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A low-dimensional dynamical systems approach for ensemble design and interpretation in climate science

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Abstract. The design and use of conceptual models have been a longstanding and successful practice of mathematicians and scientists to study and gain insights about complex phenomena in climate and beyond. Here, we demonstrate how low-dimensional dynamical systems can also be useful in the design and interpretation of climate model ensembles. We argue that, provided they possess a small number of key characteristics, such systems can serve as computationally inexpensive laboratories for investigating questions of ensemble methodology that would be difficult to explore systematically in comprehensive Earth System Models. To this end, we identify the characteristics required of informative low-dimensional systems, formalise a framework for their use, and illustrate the approach through a series of examples.

1 Introduction

Climate projections and assessments of future climate change rely heavily on numerical models of the Earth system, so called Earth System Models (ESMs; often called simply by 'climate models'. In this paper we refer to both interchangeably as synonyms) (Flato, 2011; Eyring et al., 2016). Since future climate states cannot be directly observed and controlled experiments on the climate system are generally impossible, ESMs provide the primary means through which hypotheses can be tested, uncertainty studied, and potential future pathways explored. The most prominent examples of ESMs are found in the models contributing to the Climate Model Intercomparison Project (CMIP), which itself is a significant source of information for the Assessment Reports produced by the Intergovernmental Panel on Climate Change (IPCC) (Eyring et al., 2016; IPCC, 2023).

As with experiments in a laboratory, the design of climate model experiments requires careful consideration of objectives, methodology, and interpretation (Stainforth et al., 2007a, b). In particular, many scientific questions are addressed through ensembles, in which multiple model realisations are generated to characterise uncertainty (Lehner et al., 2020; de Melo Viríssimo, 2026), internal variability (Deser et al., 2012; Böhnisch et al., 2020), and/or sensitivity to model assumptions (Murphy et al., 2004; Stainforth et al., 2005), as well as for the purposes of calibration (Peatier et al., 2024), extreme event prediction (Sauer et al., 2025), and other research aims (see e.g. Maher et al. (2021) for a comprehensive review).

Despite their central role, the design and interpretation of climate model ensembles remain challenging. Questions concerning ensemble size, sampling strategies, perturbation methodologies, experimental protocols, and the interpretation of ensemble statistics are often difficult to investigate systematically (Daron and Stainforth, 2013; Milinski et al., 2020). A key limitation is

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computational cost. State-of-the-art ESMs are among the most complex numerical models used in science and require substantial computational resources, storage capacity, and postprocessing and analysis time (Balaji et al., 2017; Govett et al., 2024). Consequently, the number of experiments that can be performed is often severely constrained, making it difficult to explore a wide range of experimental designs or to assess systematically how methodological choices affect the information extracted from an ensemble. In practice, understanding ensemble behaviour frequently competes with the resources required to generate the ensemble itself. For example, a single well-designed experiment with an ESM can consume substantial computational resources, often limiting the number of alternative experimental designs that can be explored in practice. Indeed, even a single CMIP-like run may require months of planning and computation in a high performance computing environment (Eyring et al., 2016). This raises a natural question: can important aspects of ensemble design and interpretation be investigated in a simpler setting before resources are committed to expensive ESM experiments?

1.1 Low-dimensional systems in climate science

The use of simple and low-dimensional systems to investigate complex behaviour has a long and successful history in mathematics, physics, and climate science (Dijkstra, 2013, 2024). Over the past 60 years, ever since Edward Lorenz's foundational works (Lorenz, 1960, 1963), conceptual models of all sorts, ranging from simple box models (e.g. Stommel (1961); Welander (1982)), to energy balance models (EBMs; e.g. Budyko (1969); Sellers (1969)), to Lorenz-type systems (e.g. Lorenz (1984, 1996)), have provided fundamental insights into nonlinear dynamics (Ghil, 2019), predictability (Drótos et al., 2016), feedbacks (Bódai and Tél, 2012), bifurcations and tipping (Ashwin and Newman, 2021; Mehling et al., 2024), and many other aspects of climate and weather.

In this manuscript, we argue that low-dimensional systems can (and should) play an additional and complementary role. Rather than just using them to understand (or gain insights about) the behaviour of the climate system itself, we propose that they should play a much more prominent role in understanding the behaviour of ensemble methodologies themselves. The distinction is important. Our objective is not to understand the model properties, nor determine whether a particular low-dimensional system represents the climate system well, nor to use it as a cheap ESM surrogate; rather, we seek to understand how different experimental designs affect the information that can be extracted from systems possessing a number of key characteristics shared with climate models. In practice, of course, not every low-dimensional system is suitable for such investigations and we argue that a system must satisfy a small number of conceptual requirements that make insights transferable to more complex climate models.

Although the approach is presented here in a general form, many of its elements have already been employed implicitly in previous studies investigating uncertainty (de Melo Viríssimo et al., 2024), ensemble construction (de Melo Viríssimo and Stainforth, 2025), and predictability (Daron and Stainforth, 2013) in low-dimensional climate-like systems, some of which we will revisit in Section 4. Building on a presentation at the EGU General Assembly 2023 (de Melo Viríssimo and Stainforth, 2023), the present work seeks to formalise these ideas within a unified framework and clarify the conditions under which insights obtained from such systems may inform the design and interpretation of climate model ensembles.

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The remainder of the paper is organised as follows. In Section 2 we discuss the requirements for climate ensembles to be informative, and identify and discuss the key characteristics of climate models, as well as their implications for the design and interpretation of ensembles. Section 3 introduces a proposed approach and defines when low-dimensional systems are suitable for studying ensemble design and interpretation. Section 4 presents illustrative examples, showcasing how low-dimensional systems can provide insight into ensemble methodology and guide the efficient use of computational resources in climate modelling. Section 5 discusses the methodology, limitations and implications. Conclusions are provided in Section 6.

2 The key characteristics of climate models and their implications for ensemble design and interpretation

ESMs are among the most complex models used in science, making climate prediction one of the most difficult challenges in contemporary geoscience research. However, from the perspective of ensemble design, a small number of characteristics are particularly important. These characteristics shape both the information contained within an ensemble and the methodological choices required to obtain that information.

At a first level, four characteristics are especially important.

1. **Complexity.** Contemporary climate models represent a large number of interacting physical, chemical, and biological processes through extensive systems of equations, parameterisations, and numerical algorithms. This complexity is reflected not only in the number of degrees of freedom of the resulting dynamical system, but also in the large number of interacting feedback mechanisms embedded within the model.

2. **Multiple scales.** Climate variability and change emerge from processes operating across a wide range of spatial and temporal scales. These range from turbulent and convective processes occurring over less than minutes and kilometres to oceanic and cryospheric processes whose characteristic timescales extend over decades, centuries, and millennia. Climate models must therefore represent and couple phenomena spanning many orders of magnitude in both space and time.

3. **Multiple components.** ESMs are not single dynamical systems in a narrow sense, but collections of interacting subsystems. The atmosphere, ocean, land surface, cryosphere, and biogeochemical components possess distinct dynamics and characteristic timescales, while continuously exchanging energy, momentum, water, and chemical constituents. Many of the feedbacks relevant to climate prediction emerge from these interactions.

4. **Nonlinearity and chaos.** The climate system contains nonlinear components whose evolution exhibits sensitive dependence on initial conditions. Small uncertainties in the model state can grow over time, limiting deterministic predictability and necessitating probabilistic approaches to prediction and projection.

For questions concerning climate change, a fifth characteristic also becomes important:

5. **Non-autonomy.** Under anthropogenic climate change, the climate system is subject to time-dependent external forcing. The governing dynamics therefore evolve in time rather than remaining associated with a fixed attractor. Future climatic

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conditions cannot be regarded simply as samples drawn from the same statistical distribution as the past, making climate projection fundamentally a problem of extrapolation.

These characteristics have important implications for climate prediction and for the design and interpretation of climate-model ensembles.

First, **nonlinearity and chaos** imply that uncertainties in the initial state and in the model formulation can grow over time. Consequently, individual model trajectories are generally insufficient to characterise future evolution, and climate predictions must instead be expressed probabilistically. Ensembles therefore become a fundamental tool for quantifying uncertainty and characterising the range of plausible outcomes.

Second, the **multiscale, multicomponent, and highly interconnected** nature of the climate system means that responses to perturbations and external forcing can emerge through interactions occurring across a broad range of temporal and spatial scales. Capturing these responses may require large ensembles, long simulations, and diagnostic analyses targeted at different scales of variability. Furthermore, the presence of multiple interacting feedbacks can complicate the interpretation of ensemble statistics and make it difficult to distinguish robust signals from internally generated variability.

Third, the **complexity and high dimensionality** of contemporary ESMs make them computationally expensive. Long simulations, large ensembles, sensitivity studies, and systematic investigations of alternative ensemble designs may therefore be prohibitively costly. As a result, important methodological questions often remain only partially explored because the resources required to investigate them directly compete with the resources needed to generate the simulations themselves.

Fourth, different climate models may exhibit different responses to the same forcing owing to **differences in their structure, parameterisations, and representation** of physical processes. Structural uncertainty (de Melo Viríssimo, 2026) therefore constitutes an additional source of uncertainty that cannot be quantified through single-model ensembles alone and motivates the widespread use of multi-model approaches.

Finally, **non-autonomous forcing** implies that future climate states cannot be inferred solely from observed historical variability. Since past and future climates are associated with different dynamical regimes, projections require the use of models capable of representing the system's response to changing external forcing.

These characteristics, together with their implications for climate prediction and ensemble design, are summarised in Table 1. Taken together, these characteristics explain why climate model ensembles are both indispensable and difficult to design. They motivate the need for carefully constructed ensemble methodologies while simultaneously limiting our ability to investigate those methodologies directly within comprehensive ESMs. In other words, this tension can be summarised in the following question: How to design and interpret ensembles that are informative, given the (resource) limitations in computational power and time?

Here, we argue that the five characteristics above therefore provide a useful basis for identifying low-dimensional systems that can act as informative experimental laboratories for ensemble studies, as discussed in the next session.

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Table 1. Key characteristics of climate models and their implications for climate prediction and ensemble design.

Characteristic	Meaning	Implications for prediction and ensembles
Complexity	Large numbers of interacting processes, parameterisations, nonlinearities, and feedbacks.	Complex behaviour emerges; substantial computational costs and limited opportunities for systematic experimentation.
Multiscale behaviour	Processes operating across a wide range of spatial and temporal scales, from fast atmospheric variability to slow oceanic and cryospheric adjustment.	Sampling requirements depend on the timescales of interest; long simulations and large ensembles may be needed to isolate signals and variability.
Multiple components	Interactions among atmosphere, ocean, cryosphere, land, and biogeochemical systems.	Feedbacks emerge through component interactions, complicating attribution, uncertainty quantification, and interpretation of ensemble statistics.
Nonlinearity and chaos	Sensitive dependence on initial conditions and model formulation, alongside limits on the predictability of individual trajectories.	Necessitates probabilistic prediction and motivates ensemble methods to characterise uncertainty and internal variability.
Non-autonomy	Time-dependent, non-periodic external forcing modifies the system dynamics and can alter attractors over time.	Future climates cannot be regarded as samples from past variability alone; projections require model-based extrapolation and ensemble methods.

3 A low-dimensional dynamical systems approach

The five characteristics above suggests that conceptual questions regarding the design and interpretation of climate model experiments can be successfully studied in systems possessing these properties. Indeed, the usefulness of a low-dimensional system for studying questions of ensemble design does not depend on its ability to reproduce the full complexity of an ESM. Rather, the relevant question is whether a low-dimensional system possesses the particular characteristics relevant to the specific challenges encountered in climate prediction, projection, and ensemble design and interpretation.

Provided that a system possesses the characteristics responsible for the ensemble behaviour of interest, it may be possible to study methodological questions in a low-dimensional and computationally inexpensive setting. Such systems can then serve as computationally inexpensive experimental laboratories in which these questions may be explored systematically, enabling us to understand how ensemble design choices influence the information that can be extracted from model simulations, before resources are committed to more expensive climate model experiments.

The proposed framework consists of four steps:

1. **Formulate a question concerning ensemble design or interpretation.** This could be, for instance, a question on the required size of an initial condition ensemble to properly sample extreme events in slow dynamical components such as large-scale ocean circulations.

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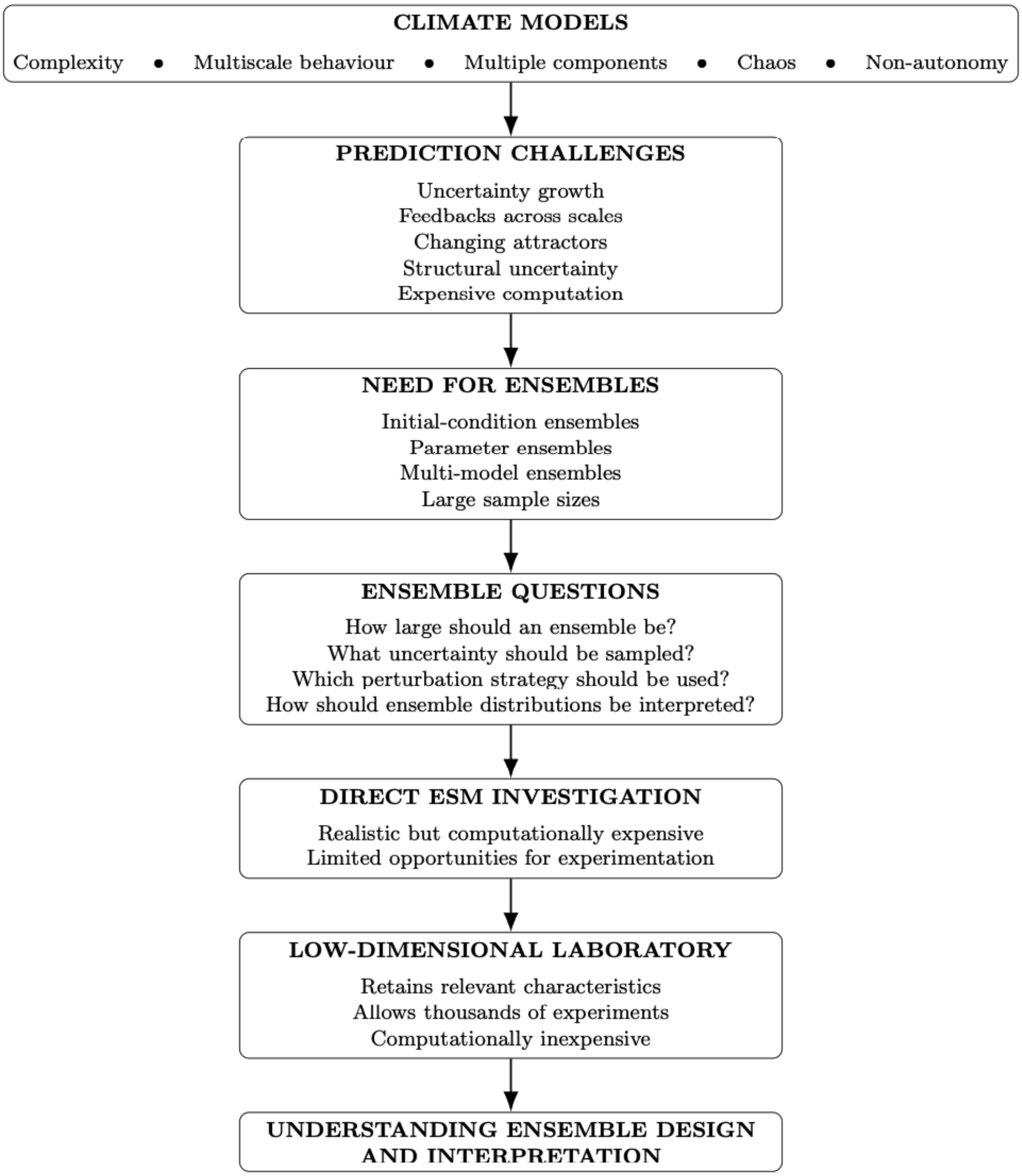

Figure 1. Conceptual framework of the low-dimensional approach proposed in this study. The key characteristics of climate models give rise to challenges for prediction and projection, motivating the use of ensembles. Questions concerning ensemble design and interpretation are, however, often difficult to investigate systematically within ESMs because of computational constraints. Low dimensional systems that retain the relevant characteristics may therefore serve as computational laboratories for studying ensemble methodology and informing the design of more expensive climate-model experiments.

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2. **Identify a low-dimensional system possessing the characteristics responsible for the phenomenon of interest.** For example, a system that includes both ocean-like variables and fast atmosphere-like variables would be enough to study questions related to ensemble initialisation from spinup and observations (see also Section 4.1).

3. **Design and perform a large set of experiments.** Since the computational power barrier has been largely removed, it is now possible to perform a very large number of experiments, including very large ensembles of thousands to hundreds of thousands of members.

4. **Analyse the results and extract lessons relevant to more comprehensive climate models.** Such ensembles can be analysed using a variety of advanced techniques in nonlinear and stochastic dynamics for example (Dijkstra, 2013; Lucarini and Chekroun, 2023).

The overall rationale of the framework is illustrated schematically in Fig. 1. The figure highlights the central argument of this manuscript: the objective is to exploit low-dimensional systems possessing key climate model characteristics as cheap computational laboratories in which questions of ensemble design and interpretation can be investigated systematically in an "unlimited computational power" setting.

3.1 Examples of useful systems

Different questions may motivate different low-dimensional systems. Transferability depends not upon reproducing all aspects of an ESM, but upon reproducing the conceptual ingredients responsible for the ensemble behaviour of interest, such as multiple timescales, interacting components, and uncertainty sources.

A prime example in this setting is the Lorenz 84-Stommel 61 (L84-S61) system (Veen et al., 2001; Roebber, 1995). The model couples the Lorenz 84 atmospheric model (Lorenz, 1984, 1990), which describes the interaction between the mid-latitude westerlies and large-scale eddies, with the Stommel 61 ocean model (Stommel, 1961), which represents thermohaline circulation through the exchange of heat and salt between two idealised ocean reservoirs.

Mathematically, the model consists of five coupled ordinary differential equations:

$$X' = -Y^2 - Z^2 - aX + a(F_0(t) + F_1 T), \quad (1)$$

$$Y' = XY - bXZ - Y + G_0 + G_1(T_{av} - T), \quad (2)$$

$$Z' = bXY + XZ - Z, \quad (3)$$

$$T' = k_a(\gamma X - T) - |f(T,S)|T - k_w T, \quad (4)$$

$$S' = \delta_0 + \delta_1(Y^2 + Z^2) - |f(T,S)|S - k_w S, \quad (5)$$

where

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$$f(T,S) = \omega T - \epsilon S, \tag{6}$$

$$F_0(t) = F_m + M\cos\left(\frac{2\pi t}{K} - \frac{\pi}{12}\right) + F_{\mathrm{CC}}(t). \tag{7}$$

The variables X, Y, and Z describe the fast atmospheric component of the model, where X represents the intensity of the westerly circulation and Y and Z represent the amplitudes of large-scale eddies. The variables T and S describe the slow ocean component, representing pole-equator temperature and salinity differences, respectively. The function $f(T,S)$ represents the strength of the thermohaline circulation, while the forcing term $F_0(t)$ represents the equator-to-pole heating contrast through a combination of a mean forcing, seasonal variability, and an optional time-dependent forcing component associated with climate change.

Together, these form a conceptual atmosphere-ocean model that exhibits many of the characteristics discussed in Section 2. First, Equations (1) to (7) represent two distinct but interacting components, namely an atmosphere and an ocean, thereby providing a simple example of a multicomponent climate system. Second, the atmospheric variables evolve on substantially shorter timescales than the ocean variables, creating a clear separation between fast and slow dynamics and hence a multiscale system. Third, the nonlinear interactions among the variables generate complex behaviour despite the small number of degrees of freedom. Fourth, for standard parameter choices, the atmospheric component exhibits chaotic dynamics and consequently sensitive dependence on initial conditions. Finally, the inclusion of the time-dependent forcing term $F_{\mathrm{CC}}(t)$ allows the system to become non-autonomous, providing a simple analogue of externally forced climate change.

Hence, although idealised, the L84-S61 model therefore reproduces many of the characteristics that make climate model ensembles necessary and simultaneously difficult to interpret. Its low computational cost, however, permits the generation of very large ensembles and extensive sensitivity studies, making it a useful laboratory for investigating questions of ensemble design and interpretation.

Other existing examples range from an L84 model coupled to idealised oceanic dynamics (Goswami and Selvarajan, 1991; Goswami et al., 1993; Knopf et al., 2005), to a reduced order, coupled tropical-extratropical model (Vannitsem et al., 2021), to a variety of stochastic models (e.g. Cessi (1994); Del Sarto and Flandoli (2024)) to mention just a few (see also Dijkstra (2024); Ghil (2019) for further examples). Since computational power is a relative concept, even some of the so-called "Earth systems models of intermediate complexity" (EMICs) such as OSCAR (Gasser et al., 2017; Quilcaille et al., 2023) could be considered feasible as part of this approach, depending on the computer power available; while they may be too compute expensive for a standard research laptop today, they could potentially be used in large ensemble mode on a multi-core cluster.

4 Examples

This approach can be illustrated with a few examples. Variants of this approach have already been used in studies of climate change projections, uncertainty quantification, and ensemble design. We discuss three of them below.

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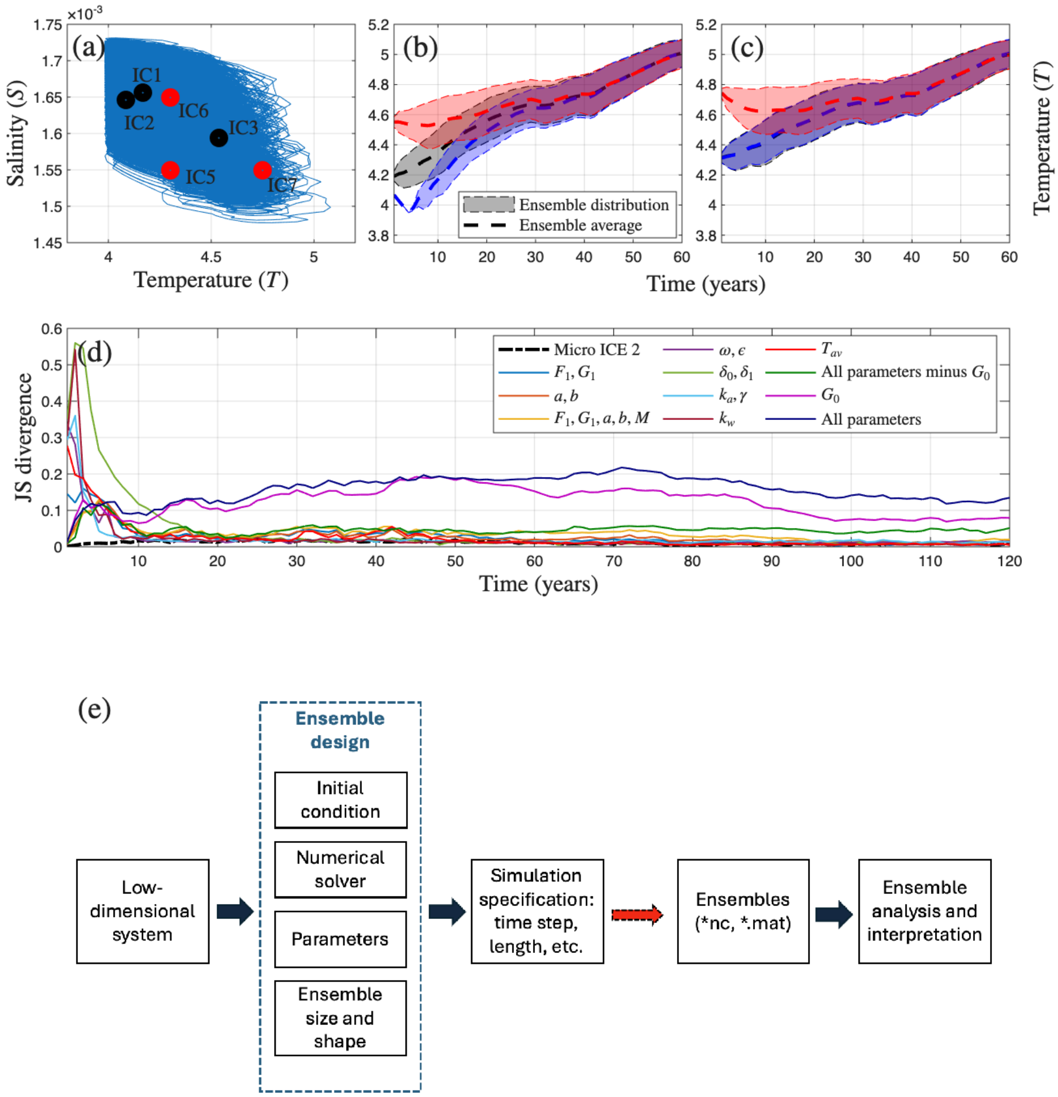

Figure 2. Illustrative applications of the low-dimensional dynamical systems approach. Panels (a) to (c) illustrate the effect of macro initial condition uncertainty and long-term memory (Example 1). In (a), a projection of attractor (on the ocean-like variables) for the L84-S61 system at $F_m = 7$ is shown. ICs 1-3 (in black) are taken from a spinup trajectory, while ICs 5-7 (in red) are chosen to represent uncertainty from observations in one variable. The effect of this macro uncertainty is shown in (b) for ICs 1-3 and in (c) for ICs 5-7 (in black, blue and red respectively for both). In (c), the grey IC5 trace is not visible because it is effectively superimposed on the blue IC6 trace, indicating near-identical values across the plotted range. Panel (d) illustrates the effect of micro parametric uncertainty on ensemble design (Example 2). It shows the JS divergence computed with respect to an initial condition ensemble. The fact that the JS divergence is substantially greater when parameters are perturbed than the values for another micro-initial-condition-ensemble (black dashed line), demonstrates the substantially different impact of micro-parameter perturbation ensembles on model-based projections. Panel (e) presents a schematic for the MATLAB-based computational framework for systematic ensemble experimentation (Example 3).

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4.1 Example 1: Ensemble initialisation and long-term memory

One of the most common sources of uncertainty in climate model ensembles arises from uncertainty in the initial state of the system. In practice, ensembles may be initialised in a variety of ways. For example, initial conditions may be selected from a long control or spin-up simulation, where different points on the same trajectory are considered plausible states of the system. Alternatively, ensembles may be initialised using observations, in which some variables may be relatively well constrained while others remain poorly observed and therefore considerably more uncertain. The choice of initialisation strategy is consequently an important element of ensemble design.

Questions of this type are difficult to investigate systematically within comprehensive ESMs because only a limited number of experiments can usually be performed. By contrast, a low-dimensional system allows many alternative initialisation strategies to be explored and compared. The L84-S61 system provides a useful example because it combines a chaotic atmospheric component with a slowly evolving oceanic component, thereby reproducing both the multiscale and multicomponent characteristics identified in Section 2.

To investigate the consequences of ensemble initialisation, de Melo Viríssimo et al. (2024) examined a series of macro-initial condition uncertainty experiments in which ensembles were generated from substantially different initial states of the system. Some of these initial states were selected from different locations along a long spin-up trajectory, thereby representing plausible states that might arise naturally in a control simulation. Others were constructed by modifying only the slowly evolving ocean variables while leaving the atmospheric variables unchanged. The resulting ensembles were then subjected to identical external forcing and their subsequent evolution compared. This is shown in Figure 2(a-c).

The experiments revealed that the influence of the initial state can persist for remarkably long periods of time. While uncertainties in the fast atmospheric variables decay relatively rapidly, the slowly evolving ocean variables retain memory of their initial values for several decades. In the L84-S61 system, characteristic decay times of approximately 26-38 years were found for the ocean variables, compared with approximately 8 years for the atmospheric variable considered.

From the perspective of ensemble design, the important result is not the specific timescales themselves, which are model-dependent, but the demonstration that slowly evolving components can exert a long-lasting influence on ensemble evolution. Consequently, initialisation choices that appear equivalent from the perspective of fast variables may nevertheless lead to substantially different ensemble outcomes on climate-relevant timescales. This finding is consistent with the earlier literature on climate model ensembles, where the role of internal variability and initial-condition uncertainty has been shown to persist for decades in some variables and regions (Hawkins et al., 2016), and later confirmed by Deser et al. (2025) using a 100-member state-of-the-art ensemble.

This example illustrates how a low-dimensional climate-like system can be used to investigate a practical question of ensemble methodology: namely, how different initialisation strategies influence the information subsequently contained within an ensemble. Because the experiments can be repeated systematically for large numbers of alternative macro initial conditions, the framework provides a computationally inexpensive means of exploring questions that would be difficult to address comprehensively in ESM. More generally, it demonstrates how the interaction of chaos, multiple timescales, and multiple system

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components can lead to methodological challenges in ensemble interpretation that are directly relevant to contemporary climate prediction and projection.

4.2 Example 2: Parametric uncertainty and ensemble construction

A second example concerns the representation of uncertainty arising from imperfect knowledge of model parameters. While considerable effort has been devoted to the study of initial-condition uncertainty, uncertainties associated with model parameters remain more difficult to characterise systematically, particularly within computationally demanding ESMs.

Using a low-dimensional atmosphere-ocean model, de Melo Viríssimo and Stainforth (2025) examined the consequences of different forms of parametric uncertainty through very large ensembles. The study distinguished between micro-parametric uncertainty, corresponding to small residual uncertainties in parameter values, and macro-parametric uncertainty, corresponding to larger uncertainties associated with alternative plausible representations of the system. The resulting ensembles were then compared with traditional initial-condition ensembles. This is shown in Figure 2(d).

Specifically, the study explored what information is gained or lost when different uncertainty sources are represented within an ensemble, and how alternative ensemble constructions affect the resulting distributions of plausible future states. Because the experiments could be repeated systematically and at negligible computational cost, it was also possible to investigate the effects of perturbing multiple parameters simultaneously and to assess the consequences of different sampling strategies.

More broadly, this example demonstrates how low-dimensional systems can be employed to evaluate competing ensemble methodologies and to investigate whether commonly used approaches adequately characterise uncertainty.

4.3 Example 3: Towards a framework for ensemble experimentation

The previous examples focused on specific scientific questions. A natural extension of the approach is the development of computational tools designed explicitly for ensemble experimentation in low-dimensional systems.

One such development is the Low-EFFourth (LEF4) (de Melo Viríssimo, 2025a), a MATLAB-based framework freely available on Zenodo (de Melo Viríssimo, 2025b), which provides a computational environment for generating and analysing ensembles arising from uncertainties in initial conditions, parameters, numerical methods, and model structure. A schematic representation of the framework is shown in Figure 2(e). Additionally, although originally motivated by questions in climate science, the framework can equally be applied to other disciplines involving nonlinear dynamical systems and uncertainty quantification.

The significance of developments such as LEF4 is that they lower the barrier to performing large ensemble studies and facilitate systematic experimentation across a wide range of low-dimensional models and uncertainty sources. They therefore illustrate how the low-dimensional approach outlined in this manuscript can evolve from a conceptual framework into a practical methodology that is accessible to a broader scientific community.

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5 Discussion

The framework proposed in this paper is motivated by a simple observation: many questions concerning the design and interpretation of climate-model ensembles are difficult, or virtually impossible, to investigate directly within comprehensive ESMs. Although ESMs provide the highest level of process realism currently available, their computational expense often limits the number of experiments that can be performed. As a consequence, methodological questions are frequently constrained by practical considerations of computational cost, storage, and postprocessing and analysis efforts.

The low-dimensional approach advocated here seeks to address this challenge by shifting attention away from the full reproduction of Earth system complexity and towards the specific characteristics responsible for a given ensemble design problem. The central premise is that if a low-dimensional system possesses (to a degree) all, or a suitable set of, the properties introduced in Section 2, then it may act as an informative laboratory within which methodological questions can be explored systematically. The examples discussed in Section 4 illustrate the potential of this idea in the context of uncertainty quantification, and initial condition and parametric ensemble construction, for instance.

Within a low-dimensional setting, it becomes possible to perform experiments that would be prohibitively expensive in an ESM. Ensemble sizes can be increased by orders of magnitude, uncertainty sources can be isolated and combined systematically, and sensitivity analyses can be repeated many times under controlled conditions. Such capabilities allow researchers to explore methodological questions more comprehensively than would typically be feasible in state-of-the-art climate models.

5.1 Limitations of this approach

The approach has important limitations. Since the purpose of low-dimensional systems is not to reproduce the quantitative behaviour of the climate system, the results obtained within a particular low-dimensional model should not be interpreted as direct predictions of climate behaviour, nor should they be assumed to transfer automatically to ESMs. The value of the approach lies instead in identifying conceptual mechanisms, methodological sensitivities, and general principles. Judgement is therefore required when assessing whether a given result is sufficiently robust and generic to inform the design or interpretation of ensembles in more complex models.

Similarly, not every ensemble design question can be addressed adequately using a low-dimensional framework. Questions that depend strongly on detailed process representations, spatial interactions, regional dynamics, or specific parameterisations may require direct investigation within more comprehensive climate models. The approach should therefore be viewed as complementary to, rather than a replacement for, ESM-based studies - although the two are related since the former can help design and optimise those ESM-based studies. Its greatest value arises in situations where methodological understanding can be advanced through systematic experimentation, and where computational constraints would otherwise limit such investigations.

5.2 Implications

One central implication of this perspective concerns the way computational resources are used within climate science. Over recent decades, advances in the field have been accompanied by a continual expansion in computing power, enabling the

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development of increasingly sophisticated ESMs and larger ensemble experiments. This progression has delivered substantial scientific benefits, but it has also reinforced a tendency to address methodological questions through ever more computationally demanding simulations (Stainforth, 2023).

The approach proposed here suggests a complementary approach. Rather than investigating every question directly within a comprehensive ESM, many issues concerning ensemble design and interpretation can first be explored systematically in low-dimensional systems at negligible computational cost. Such systems make it possible to perform large numbers of experiments, investigate alternative ensemble strategies, and develop methodological understanding before committing limited and expensive computational resources to more targeted investigations in high-dimensional models.

Viewed in this way, low-dimensional dynamical systems do not provide an alternative replacement to high-performance computing, nor do they diminish the importance of comprehensive climate models. Instead, they complement and support these resources by helping to identify informative experimental designs, clarify the questions that require detailed investigation, and reduce the amount of exploratory experimentation that must be performed within computationally expensive modelling frameworks. The resulting workflow combines the strengths of both approaches: the flexibility and experimental freedom of low-dimensional systems with the realism and process representation of state-of-the-art ESMs.

The framework may also have implications beyond climate-model development itself. Increasingly, climate information is used to support adaptation planning (IPCC, 2022), climate-risk assessment (Ranasinghe et al., 2021), and decision-making under uncertainty (Mankin et al., 2020). Approaches such as climate storylines (Shepherd, 2019; Baulenas et al., 2023), exploratory scenarios (Star et al., 2016; Serrao-Neumann and Low Choy, 2018), and risk-based assessments (Simpson et al., 2021; Jones et al., 2019) depend critically on how uncertainty is represented, sampled, interpreted and communicated. Consequently, improving our understanding of ensemble design may ultimately contribute to improving the quality and transparency of the information provided to stakeholders and decision-makers (Stainforth et al., 2007a; Stainforth, 2023). While the connection between ensemble methodology and practical decision support is indirect, it is nevertheless important, since the credibility and usefulness of climate information depend in part on the robustness of the ensemble methods from which that information is derived (Stainforth, 2023).

More generally, the ideas developed here are not unique to climate science. Similar challenges arise throughout computational science whenever complex, computationally intensive models are used to make probabilistic predictions. Epidemiology, ecology, economics, engineering, and Earth-system science all rely increasingly on large-scale simulations and ensemble methods (Palmer, 2022). The low-dimensional approach described here therefore has the potential to provide a more general framework for investigating uncertainty quantification, ensemble construction, and model interpretation in a variety of scientific disciplines.

The examples presented in Section 4 suggest that this approach is already beginning to mature beyond a conceptual proposal. Applications to aspects such as ensemble size and parametric uncertainty have demonstrated its practical utility (Daron and Stainforth, 2013; de Melo Viríssimo and Stainforth, 2025), while the development of dedicated computational (de Melo Viríssimo, 2025a) tools further lowers the barriers to systematic exploration. Future work should seek both to broaden the range of methodological questions investigated using this approach and to establish more clearly the circumstances under which

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conclusions obtained from low-dimensional systems remain informative for higher-dimensional climate models - including through mathematical investigations.

Ultimately, this perspective suggests a broader role for conceptual models within computational science. As discussed in Section 1, low-dimensional systems have traditionally been used to improve our understanding of the systems being modelled. The framework proposed here points to an additional role: understanding the methodologies through which scientific knowledge is generated from models. In this view, low-dimensional systems become not only laboratories for exploring dynamical behaviour, but also laboratories for exploring the strengths, limitations, and consequences of the tools used to study complex systems.

This suggests a broader conception of modelling itself: one in which the objective is not only to learn about a system, but also to learn about the methods through which we investigate that system. As computational models continue to grow in complexity and computational cost, such methodological laboratories may become increasingly important for ensuring that expensive and limited resources available for ESMs are efficiently spent, and that the information extracted from those models is both robust and scientifically informative.

6 Conclusions

Climate science relies heavily on ensembles to characterise uncertainty, quantify internal variability, and support projections of future climate change. Yet many fundamental questions concerning ensemble design and interpretation remain difficult to investigate systematically because of the computational expense of contemporary ESMs. This paper has argued that low-dimensional dynamical systems can provide a useful and complementary framework for addressing such questions.

Unlike their traditional use in mathematics, physics, and climate science, where the goal is often to understand the behaviour of the system itself, the focus here is different. The climate system is not the object of study; ensemble methodology is. Hence, their greatest value may not lie in what they tell us about the climate system itself, but in what they allow us to learn about the ways in which we generate, analyse, and interpret information from computational models. In this sense, low-dimensional systems can be viewed as methodological laboratories: environments in which the strengths, limitations, and consequences of ensemble methodologies can be explored systematically before those methodologies are deployed within more computationally demanding modelling frameworks.

We identified a small number of key characteristics that make climate models distinctive from the standpoint of prediction and ensemble construction, and argued that low-dimensional systems possessing these characteristics can provide informative environments in which methodological questions may be explored systematically. We also presented examples demonstrating how this approach has been emerging over the years as a tool to investigate uncertainty quantification and ensemble design questions, with answers that are transferable to higher dimensional ESMs.

The potential benefits extend beyond model development and efficient use of computational resources. Informative ensembles are increasingly in demand in climate services by a variety of industries and actors. Improving our understanding of

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ensemble design therefore contributes directly to improving the information available to downstream users, such as policy and decision makers.

Finally, although our discussion has focused on climate science, the underlying ideas are not climate-specific. Similar challenges arise throughout computational science whenever high-dimensional and computationally expensive models are used to make probabilistic predictions, where low-dimensional dynamical systems should have an important role to play in the design and interpretation of ensembles as well.

Code and data availability. No data was produced for this paper. All data used in this work is freely available online on Zenodo de Melo Viríssimo et al. (2023); de Melo Viríssimo and Stainforth (2024). All figures, diagrams and tables in this paper were generated by the authors. Panels (a) to (c) in Figure 2 were generated using the data from ICs 1, 2, 3 and ICs 5, 6, 7 from de Melo Viríssimo et al. (2024), which are available in de Melo Viríssimo et al. (2023); Panel (d) was generated using the data from the micro- and macro-PPEs (for the parameters shown in the caption) from de Melo Viríssimo and Stainforth (2025), which are available in de Melo Viríssimo and Stainforth (2024). Similar versions of these panels are available in the aforementioned manuscripts.

Appendix A: Parameters for L84-S61 model

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Parameter	Value	Description
F_1	0.02	Coupling parameter for the equator-pole temperature difference
G_0	1	Reference value for the land-sea temperature difference
G_1	0.01	Coupling parameter for the land-sea temperature difference
a	0.25	Damping coefficient of the westerly winds
b	4	Displacement of the waves due to interaction with the westerly wind
T_{av}	30	Standard temperature contrast between the polar and the equatorial box
γ	30	Proportionality constant between the westerly wind and non-homogeneous forcing by solar heating
k_w	$1.8 \cdot 10^{-5}$	Coefficient of internal diffusion in the ocean
k_a	$1.8 \cdot 10^{-4}$	Coefficient of heat exchange between the ocean and atmosphere
ω	$1.3 \cdot 10^{-4}$	Coefficient derived from the linearised equation of state
ϵ	$1.1 \cdot 10^{-3}$	Coefficient derived from the linearised equation of state
δ_0	$7.8 \cdot 10^{-7}$	Coefficient for the atmospheric water transport
δ_1	$9.6 \cdot 10^{-8}$	Coupling parameter for the wind dependent atmospheric water transport
M	1	Magnitude of the seasonal cycle
F_m	7	1-year mean value of the seasonal variation function $F_0(t)$ when $H = 0$
H	0.01	Externally forced rate of change (per year) of the 1-year mean of $F_0(t)$

Table A1. Description of the parameters and their reference values used in the L84-S61 model. The three parameters in the bottom controls the forcing function $F_0(t)$. All parameter values presented here were taken from Daron and Stainforth (2013) and are the same used in de Melo Viríssimo et al. (2024); de Melo Viríssimo and Stainforth (2025).

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Competing interests. The authors have no competing interests to declare.

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