Supplementary Information for

Purely light-scattering aerosols could induce positive shortwave radiative forcing over high-albedo surfaces

Longrui Zhang[1], Taveen S. Kapoor[1], Prabhav Upadhyay[1], and Rajan K. Chakrabarty[1]

[1]Department of Energy, Environmental and Chemical Engineering, Washington University in St. Louis, St. Louis, MO 63130, USA

Correspondence: Taveen S. Kapoor (taveensk@gmail.com), Rajan K. Chakrabarty (chakrabarty@wustl.edu)

Supplementary Information Text

S1 Particle Optical Properties Calculated by Mie Theory

Scattering phase function $P_{Mie}(\theta)$ was calculated at discrete angles $\theta_i = i\Delta\theta$, with $\Delta\theta = 2^{\circ}$ and i = 0, 1, …, 90. The phase function was normalized to satisfy

$$\int_0^{\pi} P_{Mie}(\theta) \sin\theta \, d\theta = 1 \tag{S1}$$

We utilized Legendre polynomials to fit this phase function for numerical compatibility:

$$P_{Mie}(\theta_i) \approx \sum_{i=0}^{N} k_l P_l(\theta_i) \tag{S2}$$

Let $\mu = \cos\theta$, the Legendre moments k_l were then approximated using the composite trapezoidal rule:

$$k_l = \int_{-1}^{1} P_{Mie}(\mu) P_l(\mu) d\mu \approx \sum_{i=0}^{N-1} \frac{1}{2} [P_{Mie}(\mu_i) P_l(\mu_i) + P_{Mie}(\mu_{i+1}) P_l(\mu_{i+1})](\mu_{i+1} - \mu_i) \tag{S3}$$

for l = 0,1, . . ., 16, where N= 90 and nmom = 17. The Legendre polynomials were generated via the recurrence relation:

$$P_l(\mu) = \frac{(2l-1)\mu P_{l-1}(\mu) - (l-1) P_{l-2}(\mu)}{l} \tag{S4}$$

starting from $P_0(\mu) = 1$ and $P_1(\mu) = \mu$. These moments were computed following the pmom function from libRadtran (and written into the aerosol input file as required by libRadtran's format.

S2 Calculation of Averaged ADRF

Let $\Delta F(A_s, \theta)$ be the instantaneous ADRF as a function of surface albedo A_s and SZA θ. We can calculate the averaged ADRF in a specific latitude band in a specific day:

$$\overline{\Delta F}(A_s; \phi_{eff}, d) \equiv \frac{1}{T_{day}} \int_{t:\theta(t)<90^\circ} \Delta F(A_s, \theta(t)) dt \tag{S5}$$

Where d is the specific day of the year and T_{day} is the total daylight duration for that day at 'effective latitude' ϕ_{eff}. Effective latitude is one latitude chosen to represent the range of latitude of region of interest, like between ϕ_1 and ϕ_2. It is calculated by

$$\phi_{eff} = \frac{\int_{\phi_1}^{\phi_2} \phi \cos\phi d\phi}{\int_{\phi_1}^{\phi_2} \cos\phi d\phi}$$

(S6)

One can treat ϕ_{eff} as the cosine-area-weighted mean latitude (It's a standard way in climate data averaging because the surface-area element on a sphere is $dA = R^2 \cos\phi \, d\phi d\lambda, \ \lambda \ is \ latitude$). By using ϕ_{eff} we can represent the latitude band range of interest by a single latitude band that reflects where 'most of the area' lies. $\theta(t)$ is calculated by:

$$h(t) = 15^\circ \cdot t_{hours,} \ t_{hours} \in [-12.12)$$

(S7)

$$\cos\theta(t) = \sin\phi_{eff} \sin\delta + \cos\phi_{eff} \cos\delta \cos h(t) \tag{S8}$$

$$\theta(t) = \arccos\left(\cos\theta(t)\right) \tag{S9}$$

Here δ is the sun declination which is the latitude of the subsolar point, and we use NOAA's widely used Fourier-series approximation in the 'fractional year' angle (Spencer, 1971):

$$\delta(\gamma) = 0.006918 - 0.399912 \cos\gamma + 0.070257 \sin\gamma - 0.006758 \cos 2\gamma + 0.000907 \sin(2\gamma) - 0.002697 \cos(3\gamma) + 0.00148 \sin(3\gamma) \tag{S10}$$

$$\gamma = \frac{2\pi}{365}(d - 1) \tag{S11}$$

Assumptions of the single-layer adding model:

1. Plane-parallel, horizontally homogeneous slab over a Lambertian surface of albedo A_s.
2. Non-absorbing layer ($\omega = 1$), giving the flux closure $T = 1 - R$ per direction (Eq. 5).
3. Two effective pairs: ($R\downarrow$, $T\downarrow$) for the collimated solar beam, slant path $\Delta z/\mu$ — hence μ-dependent; ($R\uparrow$, $T\uparrow$) for the diffuse upwelling field.
4. The Lambertian surface re-emits isotropically regardless of incident direction, so the field at the layer base is isotropic and $R\uparrow$, $T\uparrow$ depend on neither μ nor A_s.
5. Layer–surface multiple reflection sums as a geometric series with ratio $R\uparrow A_s$, convergent for $0 \le R\uparrow A_s < 1$.
6. $R\downarrow$, $R\uparrow$, A_s are independent arguments of $E(R\downarrow, R\uparrow, A_s)$; the partials hold the other two fixed, and the μ-dependence of $R\downarrow$ enters the total derivative through $dR\downarrow/d\mu$, not the partial.

Figures

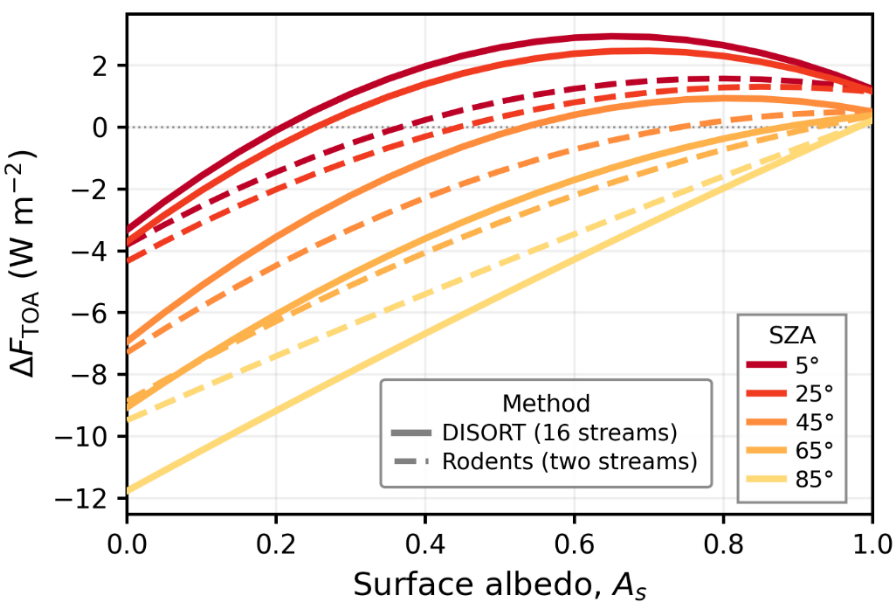

Figure S1. It illustrates the sensitivity of ADRF to surface albedo, SZA, and the multiple scattering treatment (DISORT vs. RODENTS) following the injection of alumina aerosols with imaginary part from Kapoor et al. (2026). Injected alumina follows the lognormal distribution with mean diameter 430nm, geometric standard deviation 1.5 and number concentration 11 $\#/cm^3$. The injection height is between 18km to 23km (stratosphere).

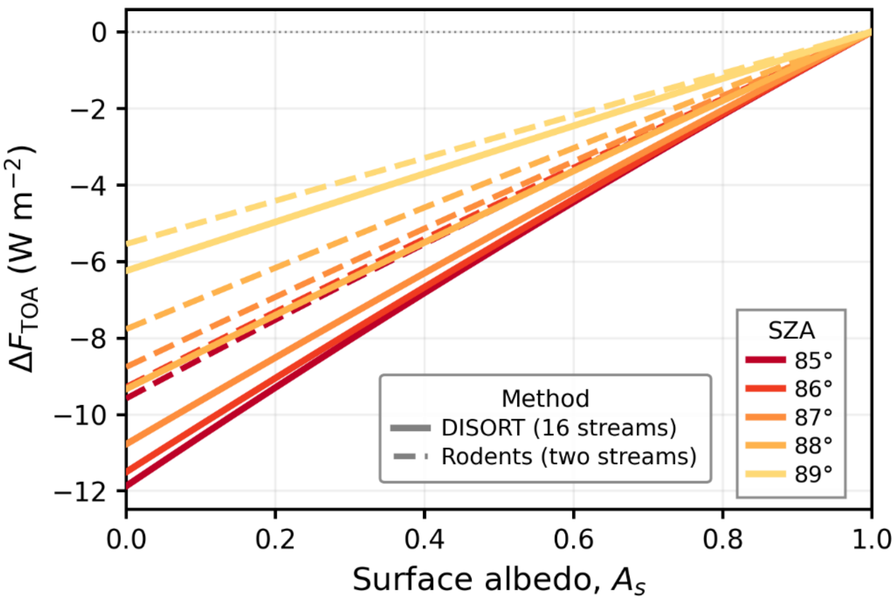

Figure S2. An extension of Fig. 1 of the main text to the high-SZA range $85° - 89°$. It illustrates that when SZA > $85°$, ADRF will diminish to zero.

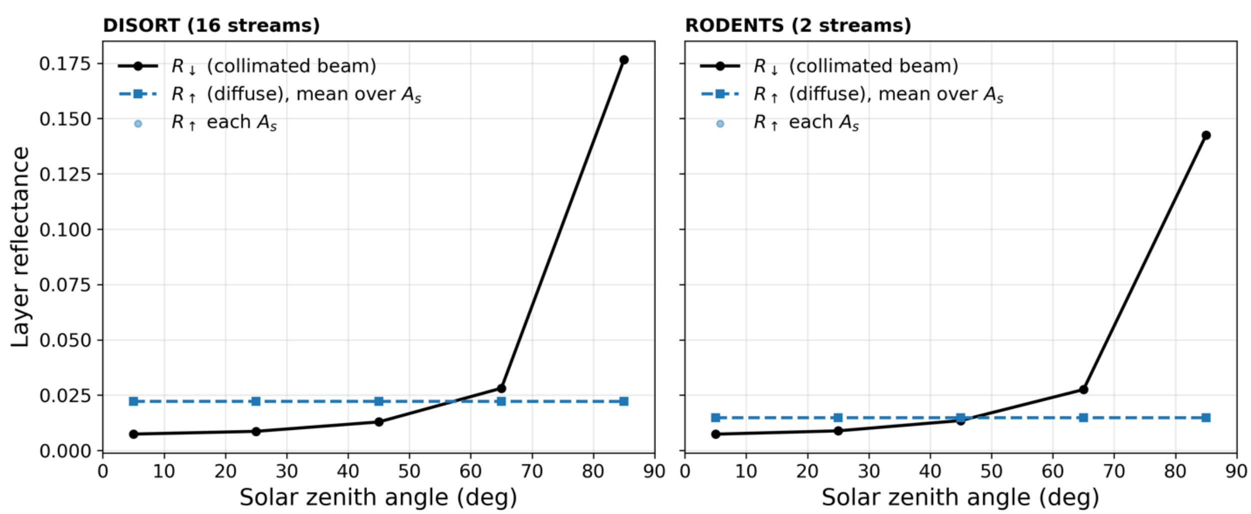

Figure S3. Downwelling and upwelling layer reflectances $R\downarrow$ and $R\uparrow$ as functions of solar zenith angle, extracted from the same simulations as Fig. 4 via Eqs. (10) and (11), for (a) DISORT (16 streams) and (b) RODENTS (2 streams). $R\downarrow$ (black circles), the layer reflectance to the collimated solar beam, increases monotonically with SZA as the beam's slant path through the layer ($\propto 1/\cos\theta$) lengthens. $R\uparrow$ (blue squares), the layer reflectance to the diffuse upwelling field, is shown as the mean over surface albedos $A_s \in [0.3, 0.95]$; light-blue points show the individual estimates extracted independently at each A_s, which coincide at every SZA. $R\uparrow$ is independent of both SZA and A_s, consistent with the model assumption that the upwelling field incident on the layer base is isotropic and therefore carries no memory of the solar geometry. Estimates at small A_s are excluded from the mean because Eq. (11) is ill-conditioned as $A_s \rightarrow 0$. The SZA-dependence of Φ in Fig. 4a therefore originates entirely in $R\downarrow$; the crossing $R\downarrow = R\uparrow$ marks $\Phi = 1$.

References

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