computational boundaries, $V(t)$ should equal $V_{\text{in}}(t)$ up to numerical truncation error. If this condition is not satisfied, the simulation is not used for interpretation. This check is especially important in two dimensions, where lateral spreading can cause artificial boundary effects if the computational domain is too narrow.

We also perform numerical-consistency checks by repeating representative simulations with smaller Δx and Δt , while keeping the explicit stability condition in Sect. 4.6 satisfied. Because the thresholded transport law creates moving active and inactive regions, these tests are used to assess practical grid sensitivity rather than to estimate a formal convergence order.

5.2 Exact one-dimensional verification with the three-moving-boundary solution

We first verify the 1D model against a closed-form solution for the three-moving-boundary problem with an ABT, SH and TOE. This solution applies to the active form of the thresholded transport law, in which the positive-part operator is inactive throughout the topset and foreset. It therefore provides an exact verification test for parameter combinations satisfying the flux-admissibility conditions defined below.

The solution is formulated for fixed sea level and square-root sea-level histories,

$$Z(t) = 2\lambda_Z \sqrt{t}, \quad (59)$$

where $\lambda_Z = 0$ corresponds to fixed sea level, $\lambda_Z > 0$ to sea-level rise, and $\lambda_Z < 0$ to sea-level fall. The moving boundaries are written as

$$r(t) = -2\lambda_{ab} \sqrt{t}, \quad (60a)$$

$$s(t) = 2\lambda_{sh} \sqrt{t}, \quad (60b)$$

And

$$u(t) = 2\lambda_u \sqrt{t}. \quad (61)$$

The topset and foreset are described using separate similarity coordinates,

$$\xi = \frac{x}{2\sqrt{t}} \quad (62)$$

and



$$\mu = \frac{x}{2\sqrt{R_v t}}, \quad (63)$$

where

$$R_v = \frac{D_{\text{fore}}^x}{D_{\text{top}}^x} \quad (64)$$

is the foreset-to-topset transport-coefficient ratio. The full analytical profiles and algebraic system for λ_{ab} , λ_{sh} , and λ_u are given in Appendix A.

The solution is physically admissible only where the active transport assumptions are satisfied. Thus, after solving the algebraic system, we require

$$q_{\text{top}}(\xi) \geq 0 \quad \text{for} \quad -\lambda_{ab} \leq \xi \leq \lambda_{sh}, \quad (65)$$

and

$$q_{\text{fore}}(\mu) \geq 0 \quad \text{for} \quad \mu_{sh} \leq \mu \leq \mu_u. \quad (66)$$

If either inequality is violated, the closed-form active solution still exists mathematically, but it is not the physical solution of the clipped threshold-transport problem solved by the numerical model. Such cases identify the boundary between the active analytical regime and the more general clipped numerical regime, but they are not used as exact verification cases.

Figure 4 compares the numerical and analytical solutions for admissible active-transport cases. Panels (a) and (b) compare the trajectories of the three moving boundaries under square-root sea-level rise and square-root sea-level fall, respectively. In both cases, the numerical ABT, SH and TOE closely follow the analytical predictions. This verifies that the fixed-grid update reproduces the correct self-similar motion of all three boundaries when the active-branch assumptions are satisfied.

Panels (c) and (d) compare the sedimentary-prism geometry for two threshold choices. The low-threshold case illustrates the geometry when the topset threshold is zero and the foreset threshold is close to its minimum admissible value. A literal zero foreset threshold is not admissible for a finite-TOE solution, because the foreset threshold slope must exceed the subaqueous basement slope for the foreset to re-intersect the basement at a finite TOE. The finite-threshold case uses larger values of C_{top}^x and C_{fore}^x , producing a steeper topset–foreset geometry and a shorter SH–TOE separation. The close match between the numerical and analytical profiles demonstrates that the model correctly represents threshold-controlled transport and finite-foreset storage within the active regime.

These experiments also clarify the process-level role of the transport thresholds in setting prism geometry. Increasing C_{top}^x increases the local slope required for active transport across the topset, thereby changing the topset gradient needed to transmit a prescribed sediment input (see equation (18)). Increasing C_{fore}^x forces the foreset to steepen and reduces the SH–TOE separation, approaching the vertical-foreset limit discussed in Sect. 5.3. The ratio R_v controls how efficiently sediment is redistributed across the foreset relative to the topset; for the



parameter combinations shown here, its effect on the large-scale geometry is smaller than that of C_{fore}^x . Finally, decreasing R_{ab} or increasing λ_z reduces the sediment flux available at the SH and can eventually move the system toward the active-flux admissibility boundary.

The input parameters for the Fig. 4 three-moving-boundary verification cases and Fig. 5 limiting-case comparisons are listed in Table 3.

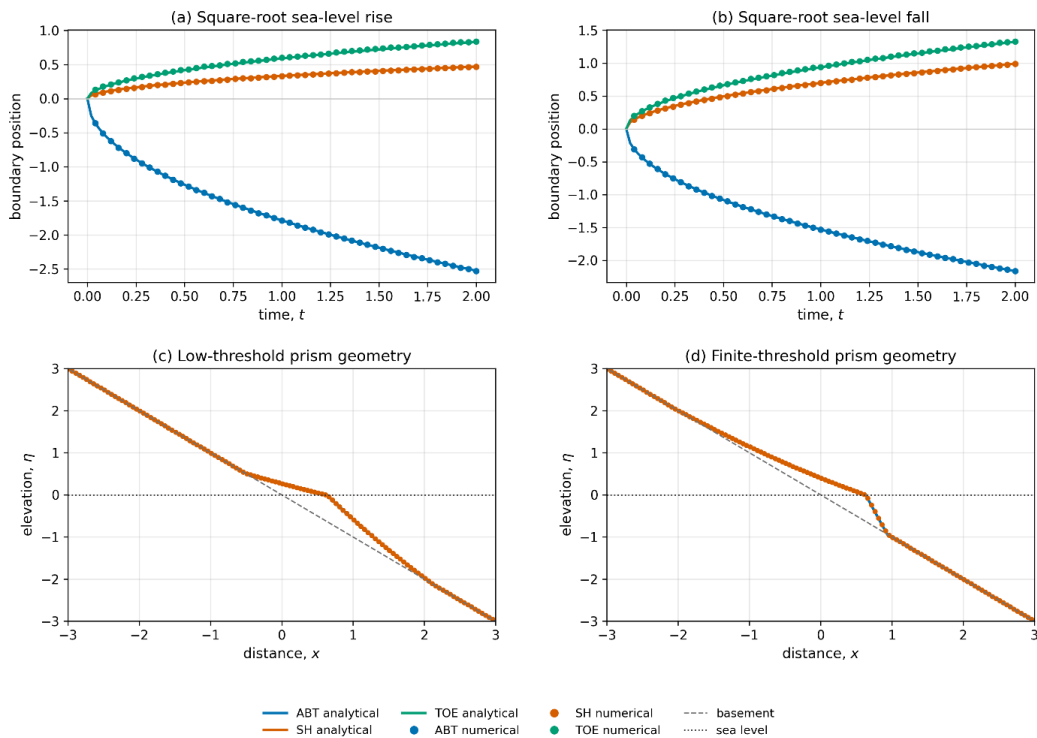


Figure 4. Comparison between the fixed-grid numerical model and the closed-form three-moving-boundary solution. (a) Boundary trajectories for square-root sea-level rise. (b) Boundary trajectories for square-root sea-level fall. In panels (a) and (b), solid lines show the analytical trajectories of the alluvial-bedrock transition (ABT), shoreline (SH), and toe (TOE), and circles show the corresponding numerical trajectories. (c) Sedimentary-prism geometry for a low-threshold case. (d) Sedimentary-prism geometry for a finite-threshold case. In panels (c) and (d), solid lines show the analytical surface profile and circles show the numerical solution; the basement and sea-level position are also shown for reference. The low-threshold case uses a near-minimal admissible foreset threshold because a finite TOE requires the foreset threshold slope to exceed the subaqueous basement slope. The close agreement in boundary trajectories and surface profiles verifies the numerical update, diagnostic SH and TOE extraction, and thresholded flux implementation for admissible active-transport cases.



745 **5.3 Limiting cases of the three-moving-boundary solution**

746 The closed-form three-moving-boundary solution also provides useful limiting cases for
 747 evaluating the numerical model. These limits test whether the finite-foreset implementation
 748 approaches simpler moving-boundary behavior when the parameter regime approaches a known
 749 asymptotic form, particularly previously derived vertical-foreset and graded-river limits.

750 The vertical-foreset limit is approached when the foreset threshold slope becomes large relative
 751 to the subaqueous basement slope. In this limit, the TOE approaches the SH (i.e., $\lambda_u \rightarrow \lambda_{sh}$).

752 Figure 5a shows this limit for square-root sea-level rise. The three-moving-boundary analytical
 753 solution, the numerical solution, and the two-boundary vertical-foreset limit all predict nearly
 754 coincident SH and TOE trajectories when C_{fore}^x is large. Figure 5b shows the corresponding
 755 profile evolution at three times. The finite-foreset solution approaches the vertical-foreset
 756 geometry, but retains a small resolved SH–TOE separation. This small separation is the finite-
 757 foreset analogue of the storage term in the vertical-foreset Stefan condition.

758 A related limiting case is the stationary-ABT or graded-river condition under square-root sea-
 759 level fall. In the vertical-foreset similarity solution, the ABT remains fixed when $\lambda_{ab} = 0$. For
 760 $C_{top}^x = 0$, this condition gives the graded-river base-level fall rate previously described by
 761 Lorenzo-Trueba et al. (2013) and equivalent to the graded-river condition discussed by Swenson
 762 and Muto (2007),

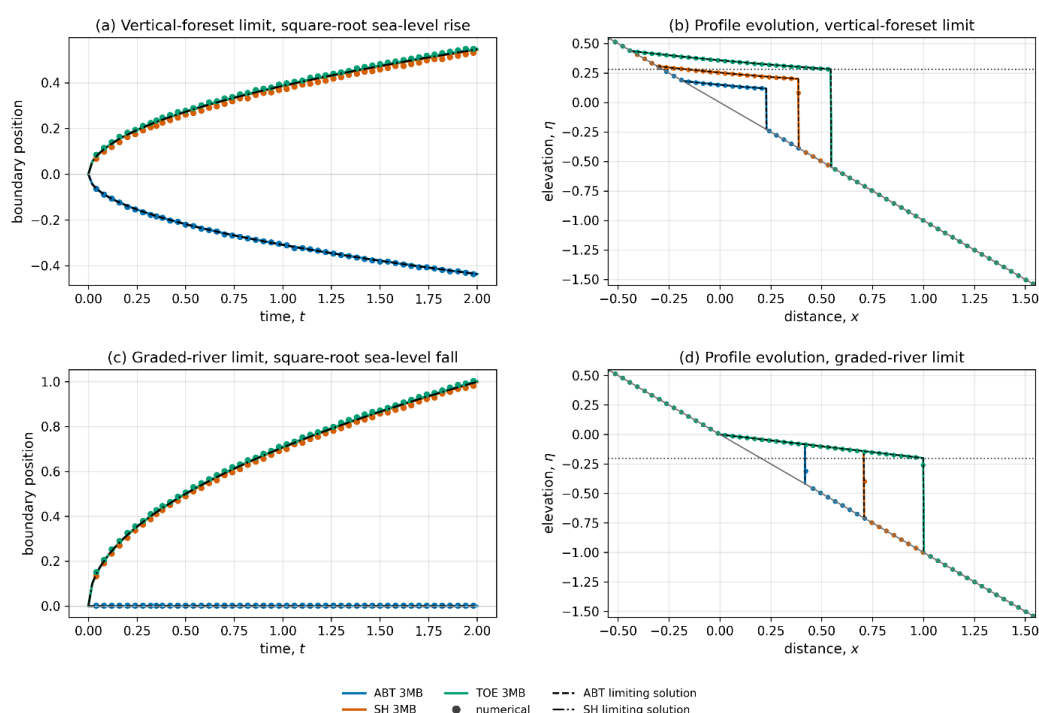
$$763 \quad \lambda_Z = -\frac{1}{2} \left(\frac{2R_{ab}^3}{1-R_{ab}} \right)^{1/2} \quad (67)$$

764 Figures 5c and 5d compare the finite-foreset numerical and three-moving-boundary analytical
 765 solutions with this graded-river limit. The ABT remains fixed at the origin, while the SH and
 766 TOE continue to migrate seaward. The close agreement among the numerical solution, the finite-
 767 foreset analytical solution, and the graded-river limiting solution shows that the model recovers
 768 the expected fixed-ABT behavior when the corresponding asymptotic conditions are approached.

769 The limiting cases in Fig. 5 also help interpret when the full finite-foreset model is required.
 770 When C_{fore}^x is moderate, the SH and TOE are separated and the full three-moving-boundary
 771 solution provides the appropriate reference. When C_{fore}^x is large, the SH–TOE separation becomes
 772 small and the solution approaches the classical two-boundary vertical-foreset form. The
 773 numerical results show the same transition, provided that the grid resolution is sufficient to
 774 resolve the small but finite SH–TOE separation.

775

776



777

778 **Figure 5.** Comparison among the finite-foreset three-moving-boundary solution, the numerical
 779 model, and the corresponding vertical-foreset limiting solutions. (a) Boundary trajectories for
 780 square-root sea-level rise in the large-foreset-threshold limit. Solid colored lines show the three-
 781 moving-boundary analytical solution, circles show the numerical solution, and black
 782 dashed/dash-dotted lines show the corresponding two-moving-boundary vertical-foreset limit.
 783 (b) Profile evolution for the same sea-level-rise case at three times, comparing numerical
 784 profiles, the three-moving-boundary analytical profiles, and the vertical-foreset limiting profiles.
 785 (c) Boundary trajectories for the square-root sea-level-fall case that recovers the graded-river
 786 condition with a fixed ABT. The numerical and three-moving-boundary analytical solutions are
 787 compared with the graded-river analytical limit. (d) Profile evolution for the graded-river case at
 788 three times. Together, the panels show that the finite-foreset numerical solution approaches the
 789 classical vertical-foreset and graded-river limits when the foreset threshold is large and the SH-
 790 TOE separation becomes small.

791 5.4 Exact one-dimensional verification under constant sea-level rise

792 We next evaluate the 1D model under constant sea-level rise. This test is distinct from the
 793 square-root sea-level histories considered above because the solution does not remain in a single
 794 progradational similarity state. Instead, the numerical solution passes through three stages: (i)
 795 initial progradation, during which the SH and TOE both move seaward; (ii) autoretreat, during
 796 which the SH retreats while the TOE continues to prograde; and (iii) post-autobreak retreat,



797 during which sediment supply to the foreset ceases, the TOE is abandoned, and the active topset
798 retreats landward (Muto, 2001; Parker and Muto, 2003).

799 The analytical target used here applies to the final post-autobreak state. In this state, the sediment
800 flux reaching the SH is zero and the incoming sediment is deposited across the active topset. Sea
801 level is prescribed as

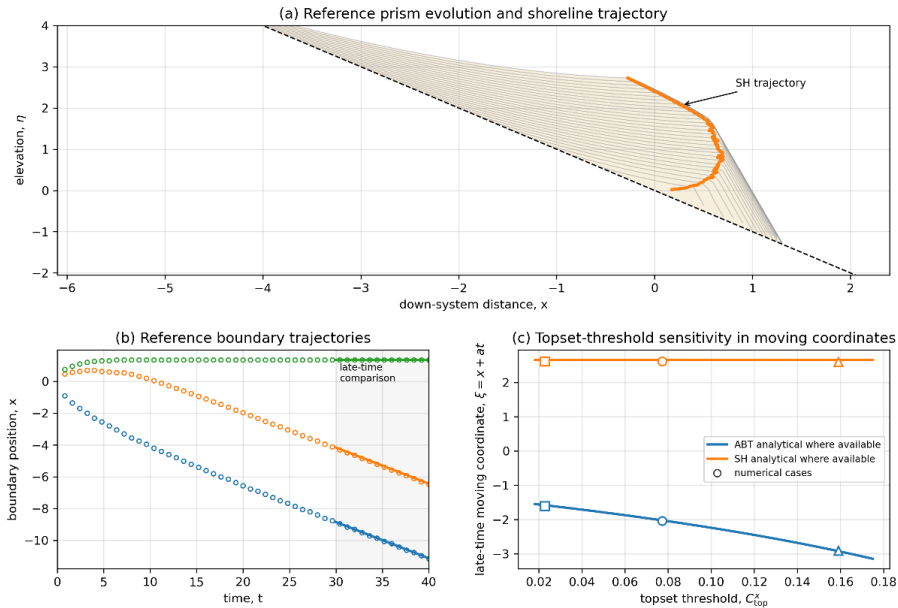
$$802 \quad Z(t) = at \quad (68)$$

803 where a is the nondimensional and β -normalized sea-level-rise rate. For a planar basement, the
804 sea-level intersection on the basement migrates landward at speed $U = a$. The post-autobreak
805 solution is steady in the moving coordinate $\xi = x + Ut$. The defining downstream condition is
806 zero sediment flux at the SH, $q_s(x_{SH}, t) = 0$. Thus, once the post-autobreak translating state has
807 been reached, the analytical boundary trajectories in physical coordinates are

$$808 \quad x_{ABT}(t) = \xi_{ABT} - Ut, \quad x_{SH}(t) = \xi_{SH} - Ut, \quad x_{TOE}(t) = x_{TOE,AB}. \quad (69)$$

809 Here ξ_{ABT} and ξ_{SH} are the steady moving-coordinate positions of the alluvial-bedrock transition
810 and SH, and $x_{TOE,AB}$ is the abandoned-TOE position. Appendix B gives the full analytical
811 profile. The main-text verification is therefore intentionally simple: after autobreak, the
812 numerical ABT and SH should approach straight landward-migrating trajectories with slope
813 $U = -a$, while the TOE should remain fixed in physical space.

814 Figure 6 uses a reduced layout to emphasize this behavior. Panel (a) shows a numerical
815 sedimentary prism from the reference case, with stacked profile snapshots and the SH trajectory
816 used to identify progradation, autoretreat, and post-autobreak retreat. Panels (b)-(d) compare the
817 numerical boundary positions with the post-autobreak analytical translating solution for a
818 reference case and two topset-threshold sensitivity cases. Numerical results are plotted as
819 symbols only, and analytical post-autobreak trajectories are plotted as solid lines. In all three
820 cases, the ABT and SH converge toward the expected landward translation after autobreak, while
821 the TOE approaches a fixed abandoned position. The input parameters for the constant-SLR
822 cases are listed in Table 3.



823

824 **Figure 6.** One-dimensional constant-sea-level-rise transition and post-autobreak verification. (a)
 825 Reference numerical simulation showing the evolution of the sedimentary prism through
 826 progradation, autoretreat, and early post-autobreak retreat. Thin gray lines show reconstructed
 827 sediment surfaces through time, the shaded region shows the final preserved deposit for the
 828 plotted interval, the dashed line shows the basement, and the orange curve shows the diagnostic
 829 shoreline (SH) trajectory. (b) Boundary trajectories for the reference case. Circles show raw
 830 numerical positions of the alluvial-bedrock transition (ABT), shoreline (SH), and toe (TOE),
 831 extracted by interpolation from the fixed-grid solution. Solid lines show the analytical post-
 832 autobreak translating solution for the ABT and SH over the late-time comparison interval; the
 833 TOE is shown as the numerically abandoned offshore position because its final location is
 834 inherited from the earlier progradation-autoretreat history. (c) Sensitivity of the late-time
 835 translating state to the topset threshold C_{top}^x , shown in the moving coordinate $\xi = x + at$. Solid
 836 lines show the analytical post-autobreak ABT and SH positions, and symbols show late-time
 837 numerical medians for the lower-threshold, reference, and higher-threshold cases. Input
 838 parameters are listed in Table 3.

839 5.5 Two-dimensional constant-SLR benchmark demonstrations

840 Finally, we evaluate the full 2D model under constant sea-level rise. The free fan-delta problem
 841 considered here does not have a closed-form counterpart because the ABT, SH, TOE, and lateral
 842 fan margins all evolve as unknown 2D contours. We therefore use this experiment as a
 843 numerical-consistency demonstration rather than an exact analytical verification.



844 The expected late-time behavior follows directly from the 1D post-autobreak result. With $Z(t) =$
 845 at over a planar basement, the active topset should translate landward at speed $U = a$ after
 846 autobreak. We use the same moving coordinate as in 1D, $\xi = x + at$. If the active fan
 847 approaches a steadily translating post-autobreak form, then the ABT and SH contours should
 848 become approximately stationary in ξ ,

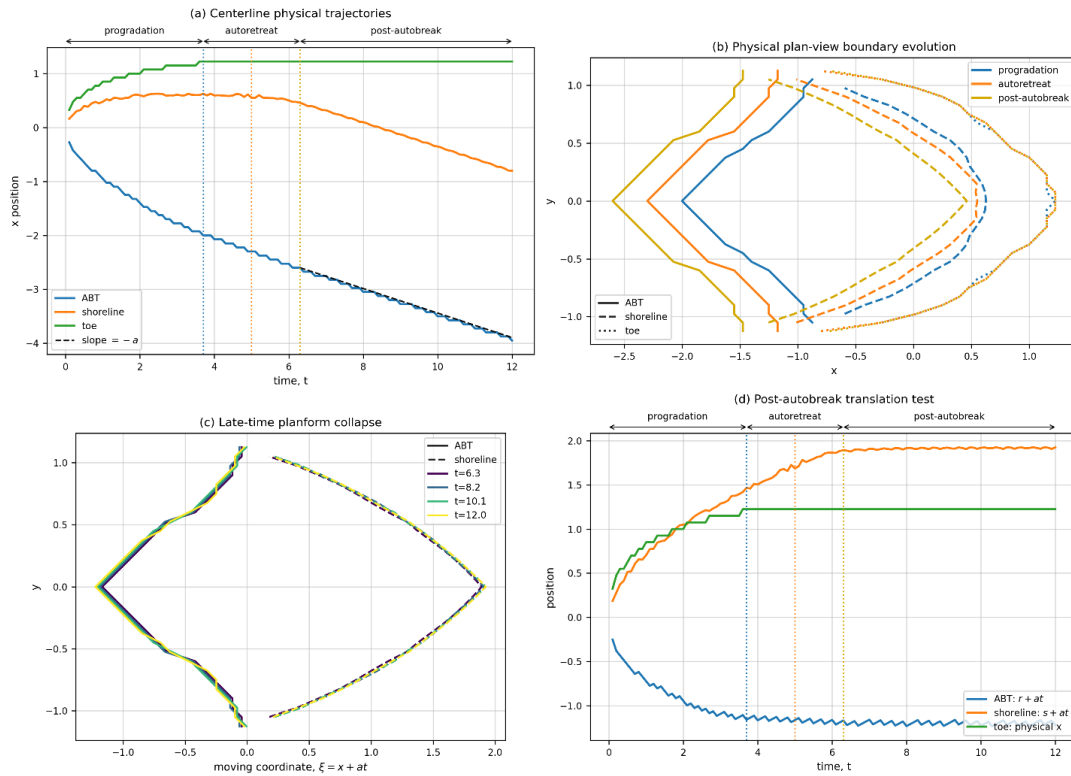
$$849 \quad x_{\text{ABT}}(y, t) + at \rightarrow X_{\text{ABT}}(y), \quad x_{\text{SH}}(y, t) + at \rightarrow X_{\text{SH}}(y). \quad (70)$$

850 The TOE behaves differently because it belongs to the abandoned foreset. After sediment flux to
 851 the foreset ceases, the TOE should remain approximately fixed in physical space,

$$852 \quad x_{\text{TOE}}(y, t) \rightarrow X_{\text{TOE}}(y). \quad (71)$$

853 Figure 7 shows the baseline 2D constant-SLR experiment. Panel (a) shows the physical
 854 centerline trajectories of the ABT, SH, and TOE. The SH and TOE initially prograde; the SH
 855 then retreats while the TOE continues to prograde during autoretreat; and after autobreak the
 856 TOE becomes fixed while the ABT and SH retreat landward with the expected slope $-a$. Panel
 857 (b) shows physical plan-view ABT, SH, and TOE contours during progradation, autoretreat, and
 858 post-autobreak retreat. Panel (c) plots late-time ABT and SH contours in the moving coordinate
 859 $\xi = x + at$; the collapse of successive contours indicates that the active fan approaches a
 860 coherent translating form. Panel (d) shows the centerline translation test based on Eq. (70), with
 861 $x_{\text{ABT}} + at$, $x_{\text{SH}} + at$, and the physical TOE position. The approach to constant late-time values
 862 in panel (d), together with the planform collapse in panel (c), shows that the active topset
 863 translates landward while the abandoned foreset and TOE remain fixed in physical space. The
 864 input parameters for the 2D constant-SLR experiment are listed in Table 4.

865 Together, the analytical comparisons, limiting cases, conservation tests, and 2D constant-SLR
 866 demonstration provide complementary support for the numerical framework. The 1D
 867 experiments verify the implementation where exact solutions are available, while the 2D
 868 experiment shows that the same fixed-grid conservation and diagnostic-boundary logic produces
 869 coherent planform evolution when the moving interfaces become free curves. Direct validation
 870 against laboratory or field observations remains outside the scope of this study.



871

872 **Figure 7.** Two-dimensional constant-sea-level-rise transition and post-autobreak translation. (a)
 873 Physical centerline trajectories of the ABT, SH, and TOE. Phase labels identify progradation,
 874 autoretreat, and post-autobreak retreat. After autobreak, the ABT and SH retreat landward at the
 875 expected beta-normalized speed a , while the TOE remains fixed in physical space. (b) Physical
 876 plan-view ABT, SH, and TOE contours during progradation, autoretreat, and post-autobreak
 877 retreat. (c) Late-time ABT and SH contours in the moving coordinate $\xi = x + at$, showing
 878 collapse toward a translating active-fan geometry. (d) Centerline translation test showing $x_{ABT} +$
 879 at , $x_{SH} + at$, and the physical TOE position. The approach to constant late-time values confirms
 880 post-autobreak translation of the active topset and abandonment of the foreset toe.

881 **Table 3.** Dimensionless input parameter values used in the 1D model-evaluation and model-
 882 exploration experiments. All quantities are nondimensional and beta-normalized following Sect.
 883 4.3, with the single-slope basement written as $E = -x$ after nondimensionalization. Unless
 884 otherwise stated, $D_{top}^x = 1$, $R_v = D_{fore}^x/D_{top}^x = 1$, and the numerical model uses the raw ABT,
 885 SH, and TOE diagnostics described in Sects. 4.7–4.8. For square-root forcing, $Z(t) = 2\lambda_z\sqrt{t}$; for
 886 constant sea-level rise, $Z(t) = at$; for cyclic forcing, $Z(t) = 0$ before t_{spin} and $Z(t) =$
 887 $A\sin[2\pi(t - t_{spin})/T]$ afterward.

Figure	Sea-level forcing	R_{ab}	C_{top}^x	C_{fore}^x
Fig. 4a	$\lambda_z = 0.1$	0.50	0.40	3.00



Fig. 4b	$\lambda_z = -0.1$	0.50	0.40	3.00
Fig. 4c	$\lambda_z = 0$	0.50	0.00	1.25
Fig. 4d	$\lambda_z = 0$	0.50	0.40	3.00
Fig. 5a,b	$\lambda_z = 0.1$	0.20	0.00	200
Fig. 5c,d	$\lambda_z = -0.071$	0.20	0.00	200
Fig. 6a,b	$a = 0.23$	0.6	0.077	4.09
Fig. 6c , lower threshold	$a = 0.23$	0.6	0.023	4.09
Fig. 6c , higher threshold	$a = 0.23$	0.6	0.159	4.09
Fig. 8	$A = 1.10; T = 5;$ $t_{\text{spin}} = 3$	0.26	0.2	2.60

888

889 **Table 4.** Dimensionless input parameter values used for the 2D model-evaluation and model-
 890 exploration experiments. All values are nondimensional and beta-normalized following Sect. 4.3.
 891 All cases use the non-erodible single-slope basement $E = -x$, $D_{\text{top}}^x = D_{\text{top}}^y = 1$, and $D_{\text{fore}}^x =$
 892 $D_{\text{fore}}^y = 1$.

Figure	Z(t)	Q_in	w_f	C_{top}^x	C_{top}^y	C_{fore}^x	C_{fore}^y
Fig. 7	0.227 t	0.636	1.050	0.077	0.386	4.091	4.091
Fig. 9a	0	0.636	1.050	0.077	0.136	4.091	2.045
Fig. 9b	0	0.636	1.050	0.077	0.727	4.091	5.455
Fig. 9c	0	0.636	1.050	0.077	0.386	2.045	4.091
Fig. 9d	0	0.636	1.050	0.077	0.386	5.455	4.091
Fig. 9e	0	0.364	1.050	0.077	0.386	4.091	4.091
Fig. 9f	0	0.818	1.050	0.077	0.386	4.091	4.091
Fig. 9g	0	0.636	0.800	0.077	0.386	4.091	4.091
Fig. 9h	0	0.636	1.600	0.077	0.386	4.091	4.091
Fig. 10a	-0.273 t	0.636	1.050	0.077	0.386	4.091	4.091
Fig. 10b	0	0.636	1.050	0.077	0.386	4.091	4.091
Fig. 10c	0.273 t	0.636	1.050	0.077	0.386	4.091	4.091

893

894 6. Model behavior beyond the analytical benchmarks

895 The previous section evaluated GeoEnthalpy-Delta v1.0 against analytical solutions, limiting
 896 cases, mass-conservation checks, and numerical-consistency tests. Here, we use the model to
 897 examine behavior that is not readily represented by closed-form moving-boundary solutions. We
 898 first consider 1D evolution under repeated sea-level cycles with a finite diffusive foreset. We
 899 then examine how transport thresholds, sediment input, feeder width, and constant sea-level
 900 change affect 2D fan-delta planform morphology.

901 These experiments are controlled model demonstrations rather than validation against
 902 independent field or laboratory observations. Their purpose is to illustrate behavior that emerges



when the ABT, SH, TOE, and lateral fan margins are diagnosed from the evolving sediment-thickness and surface-elevation fields rather than prescribed independently.

6.1 One-dimensional sea-level cycles with a finite foreset

We first examine the response of the 1D model to periodic sea-level forcing (Fig. 8). This experiment extends the enthalpy-based sea-level-cycle simulations of Anderson et al. (2019), which used a vertical or otherwise prescribed foreset geometry, by explicitly resolving transport and storage across a finite foreset and diagnosing the TOE independently of the SH.

The model is first allowed to develop a prograding sedimentary prism under fixed sea level during an initial spin-up period, and then a sinusoidal function as follows:

$$Z(t) = \begin{cases} 0, & t < t_{\text{spin}}, \\ A \sin \left[\frac{2\pi}{T} (t - t_{\text{spin}}) \right], & t \geq t_{\text{spin}}. \end{cases} \quad (72)$$

where A is the nondimensional cycle amplitude, T is the cycle period, and t_{spin} is the duration of the fixed-sea-level initialization. The simulations include multiple complete cycles and terminate before the deposit interacts with the computational boundaries. We distinguish intervals of autoretreat by the simultaneous conditions $ds/dt < 0$ and $du/dt > 0$.

During these intervals, sediment continues to reach the foreset even though the SH is transgressing. This behavior cannot be distinguished in a vertical-foreset model in which the SH and TOE are effectively coincident.

Figure 8a shows the imposed sea-level forcing and the diagnosed boundary trajectories. The SH undergoes repeated reversals, whereas the TOE responds asymmetrically and progrades episodically. The ABT also oscillates, reflecting alternating expansion and landward reworking of the topset. Shaded intervals mark times when SH retreat occurs concurrently with TOE progradation.

Figure 8b shows erosion and deposition during the final cycle using the sediment-thickness rate $\partial H / \partial t$. The migrating ABT, SH, and TOE delimit changing erosional and depositional domains. During portions of the rising limbs, erosion develops on the active topset rather than instantaneous aggradation occurring everywhere. This lagged response is consistent with the behavior identified by Anderson et al. (2019), while the finite-foreset model additionally shows that topset erosion can coincide with continued offshore storage or reworking.

Figure 8c shows the preserved stratigraphic architecture. Time surfaces introduced at quarter-cycle intervals are truncated wherever later erosion lowers the sediment surface. The resulting erosional surfaces and lens-shaped packages record both topset reworking and the finite separation between the SH and TOE.

To quantify sediment partitioning, we calculate topset and foreset sediment volumes as

$$V_{\text{top}}(t) = \int_{r(t)}^{s(t)} H(x, t) dx, \quad (73a)$$



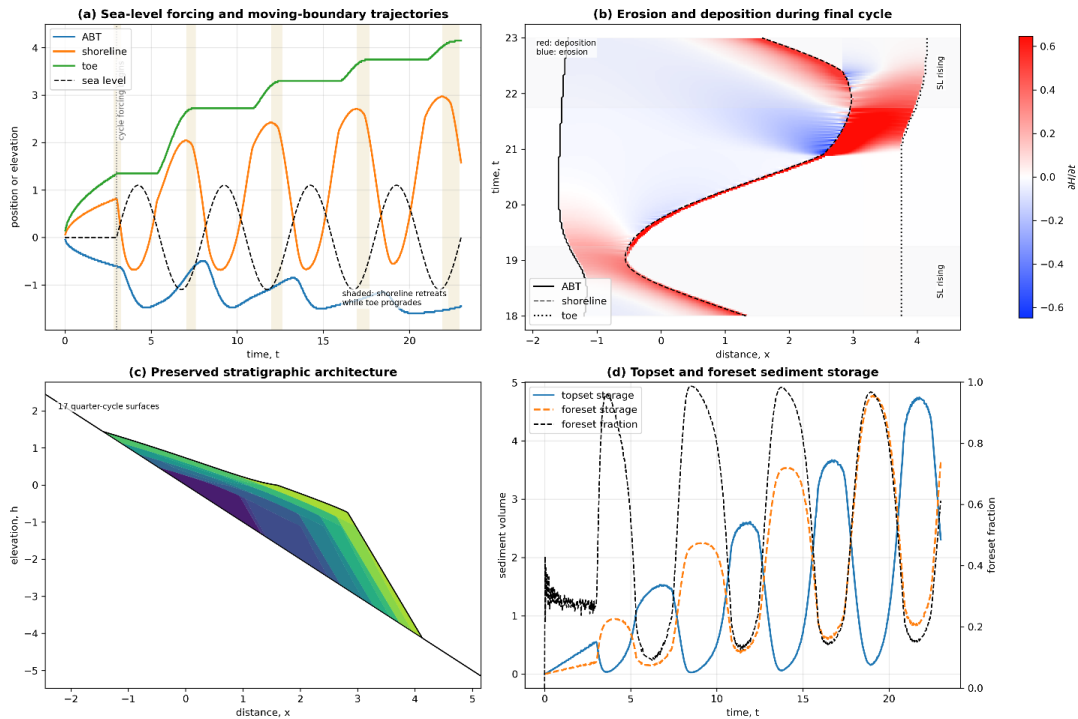
937 and

$$938 \quad V_{\text{fore}}(t) = \int_{s(t)}^{u(t)} H(x, t) dx. \quad (73b)$$

939 and the foreset storage fraction,

$$940 \quad f_{\text{fore}}(t) = \frac{V_{\text{fore}}(t)}{V_{\text{top}}(t) + V_{\text{fore}}(t)}. \quad (74)$$

941 Figure 8d shows that topset and foreset storage oscillate strongly, with their maxima generally
 942 offset in phase. Topset storage decreases during intervals of reworking, whereas foreset storage
 943 increases when sediment bypasses or is redistributed seaward of the SH. The finite-foreset
 944 formulation therefore distinguishes SH migration from cyclic exchange and preservation of
 945 sediment within the subaqueous deposit. Input parameter values are listed in Table 3.



946

947 **Figure 8.** One-dimensional response to cyclic sea-level forcing with a finite diffusive foreset.
 948 (a) Sinusoidal sea-level forcing and diagnostic trajectories of the alluvial-bedrock transition
 949 (ABT), shoreline (SH), and toe (TOE) following an initial fixed-sea-level spin-up. Shaded
 950 intervals identify autoretreat, defined here as SH retreat concurrent with continued TOE
 951 progradation. Boundary velocities used to identify these intervals are estimated from the raw
 952 diagnostic trajectories using a short moving-window average equal to 0.1 cycle periods; the
 953 plotted boundary positions themselves are unsmoothed. (b) Erosion and deposition during the
 954 final sea-level cycle, shown as the sediment-thickness rate $\partial H / \partial t$. Red indicates deposition and
 955 blue indicates erosion. Solid, dashed, and dotted black curves show the ABT, SH, and TOE



trajectories, respectively. Gray bands mark intervals of rising sea level during the final cycle. (c) Preserved stratigraphic architecture reconstructed from quarter-cycle time surfaces and truncated by subsequent erosion. (d) Sediment storage in the topset and foreset through time, together with the foreset fraction $f_{\text{fore}} = V_{\text{fore}}/(V_{\text{top}} + V_{\text{fore}})$. All quantities are nondimensional and beta-normalized as defined in Sect. 4.3; parameter values are listed in Table 3.

6.2 Two-dimensional planform response to transport thresholds, sediment input, feeder geometry, and sea-level change

We next use the 2D model to examine how directional transport thresholds (i.e., $C_{\text{top}}^x, C_{\text{top}}^y, C_{\text{fore}}^x$, and C_{fore}^y), together with sediment input, feeder width, and sea-level change, control fan-delta planform morphology. Figure 9 isolates transport and source controls under fixed sea level. Figure 10 retains the baseline transport and source parameters and varies only the constant rate of sea-level change.

For direct comparison, the simulations in Figs. 9 and 10 are evaluated at the same cumulative sediment input, $V_{\text{target}} = Q_{\text{in}} t_f$. Thus, $t_f = V_{\text{target}}/Q_{\text{in}}$, where Q_{in} is the total sediment input integrated across the feeder. Cases with different sediment supply therefore have different final times, whereas the other paired experiments have the same final time. This design separates geometric responses from trivial differences in supplied sediment volume.

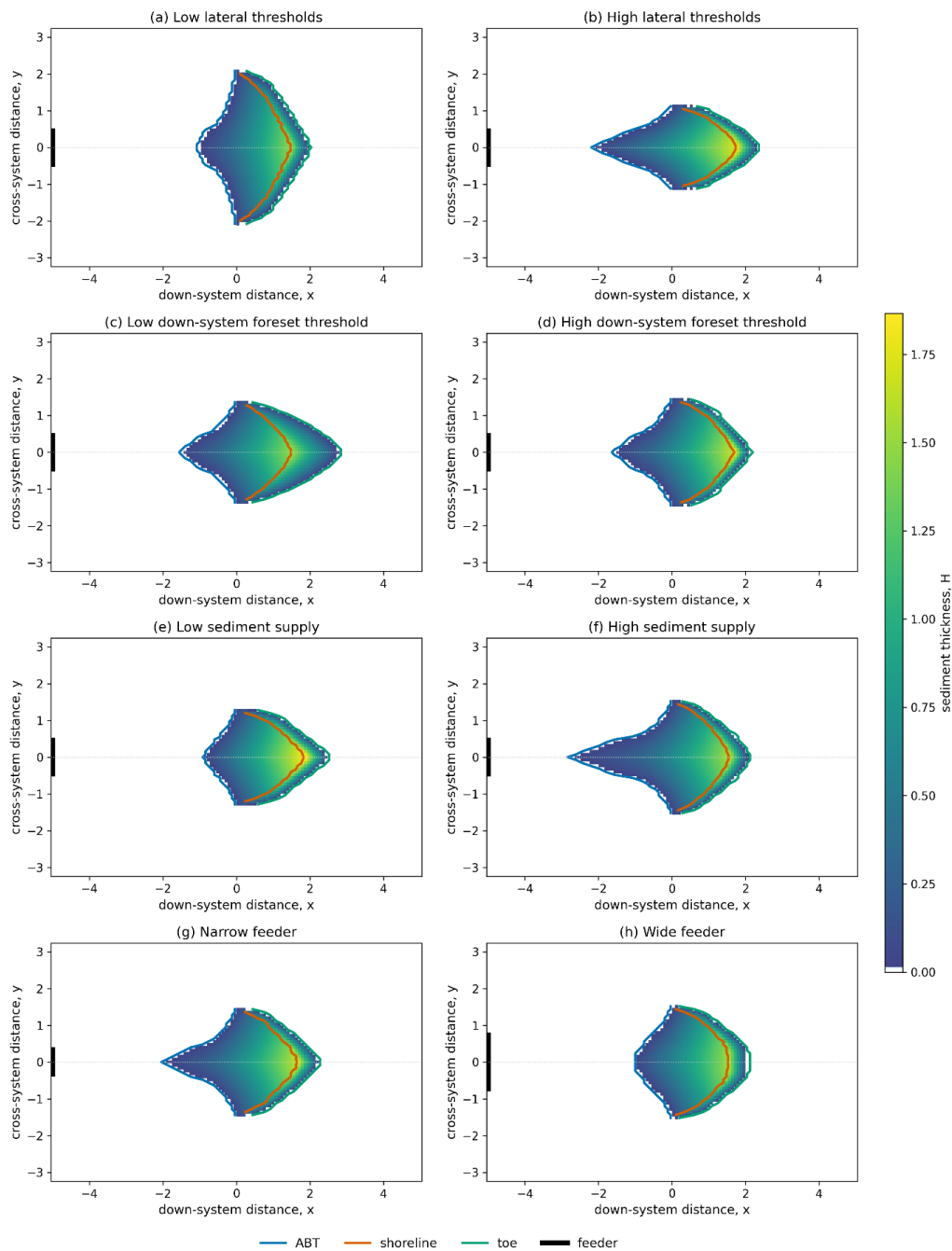
Figure 9 compares four pairs of fixed-sea-level experiments. Panels a-b examine lateral transport thresholds. In the low-threshold case, sediment moves efficiently away from the feeder axis across both the topset and foreset. The resulting deposit is broad and rounded, with gently curved ABT, SH, and TOE contours. Increasing the lateral thresholds suppresses cross-system redistribution and produces a narrower, more elongate deposit. The opening angle is therefore an emergent consequence of the balance between down-system transport, lateral transport resistance, and feeder geometry.

Figure 9c-d isolates the effect of the down-system foreset threshold. When C_{fore}^x is low, sediment moves farther across the submerged portion of the fan, producing diffuse offshore storage and a larger separation between the SH and TOE. Increasing C_{fore}^x requires a steeper foreset gradient for active transport. The foreset consequently becomes more compact, the TOE lies closer to the SH, and the SH-TOE separation decreases. This response is the planform counterpart of the 1D threshold sensitivity discussed above.

Figure 9e-f compares sediment-input rates at equal cumulative sediment volume. Because the low-input case requires a longer run time to reach V_{tar} , sediment has more time to diffuse and redistribute before the comparison time. It therefore develops greater down-system reworking relative to the higher-input case. The high-input case reaches the target volume more rapidly and retains a stronger imprint of focused deposition. These equal-volume experiments show that sediment-input rate affects morphology not only through total mass, but also through the competition between delivery and redistribution timescales.



1000 Figure 9g–h examines feeder width while holding total sediment input fixed. A narrow feeder
1001 concentrates the imposed flux near the fan axis and produces a more pronounced axial lobe. A
1002 wider feeder distributes the same total input over a larger upstream width, producing a broader
1003 fan head and smoother lateral gradients. The difference diminishes away from the source because
1004 lateral transport redistributes sediment after it enters the depositional domain. The persistence of
1005 the feeder-width signal therefore depends on the lateral thresholds: high lateral resistance
1006 preserves source geometry more strongly, whereas low lateral resistance allows the fan to
1007 smooth the imposed input pattern. The input parameter values for the 2D planform experiments
1008 are listed in Table 4.
1009



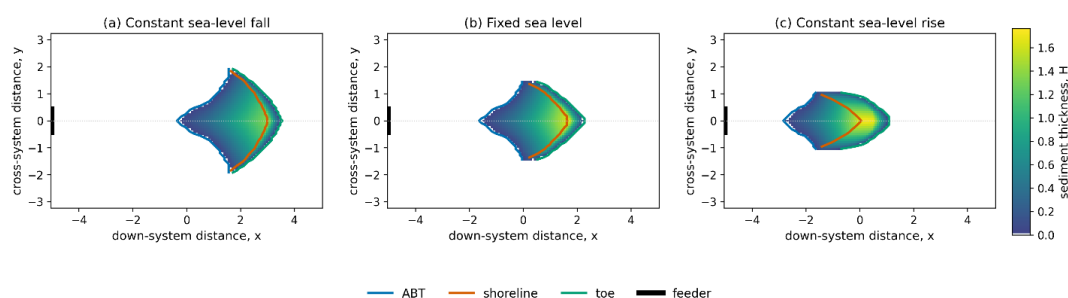
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Figure 9. 2D planform response to transport thresholds, sediment supply, and feeder geometry. Final sediment-thickness fields for eight fixed-sea-level simulations compared at equal



1015 cumulative sediment input V_{target} . Blue, orange, and green curves show the diagnosed alluvial-
1016 bedrock transition (ABT), shoreline (SH), and toe (TOE), respectively; the thick black segment
1017 at the upstream boundary shows the feeder. (a–b) Low and high lateral topset and foreset
1018 thresholds. (c–d) Low and high down-system foreset thresholds. (e–f) Low and high total
1019 sediment-input rates; because V_{target} is fixed, the low-input case has the longer simulation time.
1020 (g–h) Narrow and wide feeders at fixed total sediment input. All panels use a common sediment-
1021 thickness color scale. Input parameter values are listed in Table 4.

1022 We then repeat the baseline 2D experiment under constant sea-level fall, fixed sea level, and
1023 constant sea-level rise while keeping cumulative sediment input fixed. Sea level is prescribed as
1024 $Z(t) = \dot{Z}t$, where $\dot{Z}$ is the nondimensional rate of sea-level change. Figure 10 shows the
1025 resulting planform response. Sea-level fall reduces accommodation and shifts the SH seaward
1026 relative to the fixed-sea-level case. The deposit is broader and more topset dominated, because a
1027 larger fraction of the supplied sediment can remain above sea level or be redistributed across a
1028 shallower subaqueous surface. Fixed sea level produces an intermediate geometry. Sea-level rise
1029 creates accommodation, shifts the SH landward relative to the TOE, increases SH-TOE
1030 separation, and stores a larger fraction of sediment in the foreset. The sea-level-rise case is
1031 therefore more compact in exposed planform but has a larger subaqueous component. These
1032 comparisons show that the same thresholded transport closure can represent both autogenic
1033 planform sensitivity and externally forced changes in accommodation.



1034

1035 **Figure 10.** 2D sedimentary-prism response to constant sea-level change. Final sediment-
1036 thickness fields under (a) constant sea-level fall, (b) fixed sea level, and (c) constant sea-level
1037 rise. All simulations have identical transport parameters, feeder geometry, total sediment-input
1038 rate, and cumulative sediment input; only the sea-level-change rate $\dot{Z}$ differs. Blue, orange, and
1039 green curves show the diagnosed alluvial–bedrock transition (ABT), shoreline (SH), and toe
1040 (TOE), respectively; the thick black segment marks the feeder, and all panels use a common
1041 sediment-thickness color scale. Relative to fixed sea level, sea-level fall shifts the depositional
1042 system seaward, broadens the fan laterally, and reduces SH–TOE separation. Sea-level rise shifts
1043 the ABT and SH landward, narrows the fan laterally, and increases SH–TOE separation. Input
1044 parameter values are listed in Table 4.

1045 Together, Figs. 9 and 10 show that the 2D model separates three classes of controls on fan-delta
1046 morphology. Directional thresholds govern the relative efficiency of longitudinal progradation
1047 and lateral spreading. Source parameters control how sediment is introduced to the domain and
1048 how much time is available for redistribution before a fixed cumulative volume is reached. Sea-
1049 level change alters accommodation and therefore changes the partitioning between topset and



foreset storage. Because the geomorphic boundaries are diagnosed rather than prescribed, these controls are expressed through changes in fan shape, boundary positions, and SH–TOE separation.

7. Discussion

The term enthalpy is used here in a numerical rather than strictly thermodynamic sense. In classical one-phase Stefan formulations, and in earlier vertical-foreset delta applications, storage in the advancing domain can be collapsed into a moving-boundary condition, producing a close analogy between alluvial elevation and temperature and between accommodation filling and latent heat. GeoEnthalpy-Delta instead resolves sediment thickness and transport throughout both the topset and foreset. Therefore, H is not a thermodynamic enthalpy assembled from distinct sensible and latent contributions. The retained enthalpy character is the fixed-grid conservation strategy: sediment volume per unit area is evolved over the full domain, and the moving geomorphic interfaces are recovered by interrogating H , η , and $Z(t)$ (Crank, 1984; Swenson et al., 2000; Voller et al., 2004, 2006; Lorenzo-Trueba and Voller, 2010; Anderson et al., 2019).

The principal contribution of the model is a unified treatment of the ABT, SH, and TOE as outcomes of one sediment-conservation calculation. In 1D, this structure permits direct comparison with analytical moving-boundary solutions. In 2D, the same logic applies when the boundaries and fan margins become unknown curves. The model is therefore best viewed as a bridge between idealized moving-boundary theory and numerical experiments in which finite foreset storage, boundary reversal, and planform geometry emerge from the solution.

7.1 Diagnostic moving boundaries and finite-foreset storage

A central implication is that the SH and TOE are not required to coincide. Vertical-foreset and shoreline-shock formulations have been highly useful for theory, but they collapse subaqueous storage into a prescribed geometry or boundary condition (Swenson et al., 2000; Lorenzo-Trueba et al., 2009; Lorenzo-Trueba and Voller, 2010; Anderson et al., 2019). In GeoEnthalpy-Delta, the SH is diagnosed from $\eta = Z(t)$, whereas the TOE is the seaward boundary of the finite-thickness deposit. The two boundaries can therefore migrate at different rates, reverse at different times, or become dynamically decoupled.

This distinction changes the interpretation of both morphology and stratigraphy. During cyclic sea-level forcing, the SH can retreat while the TOE continues to prograde. The subaerial delta is then transgressive even though sediment continues to accumulate or be reworked offshore. This behavior is consistent with the broader insight that shoreline and subaqueous clinoform motion need not be identical (Swenson et al., 2005), but the present framework resolves their separation through a single evolving thickness field. Consequently, preserved stratigraphy records both shoreline migration and time-dependent sediment partitioning between the topset and foreset.

The distinction also clarifies the transition from autoretreat to autobreak under constant sea-level rise. During progradation and autoretreat, sediment delivery couples SH and TOE motion. After autobreak, flux to the foreset ceases, the active topset translates landward, and the TOE is abandoned in physical space. This connects the finite-foreset numerical model to established theory for autogenic shoreline retreat and autobreak (Muto, 2001; Parker and Muto, 2003; Muto and Swenson, 2005; Swenson and Muto, 2007). In 2D, the same diagnostic approach allows fan



width, SH curvature, SH–TOE separation, and opening angle to emerge without remeshing or prescribed planform kinematics.

7.2 Interpretation of effective transport mobility

The thresholded transport closure should be interpreted at the scale of this reduced-order model. The parameters C_{top}^x , C_{top}^y , C_{fore}^x , and C_{fore}^y are effective mobility parameters rather than direct measurements of a single grain-scale threshold of motion. They aggregate the influence of processes not represented explicitly, including discharge variability, grain size, bedforms, cohesion, vegetation, channelization, flow expansion, and transport intermittency. Likewise, the transport-coefficient ratios describe relative rates of sediment redistribution within the model rather than uniquely measurable geomorphic diffusivities.

The directional parameters provide an anisotropic closure for unresolved sediment routing. The down-system direction represents the dominant feeder-to-basin transport pathway, whereas the cross-system direction represents resistance to lateral redistribution across the topset and foreset. In a process-resolving model, such anisotropy would emerge from the hydrodynamic field, channel geometry, overbank transport, sediment sorting, and avulsion dynamics. Here, it is represented explicitly so that its influence on planform morphology can be isolated in controlled experiments. This use of effective transport closures is consistent with the broader purpose of reduced-order morphodynamic models: to preserve conservation constraints and dominant feedbacks while parameterizing unresolved processes (Paola, 2000; Paola and Voller, 2005; Paola et al., 2011; Edmonds et al., 2022).

The sensitivity experiments should therefore be interpreted comparatively rather than as calibrated predictions. Lower lateral thresholds permit more efficient cross-system redistribution and broader fan growth, whereas higher lateral thresholds preserve a narrower and more elongate feeder imprint. Lower down-system foreset thresholds promote offshore redistribution and greater SH–TOE separation, whereas higher values produce a more compact foreset. Sediment-input rate and feeder width further influence the competition between source delivery and redistribution. These experiments identify the morphological consequences of relative mobility within the chosen closure; site-specific application would require dimensional scaling, independent parameter constraints, and comparison with observations or a more process-based transport formulation.

7.3 Model scope and numerical limitations

GeoEnthalpy-Delta v1.0 is intentionally a reduced-order model. It does not solve for water depth, velocity, turbulence, shear stress, or grain-size-specific transport. In low-gradient deltas, backwater hydrodynamics and discharge variability can substantially alter erosion, deposition, and sediment partitioning relative to a local slope-driven closure (Lamb et al., 2012; Chatanantavet and Lamb, 2014; Chadwick et al., 2019; Edmonds et al., 2022). The model should therefore be viewed as complementary to process-resolving morphodynamic models rather than as a substitute for them.

The present formulation also omits wave-, tide-, and storm-driven redistribution; assumes a prescribed non-erodible basement; and supplies sediment through a fixed upstream flux or feeder. These assumptions exclude alongshore transport, tidal exchange, storm reworking,



evolving shoreface adjustment, subsidence, compaction, sediment loading, catchment–floodplain coupling, avulsion, and distributary-network evolution. Wave-driven shoreface models show that the offshore response is depth- and timescale-dependent (Lorenzo-Trueba and Ashton, 2014; Ortiz and Ashton, 2016), while distributary-network models demonstrate that channel geometry, discharge, and sediment-transport nonlinearity can strongly influence routing and progradation (Ke et al., 2019). The conclusions here consequently apply to idealized deltas whose long-term behavior can be represented by effective slope-driven redistribution over a fixed substrate.

There are also numerical limits to the diagnostic-boundary approach. The ABT and TOE depend on the thickness tolerance ε_H , and the SH is extracted from $\eta = Z(t)$ within the feeder-associated connected component of the deposit. The connected-component restriction changes only the diagnostic interpretation: detached sediment, if present, remains in H and in the sediment budget but is excluded from the active-boundary metrics. Boundary positions are accurate only to the scale permitted by the grid and interpolation, with the greatest sensitivity occurring when the SH–TOE separation approaches the cell size. The explicit finite-volume scheme also requires a stable time step and a domain large enough that the deposit does not interact with artificial downstream or lateral boundaries. These requirements are numerical controls rather than additional geomorphic parameters.

The tests presented here establish conservation, implementation consistency, and agreement with available analytical solutions. They do not constitute validation against independent field or laboratory observations. Applying the framework to a specific delta would require dimensional scaling, parameter estimation, sensitivity analysis, and comparison with observed boundary trajectories, morphology, or stratigraphy.

7.4 Future model development

The most direct process extension is to replace or augment the present flux law while retaining the fixed-grid boundary-diagnostic structure. Nonlinear transport, variable and intermittent discharge, and backwater effects could be incorporated on the fluvial side. Wave-, tide-, and storm-driven terms could represent cross-shore adjustment, alongshore redistribution, and exchange across the SH. Reduced-order dynamic-shoreface formulations provide one possible starting point for linking wave-driven transport to a time-dependent TOE or morphodynamic depth of closure (Lorenzo-Trueba and Ashton, 2014; Ortiz and Ashton, 2016). Each extension would require new benchmarks because modifying the flux closure may change the analytical regime structure.

A second priority is to improve sediment sourcing, routing, and accommodation. Erodeable upstream reaches, mobile feeder locations, avulsion rules, and partitioning among distributaries would allow feedbacks between channel dynamics and the evolving sedimentary prism. Coupling to a moving-boundary representation of distributary-network growth offers one possible route (Ke et al., 2019). Likewise, a time-dependent basement or externally supplied vertical-displacement field could represent uplift, subsidence, compaction, or sediment loading (Paola and Voller, 2005).

Numerically, a control-volume finite-element formulation could retain local conservation while allowing unstructured meshes that conform to irregular coastlines, feeders, basin boundaries, and



basement topography. The CVFEM approach used for critical-slope depositional flows by Chen et al. (2023) provides a relevant precedent, although the donor-based regime switch, positivity limiter, and feeder-associated connectivity diagnostics would all need to be generalized to an unstructured control-volume graph.

8. Conclusions

We present GeoEnthalpy-Delta v1.0, an enthalpy-based model for coupled subaerial and subaqueous delta evolution with diagnostic moving boundaries. The model evolves sediment thickness H on a fixed grid, reconstructs the sediment surface $\eta = E + H$, and computes sediment redistribution using a thresholded, slope-driven transport law. The alluvial–bedrock transition, SH and TOE are not tracked explicitly as computational boundaries. Instead, they are diagnosed from the evolving sediment-thickness field, reconstructed surface, and prescribed sea-level history.

The main contribution of the model is a unified fixed-grid framework for one-, two-, and three-moving-boundary delta problems. In the finite-foreset configuration, the SH and TOE are distinct boundaries, allowing offshore sediment storage, TOE migration, TOE abandonment, and shoreline reversal to be represented explicitly. The same numerical structure extends naturally to two dimensions, where the ABT, SH, TOE, and lateral fan margins emerge as evolving contours rather than prescribed curves.

The 1D implementation is verified against analytical benchmarks where exact comparisons are available. These include the closed-form three-moving-boundary solution for fixed sea level and square-root sea-level histories within the active threshold-transport regime, limiting vertical-foreset and graded-river cases, and the post-autobreak translating solution under constant sea-level rise. These tests show that the numerical model reproduces the expected ABT, SH, and TOE trajectories when the assumptions of the analytical solutions are satisfied. Mass-conservation and grid-consistency checks provide additional support for the finite-volume implementation.

The 2D implementation is evaluated through controlled benchmark demonstrations rather than exact analytical verification, because the free fan-delta problem does not have a closed-form solution when the SH, TOE, fan margins, and opening angle are all unknown. Under constant sea-level rise, the model produces the expected transition from progradation to autoretreat and post-autobreak retreat: the active topset translates landward in the moving coordinate, while the abandoned TOE remains approximately fixed in physical space. Fixed-sea-level planform experiments show that lateral thresholds, down-system foreset thresholds, sediment-input rate, and feeder width control fan width, SH-TOE separation, source imprint, and sediment partitioning.

The model-exploration experiments demonstrate behaviors that are difficult to capture with analytical or vertical-foreset models alone. In one dimension, cyclic sea-level forcing can produce SH retreat while the TOE continues to prograde, showing that SH motion and offshore sediment storage need not be equivalent. In two dimensions, the diagnostic-boundary framework allows fan shape, opening angle, SH curvature, and TOE position to emerge from sediment conservation and effective transport mobility rather than from prescribed geometry.

GeoEnthalpy-Delta v1.0 is intentionally idealized. It does not include backwater hydrodynamics, waves, tides, storms, grain-size sorting, vegetation, compaction, subsidence, erodible basement, or distributary-channel dynamics. These limitations define clear directions for future development. The value of the present version is that it provides a transparent, conservative, and



reproducible baseline for testing moving-boundary delta theory and for building future models that incorporate more detailed hydrodynamic, coastal, sedimentological, and basin-evolution processes.

Code and data availability. The exact version of GeoEnthalpy-Delta used to produce the results presented in this manuscript (software release v1.0.0) is archived on Zenodo at <https://doi.org/10.5281/zenodo.21435222> (Lorenzo-Trueba et al., 2026). The development repository is available at <https://github.com/GeoJorge/GeoEnthalpy-Delta/tree/main>. The archived release contains the shared 1D and 2D MATLAB and Python model cores, scripts used to reproduce Figs. 4–10, dependency and license files, user documentation, and a compact 1D pedagogical example. No specialized hardware is required; the tested MATLAB and Python versions and the Python package dependencies are documented in the archived release. All simulations presented here are idealized and require no external observational data. The software is distributed under the MIT License.

Appendix A. Closed-form three-moving-boundary similarity solution

This appendix gives the 1D active-branch analytical solution used for verification in Sect. 5.2 and for the limiting cases discussed in Sect. 5.3. The solution applies when the positive-part operator in the threshold transport law is inactive throughout the active topset and foreset. The resulting profiles are therefore exact solutions of the thresholded problem only if the flux-admissibility conditions at the end of the appendix are satisfied.

Unless otherwise stated, Appendix A is written in the beta-normalized nondimensional variables introduced in Sect. 4.3. Thus, the planar basement used for the verification cases is $E = -x$, the imposed upstream flux is R_{ab} , the threshold slopes $C_T = C_{top}^x$ and $C_F = C_{fore}^x$ denote beta-normalized topset and foreset thresholds, and $R_v = D_{fore}^x/D_{top}^x$. The topset transport coefficient is the reference coefficient used in the time scaling and is therefore $D_{top}^x = 1$. For the single-slope cases considered in the manuscript, the subaqueous basement slope in nondimensional variables is one.

A1. Active equations and boundary conditions

The prescribed β -normalized basement is

$$E(x) = -x. \quad (A1)$$

The active, unclipped transport laws are

$$q_T = -\frac{\partial \eta_T}{\partial x} - C_T, \quad (A2)$$

$$q_F = R_v \left(-\frac{\partial \eta_F}{\partial x} - C_F \right). \quad (A3)$$

Because the basement is fixed, the reconstructed surface satisfies diffusion equations in the active topset and foreset:

$$\frac{\partial \eta_T}{\partial t} = \frac{\partial^2 \eta_T}{\partial x^2}, \quad r(t) < x < s(t), \quad (A4)$$

$$\frac{\partial \eta_F}{\partial t} = R_v \frac{\partial^2 \eta_F}{\partial x^2}, \quad s(t) < x < u(t). \quad (A5)$$



1270 For fixed sea level and square-root sea-level histories, the forcing and moving boundaries are
 1271 written as

$$1272 \quad Z(t) = 2\lambda_z\sqrt{t}, \quad (\text{A6})$$

$$1273 \quad r(t) = -2\lambda_{ab}\sqrt{t}, \quad (\text{A7})$$

$$1274 \quad s(t) = 2\lambda_{sh}\sqrt{t}, \quad (\text{A8})$$

$$1275 \quad u(t) = 2\lambda_u\sqrt{t}. \quad (\text{A9})$$

1276 The physical branch considered here satisfies $\lambda_{ab} \geq 0$ and $\lambda_u > \lambda_{sh}$. The boundary conditions
 1277 are sediment-thickness closure at the ABT and toe, shoreline elevation continuity, imposed
 1278 upstream sediment flux, shoreline flux continuity, and zero foreset flux at the toe:

$$1279 \quad \eta_T(r, t) = -r(t), \quad (\text{A10})$$

$$1280 \quad \eta_T(s, t) = Z(t), \quad (\text{A11})$$

$$1281 \quad q_T(r, t) = R_{ab}, \quad (\text{A12})$$

$$1282 \quad \eta_F(s, t) = Z(t), \quad (\text{A13})$$

$$1283 \quad q_T(s, t) = q_F(s, t), \quad (\text{A14})$$

$$1284 \quad q_F(u, t) = 0, \quad (\text{A15})$$

$$1285 \quad \eta_F(u, t) = -u(t). \quad (\text{A16})$$

1286 Equation (A16) is the toe-elevation condition: the foreset toe is a finite-thickness boundary, so
 1287 the reconstructed surface must re-intersect the prescribed basement there.

1288 A2. Similarity reduction

1289 For the topset, define

$$1290 \quad \xi = \frac{x}{2\sqrt{t}} \quad (\text{A17})$$

$$1291 \quad \eta_T(x, t) = 2\sqrt{t} \Phi_T(\xi). \quad (\text{A18})$$

1292 For the foreset, use the diffusivity-scaled coordinate

$$1293 \quad \mu = \frac{x}{2\sqrt{R_v t}}, \quad (\text{A19})$$

$$1294 \quad \eta_F(x, t) = 2\sqrt{R_v t} \Phi_F(\mu). \quad (\text{A20})$$

1295 Both similarity profiles satisfy the same ordinary differential equation, with χ denoting either ξ
 1296 or μ :

$$1297 \quad \frac{d^2\Phi}{d\chi^2} + 2\chi \frac{d\Phi}{d\chi} - 2\Phi = 0. \quad (\text{A21})$$

1298 A convenient basis function is

$$1299 \quad F(\chi) = \exp(-\chi^2) + \sqrt{\pi} \chi \operatorname{erf}(\chi), \quad (\text{A22})$$

$$1300 \quad F'(\chi) = \sqrt{\pi} \operatorname{erf}(\chi). \quad (\text{A23})$$

1301 where erf denotes the error function.



1302 A3. Topset profile

1303 The general topset solution is a linear combination of the basis function $F(\xi)$ and the coordinate
 1304 ξ . Applying the imposed-flux condition at the ABT gives the following compact definitions:

$$1305 \quad Q_T = R_{ab} + C_T, \quad (A24)$$

$$1306 \quad G_T = \text{erf}(\lambda_{ab}) + \text{erf}(\lambda_{sh}), \quad (A25)$$

$$1307 \quad \Delta_T = \exp(-\lambda_{sh}^2) + \sqrt{\pi} \lambda_{sh} G_T, \quad (A26)$$

$$1308 \quad A_T = \frac{\lambda_Z + Q_T \lambda_{sh}}{\Delta_T}, \quad (A27)$$

$$1309 \quad B_T = A_T \sqrt{\pi} \text{erf}(\lambda_{ab}) - Q_T. \quad (A28)$$

1310 The topset similarity profile can then be written as

$$1311 \quad \Phi_T(\xi) = A_T [\exp(-\xi^2) + \sqrt{\pi} \xi \{\text{erf}(\xi) + \text{erf}(\lambda_{ab})\}] - Q_T \xi. \quad (A29)$$

1312 The remaining ABT elevation condition gives the first algebraic equation:

$$1313 \quad A_T \exp(-\lambda_{ab}^2) = \lambda_{ab} (1 - Q_T). \quad (A30)$$

1314 A4. Foreset profile

1315 The shoreline and toe coordinates in the foreset similarity variable are

$$1316 \quad \mu_s = \frac{\lambda_{sh}}{\sqrt{R_v}}, \quad (A31)$$

$$1317 \quad \mu_u = \frac{\lambda_u}{\sqrt{R_v}}. \quad (A32)$$

1318 The general foreset solution is again a linear combination of the basis function and the
 1319 coordinate μ . The zero-flux condition at the toe determines the coefficient multiplying μ , and the
 1320 shoreline elevation condition gives

$$1321 \quad \Delta_F = \exp(-\mu_s^2) + \sqrt{\pi} \mu_s \{\text{erf}(\mu_s) - \text{erf}(\mu_u)\}, \quad (A33)$$

$$1322 \quad A_F = \frac{\lambda_Z + C_F \lambda_{sh}}{\sqrt{R_v} \Delta_F}. \quad (A34)$$

1323 The foreset similarity profile is therefore

$$1324 \quad \Phi_F(\mu) = A_F [\exp(-\mu^2) + \sqrt{\pi} \mu \{\text{erf}(\mu) - \text{erf}(\mu_u)\}] - C_F \mu. \quad (A35)$$

1325 Applying the toe-elevation condition, Eq. (A16), gives the third algebraic equation:

$$1326 \quad \frac{(\lambda_Z + C_F \lambda_{sh}) \exp(-\mu_u^2)}{\Delta_F} = (C_F - 1) \lambda_u. \quad (A36)$$

1327 Thus a finite toe in this active-branch solution requires the foreset threshold to exceed the
 1328 basement slope in the relevant nondimensional units, $C_F > 1$, for the usual case in which the left-
 1329 hand side of Eq. (A36) is positive.

1330 A5. Shoreline flux continuity and final nonlinear system

1331 The topset and foreset fluxes evaluated at the shoreline are

$$1332 \quad q_T(s) = R_{ab} - A_T \sqrt{\pi} G_T, \quad (A37)$$



$$q_F(s) = R_v A_F \sqrt{\pi} \{\text{erf}(\mu_u) - \text{erf}(\mu_s)\}. \quad (\text{A38})$$

Flux continuity gives the second algebraic equation:

$$R_{ab} - A_T \sqrt{\pi} G_T = R_v A_F \sqrt{\pi} \{\text{erf}(\mu_u) - \text{erf}(\mu_s)\}. \quad (\text{A39})$$

The three unknown similarity constants are obtained by solving Eqs. (A30), (A39), and (A36), with the auxiliary quantities defined above. Once these constants are known, the surface profiles in the original nondimensional variables are

$$\eta_T(x, t) = 2\sqrt{t} \Phi_T \left(\frac{x}{2\sqrt{t}} \right), \quad (\text{A40})$$

$$\eta_F(x, t) = 2\sqrt{R_v t} \Phi_F \left(\frac{x}{2\sqrt{R_v t}} \right). \quad (\text{A41})$$

A6. Flux-admissibility conditions and limiting cases

The active analytical branch is the physical solution of the clipped threshold-transport problem only if the computed fluxes remain non-negative across the active topset and foreset. The fluxes are

$$q_T(\xi) = R_{ab} - A_T \sqrt{\pi} \{\text{erf}(\xi) + \text{erf}(\lambda_{ab})\}, \quad (\text{A42})$$

$$q_F(\mu) = R_v A_F \sqrt{\pi} \{\text{erf}(\mu_u) - \text{erf}(\mu)\}. \quad (\text{A43})$$

Therefore the admissibility conditions are

$$q_T(\xi) \geq 0 \quad \text{for} \quad -\lambda_{ab} \leq \xi \leq \lambda_{sh}, \quad (\text{A44})$$

$$q_F(\mu) \geq 0 \quad \text{for} \quad \mu_s \leq \mu \leq \mu_u. \quad (\text{A45})$$

If either condition is violated, the algebraic solution remains a mathematical solution of the unclipped equations but is not the solution of the thresholded numerical model, because the numerical model sets negative candidate fluxes to zero.

The large-foreset-threshold limit recovers the vertical-foreset behavior asymptotically. Let $C_F = C$ and take C to be large while holding the other parameters fixed. The toe approaches the shoreline, but the separation must not be set to zero before the limit is taken. From Eq. (A36), the leading-order separation is

$$\lambda_u - \lambda_{sh} \sim \frac{\lambda_z + \lambda_{sh}}{C-1}. \quad (\text{A46})$$

Therefore

$$C(\lambda_u - \lambda_{sh}) \rightarrow \lambda_z + \lambda_{sh}. \quad (\text{A47})$$

Eq. (A46) reduces to the form used in Sect. 5.3. The finite product in Eq. (A47) is the storage term that appears in the vertical-foreset Stefan condition. Directly setting $\lambda_u = \lambda_{sh}$ inside the three algebraic equations would remove this finite contribution and would not recover the correct limiting problem.

The fixed-ABT, graded-river case used in the limiting comparison is recovered from the vertical-foreset, no-topset-threshold limit by setting $C_T = 0$ and $\lambda_{ab} = 0$. The required square-root sea-level-fall coefficient is



$$\lambda_Z = -\frac{1}{2} \left(\frac{2R_{ab}^3}{1-R_{ab}} \right)^{1/2}. \quad (\text{A48})$$

The corresponding shoreline similarity constant is $\lambda_{sh} = -\lambda_Z/R_{ab}$, with $0 < R_{ab} < 1$.

Appendix B. Post-autobreak translating solution for constant sea-level rise

This appendix gives the 1D post-autobreak solution used for the constant-sea-level-rise verification in Sect. 5.4. The solution describes the active topset after sediment flux to the foreset has ceased. It does not determine the abandoned toe position; that position is inherited from the preceding progradation and autoretreat history. The derivation applies to the verification cases considered in Sect. 5.4, for which $D_{top}^x = 1$.

B1. Moving coordinate and active-topset equation

Let sea level rise at a constant nondimensional rate

$$Z(t) = at. \quad (\text{B1})$$

For a planar basement $E(x) = -x$, the sea-level intersection with the bare basement migrates landward at speed

$$U = a. \quad (\text{B2})$$

We use the moving coordinate and relative surface elevation

$$\xi = x + Ut, \quad (\text{B3})$$

$$\psi(\xi) = \eta(x, t) - Z(t). \quad (\text{B4})$$

In the translating post-autobreak state, $\eta(x, t) = at + \psi(\xi)$. Substitution into the active topset diffusion equation gives

$$\frac{d^2\psi}{d\xi^2} - U \frac{d\psi}{d\xi} - a = 0. \quad (\text{B5})$$

B2. Boundary conditions

Let ξ_r and ξ_s denote the fixed ABT and shoreline positions in the moving coordinate. The shoreline is the zero relative-elevation contour and the post-autobreak shoreline flux is zero:

$$\psi(\xi_s) = 0, \quad (\text{B6})$$

$$-\frac{d\psi}{d\xi}(\xi_s) - C_T = 0. \quad (\text{B7})$$

At the ABT, sediment thickness is zero and the imposed upstream flux is R_{ab} :

$$\psi(\xi_r) = -\xi_r, \quad (\text{B8})$$

$$-\frac{d\psi}{d\xi}(\xi_r) - C_T = R_{ab}. \quad (\text{B9})$$

B3. Profile and boundary locations

Solving Eq. (B5) and applying Eqs. (B6) and (B7) gives the relative surface profile

$$\psi(\xi) = -(\xi - \xi_s) + \frac{1-C_T}{U} [\exp\{U(\xi - \xi_s)\} - 1]. \quad (\text{B10})$$



1398 The derivative is

$$1399 \quad \frac{d\psi}{d\xi} = -1 + (1 - C_T)\exp\{U(\xi - \xi_s)\}. \quad (B11)$$

1400 The ABT flux condition, Eq. (B9), gives

$$1401 \quad \exp\{U(\xi_r - \xi_s)\} = 1 - \frac{R_{ab}}{1 - C_T}. \quad (B12)$$

1402 The ABT elevation condition, Eq. (B8), then gives the shoreline position

$$1403 \quad \xi_s = \frac{R_{ab}}{a}. \quad (B13)$$

1404 Combining Eqs. (B12) and (B13) gives

$$1405 \quad \xi_r = \frac{R_{ab}}{a} + \frac{1}{a} \ln \left(1 - \frac{R_{ab}}{1 - C_T} \right). \quad (B14)$$

1406 The active topset length in the moving coordinate is therefore

$$1407 \quad L_T = \xi_s - \xi_r = -\frac{1}{a} \ln \left(1 - \frac{R_{ab}}{1 - C_T} \right). \quad (B15)$$

1408 The reconstructed surface and sediment thickness are

$$1409 \quad \eta(x, t) = at + \psi(x + at), \quad (B16)$$

$$1410 \quad H(x, t) = \eta(x, t) + x. \quad (B17)$$

1411

1412 B4. Physical-coordinate trajectories and admissibility

1413 Transforming the fixed moving-coordinate positions back to physical coordinates gives

$$1414 \quad x_{ABT}(t) = \xi_r - at, \quad (B18)$$

$$1415 \quad x_{SH}(t) = \xi_s - at. \quad (B19)$$

1416 The toe is abandoned after autobreak and is not part of the active translating topset solution:

$$1417 \quad x_{TOE}(t) = x_{TOE,AB}. \quad (B20)$$

1418 The logarithm and the zero-shoreline-flux condition require

$$1419 \quad 0 < R_{ab} < 1 - C_T, \quad C_T < 1, \quad a > 0. \quad (B21)$$

1420 In the special case $C_T = 0$ the moving-coordinate boundary positions reduce to

$$1421 \quad \xi_s = \frac{R_{ab}}{a}, \quad (B22)$$

$$1422 \quad \xi_r = \frac{R_{ab} + \ln(1 - R_{ab})}{a}. \quad (B23)$$

1423 Equations (B18)–(B20) are the analytical trajectories used in the main text: after autobreak, the
 1424 ABT and shoreline translate landward with slope $-a$ in physical coordinates, while the
 1425 abandoned toe remains fixed at $x_{TOE,AB}$.

1426

1427 **Author contribution**



JLT designed the study, developed the model formulation, prepared the analytical and numerical experiments, generated the manuscript figures, and wrote the manuscript draft. WA contributed to the analytical moving-boundary solutions, numerical verification strategy, and interpretation of the enthalpy formulation. VB contributed to the development and testing of the 2D numerical implementation and figure-reproduction workflow. VV contributed to the conceptual development of the enthalpy approach, numerical-method interpretation, and manuscript review. All authors discussed the results and contributed to revision of the manuscript.

Competing interests

The authors declare that they have no conflict of interest.

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