General Comments:

This manuscript proposes a random-coefficient merging particle filter (RMPF) to improve the conventional MPF. The mathematical approach is interesting and potentially useful. However, the manuscript needs major improvements in structure, clarity, and reproducibility. Many explanations, equations, and results are repeated, especially in Sections 8–10, making the main contributions difficult to understand. These sections should be reorganized and shortened. In addition, many numerical results appear to be based on single trials. The authors should repeat the experiments and report the mean and variability across multiple independent trials. Because these issues can be addressed through revision, I recommend major revision.

Major Comments:

1. Reduce redundancy throughout the manuscript
   a. Overall
      - The same results, explanations, and equations are repeated across multiple sections, making the main contributions unclear. In particular, please consolidate the following:
        - The superiority of pre-merging estimates: Sections 6.2, 7.2, 9.1, and 10.1
        - The claim that the required number of particles can be reduced to one-quarter or one-eighth: Sections 7.2, 9.1, and 10.1
        - Equations (24) and (43)
        - The role of beta: Sections 5.1 and 9.1

   b. Reorganize Sections 8–10
      - Sections 8–10 should be reorganized. The MISE-based analysis in Section 8.1 could be moved to Section 5. Section 9.1 mostly repeats earlier sections and should be removed or shortened. The Discussion should focus on comparisons with existing methods, computational cost, applicability, and limitations. Section 10 should be shortened to one or two paragraphs covering the main conclusions and future work.

   c. Simplify Sections 5–7
      - In Section 5, clearly explain what was tested, what the main results were, and what they mean. Remove repeated explanations. Also, consider moving P13, L284–286 to Section 5.2.1.
      - In Sections 6 and 7, do not describe every value shown in the tables and figures. Focus only on the main trends and important exceptions.

2. Demonstrate the reproducibility of the numerical experiments

If the results in Tables 3, 4, 5, and 7, Figure 4, and Sections 6 and 7 are based on only one trial, their reliability cannot be properly evaluated. Please repeat each experiment using different random seeds and report the number of trials, the mean or median, and a measure of variability, such as the standard deviation or confidence interval. Please also provide enough information to reproduce the experiments.

For example, the sharp increase at (n=7) in the upper-left panel of Figure 4 may reflect either a consistent pattern or random variation. This should be confirmed through repeated experiments. The reported improvements in performance and reductions in the required number of particles should also be reevaluated using results from multiple trials.

3. Clarify the Choice and Effects of the Number of Particles

P16, L329 shows that at least 32 particles were used. Please explain why this range was selected and compare it with particle numbers used in previous studies. Smaller ensembles, such as 3, 5, or 10 particles, should also be tested, or the authors should explain why they are not appropriate. If RMPF does not work well with small ensembles, please explain why.

In addition, Section 6.2.1 uses 256 or more particles. Please examine whether outliers or extreme particles affect the merging process and the estimates when many particles are used. If their influence is limited, explain how the proposed method reduces it. Otherwise, discuss this issue as a limitation of RMPF.

To further evaluate the robustness and applicability of RMPF, the authors should also consider an experiment with a nonlinear observation operator. This would help clarify whether the method remains effective under more challenging observation conditions.

4. Verify the Theoretical Derivations

Please carefully check the derivation of the merged-particle distribution in Section 4 and Appendix B. The rank assumption for (n=d+1), the Jacobian argument, and the conditions on (n), (r), and (beta) appear inconsistent. Please provide a corrected theorem with clear assumptions, boundary cases, and a normalization check.

5. Evaluate Computational Efficiency Fairly

A reduction in the number of particles does not necessarily mean a reduction in computational cost, because RMPF requires additional merging operations involving particles. Please report computation time and compare PF, MPF, and RMPF under similar computational budgets. Also clarify whether the random coefficients were newly generated or reused at each assimilation step.

6. Clarify How the Merging Size (n) Is Selected

The relationship between likelihood and RMSE is interesting, but the manuscript does not provide a practical procedure for selecting (n) when the true state is unknown. Please clearly describe how (n) is selected and validate the selected value using independent experiments or time periods. The uncertainty and additional computational cost of this procedure should also be discussed.

Detailed Comments:

1. **P2, L38–41**
   This paragraph is repetitive and should be shortened. Please also reconsider whether it is needed between Sections 2 and 2.1.
2. **P2, L38 and throughout the manuscript**
   Please consistently use “PF” and “PFs” instead of “particle filter” and “particle filters” after the abbreviations have been defined.
3. **P4, L80: Organization of Section 2**
   Does the discussion shift from PFs to the MPF at this point? The relationship between them is unclear. Please reorganize Sections 2 and 2.1 and use subsection titles that accurately describe their contents.
4. **P4, L63–84: Previous studies and the position of the MPF**
   Do any of the PF methods discussed here use a resampling approach similar to that of the MPF? For example, the resampling method of Poterjoy (2016) appears to share some features with the MPF. Although I am not necessarily requesting an extensive literature review, the manuscript relies heavily on Nakano et al. (2007) when discussing the MPF. Please reconsider whether this emphasis is appropriate and clarify how the MPF relates to other relevant methods.
5. **P5, L97**
   Is it necessary to provide specific numerical values here? If they are explained in Section 3.2, they could be omitted here.
   In addition, the discussion focuses heavily on Nakano et al. (2007), with little mention of other studies or subsequent developments. Please provide a more balanced account of the relevant literature.
   This material also seems out of place in Section 2.1, “Particle Filter for State Space Models,” so its location should be reconsidered.
6. **P6: Sections 3.1 and 3.2**
   Section 3.1, “Overview,” could be merged with Section 3.2 to make the presentation more concise. Please reconsider whether these sections need to be separate.
7. **Figure 1**
   The font size is too small. Please enlarge the text, circles, and black dots to improve readability.
8. **P13, L284–286**
   Please reconsider whether this explanation is needed here. It may fit better in Section 5.2.1.
9. **Section 5**
   The results in this section could be explained more simply and clearly.
10. **P16, L329: Number of particles**
    Even the minimum setting of 32 particles appears relatively large. Please explain why this many particles are required and compare the setting with those used in previous studies. Would the method fail with much smaller ensembles, such as 3 or 10 members? The behavior with fewer particles should be discussed or tested.
11. **Section 6.2.1: Outlier problem**
    When using a large number of particles, such as 256 or more, could outliers become a problem? If not, please explain why.

12. **Section 3.4 and Algorithm 3**
    The timing of the state estimate is described inconsistently. Please ensure that the text agrees with Algorithm 3, where the estimate is calculated before extended resampling.
13. **Resampling procedure**
    Please specify the resampling method and whether resampling is performed at every observation time.
14. **Special case of (n=1)**
    The random-coefficient construction is not defined for (n=1). Therefore, PF should be described as an algorithmic special case rather than a direct application of the proposed coefficient generator.
15. **Number of possible merged particles**
    Please check the statement about the number of possible merged particles. Because resampling is normally performed with replacement, repeated selections may produce fewer distinct particles than the stated value.
16. **Lorenz-63 results with small (N)**
    Please explain the very large errors for small (N). Also clarify whether divergent runs are included. Reporting the median RMSE and failure rate would be helpful.
17. **Reproducibility**
    Please ensure that reviewers can access the code, settings, random seeds, and data required to reproduce all tables and figures.
18. **Strength of the conclusions**
    Please avoid strong expressions such as “resolved,” “solid foundation,” and “real-world deployment” unless they are fully supported by the theoretical and numerical results.