



Part II: An initial evaluation of the QuSI-DEM Discrete Element model using idealized and realistic test cases for sea ice

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Abstract. Sea ice is a quasi-brittle material that cracks, deforms, and flows under the load of wind and ocean currents. In part I we introduced QuSI-DEM, a model designed to capture crack patterns that develop as a result of sea ice fracture that depend critically on the elastic and dissipative response of the material at the location of failure. Parameters were introduced to the contact physics in QuSI-DEM to provide control of the failure rate and timing of transitions in contact response and to constrain the amount of energy and momentum transfer as elements interact. The sensitivity of the model response to these parameters is first demonstrated here in a set of idealized experiments involving the interactions of a few elements. The model is then applied to realistic simulations of sea ice response to northerly wind stress in the southern Chukchi Sea and Bering Strait. Here again, the parameter space is explored along with sensitivity to element size and arrangement through sets of ensemble simulations to explore the phenomenology of ice arching. Post-failure parameters such as the strength of the sea ice at the transition from bonded to frictional sliding, and the residual sea ice strength for heavily impacted ice are found to significantly impact model response.



1 Background

In Part I of this paper, we introduced the QuSI-DEM model. This polygon based discrete element model, formulated in spherical coordinates, is designed to offer a complimentary approach to sea ice modeling on geophysical scales, aiming to simulate ice
15 floe interactions. The discrete element method (DEM) can simulate sea ice dynamics from fracture through post-failure granular behavior. Contact physics between elements in QuSI-DEM is designed explicitly to represent the cohesive or frictional forces between distinct floes, providing a better representation than continuum-based approaches for phenomena involving ice floe interaction. In Part I, we outlined uncertainties that remain regarding best practices in the application of DEM to sea ice on these scales. These range from appropriate choice of element shape, element scale, contact physics, choice of parameters and
20 specifics of the numerical implementation. The design of the model was outlined in Part I, explaining the implementation details and outlining the fundamental response of interaction of element-pairs undergoing bond failure or collisions.

The crack patterns that develop as a result of sea ice fracture depend critically on the elastic and dissipative response of the material at the location of failure. To simulate this process while maintaining numerical stability, parameters were introduced to the QuSI-DEM contact physics that provide control of the failure rate and timing of transitions in contact response and
25 to constrain the amount of energy and momentum transfer as elements interact. While Part I describes the model, the model tuning needs to be described. Sensitivity of element interactions to element shape, contact orientation, application of damage and other damping parameters is important to understand when evaluating whether a DEM model can realistically simulate ice interaction. Modeling brittle fracture with a DEM can produce abrupt changes in the force and energy balance at contacts. Solutions can become unstable, producing widespread shattering which is not generally observed. With this as impetus, several
30 parameters were introduced to the QuSI-DEM contact physics that assist in producing a stable solution with a computationally reasonable timestep. These parameters may also be physically justified as representing sub-element scale processes such as partial failure of joints. Here, we explore the sensitivity of the QuSI-DEM solution to these parameter choices and element arrangements.

The tests consider variation in element shape and arrangement, and the impacts of this on energy conservations in element
35 collisions, bond breakage and the impact of friction in shear along bonds. The tests presented in Sect. 2 are used to demonstrate the characteristics of the model physics along with the damage and damping parameterizations described in part I. In Sect. 3 a representation of ice flow in the Chukchi Sea towards Bering Strait is used to demonstrate how QuSI can capture a realistic ice arching event. The sensitivity of the arch formation and stability to element shape and adjustments in the contact physics is explored. We summarize the utility of these case studies for other groups developing DEMs in Sect. 4.

2 Evaluations of model contact physics

In this section, the QuSI-DEM contact physics is explored with a collection of idealized test cases using just a few elements. The objective is to demonstrate and characterize the model behavior at the element contact scale and to check that stresses, momentum, and energy transfer are reasonably modeled. The focus is on the performance of the contact physics, first with the response of a perturbed bond, followed by a basic collision contact. The behavior of joint damage and contact failure is



45 then presented. The final test case demonstrates shear failure under compression, combining numerous aspects of the previous cases.

2.1 Elastic contact response

Sea ice interactions in nature are often highly dissipative, but to start, we focus on an analysis of the energetics for a non-dissipative case to demonstrate the degree to which conservation of energy is achievable with this model. Friction and viscous
50 dissipation will be introduced in later sections. Without dissipation, energy in the system will be distributed among translational and rotational kinetic energy and the potential energy in the bond between the elements. The translational kinetic energy in the system is calculated, as

$$KE_t^{tot} = \frac{1}{2} \sum_{i=1}^{N_e} m_i v_i^2, \quad (1)$$

where N_e is the number of elements in the system, m_i is element mass and v_e is element velocity along the surface of the
55 sphere. The rotational kinetic energy is

$$KE_r^{tot} = \frac{1}{2} \sum_{i=1}^{N_e} I_i \omega_i^2, \quad (2)$$

where I_i is the moment of inertia about the radial axis, from the center of the sphere to the centroid of the element, and ω_i is the angular velocity about that axis. For joint contacts in QuSI-DEM, the bond energy can be represented as a function of the distance δ between associated vertices of the element polygons. We define a translational component of the bond energy BE_t
60 as,

$$BE_t^{tot} = \frac{1}{2} \sum_{j=1}^{N_b} |\overline{\mathbf{F}^b} \cdot \delta|_j, \quad (3)$$

where the over-bar in this equation indicates averaging between the two vertex pairs associated with each bond, and N_b refers to the total number of bonds in the system. We specify the rotational component of the bond energy as the difference between the total bond energy and the translational component, which can be written as

$$65 \quad BE_r^{tot} = \frac{1}{2} \sum_{j=1}^{N_b} (\mathbf{F}_1^b - \overline{\mathbf{F}^b}) \cdot (\delta_1 - \bar{\delta}) + (\mathbf{F}_2^b - \overline{\mathbf{F}^b}) \cdot (\delta_2 - \bar{\delta}). \quad (4)$$

This potential energy is associated with joint torsion.

2.1.1 Bonded contact

The response of a pair of bonded elements to perturbations in relative velocity and translation on the surface of the sphere are explored as a first test of model stability and energy conservation. The elements in these experiments are 10 cm thick regular
70 hexagons with 1 km² horizontal area (Fig. 1a). The ice properties specified for the elements and their bonds are listed in table



1. The three experiments undertaken in this section do not include physical dissipation mechanisms. The stresses caused by the relative motion of the elements at no point exceed the joint failure criteria. In the first (Case A, Fig. 1a), one element is immobile while the other element is given an initial velocity at an arbitrary angle that would initially produce compression and shear between the elements. In the second (Case B, Fig. 1b), the experiment is repeated with both elements free to move. In the
75 third (Case C, Fig. 1c) both elements are given a much larger initial velocity, but with the same difference in velocity as was present in the first two cases. All cases are run for 12 hours with a timestep of 25 ms.

Physical Parameters for Idealized Test Cases		
Name	Variable	Value
Modulus of Elasticity	E	1 GPa
Poisson's ratio	ν	0.3
Compressive strength	σ_c	125 kPa
Tensile strength	σ_t	12.5
Coefficient of Friction	μ	0.6
Density of ice	ρ	910 kg m ⁻³
Ice thickness	h	100 cm

Table 1. Table of physical parameters used in idealized experiment test cases

In Case A, e_2 is held fixed while e_1 is initialized with a velocity magnitude of 0.75mm/s that stresses the bond to a point close to but not exceeding the failure criteria. The resulting motion involves both translational and rotational oscillations, as can be diagnosed through time series of energy budget terms (Fig. 1d). Energy is exchanged between the initial pure translational
80 kinetic energy and bond energy as the bond is stretched. Much of the transfer is between translational kinetic and translational bond energy. However, because the initial velocity is at an angle relative to the normal direction of the joint, one end of the joint may be in tension while the other is in compression, so some initial translational KE of the movement is transferred into rotational bond energy. Subsequently, the energy is returned to the free element through the force couple at the joint as rotational kinetic energy. Throughout the 20-minute simulation the total energy in the system is well conserved as can be seen
85 by comparing the dashed green (KE_{tot}) and black lines (KE_{init}) in Fig. 1d. This offers reassurance that for this simple test, the numerical implementation is well-coded and reasonably accurate.

In Case B (Fig. 1b,e), because the second element is free to move, very little energy goes into torsion at the bond and less energy goes into BE_t . Rather, the initial translational kinetic energy of element 1 is transferred through the bond into KE_t of the second element, along with KE_r of both elements. Because the frequency of oscillation of the different modes is not
90 commensurate over time, the distribution of energy between the different components varies, but as in case A, there is no significant loss or gain in energy due to the numerical inaccuracy (Fig. 1b,d).

Case C is designed to test the effect of translation over the surface of the sphere simultaneously with bond interactions. In order to do this an additional translational velocity of 7.425 cm s⁻¹ for both elements is added to the setup of the previous



case (Fig. 1c) and, planetary rotation matching that of the Earth ($\Omega_e = 86400s$) is included. The simulation is run for 12 hours
95 as in the previous cases. The elements are initially positioned at a latitude of approximately $64^\circ N$.

The translation and planetary rotation in this case cause the motion of the elements to be more complex than in the previous
two examples. The element pair undertake an inertial oscillation, moving along a nearly circular clockwise path while oscillat-
ing relative to each other (as in case B) due to the slight difference in initial velocities. The deviation from a circular path for a
given element is also due to the elements rotating about their shared center of mass. Oscillations in bond and rotational kinetic
100 energy exhibit phase and amplitude characteristics consistent with Case B, even 12 hours after model initialization (Fig. 1g).
The root mean squared difference between the bond energy between Case B and Case C at the end of the 12 hour simulations
is only approximately 0.31J while that for the total rotational kinetic energy is 0.33J.

The total energy (E) and translational kinetic energy (KE_t) in the Case C experiment is much higher than in either of the
previous cases due to the added translation specified in the initial conditions. Over the course of the 12 hour simulation the two
105 elements alternate between having higher KE_t such that at 12 hours $KE_t(e_2) > KE_t(e_1)$ (Fig. 1f). Although the difference in
initial momentum between the two elements is the same as in Case B, the difference in KE is much larger due to the translation
and it is the transfer of KE_t between the two elements that dominates the pattern, compared to the earlier cases where transfers
from KE_t to BE_t and KE_r were of a similar order of magnitude (Fig. 1 b,d,f). The total energy in the system is relatively well
conserved, decreasing by approximately 4J ($8 \times 10^{-5}\%$) over the 12 hours. A critical aspect to maintaining this level of energy
110 conservation was the utilization of an Adams-Bashforth formulation for time-stepping the Coriolis term as described in Part I.
Using a forward Euler time step for this term for this particular experiment was found to lead to amplification of energy in the
system by approximately 100J in 12 hours.

2.1.2 Collision contact

Similar experiments can be performed to test energy conservation in collision contacts. In order for conservation of energy
115 to hold, the functional relationship between energy and overlap must take the form of a potential function (Feng and Owen,
2004). With the potential function taking the form of a power law (with $n=2$) as specified in Eq. (I-19) the energy stored in the
collision is:

$$CE = \gamma k_e^* A_i^2. \quad (5)$$

Energy conservation in a collision can be demonstrated in a simple experiment in which an element with an initial velocity
120 strikes several other elements as depicted in Fig. 2a. Here again hexagonal 1 km^2 elements are utilized. In this case the elements
are unbonded and at randomly chosen orientations relative to each other. Element e_1 is given a 1 mm s^{-1} velocity directed at
the centroid of e_2 . A timestep of 25 ms is used. Two experiments are conducted, one without friction, the other with a friction
coefficient $\lambda = 0.2$.

The frictionless experiment demonstrates that the total energy is conserved through the sequence of collisions (Fig. 2) .
125 Energy is transferred from e_1 into mostly translational KE of e_2 in the first collision. e_2 subsequently contacts e_3 causing



translation and rotation of both elements, leading to a more forceful collision between e_2 and e_4 . Due to the orientation of e_4 relative to e_2 and the direction of contact, nearly as much energy goes into rotating e_4 as in translating it.

The introduction of friction between the elements limits how much the corner of e_3 slides along the contacting face of e_2 , prolonging the contact and causing an increase in the transfer of both translational and rotational kinetic energy between the two (Fig. 2). With friction, the total energy in the system decreases with each collision, and the cumulative frictional dissipation from all contacts matches the reduction in the total energy in the system.

Energy is well conserved in the frictionless case with a fractional loss of approximately $1.7 \times 10^{-5} KE_o$ through the 3 collisions. Increasing the timestep by a factor of 4 marginally increased this loss to $3.0 \times 10^{-5} KE_o$. The same experiment repeated with the initial kinetic energy one hundred times larger (and collision damage turned off) increased the fractional energy loss to $7.1 \times 10^{-5} KE_o$ with a 25 ms timestep.

2.1.3 Consistency in response to compression of bonded and unbonded elements

In general, the response of bonded elements under pure compression will not be identical to the response of unbonded elements because the bond force $\mathbf{F}_{\text{elas}}^b$ (Eq. (I-13)) takes a different functional form than the collision force $\mathbf{F}_{\text{elas}}^c$ (Eq. (I-19)). As described in Sect. I-3.1, we define a scaling parameter γ parameter that for a purely normal collision of vertex aligned elements, at the time of maximum inter-element penetration matches the collision energy with the bond energy at the same point. To demonstrate the implementation of this in the model, a pair of simulations are performed with two hexagonal elements arranged as in Fig. 3a. e_1 is given an initial velocity of 1 mm s^{-1} directly towards the centroid of e_2 . In the first experiment, the elements are bonded, and in the second they interact through collision contact. The solutions exhibit a quite similar energetic behavior for the initial contact (Fig. I-4b). Almost all the kinetic energy of e_1 is transferred to the contact (BE/CE) and to the acceleration of e_2 , with a very similar peak in energy in the contact (at approximately 1.3s). The small difference in the rate of energy transfer between the cases is again due to the difference in the forcing formulations. A small amount of energy is transferred to rotational kinetic energy in the collision case as the elements separate, whereas a negligible amount occurs in the bonded case. This results from imprecision in the specification of the initial velocity as purely normal. Whereas any tendency toward bending (relative rotation of the elements) in the bonded case results in a restoring force, no such mechanism exists under collision physics.

The two solutions diverge when energy begins to be transferred back to e_1 due to the bond (Fig. I-4b). However, total energy in the system between the two cases remains consistent and well conserved. The γ coefficient used here works with similar results for different initial e_1 velocities and different size elements. The scaling parameter used in the collision contact formulation is a function of the average side length of the two elements, so the bonded and collision contact solutions will tend to deviate from each other more when contacting side lengths differ from the average or when the orientation of $\mathbf{F}_{\text{elas}}^b$ and $\mathbf{F}_{\text{elas}}^c$ differ due to the geometry of the elements and contact.



2.2 Bond Failure

Bonds can fail in tension, shear, or compression. The energy in the bond will be redistributed and dissipated to varying degrees depending on parameter choices. This section describes a set of idealized experiments that demonstrate how these processes are represented in the model. As will be demonstrated, the model is designed to offer a fair amount of flexibility in specifying how redistribution and dissipation is parameterized, but there are subtleties that emerge based on those choices.

In each of these experiments, a control-element is used to gradually increase the normal force at a joint until a desired stress state is reached, as described in I-2.5. A set of basic tensile tests demonstrate the approach. In these simulations, three identical hexagonal elements are bonded in a row (Fig. 4a). The element e_1 is a control element that pulls directly away from e_2 along the centerline between the two elements and perpendicular to the face that connects them. e_3 is a fixed element while e_2 is free to move. The maximum tensile strength σ_t of the bond between e_2 and e_3 is set to 1285 N m^{-2} . Due to the geometry, the contact between e_2 and e_3 experiences forces essentially identical to those between e_1 and e_2 . In the following, we will use the terminology $e_a||e_b$ to refer to the contact between elements **a** and **b**. In these experiments, joint failure is allowed to occur abruptly when a maximum stress criteria is exceeded on a single timestep.

Two experiments were performed. In Case A, the control element pulls directly away from e_2 to a stress of $\sigma = 0.95\sigma_t$. In Case B, the control element attempts to reach a normal tensile stress of $1.05\sigma_t$ at $e_1||e_2$. Figure 4b shows a time series of the stress on $e_2||e_3$, demonstrating how application of the control element allows the system to smoothly approach the desired stress. In Case A, a near -steady state is reached in the system. However, abrupt failure occurs in Case B when the tensile failure stress is exceeded, immediately bringing the stress at $e_2||e_3$ to zero. In this case, e_2 remains bonded to e_1 as the control element continues to move away from e_3 .

In terms of energetics (Fig. 4c), bond energy accumulates in both the $e_1||e_2$ and $e_2||e_3$ contacts. In Case B, upon failure of $e_2||e_3$, all of the bond energy associated with that contact is released into kinetic energy of e_2 . Simultaneously $e_1||e_2$ experiences a partial relaxation causing $\text{KE}(e_2)$ to be larger than the amount of bond energy in the broken bond alone. From this point forward the Case B system oscillates with energy being transferred largely between the $e_1||e_2$ bond and kinetic energy of the one free element (e_2). The viscous damping of the control element leads to a gradual dissipation of the total energy in the system, as can be seen in the diminishing amplitude peaks in $\text{KE}(e_2)$ in Fig. 4c.

2.2.1 Tensile failure and the role of the failure rate parameter

The basic tensile stress experiment described above demonstrates how a large amount of kinetic energy is released into the system upon bond failure. In typical situations where elements share bonds with a number of neighbors, bond failure at one contact will lead to an abrupt change in stress at neighboring contacts. If this amount is large, it can lead to a chain reaction of bond failures that appear as rapid crack propagation in the model. Although this is a physical process that we want to model, the extent, speed, and orientation of crack propagation depend critically on the rate at which the failing contact unloads and the amount of local energy and momentum dissipation. The primary mechanism built into the model to control these properties



during failure is the failure rate parameter f_r introduced in Sect. I-3.2. This parameter allows for local dissipation of energy by providing a progressive weakening of the elastic modulus of the material, when the failure stress criteria are exceeded.

The Case B tensile stress experiment described in the previous section is extended here to demonstrate the effect of varying f_r . High f_r leads to rapid unloading of a failing bond. Here, f_r is varied between 0.05 and 10 s^{-1} such that with a 25ms timestep, $f_r = 10 \text{ s}^{-1}$ corresponds to unloading over 4 time steps and $f_r = 0.05$ corresponds to unloading over 100 time steps. (In the previous section, failure was allowed to occur on a single timestep, $f_r = 40 \text{ s}^{-1}$.)

The $f_r = 10$ case exhibits a response close to that of Case B in Fig. 4, with stress and strain behavior representative of brittle failure (Fig. 5b,c). For lower failure rates, energy in the bond is lost more gradually due to the softening and weakening of the material described in Eq. (I-28) and Eq. (I-29). The stress at the contact experiences a more gradual decline, and the strain rate increases post-failure representative of a more plastic material response. The kinetic energy released into the system can be significant, and the timing of peak KE occurs later, shortly after the bond fully fails (Fig. 5c). The bond energy is proportional to both the stretch between the elements and the elastic force. So, for loading rates high enough, as the bond continues to stretch and weaken, energy continues to accumulate in the failing joint until the rate of diminishment of the elastic force exceeds the rate of increasing strain.

The failure rate parameter allows the model to emulate a range of physical behavior but can also be a numerical convenience. Modeling a strong, brittle material that fails abruptly tends to produce shattering patterns with this class of models, as the energy necessary to break one bond propagates rapidly to stress other bonds with similar failure criteria. Material softening through f_r diminishes crack propagation and shattering behavior, but it is unclear to what extent this is appropriate for sea ice. At the element scale of tens of meters to kilometers being primarily considered here, it is likely that a variety of dissipative processes occur as a crack develops over these distances. So it is not unreasonable to expect that only a fraction of the energy stored in the straining of the ice pre-failure, propagates beyond these scales.

2.2.2 The effect of different normal and shear orientations at a contact

In Sect. I-2.3 two alternatives were mentioned for specifying the orientation of material properties at a joint. One option is to consider the edge orientation between the two elements to define normal and shear directions. In this case, the elastic modulus describes the stress strain relationship normal to the edge and the shear modulus describes the relationship tangential to it. Failure criteria are aligned similarly. This representation is referred to as face-oriented (FO) from here on. The other alternative considered here, is to use the centerline between the two element centers of mass as the direction of pure tension/compression and the shear direction to be normal to this (referred to as center-of-mass oriented, CO). In the CO alternative, the edge between the polygons necessarily remains in the orientation of the failure plane, but does not play a role in the failure conditions. This alternative creates a model that is more agnostic towards the edge orientation, allowing a bonded material to exhibit more consistent behavior independent of how randomly specified polygons constitute its internal joints. In ways, it is more consistent with circular-disk based discrete element methods which also use relative positions of the centers of mass for orienting material properties. The FO approach is the more popular for polygon-based models where the shape of the element is meant to reflect a physical structure. With the FO conceptual model, the polygon edges represent lines of weakness in the material, as might



be the case for a fused aggregate of previously distinct flows. In either case element shape, and relative orientation can have substantial impact on the model response. In this section, we demonstrate how these different choices affect stress-strain and failure behavior for several idealized cases.

As in the previous section, tensile failure experiments are performed using a control element to draw a free element away from a fixed element until a desired tensile stress is reached. In these simulations, the tensile stress at the $e_1||e_2$ joint is raised to $0.95\sigma_t$ then maintained. To start with, two three-hexagon element configurations are considered. For the first, the $e_1||e_2$ joint is parallel to the $e_2||e_3$ joint (Fig. 6a). This configuration is identical to Case A described above. Here, the normal to the $e_2||e_3$ edge is aligned with the line connecting the centers-of-mass of the two elements. For the second, the shared edge between e_2 and e_3 is skewed such that the face-normal is at a 30° angle to the line between the COMs (Fig. 6b). Simulations are conducted with each of these setups with contact normal and shear directions defined as either CO or FO

Not surprisingly, the model response at $e_2||e_3$ is very similar for the CO or FO formulation when the contacting edge is perpendicular to the line between the element centroids (yellow and blue-dashed lines in Fig. 6c and d). The two schemes give very similar stresses and strains and joint failure does not occur as failure criteria are not exceeded at the joint.

In the pair of companion experiments with the $e_3||e_3$ orientation skewed (Fig. 6b), the response to identical forcing differs noticeably. The CO experiment case exhibits contact failure close to the tensile failure stress observed in the Case B examples of Fig. 4. The reason why it fails here when the control element brings the stress at $e_1||e_2$ to $0.95\sigma_t$ is that the effective width of the $e_2||e_3$ bond in the direction normal to the center line is slightly less than the width of the bond in the regular hexagon case. Given the same forcing, the stress increases more rapidly and reaches a higher peak value at $e_1||e_2$. Also, the orientation of the line between the centroids e_2 and e_3 differs slightly from that between e_1 and e_2 due to the skew element shapes, leading to a non-zero shear stress (Fig. 6d). The response is basically that one might expect if considering the set of elements as a continuous material. The failure occurs at a stress close to the failure criteria, at a position close to where the material is most narrow. But the orientation of the failure plane is unrelated to the material properties.

The FO experiment with the skew e_3 hexagon in contrast leads to failure sooner and at noticeably lower stress and strain values (Fig. 6c,d). Here, because shear and tensile directions are measured with respect to the edge orientation, which is at a 30° angle to the direction of the applied force, shear stress is larger and tensile stress smaller than with the CO assumption. The early failure occurs by exceeding the shear stress failure criteria (Eq. (I-18)). The response is akin to that of two unbreakable floes with the material properties representing the characteristic of the edge between them, and the orientation associated with a preferential fracture angle.

Another aspect of element geometry that affects bond characteristics is demonstrated by repeating the experiment above (6b) but with the fixed element represented as a triangle rather than a hexagon (7b). In this situation, the elastic response and failure characteristics in the FO case only show minor differences as the face orientation of contact between e_2 and e_3 is unchanged. But due to the COM orientation between e_2 and e_3 being different from the direction of tension, the CO case exhibits higher shear than in the earlier case with less strain (7c).

Both the CO and FO approach exhibit variability due to the element geometry. The FO approach can work well for element distributions that represent distinct ice floes, but leaves more ambiguity in specifying material characteristics when distinct



260 floes are represented as clusters of elements because edge orientations cannot be chosen in general, when filling a domain with convex polygons. The CO approach is less sensitive to edge orientation but still has a dependence on the relative positions of COMs that is not based in physics. In general, for similar sized and shaped elements, the FO approach introduces more variability in characteristics at a contact because the angle of orientation of the contacting face can vary more than the angle between COMs. In either case though, the heterogeneity in properties can be justified given the likely characteristics of a realistic ice flow. It may be argued that the geometric effects of the random elements average-out over a large enough set of elements. Nonetheless, it is important to note the degree to which element geometry is determining the nature of this randomization. This highlights the value in considering simulation ensembles that employ a variety of element distributions for polygon-based DEM studies.

2.2.3 Compressive failure

270 As discussed in Sect. I-3.3 accurately representing bond failure and the post-failure behavior of elements that are under compression presents challenges not found in tensile failure because the transition between contact models can lead to numerical instability and unintentional changes in the stress and energy state of the system. In this section idealized compressive experiment akin to the tensile experiments of the previous section are presented to demonstrate the range of post failure behavior the model can exhibit as a function of parameter choices.

275 As in the tensile experiment, abrupt brittle failure of the bond can release a large amount of kinetic energy into the system. However, as failure under compressive strain tends to occur at much higher stresses in sea ice than tensile failure, the effect is larger. The amount of KE released could cause accelerations that violate numerical stability constraints and lead to cascading bond failure throughout the model domain (appearing as shattering and/or rapid acceleration of elements away from each other).

280 As in tension, the failure rate parameter (Sect. 2.2.1) is useful in compression to provide local energy dissipation and allow a more gradual failure of a joint. Because failure under compression occurs at higher stress, a proportionately lower failure rate is applied to locally dissipate more energy as the bond weakens (Eq. (I-27)). A joint that breaks under compression transitions to a collision contact as described in Sect. I-3.3. This offers additional dissipation mechanisms (through the specification of α and β). Post-failure weakening under sustained compression can also be represented (Sect. I-3.4).

285 These parameterizations are designed to provide flexibility in tuning the model as our understanding of sea ice mechanics develops. Few observations exist for quantifying the rate of failure, the amount of local energy dissipation, and the residual strength of sea ice on geophysical scales. They are likely to be functions of the level of confinement, the relative shear and normal stress at which failure occurred, the degree to which out-of-plane processes such as ridging may occur, and the particulars of the fractured ice characteristics at smaller scales. Here, we demonstrate the range of response that the model can exhibit for a simple system where failure under compression occurs.

290 This experiment repeats the simulation setup of Sect. 2.2.1 but with compressive rather than tensile forcing applied by the control element (Fig. 8a). In this case, the control element is gradually moved towards the other two elements to reach and maintain a stress of $1.05\sigma_c$ at $e_1||e_2$. The $e_2||e_3$ bond fails before this is reached (at $\sigma_n = \sigma_c$) so the control element continues



to increase the compression on the set of elements post-failure. For all experiments $f_r = 0.05 \text{ s}^{-1}$, $\alpha = 1$, and $\beta = 1$. Two parameters are varied; D_f , the fractional strength at which a bond transitions to a collision (Sect. I-3.2), and f_{rc} , the rate at which a collision contact continues to weaken under sustained compression (Sect. I-3.4).

295 The basic response at the $e_2||e_3$ contact is for stress to increase until the onset of failure occurs at σ_c (Fig. 8b). After this point, gradual weakening of the failing bond causes stress to decrease until damage to the joint has reached the failure threshold $D = D_f$. When this occurs, the contact transitions from bonded to collision. Shear stress at the contact becomes frictional rather than elastic. Post-failure weakening further reduces the ability of the material at the contact to support compressive stress. Eventually, the collision damage drops to D_{cr} after which point the contact acts to maintain a constant stress level under
300 further compression. To do this ΔA_i (the change in overlap area on a timestep) is subtracted from A_i at the end of each time step. Since ΔA_i is larger for weaker material, for a given control element forcing, the collision damage gradually erodes the overlap area between the elements, eventually dropping the energy in the contact to zero.

Reducing the value of D_f lowers the residual strength of the material at the point that the shear at a contact becomes determined by friction. It also determines the amount of energy dissipated locally in the bond prior to this transition (Fig. 8c).
305 Higher values of f_{rc} increase the rate of post-failure weakening (Fig. 8b and c). The post-failure slope of the stress versus time and energy versus time curves for the idealized experiment (depicted in Fig. 8), depend on both the rate of weakening and the rate of compression applied by the control element. Low values of f_{rc} can lead to a period of further accumulation of energy and stress in the contact post-failure before damage accumulations begins to reduce both.

An important feature of the model that is demonstrated in Fig. 8b and c is how the model maintains the normal stress and
310 the normal force contribution to contact energy through the transition from bond to collision (when $\alpha = 1$, and $\beta = 1$). For the purely compressive case, this reduces the energy transfer from contact to kinetic energy at bond failure to zero. Alternatively, the α and β coefficients in Eq. (I-32) and Eq. (I-33) could be set to values less than 1 to provide an impulsive release of momentum and/or energy at failure. Setting α and β to 1, does not prevent impulsive release of momentum or energy when a bond under compressive shear fails. This case is explored in the next section.

315 2.2.4 Shear failure

Bonds that fail by exceeding the shear failure criteria while under compression exhibit behavior that is distinct from the pure compression cases presented in the previous section. This is because *post-failure* softening in the model occurs only when the normal stress component exceeds σ_c . In this section, an idealized test case is presented that demonstrates the stress and energy characteristics of bond failure and weakening for a contact that is significantly sheared. To simulate this, three one km²
320 hexagonal elements are arranged in an arch as depicted in Fig. 9a, with a 120° angle between the center lines connecting e_1 to e_2 and e_2 to e_3 . As in the previous cases, a control element (e_1) is pressed towards a free element (e_2) which is bonded to a fixed element. The material and failure rate characteristics are consistent with earlier experiments ($D_f = 0.25$, $f_r = 0.2$, $f_c = 0.25f_r$, $\alpha = 1$, $\beta = 1$, CO contact orientation). with three variations: 1) The coefficient of friction is varied [0.3, 0.6], 2) the shape of the free and fixed elements are varied (regular and skewed hexagons Fig. 9a), and 3) the maximum compressive
325 pressure applied by the control element on e_2 is varied [$0.5\sigma_c$, $0.2\sigma_c$].



The basic stress response of the contact between e_2 and e_3 exhibits two peaks (**9b**). Before bond failure, the normal and shear stress components (as measured relative to the line connecting the COMs of the two elements) increase proportionately as a function of the angle between the elements and the forcing. Once the shear stress failure criteria is exceeded (Eq. (I-18)) the contact between the elements weakens until the failure strength D_f is reached. At this point the contact transitions from bond to collision. Given the coefficient choices both the normal stress and the repulsive component of contact energy in the collision are set to be consistent with that of the failed bond. But since there are tangential contributions to stress and energy, the total energy and the shear stress in the collision contact are discontinuous with the pre-failure state (Fig. 9b,c). In the absence of viscous damping terms the momentum and energy associated with this abrupt change in shear is propagated away from the contact.

The second peak in the stress and energy failure response appears because stress continues to accumulate in the collision contact until σ_c is exceeded. The double-peaked behavior with shear failure under compression can be interpreted to represent elements retaining much of their compressive strength despite the appearance of a shear fault. The amount of compressive strength remaining after the initial shear-failure can be tuned with D_f which determines how gradually compressive stress will increase with compressive strain under sustained forcing.

The set of experiments presented for this test case demonstrate several aspects of shear failure. In these experiments, normal and shear directions were determined relative to the line between the COMs of the elements, so the orientation of the face between e_2 and e_3 does not affect the relative magnitude of normal and shear stress. However, comparison of the experiments with regular versus skew-shaped hexagons in Fig. 9b (red and blue lines), demonstrates how a relatively small change in orientation of the shear and normal direction specification (due to the slightly different location of the COM of the skew and regular e_3) can lead to different failure behavior. In these experiments, the shear stress is slightly less relative to the normal stress for the skew hexagon configuration. But the small difference allows accumulation of approximately 50% more stress in the bond before failure compared to the regular hexagon case as the difference is enough to avoid exceeding the shear failure criteria for significantly longer. In order to match normal stress and energy post failure, D_c in the skew hexagon case was lower than that in the regular hexagon case also, which leads to a much earlier secondary failure of the contact (Fig. 9b).

Altering the friction coefficient in this experimental setup changes the result as expected. A larger coefficient of friction leads to a larger post failure shear stress and greater frictional energy dissipation (Fig. 9b,c). As lateral movement is attenuated by the friction, the normal stress (at a given time) is slightly higher and increases slightly more rapidly post-failure with higher friction as well.

The final experiment in this section illustrates an interesting case in which the maximum compressive loading applied by the control element (σ_{cntr}) falls between the normal stress at which shear failure first occurs and the σ_c at which the collision contact begins to weaken (green line in Fig. 9b). In this case the effective strain rate at the $e_2||e_3$ contact diminishes as σ_{cntr} is approached. This allows an oscillation to develop in the shear stress associated with the frictional contact between the two elements that is consistent with stick-slip behavior, where the nonlinearity of the frictional relationship causes intermittent accelerations and decelerations. Varying the frictional force parameterization slightly showed that this behavior is sensitive to the time-averaged relative velocity used to determine the frictional force. The model does not include a static friction



formulation. So, it is not immediately apparent whether this phenomena should be considered more physical or numerical. Here we just note that it is behavior that is exhibited in the model. As can be the case with real stick-slip systems, frictional dissipation is enhanced while this process persists (9c) in the model.

In this section, we have demonstrated how we design element contact physics and choose parameters such that simulations might be realistic and stable. In the following section, we explore the contact physics parameter space and identify sensitivity of solution of an arching event to parameter choice.

3 Case Study: Ice arch formation at Bering Strait

The Bering Strait located between the Seward Peninsula of Alaska in the United States and the Chukotka Peninsula of Russia is a geographic constriction that limits the exchange of sea ice between the Arctic Ocean and the Bering Sea. In typical winters, sea ice coverage develops earlier north of the Strait (in the Chukchi Sea). Predominantly northerly winter winds tend to lead to transport of seasonal sea ice through the Strait onto the Bering Sea Shelf where it supplements local production. Warming and wind reversals in the spring tend to reverse the direction of transport, causing remnant Bering Sea ice to flux back into the Chukchi. Sea ice extent over the winter season in the Bering Sea can depend significantly on the amount of ice import and export through the strait (Smith et al., 2025).

The sea ice exchange between the Chukchi and Bering Sea is simulated with QuSI-DEM (Fig. 10a). The intention is to explore model sensitivity to parameters in a geometry that has realistic aspects but also bears similarity to classic experiments of granular flow through a constriction. The Chukchi Sea and Bering Strait have ice flow that is analogous to a grain hopper Shapiro and Burns (1975), displaying frictional jamming and failure under tension and shear. We chose this case study because it tests multiple aspects of the contact physics.

A case is considered in which the model is initialized with a continuous sheet of sea ice that spans the lower Chukchi Sea attached to both the Chukchi and Seward Peninsular coasts. These coastal sea ice boundaries are approximated as consisting of smooth straight walls with the same material properties as the ice. More realistic coastline or landfast ice boundary geometries are neglected in this case study to focus attention on the ice arching behavior at the strait.

Although it is recognized that winter sea ice extends across much of the Arctic ocean, the northern edge of the ice pack is specified as an arc between the two coastlines with farthest northern extent at approximately 67.8 °N directly north of the straight. This is done primarily for computational economy. The Diomed Islands are included as boundary elements in the simulations, despite being located south of the specified ice pack because they eventually impact sea ice drift as floes advect through the strait. Double ice arches are also sometimes observed in satellite images of Bering Strait, when the sea ice pack extends south as far as the Diomedes, but this is not explored here. We focus on the formation of a single arch across the strait and the upstream ice pack behavior. In the experiments, a northerly wind stress is applied to the ice, ramped to a speed of 10 ms⁻¹ over the first hour, then sustained for the duration of the simulations which run from 6 to 18 hours. Simulations are performed with planetary rotation active (1 cycle per 86400 s). The ocean beneath the ice is considered to be motionless.



For the majority of the experiments presented here, discrete element polygons of roughly uniform 3.5 km^2 area fill the ice covered region (Fig. 10a). To generate the elements, a randomized perturbation is applied to node locations determined in the triangulation described in Sect. I-5 prior to tessellation to generate a more heterogeneous set of element shapes. A set of 12 such domain discretizations that identically fill the region, is generated so that ensemble simulations may be performed. Sensitivity to element size is explored in Sect. 3.3 below, for which an ensemble of 4 discretizations is generated with the highest density of elements at approximately 1/3rd the area ($\sim 1.2 \text{ km}^2$) of the near-uniform element size (Fig. 10b) but with approximately the same total number of elements in the domain.

In all experiments, elements are considered initially bonded to all their neighbors and to the coastlines. This state represents an idealization of conditions observed in the vicinity of Bering Strait, as often, over the course of a winter satellite imagery will show interaction of a melange of broken and breaking floes over a wide range of sizes that converge, diverge, and shear in response to varying wind intensity and direction. Here, the focus is placed on the initial break up of the continuous sheet, in order to demonstrate model behavior and better understand the function of various model parameters.

Settings for this case study are presented in Table 2. We choose as a baseline a set of parameters for which the bond properties of the ice are weak enough that a marginally unstable arch forms but not so weak that the full ice sheet rapidly fails. This allows us to explore a portion of parameter space where the model shows sensitivity to modest variation in parameter values. But the settings are not chosen to match any particular field observations of a specific ice-arching event.

Physical Parameters for Idealized Test Cases		
Name	Variable	Value (units)
Modulus of Elasticity	E	1 GPa
Shear Modulus	ν	0.3
Compressive strength	σ_c	60, 90 , 120 kPa
Tensile strength	σ_t	6, 9 , 120 kPa
friction parameter	μ	0.6
Density of ice	ρ	910 kg m^{-3}
Ice thickness	h	100 cm
Atmos-Ice drag coefficient (kinematic)	C_d^{ai}	$0.00122 \text{ m}^2 \text{ s}^{-1}$
Ocean-Ice drag coefficient (kinematic)	C_d^{oi}	$0.005 \text{ m}^2 \text{ s}^{-1}$
Coefficient of friction	λ	0.3
Viscous damping	η	0.1
Failure rate	f_r	0.01, 0.1 s^{-1}
Durability at bond failure	D_c^o	0.1, 0.25 , 0.5
Durability at residual strength	D_f	0.01, 0.05, 0.1 , 0.2

Table 2. Table of physical and model parameters used in the Bering Strait case study. Multiple values indicate parameters varied in sensitivity studies, with bold indicating the base-case.



The evolution of our base simulation case is shown in Fig. 11. A crack pattern in the shape of an arch forms within an hour of the onset of winds. Failure first occurs at the tip of the Chukotka Peninsula with crack propagation proceeding eastward approximating the shape of a southward facing parabola. The sea ice floes formed by this cracking separate from the ice sheet reaching speeds of approximately 0.35 ms^{-1} before oceanic drag balances wind forcing (Fig. 11c). Although floes are initially accelerated southward under the wind influence, they exhibit an anti-cyclonic rotation over time as a result of the Coriolis force (which appears as a westward drift in the position of the detached floes between panels b) and d) of Fig. 11. The leading edge of the drifting floes begins to interact with the Diomedes after approximately 9 hours, causing further breakup and deceleration (Fig. 11d).

By the end of the 12 hour simulation, a progression of leads is present, indicative of the continued erosion of the arch (Fig. 11d). This resembles what is frequently observed in visible spectrum satellite imagery of the strait in winter and spring. Cracking of the ice sheet occurs most ubiquitously on the eastern (Seward Peninsula) side of the arch due to the asymmetry in the orientation of the two coastlines. The Chukchi coast slopes approximately 45 degrees west of North, while the Seward Coast is approximately 55 degrees east of North. As will be presented in more detail below, this results in nearshore fracture and sliding of the arch along the Chukchi coastline, which increases compressive stresses near the pinning point on the Seward Coast. This increased stress leads to continued weakening and joint failure, causing most elements to completely disconnect from their neighbors. The lead pattern results as bands of frictionally interacting elements break off from the main floe and advect towards the strait (Fig. 11d).

3.1 Sensitivity to ice strength

The slowly eroding arch just described develops when the loading on the ice is only marginally greater than its bearing capacity. Here, we explore the model response when the ice strength is varied. If the simulation is repeated with weaker ice (i.e., ice with lower values for the compressive and tensile stress parameters, $\sigma_c = 60 \text{ kPa}$, $\sigma_t = 6 \text{ kPa}$, referred to as 'low-strength'), cracking occurs earlier and more extensively (Fig. 12a,d). In this case a large portion of the Chukchi sea ice becomes pack ice, broken into individual element-sized floes advecting under the influence of wind forcing towards the Bering Sea. In contrast, if the simulation is repeated with somewhat stronger ice than in the base case ($\sigma_c = 120 \text{ kPa}$, $\sigma_t = 12 \text{ kPa}$, referred to as 'high-strength'), a stable arch develops, with virtually no cracks north of the section near the mouth of the strait that initially becomes detached (Fig. 12c,f).

Examining the number of failed contacts as a function of time (Fig. 12g) provides a measure of the rate of ice breakup. The onset of joint failure occurs as the winds ramp up over the first hour in all three cases, occurring earlier and with more cracking the lower the ice strength. In the high-strength case, after the initial separation, no further bond failures occur until approximately 9.5 hours into the simulation, when the separated floe contacts the Diomedes Islands. But in each of the weaker cases, cracking is continuous throughout the 12 hour simulation. The number of cracks increases at a close to linear rate in the base case, as the arch gradually erodes. For the more rapidly eroding arch in the low-strength case, the joint failure rate varies, slowing down, accelerating, and slowing down again over the simulation as the motility of the ice floes brings the system into arrangements with differing resistances to breakage.



It is interesting to note that despite the failure of bonds, the continued erosion of the ice sheet in the two weaker ice simulations leads to increased domain-integrated bond energy relative to the stable arch case (Fig. 12h). This is because the remaining
445 bonds are under increasingly larger compressive and shear strains as the sheet is bent towards the strait. For the low-strength case, the rate of loss of bond energy due to bond failure eventually exceeds the effect of increasing strain, while for the base case, bond energy continues to increase until the end of the simulation. In the stable arch case, bond energy integrated over the domain stabilizes approaching an equilibrium value within an hour but exhibits gradually diminishing oscillations about that equilibrium due to the propagation of weakly damped elastic waves through the sheet, likely caused by the initial arch
450 detachment event. The beat-frequency that appears is likely due to the presence of two incommensurate oscillatory modes.

Once the initial arch forms in all three cases, the domain-integrated kinetic energy (KE) in the system (Fig. 12i) quickly exceeds the amount of energy stored in bonds. This energy is almost entirely in the floes that are in, or close to, free drift. In the two lower ice strength cases, as the ice sheet continues to erode, wind-work continues to increase. In contrast, in the high-strength case, the domain integrated KE reaches an equilibrium after approximately 5 hours due to ocean drag balancing
455 the atmospheric forcing in the southward advecting floe. The effect of floes colliding with the Diomedes causes a brief drop in KE in all cases (Fig. 12i). The scale of frictional dissipation due to contact between floes is also shown in Fig. 12i. In the lowest strength case, where there are many broken bonds, this frictional dissipation exceeds the total bond energy by approximately four hours into the simulation and becomes approximately 25% as large as the total kinetic energy in the system by the end of the simulation. The range of behavior exhibited by these three cases illustrates how important accurate representation of sea
460 ice failure and frictional characteristics can be, as air-sea exchange of momentum and heat will critically depend on the sea ice dynamics.

3.2 Sensitivity to element arrangement

Discrete element modeling introduces heterogeneity into material properties because it only determines stresses and strains at particular orientations (either relative to element faces or center-of-mass positions in this formulation). In generating the
465 initial element distributions, randomization of element centroid positions after the Delaunay triangulation step (Sect. 5) tends to produce more homogeneous material properties when quantified on scales significantly larger than the element scale. But given limitations computational costs can place on model resolution, the specific element arrangement can play a critical role. For elements arranged in the shape of an arch, if contact faces are oriented perpendicular to the curve of the optimal arch shape (usually somewhere between semi-circular and parabolic, depending on load distribution and orientation), the load bearing
470 capacity will be maximized. Each element contact face will be normal to the predominantly compressive loading. But in general, random face orientations will lead some contacts to be primarily in tension or shear and more easily prone to failure (e.g., an arch capstone that is broader at the base than top).

In order to explore the model sensitivity to element arrangement, an ensemble of twelve distinct element distributions was generated, and simulations were run with the same parameter values and forcing of the base case simulation. The solutions
475 (13) ranged from showing eroding arch behavior similar to the base case, to a persistently stable arch, reminiscent of the high-strength example ($\sigma_c=120$ kPa) in Fig. 12c,f. In the case of the eroding arch, the western portion of the ice sheet is increasingly



strained until secondary failure occurs along the Chukchi coast, leading to a strong zonal asymmetry in sea ice displacement (13a). However, other ensemble members exhibit stable Chukchi and Seward coastlines for the duration of the simulation with element displacement largest along the centerline of the arch (13d). Different ensemble members exhibit differing degrees of failure, but in all cases that did not stabilize upon separation of the initial arch cut-out, the strain on the western side of the domain increased disproportionately as the arch failed. This is likely due to the steeper angle relative to the wind of the Chukchi coast compared to the Seward coast. In the ensemble, it is not necessarily true that an eroding arch will fail indefinitely. Given the specifics of element geometry, a range of stable arch shapes with varying base widths and heights are possible. Stresses in the ice sheet evolve as floes detach and load is redistributed, providing the possibility of stabilization after initial failure. Figure 14a displays the fraction of failed joints for the ensemble of 12 simulations. Whereas the fraction steadily increases for the base-case experiment (BE, blue line in Fig. 14), numerous other members show element failure rates approaching zero later in the simulation (excluding failures associated with separated floes striking the Diomedes) and near constant domain integrated BE (14c). Similarly, some ensemble members exhibit notably later separation of the arch cutout (visible in the domain averaged KE Fig. 14b).

The distinction between cases with and without ice sheet separation from the Chukchi coast appears clearly in the domain integrated bond energy, as the greater deformation of the sheet leads to larger area integrated bond energy. It appears as enhanced inter-element frictional dissipation as well due to the greater amount of breakup and sliding friction with the Chukchi coast. Ensemble member 7 (panel c in Fig. 13, and highlighted green lines in Fig. 14) is an interesting case that exhibits a large arch, late floe separation, and high frictional dissipation. It appears to stabilize in part due to the frictional resistance between the ice sheet and the Chukchi coast, after all the element-boundary contacts along that edge have failed.

3.3 Sensitivity to element size

In order to explore the model response as a function of element size, the ensemble experiment of Sect. 3.2 is repeated with four instances of elements three times smaller than those in the base experiment in the vicinity of the arch (Fig. 10b). In these experiments, in order to save on computation, mesh refinement is used, so that while elements along the Chukchi and Seward coasts and across the Bering Strait have an average area of 1 km^2 (compared to 3 km^2 in the base case), elements near the northern edge of the domain average 10 km^2 (Fig. 15a). In addition to the element size and distribution, all physical model parameters remain the same. A model timestep of 25ms is used, compared to 50ms in the base experiment, for numerical stability. Simulations are performed for 6 hours to examine the initial arch formation process and immediate aftermath. In simulations longer than this, cracking would tend to propagate into the lower resolution portion of the variable-resolution domain, complicating the analysis.

The results of this experiment are consistent with the base case ensemble, with comparable arches forming and similar energetics (Fig. 15c,d,e). In terms of domain integrated bond energy, kinetic energy, and frictional dissipation the four members of the high resolution ensemble span the range of results in the lower resolution ensemble. The two distinct outcomes observed in the base case, of a stable arch, and arch failure with detachment of the ice sheet along the Chukchi coast, emerge with the



510 higher resolution as well. The oscillation observed in bond energy for the stable arch ensemble members in (Fig. 14c), is also present here.

One difference between the lower and higher resolution simulations is that in three of the four high resolution ensemble members the total crack length is longer compared to the lower resolution ensemble (Fig. 15b). At both resolutions, the model tends to produce damage zones where nearly all element contacts fail. The greater crack length in the high resolution ensemble 515 members may be a result of there simply being more edges that failure can occur along.

It is expected that the specific element distribution should play a diminishing role as the resolution is increased. Although there can always exist the possibility of an initial element distribution that is particularly fortuitous for maintaining the arch, as resolution increases, the likelihood that unfavorable contact orientations will exist increases. For a marginally stable configuration considered here, it is likely that the continuously eroding cases are the more likely to occur. Although the rather 520 small ensemble of higher resolution simulations spanned the behavior observed at the lower resolution, only one of the four simulations still exhibited a stable arch which may indicate we are approaching statistical convergence to the eroding arch solution. A significantly larger ensemble is necessary to resolve this further.

3.4 Sensitivity to failure rate parameter

In Sect. 2.2 the failure rate parameter was introduced which effectively provided softening of the material in the vicinity 525 of the failure by allowing the elastic moduli of the contacts to decrease over a number of time steps when failure criteria are exceeded. Lower failure rates effectively produce a more ductile, less brittle response because the load carried by the joint is more gradually transferred to neighboring contacts during weakening. This is supported by the response observed in the idealized tensile failure experiment (Sect. 2.2) which showed more elastic energy being released at higher failure rates. To explore the model sensitivity to this parameter, simulations were repeated for the four ensemble members highlighted in 530 Sect. 3.2 with $f_r = 0.01 \text{ s}^{-1}$ rather than 0.1 s^{-1} . In all cases, with either failure rate, the arch developed through crack initiation at the tip of the Chukchi Peninsula, followed by extension eastward to the tip of the Seward Peninsula (Fig. 16). But, the slower failure rate experiments experienced slower arch formation and detachment. This was due both to individual joints failing more gradually, and to differences in efficacy with which the crack propagated.

Arch formation is fundamentally a process of tensile failure. But in a discrete element model, where failure can only occur 535 relative to element contact orientations, approximating the form of an arch can be accommodated if contacts that are under compressive shear also become susceptible to failure. In the failure rate sensitivity experiments, all the lower failure rate experiments exhibited less failure at joints under compression but similar failure at joints in tension (Fig. 16i,j). Secondary damage to neighboring joints occurs with more brittle cracking in the model. More abrupt unloading of a contact as occurs with a higher failure rate, increases the strain on neighboring joints. In the higher failure rate cases, this is more often sufficient 540 to cause neighboring joints that are experiencing compressive shear to exceed their failure criteria. When failure is largely limited to tensile joints, as appears to be the case with the low failure rate, the pathway for crack propagation becomes more tortuous in the model. This results in some low failure rate experiments exhibiting broader failure zones where nearly all element contacts eventually failed in tension as the arch developed (Fig. 16f,h).



3.5 Sensitivity to material strength after joint failure

545 Material failure in this model involves softening of the contact prior to breakage of the joint. At the breakage point, the tangential contact physics changes from representing shear within a continuous material, represented by lateral vertex displacements, to friction between two discrete objects determined by the friction coefficient and normal compressive force. In addition to f_r presented above, a second important parameter in this process is D_c , the fractional softening of the joint at which the transition from bonded joint to collision contact occurs. Sensitivity to this parameter was explored in a series of simulations that differed from the base case (ensemble member 2) by specifying a higher ($D_c = 0.5$) or lower ($D_c = 0.1$) value for this parameter than in the base case ($D_c = 0.25$). The results illustrate the role that the post-failure strength of the material can have on the overall response of the system through its impact on frictional resistance to sliding.

Figure 17a, b and c show the shear stress in the sea ice from simulations with different D_c values after four hours of simulation. In all three cases, shear stress is concentrated along the coast of the Seward Peninsula close to the base of the arch. But the extent of secondary cracking and the size of the high S_s region increases as D_c decreases. With $D_c = 0.1$ erosion of the ice sheet proceeds rapidly following the initial arch formation. Secondary, much larger arches, have started to form extending out from both coasts. In contrast, the simulation with $D_c = 0.5$ has largely equilibrated after 4 hours with no secondary arch formation. The difference between these cases is related to the frictional sliding resistance of the Chukchi coastline. As mentioned earlier, bonds tend to break along the steeper Chukchi coast under the influence of wind stress. This leads to the ice sheet acting as a cantilever beam, attached at the Seward Coast, and sliding with frictional resistance along the Chukchi Coast. Higher values of D_c are associated with higher post-failure elastic and shear moduli. This allows for larger normal stresses at the Chukchi coastal boundary and consequent greater frictional resistance to sliding. This explanation is corroborated by repeating the base experiment with $D_c = 0.25$, but changing the post- to pre-failure normal force ratio (Eq. (I-32)) α coefficient from 1.0 to 0.5. The lower value of α forces the normal stress at a contact immediately post-failure to be half its pre-failure value. This simulation (Fig. 17e) shows a response most similar to the $D_c = 0.1$ simulation. There is extensive secondary cracking indicative of the higher strain associated with the bending displacement.

These effects can be quantified by examining time series of latitudinal displacement over a Chukchi coast subdomain along with average shear and normal stress along the coastal boundary (Fig. 17d and f). Note that in all cases the ice sheet entirely detaches (becomes unbonded) along this stretch of coastline. Separation occurs earliest with the highest D_c , as expected given the model design (Fig. 8). Higher values of D_c are associated with near-cessation of southward sliding after the initial adjustment period when the ice sheet separates from the coast. In contrast, the simulation with $D_c = 0.1$ and with $D_c = 0.25$, $\alpha = 0.5$ approach steady latitudinal displacement rates of approximately 5 cm min^{-1} within two hours. The average normal and shear stress magnitudes in the subdomain illustrate how normal stress along with higher shear stress, are maintained post-failure with high D_c , while a greater degree of pre-failure weakening leads to reduced frictional contact. For the lower friction cases the average S_s on the boundary is approximately λS_n . But in the high friction case, $S_s/S_n < \lambda$ because at some locations λS_n is larger than necessary to halt the relative motion of elements.



3.6 Sensitivity to residual strength

When the sea ice material properties lead to stable arch formation in the simulations, the system response is largely determined by the pre-failure material properties and the failure criteria. But in situations where the loading on the system causes continued high compressive stress on contacts past failure, the specific representation of the post-failure characteristics can play a prominent role (as demonstrated in the sensitivity to f_r and D_f above). D_t sets the final limit to the weakening of the material (Sect. I-3.4). Once a collision contact under compressive loading has weakened by a factor of $1/D_t$, the strength of the material, and consequently its elastic and shear moduli, do not weaken further. The collision force is then proportional to the increase in the area of overlap over the time step rather than the total area of overlap. This represents a residual state where strain can continue to increase without a proportional increase in stress.

In order to explore sensitivity to this parameter, a series of five 18-hour long simulations were compared. In four of them, D_t was varied ([0.01, 0.05, 0.1, 0.2]), and all other parameters are kept to their base simulation values. The fifth simulation used the base case value of D_t , but a lower peak strength ($\sigma_c = 90$). In all cases, the initial element distribution from the ensemble member 2 simulation was used (Fig. 14).

Simulations with values of D_t lower than the base case value of 0.1 exhibited much greater cracking and arch erosion (Fig. 18e) because in these cases, heavily stressed failed contacts have less load bearing capacity. At the lowest simulated parameter value, $D_t = 0.01$ nearly the entire the ice sheet is in motion after 18 hours of northerly wind forcing. A fracture forms at approximately $167^\circ W$ on the Seward coast that reaches the northern extent of the ice sheet with ice to the west of the lead, advecting towards the strait notably faster than does ice in the eastern portion of the domain (marked by a sharp color gradient in displacement in Fig. 18a). Elements tend to overlap significantly along both coasts where compressive stress on the contacts is highest and also in the shear zone of the fracture originating at $167^\circ W$ (Fig. 18c). After 18 hours of simulation, the effective ice area in the $D_t = 0.01$ case has been reduced by approximately 3000 km^2 , which is orders of magnitude larger than in the $D_t = 0.1$ case which exhibited a slightly unstable arch (2.8 km^2 reduction in area)(Fig. 18f). This degree of overlapping reduces the overall ice area enough to allow the fractured ice to fluidize.

Similar extensive cracking of the ice sheet can be produced with the base case value of $D_t = 0.1$ but with a lower initial failure criteria of $\sigma_c = 60 \text{ kPa}$ (Fig. 18e). In this case, a similar eroding arch forms, with a similar secondary fracture from the Seward coast to the northern extent of the ice sheet at approximately $167^\circ W$ (Fig. 18b). But because the failure occurs due to the weaker bond failure characteristics rather than lower residual ice loading capacity, the degree of element overlap is less (750 km^2) than in the $D_t = 0.01, \sigma_c = 90$ case, leading to less fluidization 18 hours into the simulation. In this case, regions of significant area overlap are more evenly dispersed throughout the domain. This likely results because the higher residual strength of heavily compressed contacts allows for redistribution of load as the ice sheet erodes.

4 Summary

This paper presents a set of case studies that demonstrate the characteristics of a polygon based DEM for sea ice fracturing and subsequent redistribution that is appropriate for use on scales of tens of meters to kilometers and for time periods of minutes to



610 days. The model is shown to be capable of simulating sea ice fracture without unrealistic levels of elasticity or computationally infeasible short time steps. As discussed in Part 1 of this study, reasonably approximating the full transition from continuum behavior prior to failure to granular response post-breakup requires careful alignment of the pre- and post- failure numerics. Idealized test cases were used to demonstrate the particulars of the contact physics and examine how energy and stresses adjust at the single contact level as the physics changes from representing a bond to representing a frictional collision. Although it is not expected that these properties be continuous through the fracturing process, having a model such as QuSI-DEM that allows for their conservation ensures that changes can be quantitatively associated with specified damping mechanism and that there will not be artificial amplification in the transition that causes numerical instability. To our knowledge, our approach to the development and testing of QuSI-DEM is not standard practice for DEMs, but provides transparency that should facilitate inter-model comparison.

620 Although ice arching is a popular case study that several DEM and continuum models have been applied to Dumont et al. (2009); West et al. (2022); Åström and Polojärvi (2024); Montemuro and Manucharyan (2025), there is limited understanding of the sensitivity of these simulations to element geometry and contact physics. In this paper, this sensitivity is explored in detail using an idealized setup that realistically mimics ice arching across the Bering Strait. A suite of simulations demonstrate the sensitivity of the accumulative bond failure process of ice arching to bond strength, element size, failure rate parameter, material strength at the transition from bonded to frictional sliding resistance, and residual ice strength. The importance of undertaking ensembles of simulations with consistent parameter values but different element arrangements in polygon-based DEM modeling is also highlighted.

QuSI-DEM parameters such as the elastic modulus or the material strength can only be loosely associated with sea ice properties because significant unresolved subgrid-scale variability exists at the scales being considered. Other model parameters, such as those associated with post failure characteristics, have an even more tenuous connection to observed properties due to the extreme complexity of the fracture process and the dearth of relevant observations. Despite these limitations, the model provides insight into the characteristics of sea ice fracture in the vicinity of the Bering Strait. It demonstrates how different coastline slopes relative to wind direction affect the orientation of progressively eroding arches and the role that differential sliding along the coasts can play in this. It shows how changes in sea ice area due to ridging/rafting processes (modeled as element overlap) impacts breakup and transport. Overall it shows how sensitive the evolution of break up can be to the rate at which and the way with which energy is dissipated as cracking progresses.

This work was motivated by the experiences of the authors trying to reproduce arching in a stiff material. Prior to the contact physics development described in Part I, the QuSI-DEM predecessor, Siku, would shatter under relatively low wind loading. To simulate an arch, the elasticity of bonds would need to be unrealistically high. Simulations demonstrating arches were so highly tuned we questioned if other models could demonstrate arching behavior within reasonable physical bounds. In the future, a DEM model inter-comparison using case studies like those presented in this paper would be informative to identify if individual model tuning is within reasonable bounds of model sensitivity and numerical stability, allowing realistic simulation of ice pack damage propagation.



645 *Code and data availability.* The current version of QuSI-DEM is openly available from <https://bitbucket.org/durskis/quasi-dem> under the GNU General Public License, version 3.0 or later. The exact version used to produce the results in this paper is archived on Zenodo at <https://doi.org/10.5281/zenodo.21367084> (Durski, 2026), together with the input data and run scripts needed to reproduce the simulations and figures presented in this paper. QuSI-DEM bundles the shapelib library (`deps/shapelib/`), which is distributed separately under its own GNU Lesser General Public License.

650 *Author contributions.* Conceptualization: SMD and JKH. Methodology, software, validation, formal analysis, investigation, data curation, visualization, and writing – original draft: SMD. Resources, writing – review & editing, supervision, project administration, and funding acquisition: JKH.

Competing interests. The authors declare that they have no competing interests.



References

- Dumont, D., Gratton, Y., and Arbetter, T. E.: Modeling the Dynamics of the North Water Polynya Ice Bridge, *Journal of Physical Oceanography*, 39, 1448–1461, <https://doi.org/10.1175/2008JPO3965.1>, 2009.
- Durski, S. M.: QuSI-DEM: A polygon-based discrete element sea ice model in spherical coordinates, <https://doi.org/10.5281/zenodo.21367084>, software, 2026.
- Feng, Y. and Owen, D.: A 2D polygon/polygon contact model: algorithmic aspects, *Engineering Computations*, 21, 265–277, <https://doi.org/10.1108/02644400410519785>, 2004.
- Montemuro, B. P. and Manucharyan, G. E.: The Role of Islands in Sea Ice Transport Through Nares Strait, *Journal of Geophysical Research: Oceans*, 130, e2025JC022 357, <https://doi.org/10.1029/2025JC022357>, 2025.
- Shapiro, L. H. and Burns, J. J.: Major Late-Winter Features of Ice in Northern Bering and Chukchi Seas as Determined from Satellite Imagery, Scientific Report UAG R-236, Geophysical Institute, University of Alaska Fairbanks, Fairbanks, Alaska, USA, sea Grant Report No. 75-8, 1975.
- Smith, J. C., Durski, S. M., and Kurapov, A. L.: Drivers of Bering Sea Ice Variability in Low and Average Ice Years: Inferences From a Regional CICE6 Simulation, *Journal of Geophysical Research: Oceans*, 130, e2025JC022 684, <https://doi.org/10.1029/2025JC022684>, [_eprint: https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/2025JC022684](https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/2025JC022684), 2025.
- West, B., O'Connor, D., Parno, M., Krackow, M., and Polashenski, C.: Bonded Discrete Element Simulations of Sea Ice With Non-Local Failure: Applications to Nares Strait, *Journal of Advances in Modeling Earth Systems*, 14, e2021MS002 614, <https://doi.org/10.1029/2021MS002614>, [_eprint: https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/2021MS002614](https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/2021MS002614), 2022.
- Åström, J. A. and Polojärvi, A.: High-Resolution Fracture Dynamics Simulation of Pack-Ice and Drift-Ice Formation During Sea Ice Break up Events Using the HiDEM2.0 Code, *Geophysical Research Letters*, 51, e2024GL110 552, <https://doi.org/10.1029/2024GL110552>, [_eprint: https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/2024GL110552](https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/2024GL110552), 2024.

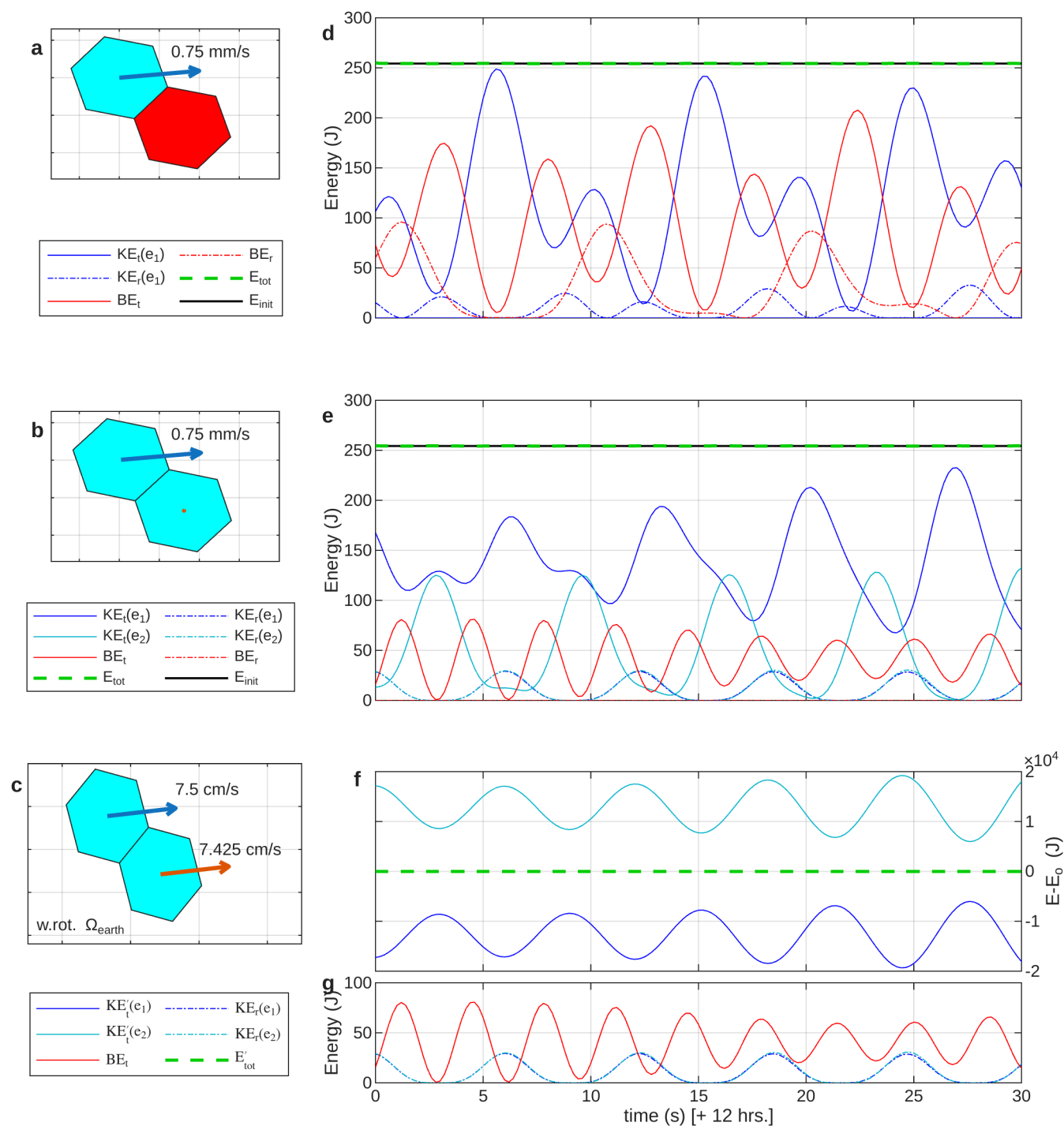


Figure 1. Panels **a**, **b**, and **c** display element configurations and initial velocities for three simulation cases. Cyan elements are free to move while the red element is fixed. In each case the two elements are bonded. Panels **d**, **e**, and **g** depict time series of different forms of energy for the corresponding experiments to the left. Because KE_t is much larger than the other terms in the Case C experiment, the translational KE for the two elements and total energy in the system are plotted as the difference from their initial values (denoted by the prime) in Panel **f**.

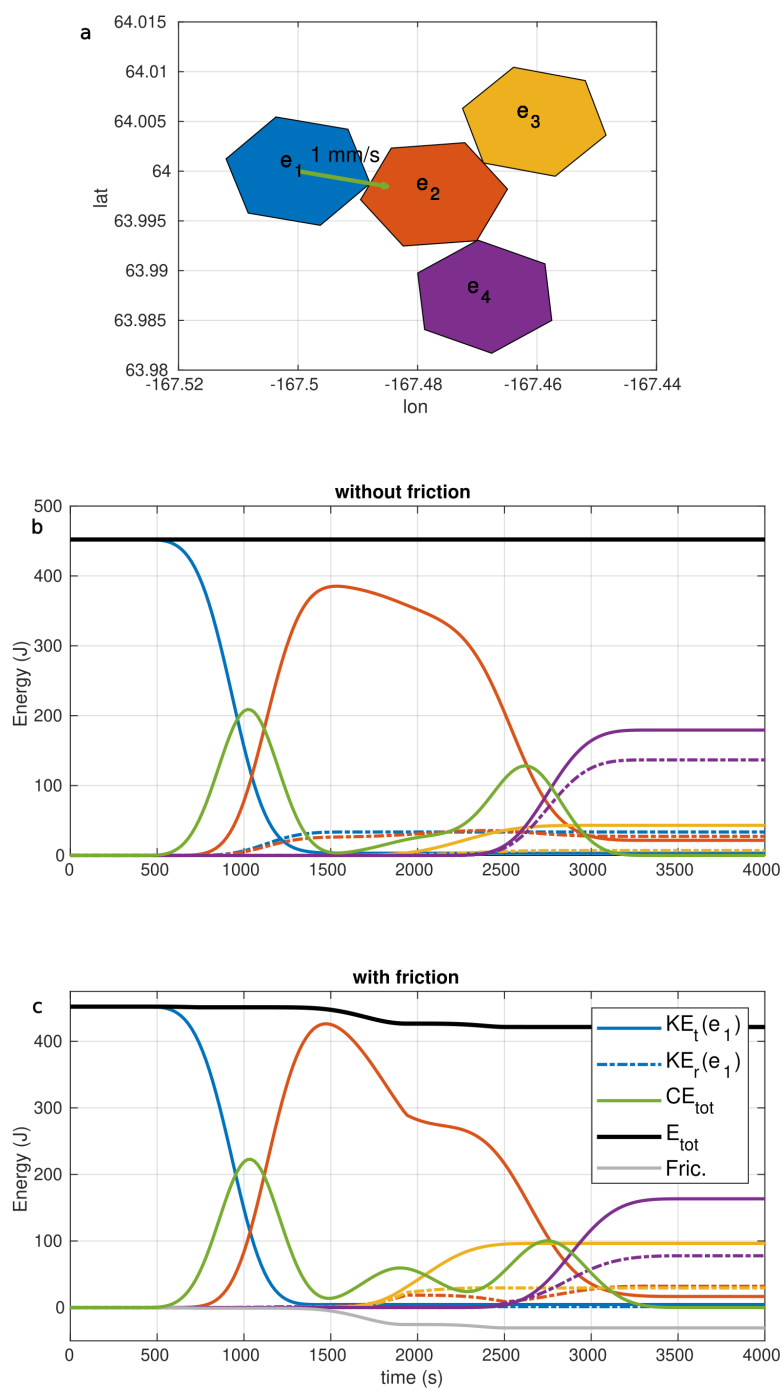


Figure 2. **a** Diagram of a collision test case where one unbonded element strikes another which strikes two others. **Panel b** displays energy as a function of time for an experiment with frictionless elements. **Panel c** depicts the same case with friction. For KE_t and KE_r , Line color in the lower two panels is matched with element color in **Panel a**. Frictional dissipation in **Panel c** is plotted as the cumulative energy dissipation over all contacts

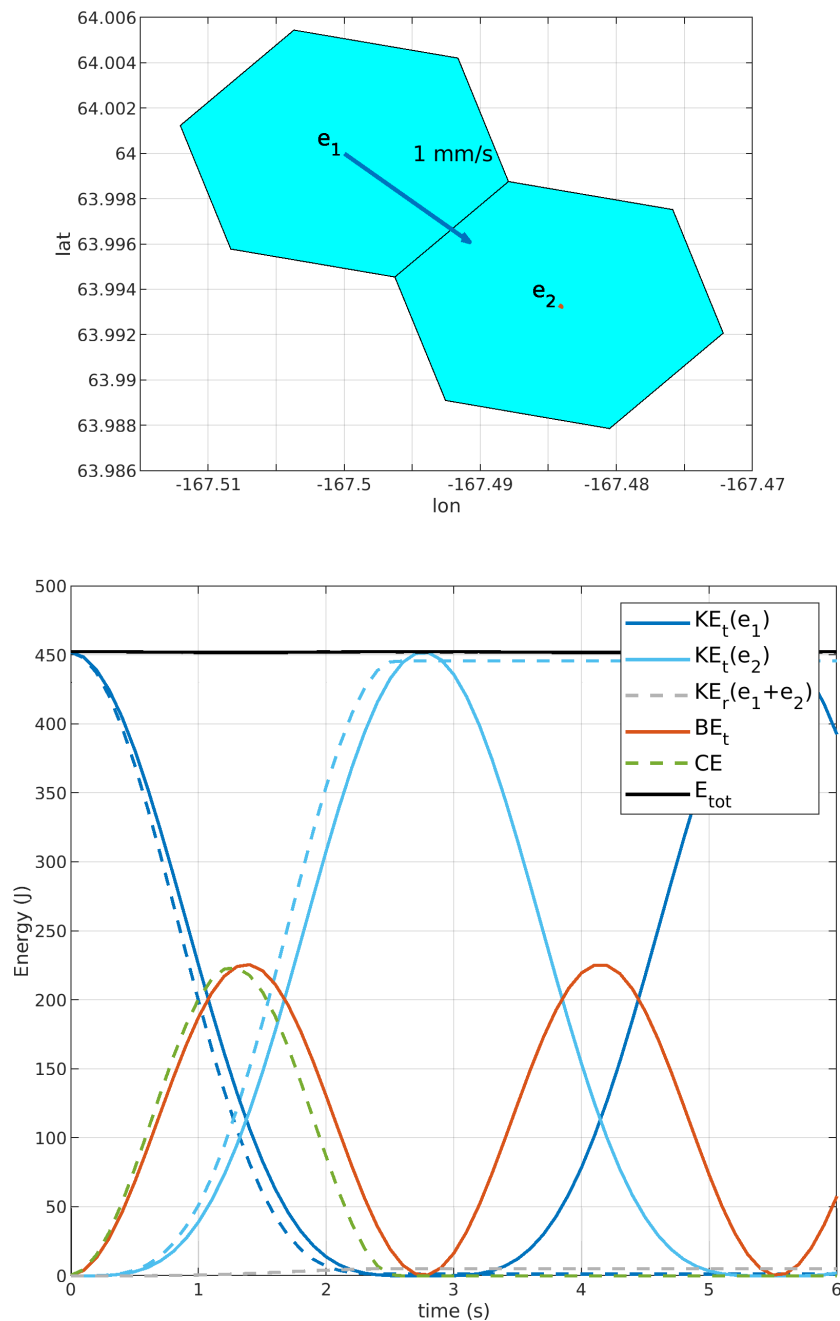


Figure 3. a Diagram of a test case where e_1 is given an initial normal velocity towards e_2 for bonded and unbonded elements. Panel **b** shows the time evolution of the energy of the elements and contact in the system. Solid lines show results for the bonded element experiment, dashed lines for the collision contact experiment. The dashed black line indicating E_{tot} for the collision case falls directly beneath the solid black line. The solid gray line indicating KE_r for the bonded case is indistinguishable from the x-axis.

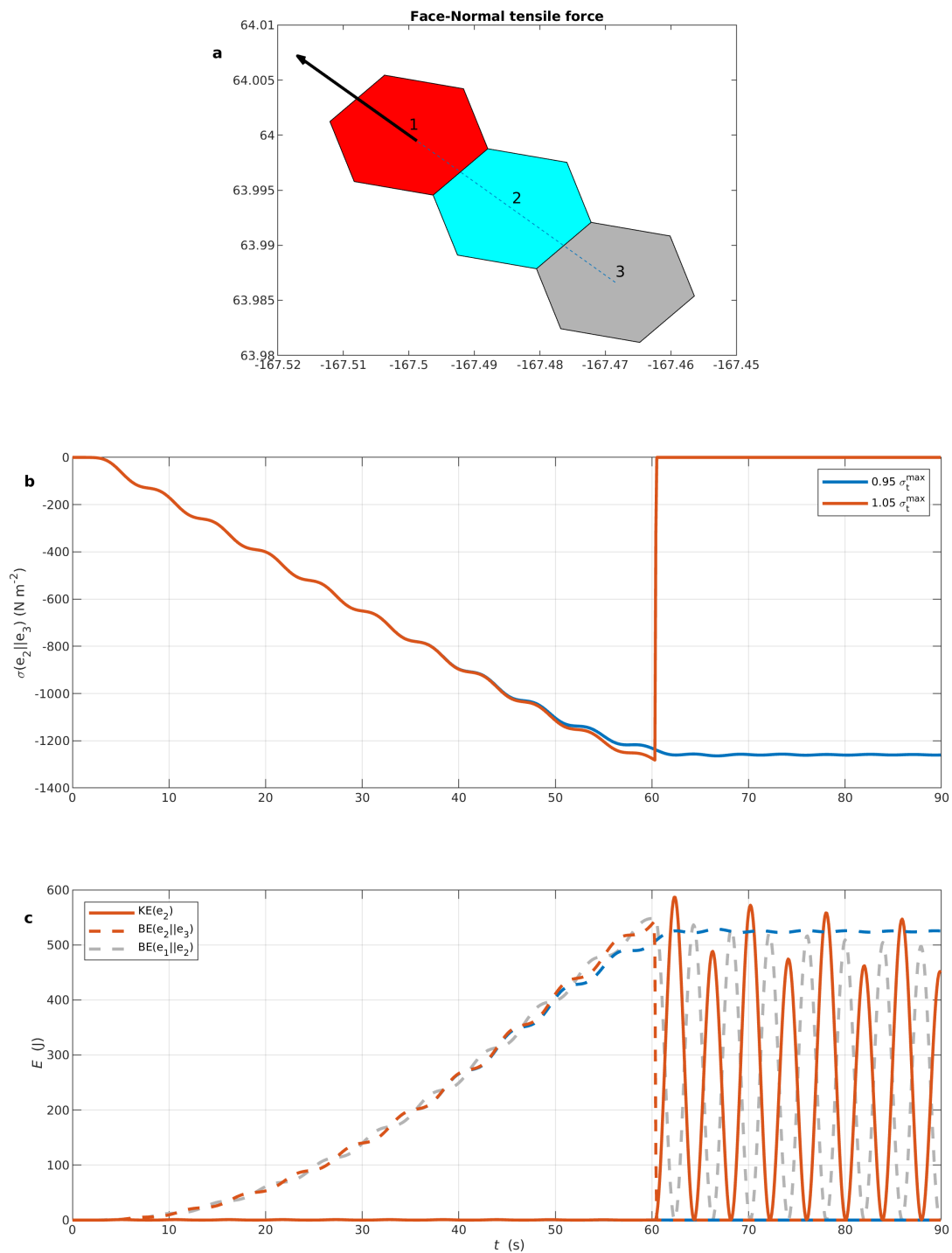


Figure 4. **a** Depiction of simulation setup. A red control elements e_1 draws free element e_2 directly away from a stationary element (gray - e_3). **b** Tensile stress as a function of time at the $e_2||e_3$ joint for simulations where the $e_1||e_2$ stress approaches a level just below and just above σ_t . **c** Displays the kinetic energy of e_2 and the bond energy at the two contacts as a function of time.

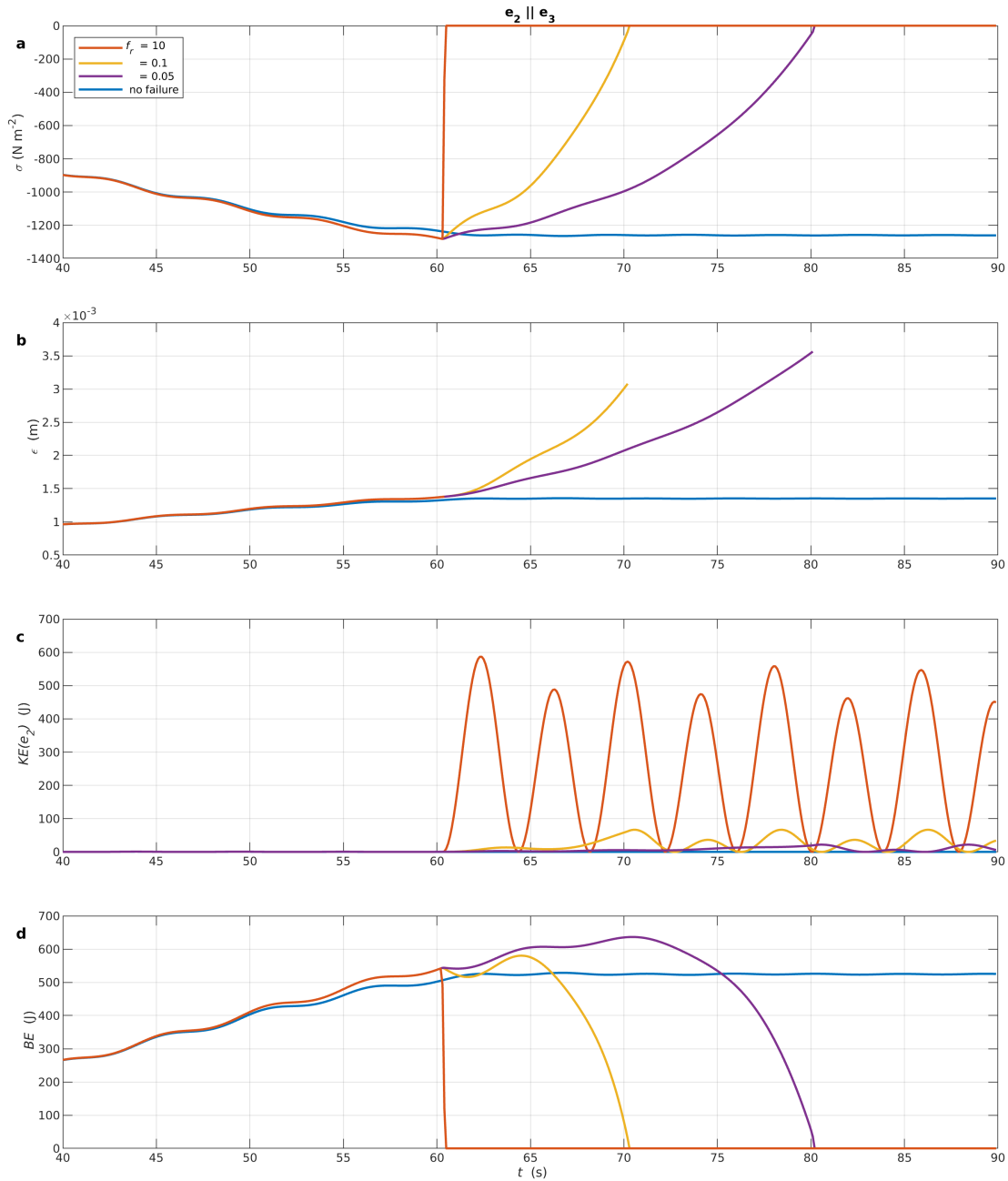


Figure 5. **a** Tensile stress as a function of time for four simulations using the setup depicted in Fig. 4a. f_r is varied in three simulations set up as in Case B above. The fourth (blue line) depicts the Case A results for comparison. **b** shows normal strain in the four experiments as a function of time **c** Displays the kinetic energy of e_2 for each experiment as a function of time **d** depicts bond energy as a function of time for the four simulations.

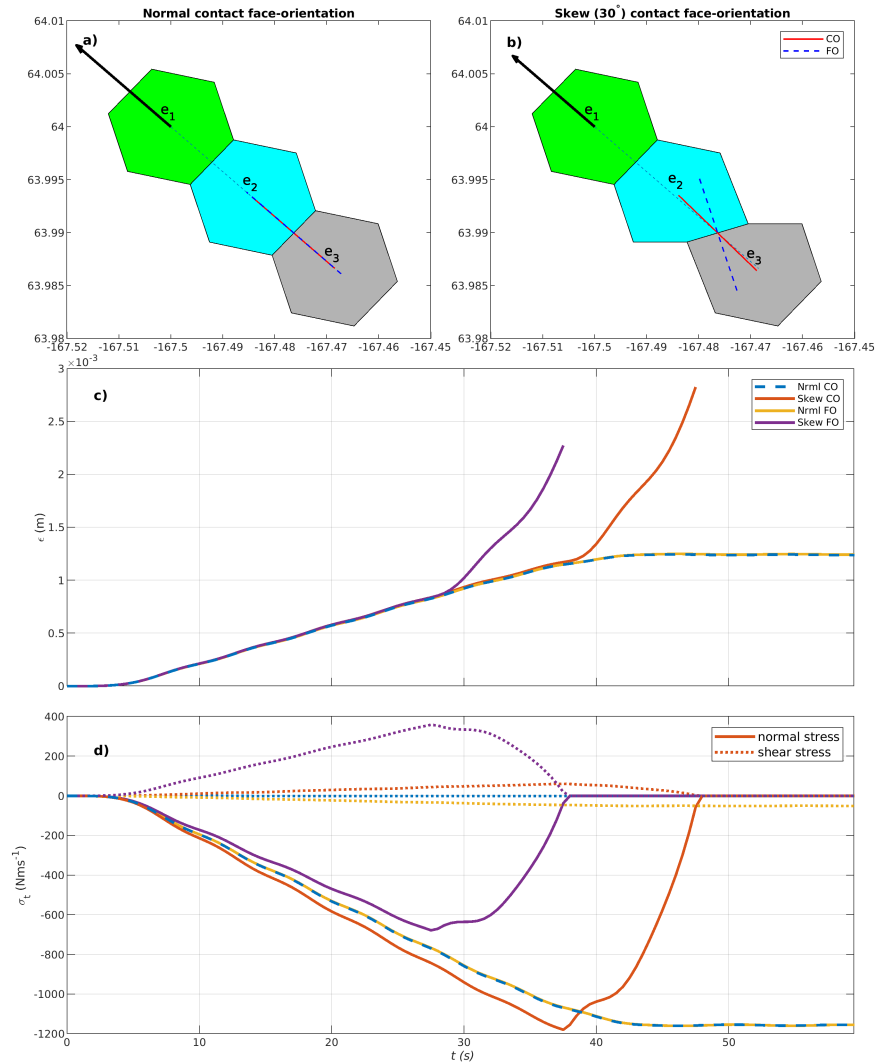


Figure 6. **a** depicts simulation setup for tensile stress experiment with normal contact orientation at $e_2||e_3$. **b** depicts simulation setup for case with skew contact orientation at $e_2||e_3$. In each case a control element e_1 (green) draws a free element e_2 (cyan) directly away from a stationary element (e_3 - gray). **c** Average normal strain at the $e_2||e_3$ joint is plotted as a function of time for 4 simulations where the $e_1||e_2$ stress approaches a level just below σ_t . The element setup (normal or skew) and the definition of normal and tangential joint orientations are varied between simulations **d** Displays normal and shear stress at the $e_2||e_3$ joint as a function of time for the four experiments.

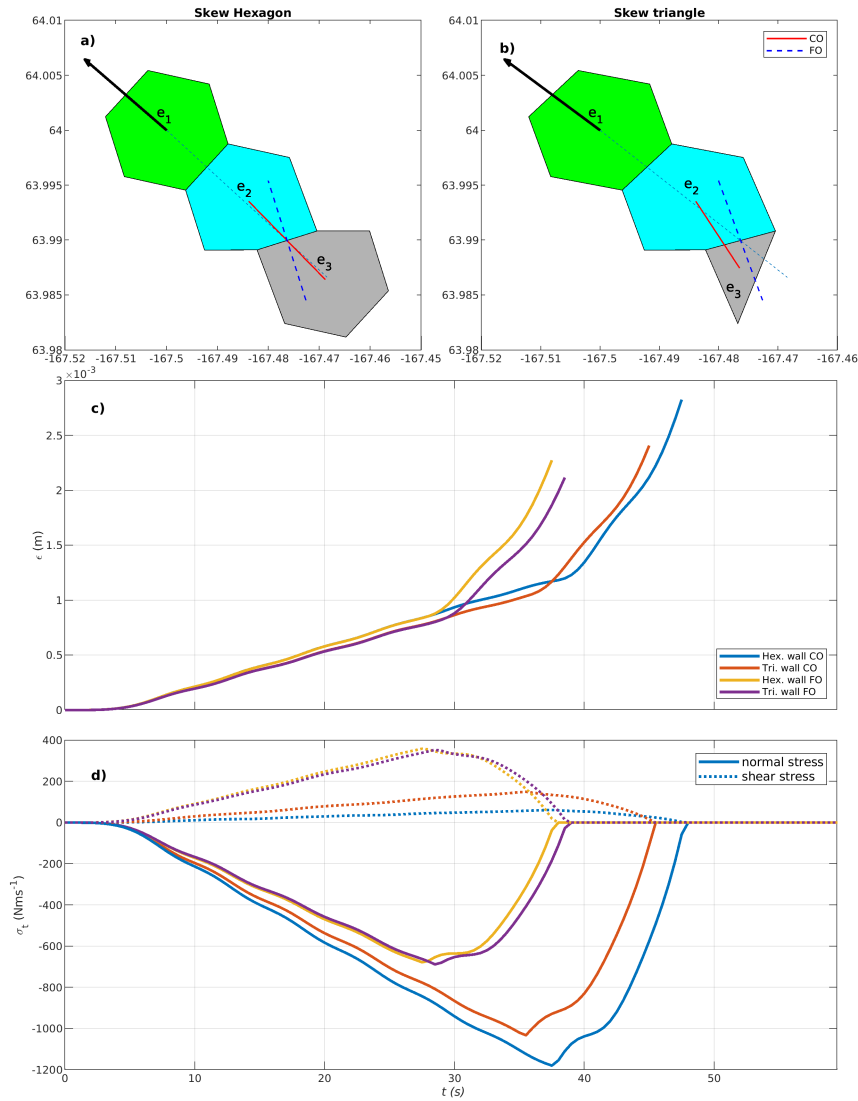


Figure 7. **a** and **b** depict simulation setups for tensile stress experiment with skew contact orientation identical to that in Fig. 6b but with a triangular shaped e_3 element in panel **b**. **c** Average normal strain at the $e_2||e_3$ joint is plotted as a function of time for 4 simulations where the $e_1||e_2$ stress approaches a level just below σ_t . The element setup (skew hexagon e_3 and skew triangle e_3) and the definition of normal and tangential joint orientations are varied between simulations **d** Displays normal and shear stress at the $e_2||e_3$ joint as a function of time for the four experiments.

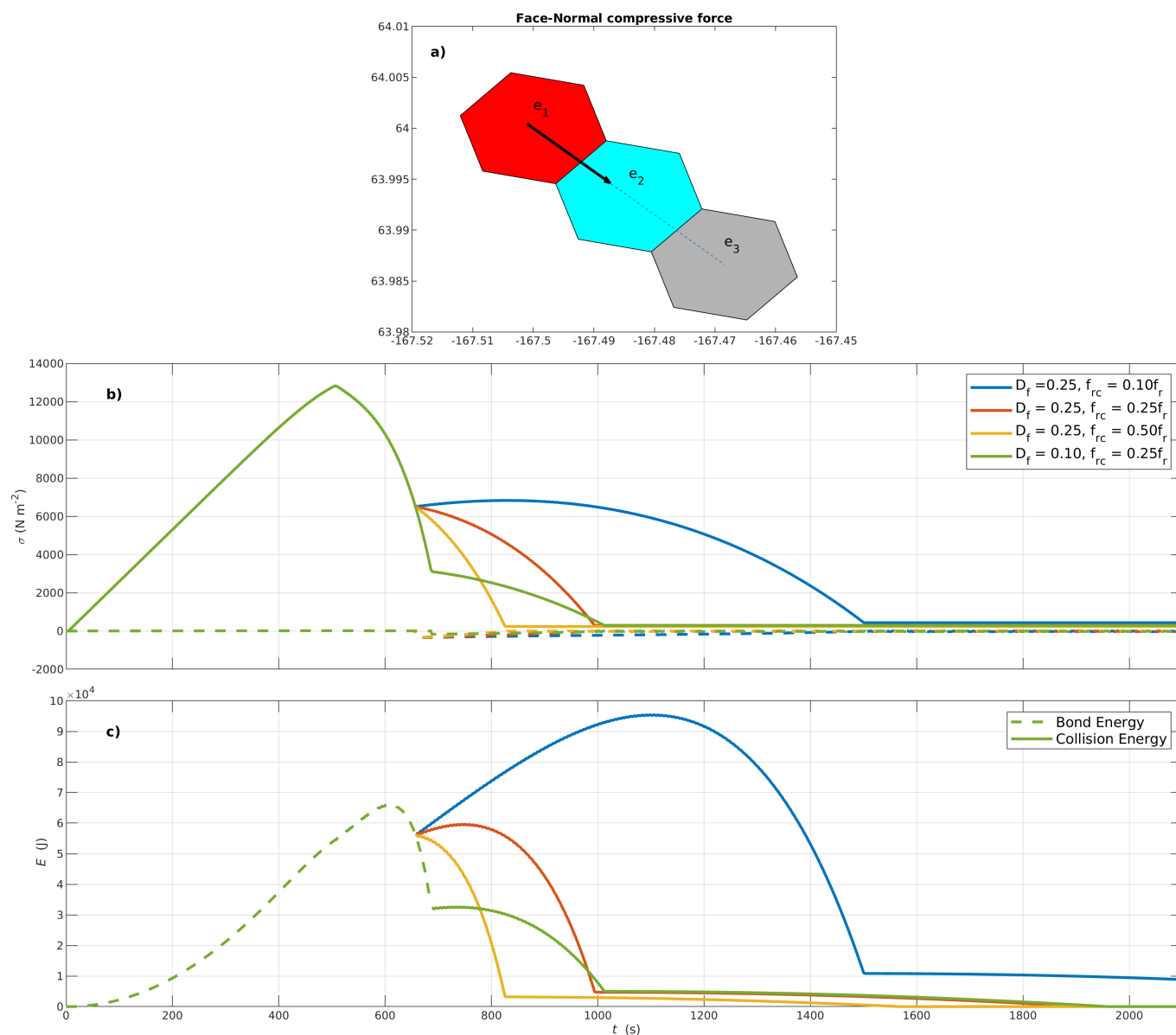


Figure 8. **a** Diagram of element arrangement for compressive stress experiment. Red element (e_1), is a control element that is pressed towards cyan element (e_2) which is bonded to the fixed gray element (e_3). **b** Normal (solid lines) and shear (dashed lines) stresses at $e_2||e_3$ versus time for a set of compressive experiments. **c** Collision (solid lines) and bond (dashed lines) energy at $e_2||e_3$ versus time for a set of compressive experiments. Green lines overlay red, yellow, and blue lines in **b** and **c** for approximately the first 625 s of the experiments.

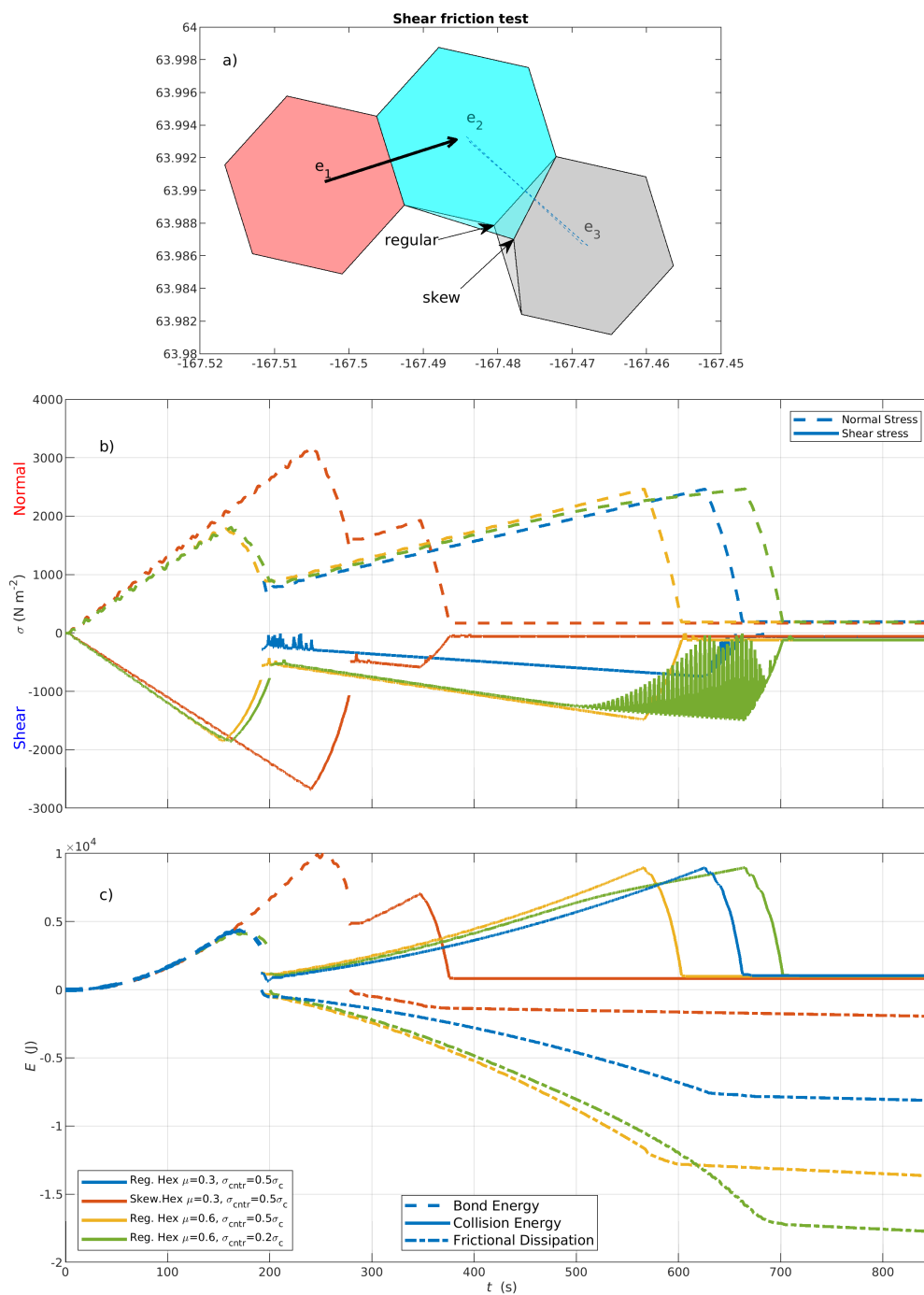


Figure 9. **a** Diagram of element arrangement for shear stress experiment. Red element (e_1), is a control element that is pressed towards cyan element (e_2) which is bonded to fixed gray element (e_3). **b** Normal (solid lines) and shear (dashed lines) stresses at $e_2||e_3$ versus time for the set of experiments. **c** Collision (solid lines) and bond (dashed lines) energy at $e_2||e_3$ versus time for a set of compressive experiments. Cumulative frictional dissipation versus time is plotted in **c** as dash-dotted lines.

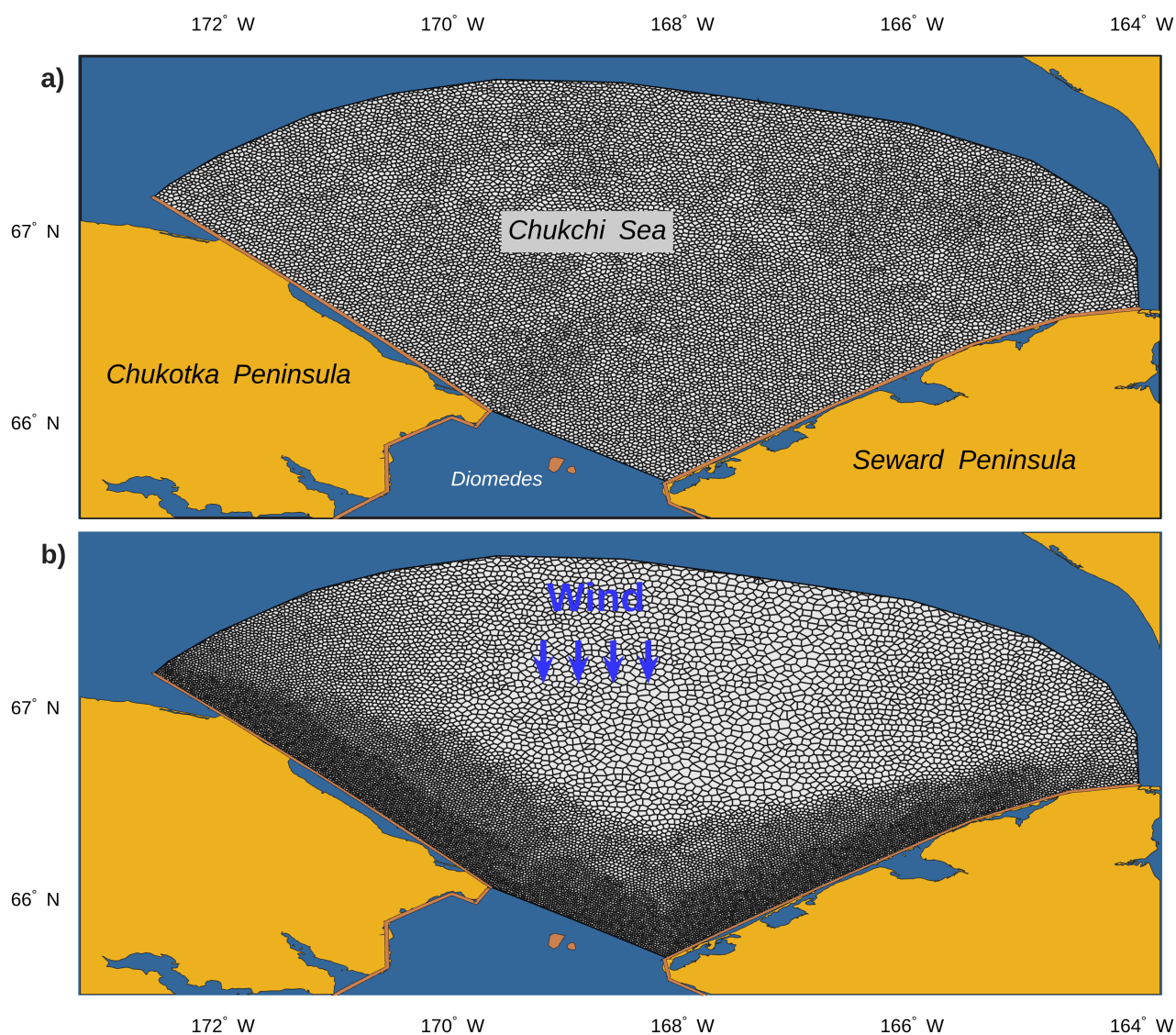


Figure 10. Map of Bering Strait model domain with examples of element distribution **a)** depicts the element distribution for one of the uniform resolution cases. All simulations are forced with uniform northerly 10 m s^{-1} winds. **b)** depicts the element distribution for a case with variable element size. Elements along the coast have approximately one-third the area of in the uniform element cases.

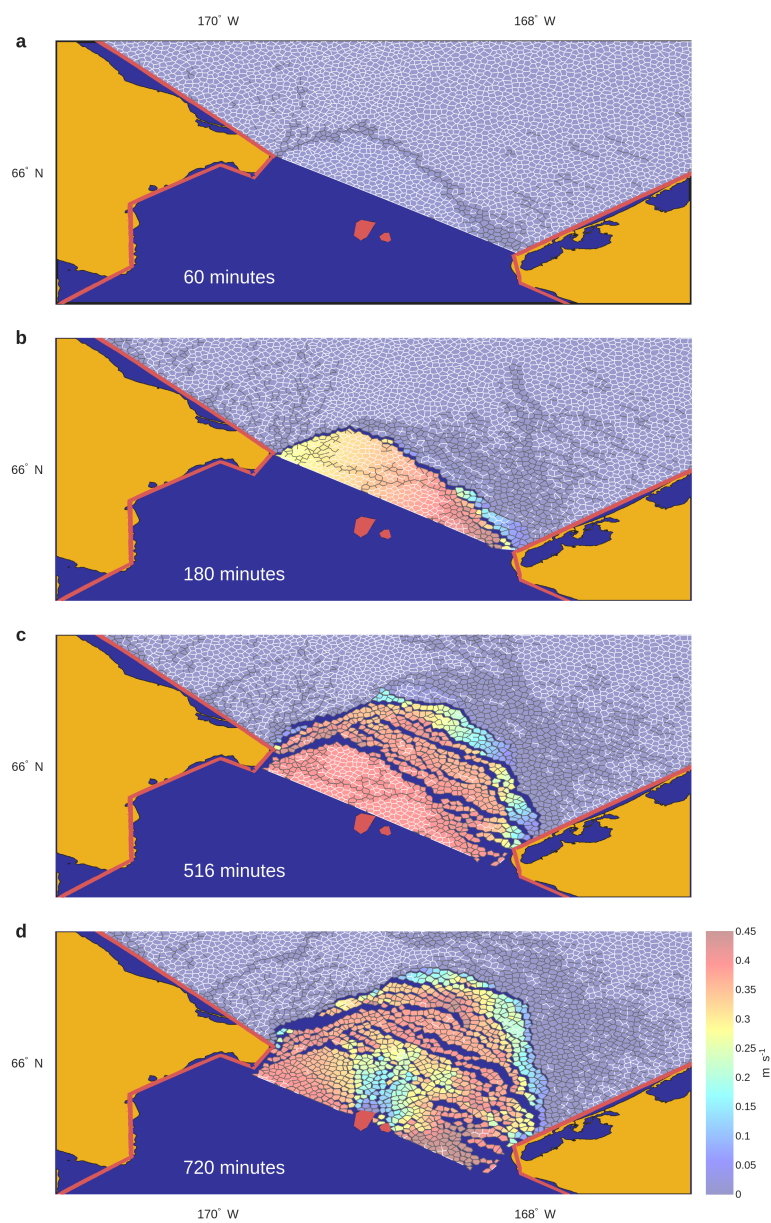


Figure 11. Base case model simulation of sea ice under the influence of a northerly wind in the Chukchi Sea. Panels **a b c d** depict a subsection of the model domain in the vicinity of the Bering Strait. Elements are colored based on their velocity magnitude. In these panels edges representing broken bond contacts between elements are displayed as dark gray, while intact contacts are white.

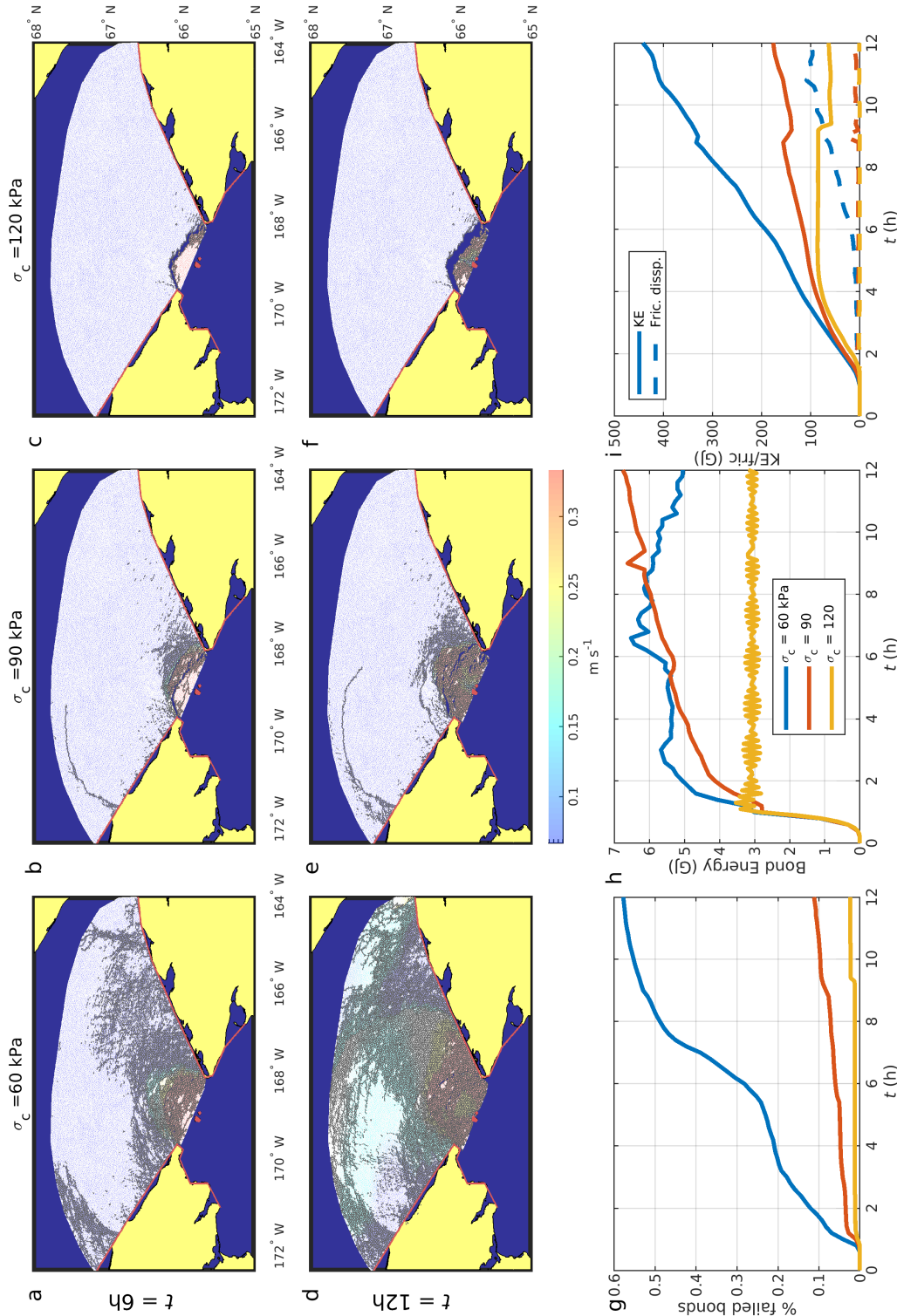


Figure 12. The effects of varying ice strength on modeled evolution. Panels a, b and c, display the model state at 6 hours into the simulation for experiments with $\sigma_c = 60$ kPa, $\sigma_c = 90$ kPa and $\sigma_c = 120$ kPa respectively. Panels d, e and f show similarly for 12 hours. These panels display cracked element edges in black and undamaged edges in white, shading indicates sea ice velocity. Panel g shows the fraction of failed bonds as a function of time, h displays the domain integrated contact bond energy and i shows domain integrated kinetic energy and frictional dissipation

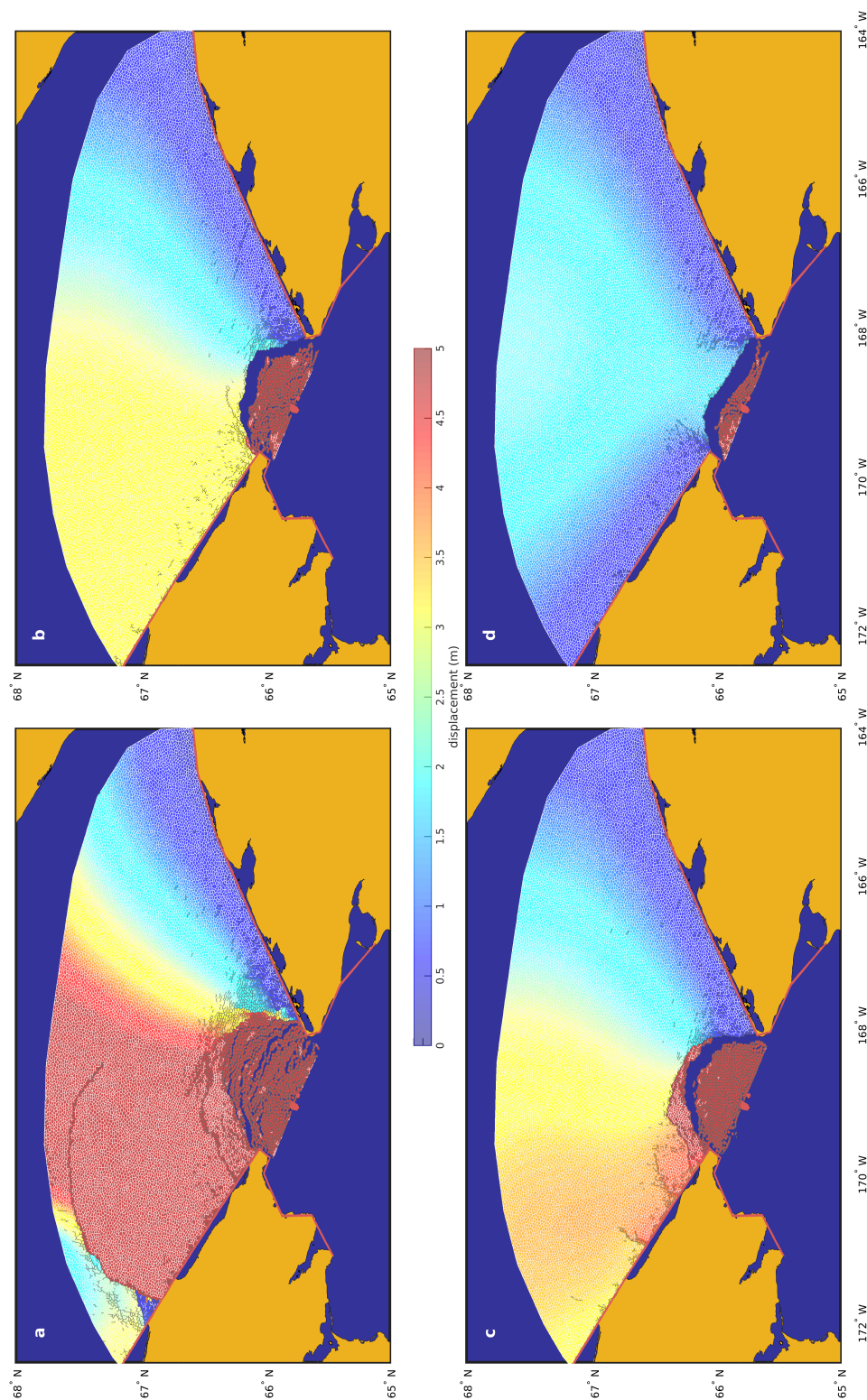


Figure 13. Element displacement after 12 hours of northerly wind stress for four ensemble members (Cases 1(a), 2(b), 7(c) and 8(d))

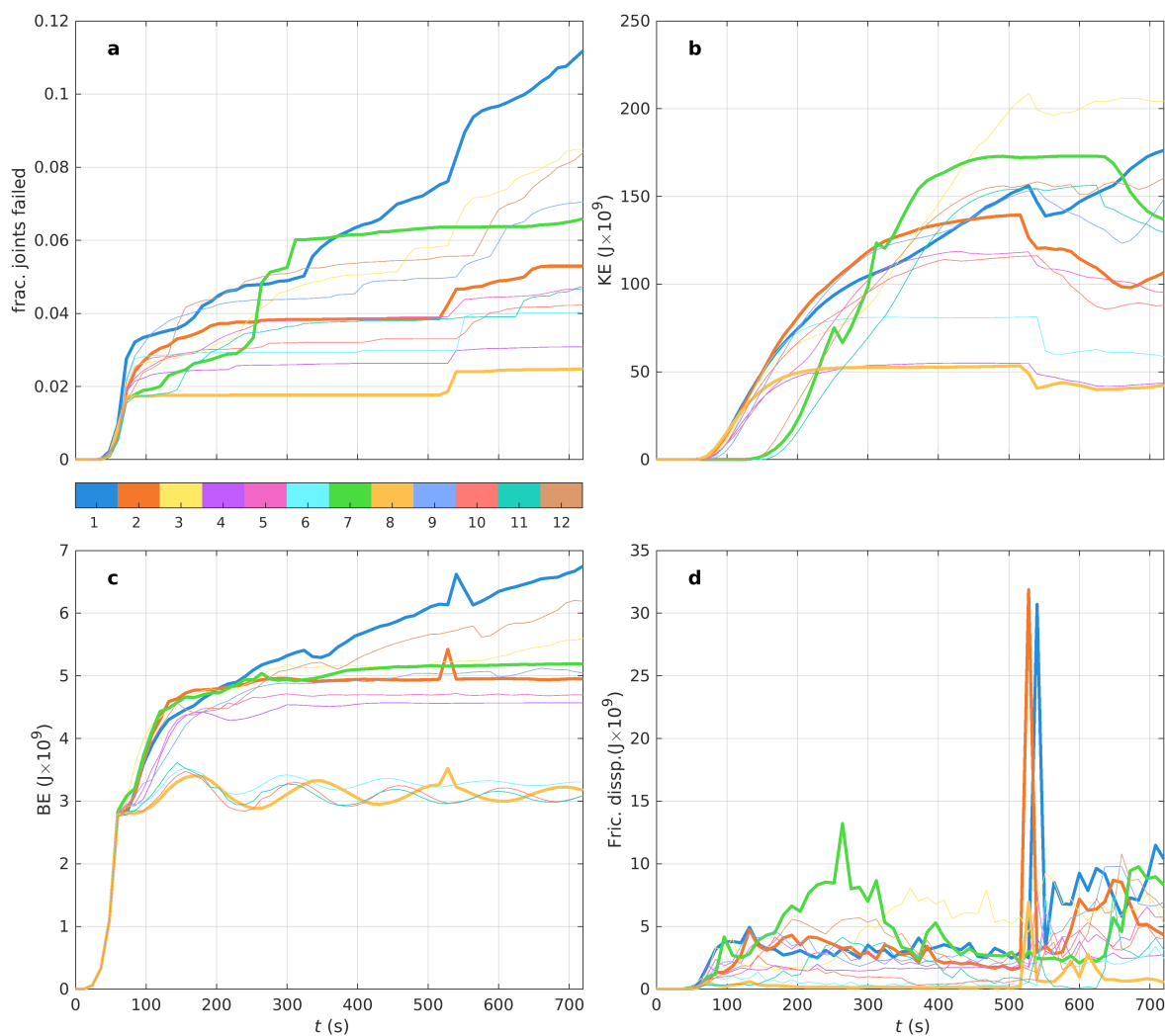


Figure 14. Time series of a) fraction of joints failed, b) domain integrated kinetic energy, c) domain integrated bond energy, and d) domain integrated frictional dissipation for twelve ensemble member simulations. Thick lines indicate simulations displayed in Fig. 13 Cases 1(a), 2(b), 7(c) and 8(d) where the associated color is specified in the colorbar.

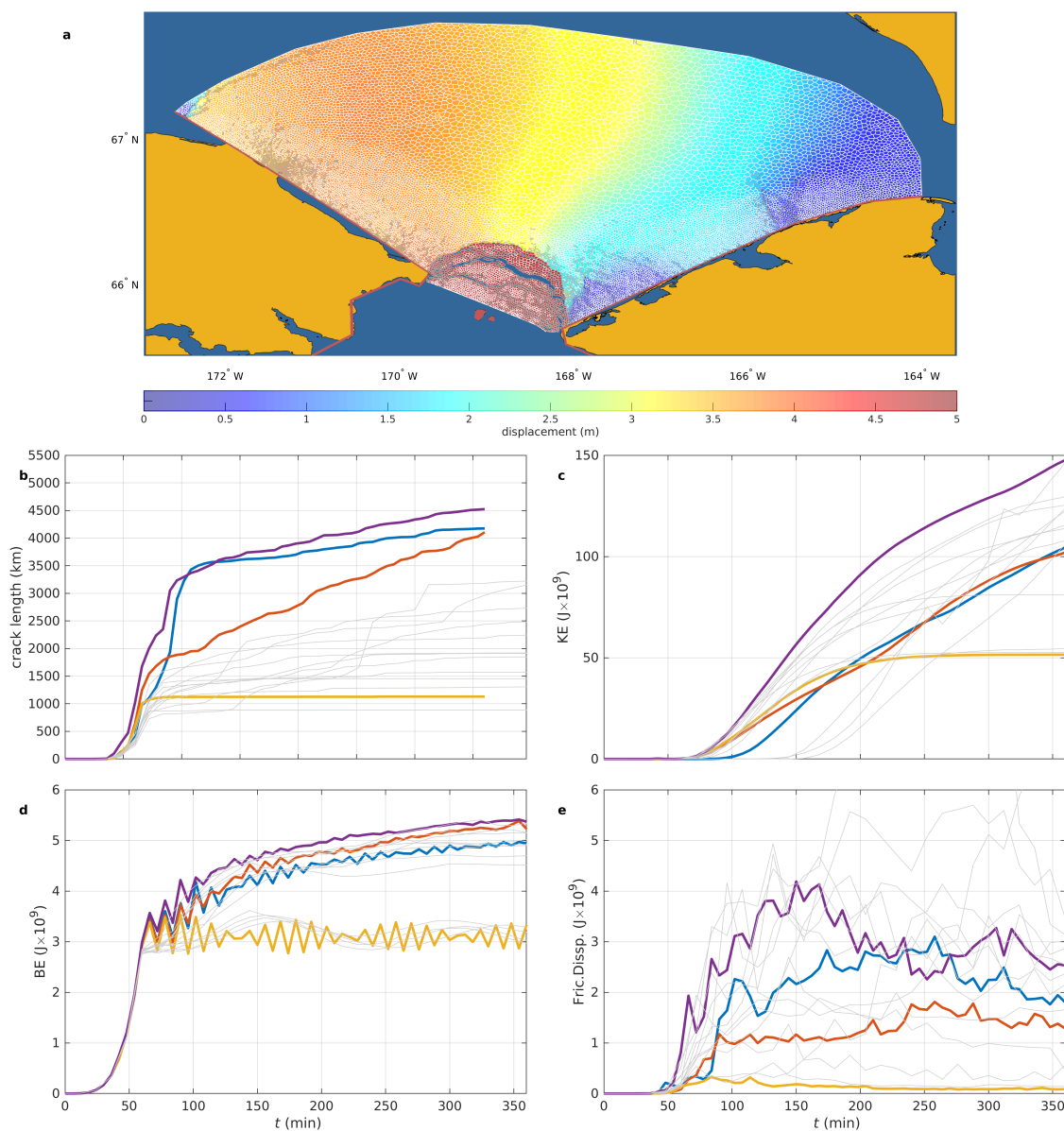


Figure 15. Panel a depicts sea ice displacement after 6 hours, for one of four simulations that match the base case simulation but with three times the spatial resolution along the coasts and in the strait. (b) Domain integrated crack length, (c) kinetic energy, (d) bond energy, and (e) frictional dissipation are displayed in the lower panels. The four variable resolution simulations are displayed with colored lines while the 12 uniform (lower) resolution simulations are depicted as light gray lines in each panel.

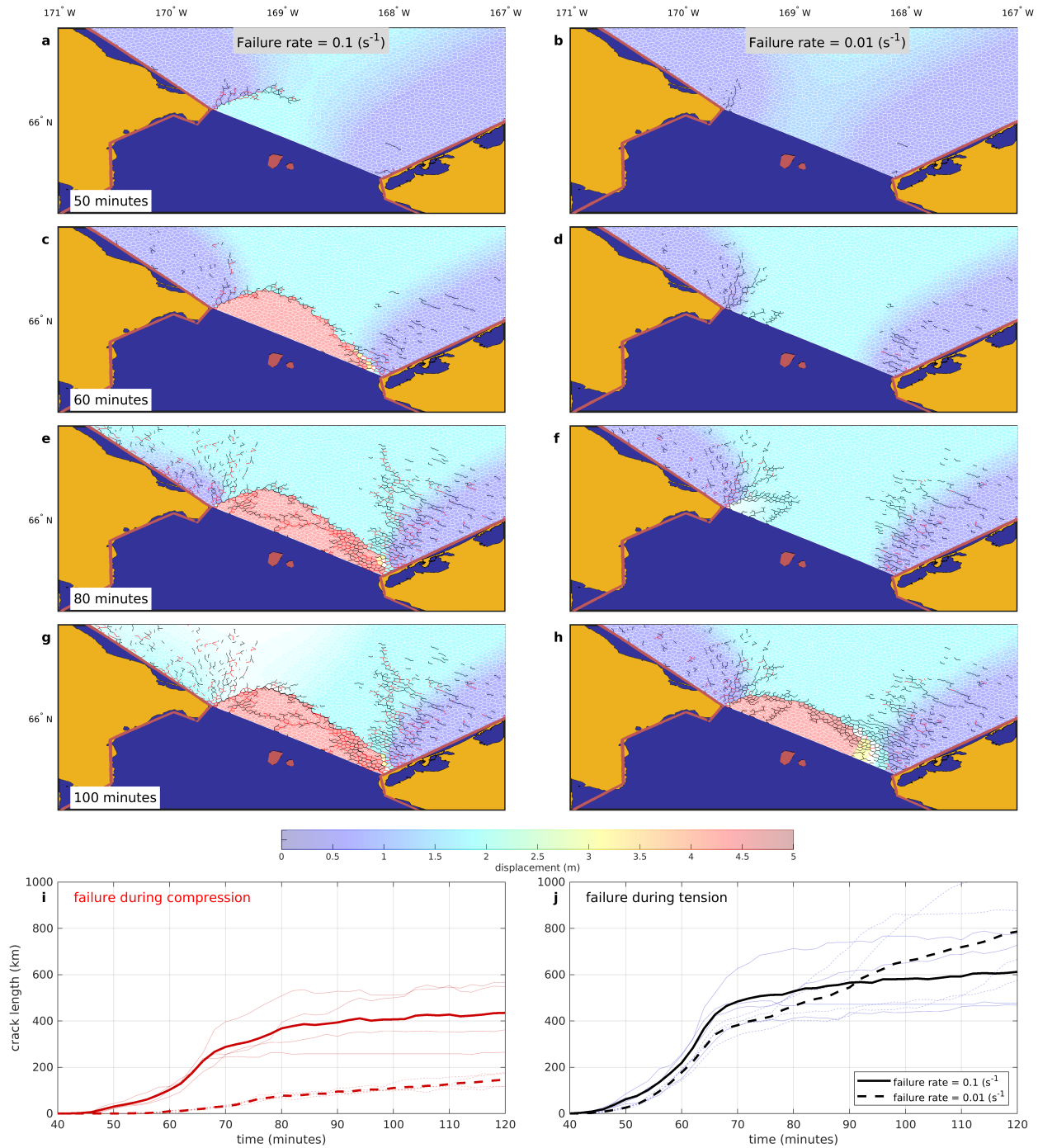


Figure 16. Displacement and cracking in simulations with higher (panels a,c,e and g) and lower (b,d,f and h) f_r parameter. Joints that failed while in tension are displayed in black, while those that failed while in compression are in red. Panel i depicts the total crack length of joints that failed under compression for the two ensembles of 4 experiments with heavier lines representing the average over the ensemble members. Panel j shows similarly for joints that failed under tension.

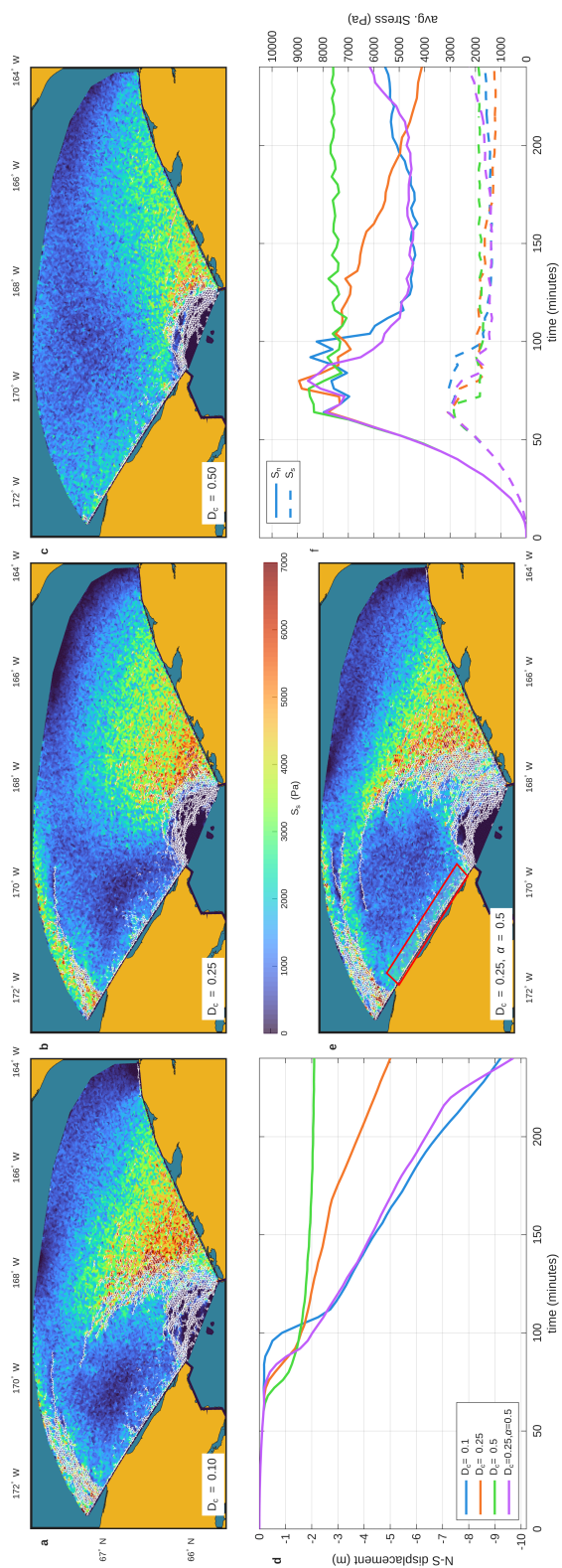


Figure 17. Cracking and distribution of shear stress at contacts after four hours for simulations with different values of the D_c parameter **a,b,c** and for a different value of α **e**. Panel **d** displays latitudinal displacement of the center of mass of elements in a subdomain defined by the red box in panel **e** for the four simulation. Panel **f** displays average normal and shear stress along the coastal boundary within that same subdomain.

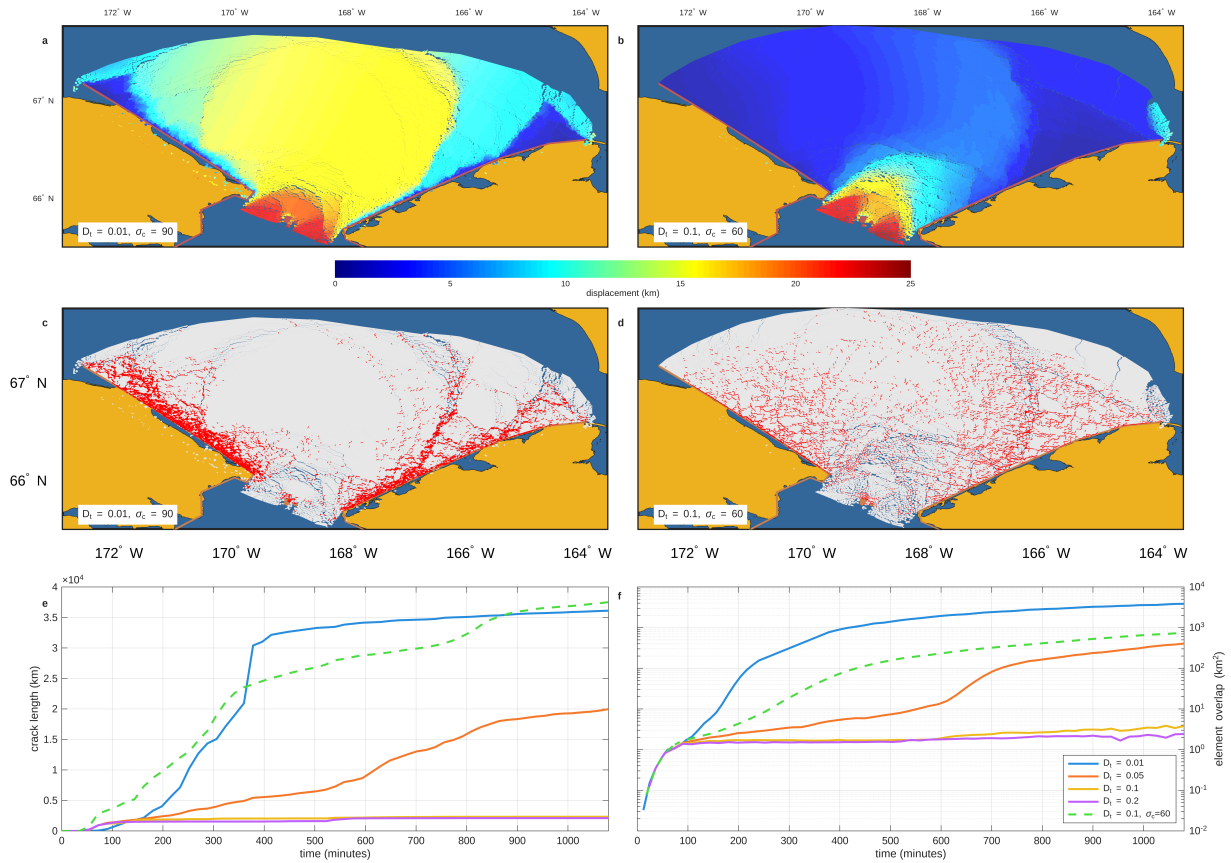


Figure 18. Panels a and b display displacement of elements after 18 hours of simulation, for a case with $D_t = 0.01, \sigma_c = 90$ and one with $D_t = 0.1, \sigma_c = 60$. In panels c and d element positions after 18 hours are displayed in white. Polygons that define element overlaps of greater than 1 km² are displayed in red. Panel e displays time series of crack length for five simulations varying D_t and σ_c . Panel f displays time series of total element overlap area for the same five simulations, on a semi-logarithmic plot.