Supplementary Material

Additional comparison with the BO-informed MPPI baseline

This supplementary material provides additional details on the BO-informed MPPI baseline introduced in our response to Referee Comment 2 and summarizes the corresponding Lorenz–96 results. Throughout this document, we refer to this baseline as **BO-MPPI**.

Experimental setup

For each fixed value of the MPPI temperature parameter λ, BO-MPPI is run with

$$\alpha \in \{100,\, 10,\, 1,\, 0.1,\, 0.01\},$$

where α is the BO-side trade-off parameter that balances the predicted threshold exceedance over the rollout horizon against the intervention magnitude (see below). On the other hand, λ is the MPPI temperature parameter. The two parameters therefore play different roles and are varied separately.

To ensure a fair comparison with EKG-MPPI, BO-MPPI uses the same value of λ and the same sampling variances for intervention location and intervention magnitude as EKG-MPPI. In the BO stage, the *Expected Improvement* (EI) acquisition function is used, and the BO budget is fixed at 7 iterations at each control decision. This budget was selected because even 7 BO iterations already lead to a substantially larger per-decision computational cost than that of the proposed method.

The objective function used in Bayesian optimization evaluates both the magnitude of extreme events and the intervention cost. Let x_t denote the system state at time step t, u_t the intervention applied at time t, and r_{th} the threshold for extreme events. The system dynamics are represented by $\text{Model}(\cdot)$, and the prediction horizon is denoted by H. The cost function is defined as

$$c(x_t, u_t) = \sum_{i=1}^{H} \sum_{j=1}^{N} \max\left(x_{t+i}^{(j)} - r_{\text{th}}, 0\right) + \alpha \|u_t\|, \tag{S1}$$

where the state trajectory is obtained by propagating the model dynamics as

$$x_{t+1} = \text{Model}(x_t + u_t), \qquad x_{t+i} = \text{Model}(x_{t+i-1}) \quad (i = 2, \ldots, H). \tag{S2}$$

The first term measures the total exceedance above the threshold during the prediction horizon, while the second term penalizes the magnitude of the intervention. The parameter α controls the trade-off between intervention effectiveness and intervention magnitude.

Rescaling used for visualization

To facilitate visual comparison in the two-dimensional performance–control plots, we rescale both the average intervention magnitude and the threshold-exceedance count (the horizontal-axis performance metric) by the corresponding mean over the five BO-MPPI runs for the same fixed value of λ. This rescaling is used only for visualization. Because each quantity is divided by a

reference mean rather than mapped to the interval $[0, 1]$, we refer to the resulting quantities as *rescaled* (or *mean-scaled*) values rather than normalized values. All qualitative comparisons reported in the text are unchanged when the original, unscaled metrics are used.

For each fixed λ, let $\mathcal{A} = \{100, 10, 1, 0.1, 0.01\}$. Let m_{EKG} and m_α denote the average intervention magnitudes obtained by EKG-MPPI and BO-MPPI, respectively. We define the BO-MPPI reference mean by

$$\bar{m}_{\text{BO}} = \frac{1}{5} \sum_{\alpha \in \mathcal{A}} m_\alpha, \tag{S3}$$

and the corresponding rescaled quantities by

$$\tilde{m}_{\text{EKG}} = \frac{m_{\text{EKG}}}{\bar{m}_{\text{BO}}}, \qquad \tilde{m}_\alpha = \frac{m_\alpha}{\bar{m}_{\text{BO}}}. \tag{S4}$$

Similarly, let e_{EKG} and e_α denote the threshold-exceedance counts used on the horizontal axis. We define

$$\bar{e}_{\text{BO}} = \frac{1}{5} \sum_{\alpha \in \mathcal{A}} e_\alpha, \tag{S5}$$

and

$$\tilde{e}_{\text{EKG}} = \frac{e_{\text{EKG}}}{\bar{e}_{\text{BO}}}, \qquad \tilde{e}_\alpha = \frac{e_\alpha}{\bar{e}_{\text{BO}}}. \tag{S6}$$

Values below 1 therefore indicates performance better than the BO-MPPI mean for the same λ on the corresponding axis. Smaller horizontal values indicate fewer threshold exceedances, and smaller vertical values indicate a smaller average intervention magnitude. Since each axis is rescaled by a common positive constant within each panel, this transformation does not change the within-panel ordering or Pareto dominance relations; it only changes the axis units for visualization.

Results

Figure S1 compares EKG-MPPI and BO-MPPI for $\lambda \in \{0.1, 1.0, 10.0\}$. For each tested value of λ, the EKG-MPPI result lies below the empirical lower envelope (Pareto frontier-like trade-off curve) of the BO-MPPI results obtained by varying α. In other words, EKG-MPPI achieves a more favorable trade-off between the intervention-effectiveness metric on the horizontal axis and the average intervention magnitude on the vertical axis. Although EKG-MPPI is not uniformly located in the lower-left of all BO-MPPI points, no tested BO-MPPI configuration achieves both a more favorable intervention-effectiveness value and a smaller intervention magnitude than EKG-MPPI. This is consistent with the conclusion in our response to Referee Comment 2 that the advantage of EKG-MPPI is robust to the choice of λ and is not attributable to a single hyperparameter setting.
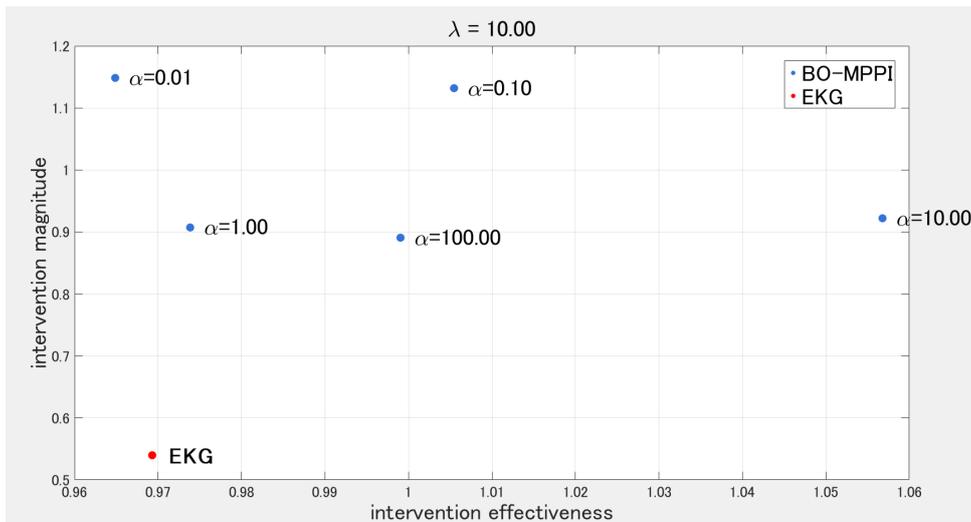
The mean computational time per decision step was $5.9953 \times 10^{-2}\,\text{s}$ for BO-MPPI (variance: $2.3513 \times 10^{-5}\,\text{s}^2$) and $2.80 \times 10^{-4}\,\text{s}$ for EKG-MPPI (variance: $3.4 \times 10^{-9}\,\text{s}^2$). Note that, since the sampling interval is $0.05\,\text{s}$, BO-MPPI slightly exceeds the available real-time budget on average, whereas EKG-MPPI remains well within this constraint.
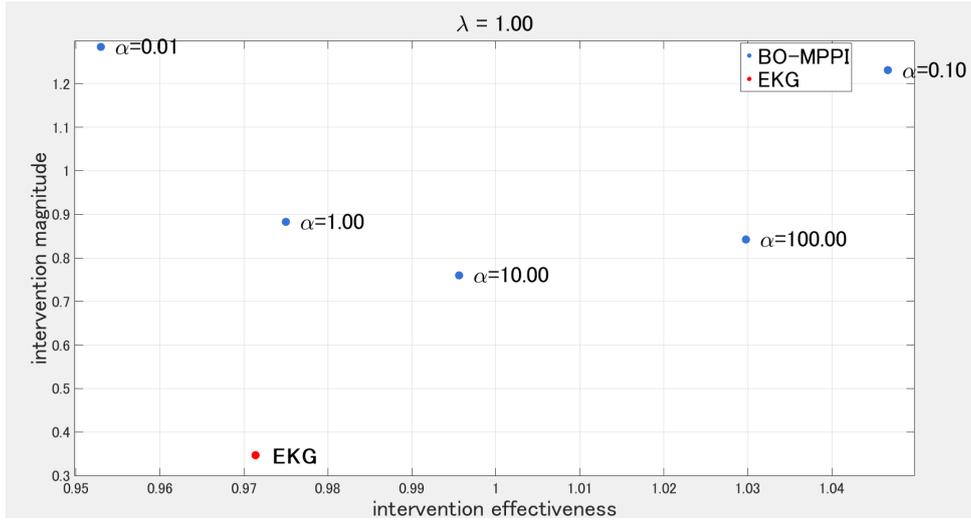
(a) $\lambda = 0.1$
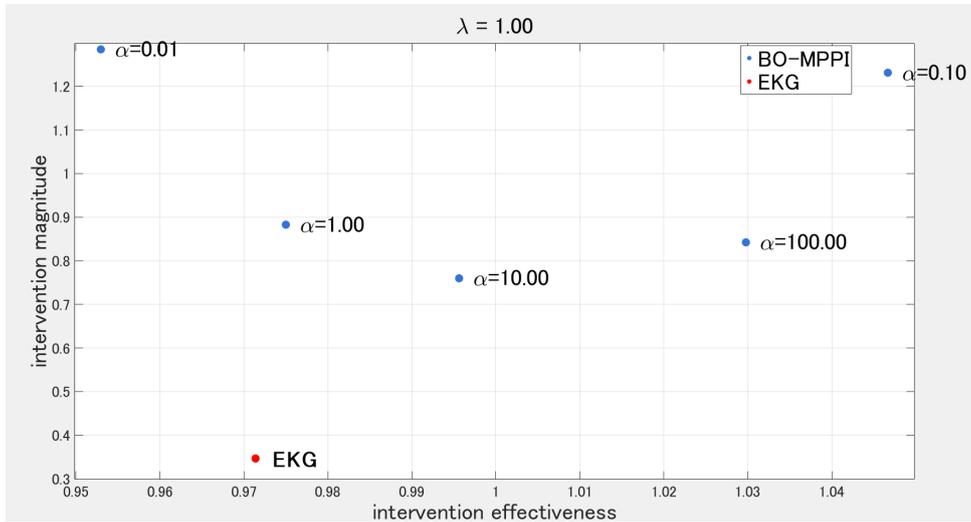
(b) $\lambda = 1.0$

(c) $\lambda = 10.0$

Figure S1: Comparison between EKG-MPPI and BO-MPPI in the Lorenz–96 experiment for three values of the MPPI temperature parameter λ. The horizontal axis represents the normalized intervention-effectiveness metric, defined here as the number of threshold exceedances, and the vertical axis represents the normalized average intervention magnitude. Blue points correspond to BO-MPPI with different values of the BO trade-off parameter α, and the red point corresponds to EKG-MPPI.

3