



Development of a GUI-based Educational Tool for the Comparison of Pillar Design Models in Underground Mining

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Abstract. The room-and-pillar method is one of the most widely used underground mining methods owing to its simplicity, safety, and low cost. Pillar design is fundamental to ensure operational stability and increase ore recovery. In recent decades, various empirical formulas have been developed to estimate pillar strength under different geometric and geomechanical conditions, each considering specific conditions of application and presenting a series of limitations. This study presents a Python-based educational software tool with a graphical user interface (GUI), designed to support the teaching and preliminary analysis of empirical pillar-design models in underground mining. The tool integrates tributary area theory, equivalent width calculation, pillar strength estimation, safety factor, and extraction ratio assessment within an interactive environment. Several empirical and semi-empirical models, including Lunder-Pakalnis, Hedley-Grant, Salamon-Munro, Krauland-Soder, Sjoberg, Laubscher, Hoek-Brown, González, USBM, Bieniawski and user-defined equations, are implemented to allow systematic comparison under identical input conditions. In addition, the software provides graphical visualization, automatic critical model identification, and parametric analyses of the safety factor as a function of We/h , including the associated extraction ratio to reach a target safety factor. A case study is presented to examine how the tool supports the visualization, comparison, and interpretation of empirical design equations. The case study and preliminary student feedback suggest that the software can support the conceptual understanding of pillar stability, model selection, and the trade-off between safety and extraction in room-and-pillar mining. The tool is intended for educational and preliminary engineering purposes and can be used in courses related to underground mining, rock mechanics, and mine design.

1 Introduction

Underground mining has gained significant relevance in recent years, mainly driven by the progressive depletion of easily accessible and shallow deposits (Fitsak et al., 2017; Ghorbani et al., 2023). This scenario has led to the adoption of various underground methods, in which pillar design is a fundamental component for ensuring the stability and safety of excavations. This consideration is particularly critical in the room-and-pillar method, where pillars are the main source of support throughout the entire operation (Jessu et al., 2022).

The room-and-pillar method is generally applied to flat, stratified mineralized bodies of limited thickness, such as copper slate deposits, coal seams, salt and potash deposits, limestone, and dolomite (Hartman and Mutmanský, 2007). Its advantages



30 include continuous production, rapid development rate, simple ventilation design, moderate operational costs, and high selectivity (Nieto, 2010).

The development of computational tools has gained great relevance in recent years, both in industrial and research applications, where Python has established itself as an accessible language that allows for the automation of calculations, the generation of interactive visualizations, and a reduction in reliance on extensive spreadsheets, thereby reducing the probability of human errors. The development of tools in Python enhances the efficiency of data processing and analysis by providing accessible data structures, visualization capabilities, and reusable computational workflows that support technical interpretation and decision-making processes (McKinney, 2010).

In this context, this study presents a Python-based graphical user interface (GUI) software developed to support the teaching and preliminary comparison of empirical pillar-design models in underground mining. The tool allows users to quickly and systematically evaluate pillar load, equivalent width, safety factors of different empirical models, extraction ratio, and parametric trends under consistent input conditions. Additionally, the tool is intended to be applicable in the preliminary phases of mine design and facilitate an understanding of the factors that influence pillar design.

To frame the development of the tool within a pedagogical research perspective, this study addressed the following research questions:

- 45 • RQ1: How can an educational graphical user interface support the comparison and interpretation of empirical pillar design models in underground mining?
- RQ2: What conceptual relationships among pillar geometry, tributary area theory, safety factor, and extraction ratio can be visualized using the proposed tool?
- RQ3: How do students perceive the usefulness of the tool as a teaching resource for understanding pillar design models, model variability, and the limitations of empirical approaches?

2. Background and Educational Motivation

2.1 Pillar design in underground mining

A pillar is defined as a mass of in situ rock left between two or more excavations (Coataes, 1981; Martin and Maybee, 2000). Interest in pillar design increased significantly after the Coolbrook coal mine disaster in South Africa in 1960, when a collapse resulted in the death of 437 workers. This event marked a turning point in research on pillar stability and highlighted the need for more systematic design methodologies (Jessu et al., 2022). A proper pillar design can determine the technical and economic feasibility of a mining project. Undersized pillars can cause local or even global collapses due to the “domino” effect (Jessu et al., 2022) or generate air blasts that affect nearby infrastructure (MSHA, 2021). However, oversized pillars may unnecessarily reduce ore recovery and significantly negatively impact the profitability of operations (Jessu et al., 2022).

60 There are different criteria for classifying pillars depending on their functionality, shape, geometry, or location within the extraction method. Some of the most common classifications are as follows:



- **According to functionality:** they differentiate between those that serve a support role, either globally or locally, or provide protection for structures of interest (Jessu et al., 2022).
- **According to geometry:** square, rectangular, barrier, or irregular pillars are distinguished depending on the geometry of their basal section (Hoek and Brown, 1980).
- **According to shape:** they are categorized according to their shape and width/height ratio into slender, intermediate, and squat pillars, which directly influence the failure mode and strength of the pillar (Mark, 1999).
- **According to their location and orientation within the mining design:** they are categorized as rib, sill, and crown pillars (Betournay, 1989).

According to Mark (2000), the classical empirical methodology for pillar design involves the following three steps:

- i. Estimating the load on the pillar using tributary area theory (TAT);
- ii. Estimating the pillar strength using one of the existing formulas; and
- iii. Calculating the design safety factor. In addition, the above should be complemented by determining mineral recovery without losing sight of the profitability of mining operations.

From an educational perspective, this design process is particularly relevant because it combines geomechanical concepts, empirical model selection, geometric interpretation, and economic trade-offs. However, students often learn these equations separately, without easily visualizing how changes in the pillar geometry, equivalent width, depth, rock strength, and extraction ratio affect the resulting safety factor. This motivated the development of interactive computational tools that allow students to compare pillar-design models under consistent input conditions.

2.2 Tributary area theory and pillar load

The determination of the load or forces acting on a pillar is carried out using tributary area theory, computational techniques, or more rigorous methods that involve measuring the forces in situ (Castro-Caicedo et al., 2019). The tributary area theory (TAT) has been universally and successfully used as the main methodology for assessing the loads acting on pillars (Vlachogiannis and Benardos, 2024).

This theory implies that the load acting on each pillar is a function of the column of rock directly above the pillar and the rock situated between an individual pillar and any of its adjacent pillars (Maybee, 2000). For a rectangular pillar layout with width W , length L , and room width and length left in each direction B_L and B_w , respectively, the following scheme reflects the theory (Figure 1).

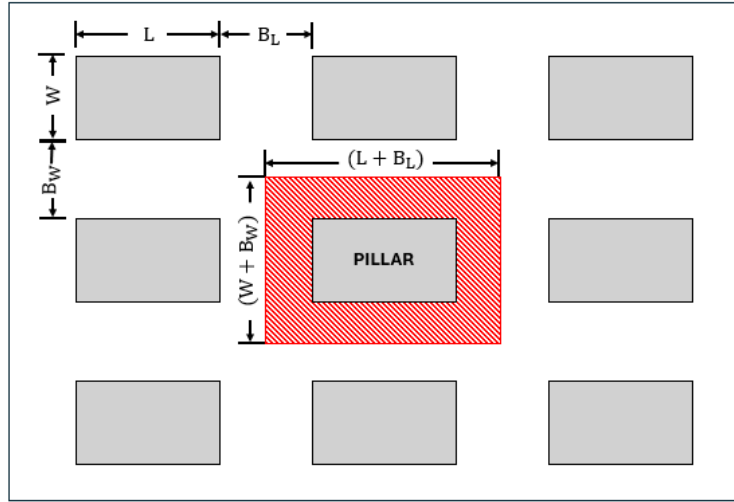


Figure 1: Diagram of rectangular pillar design.

The vertical pre-mining stress can be estimated as follow:

$$\sigma_v = \rho g H \quad (1)$$

where σ_v is the vertical stress, ρ is the density of the rock (kg/m^3), g is the gravitational constant (9.81 m/s^2), and H is the depth at which the pillar is located (m).

95 For a regular rectangular room-and-pillar layout, the load acting on the pillar can be estimated using the tributary area theory (TAT):

$$S_p = \sigma_v \frac{(W + B_w)(L + B_L)}{(W \cdot L)} \quad (2)$$

where S_p is the average pillar load, W and L are the pillar width and length, respectively, and B_w and B_L are the room width and length, respectively. For square pillars, the equation assumes that the pillar length is equal to the pillar width, simplifying the geometry while preserving the same tributary area logic.

100 In addition to stability, the profitability of the design must be ensured. For this purpose, the extraction ratio (e) was used, which represents the amount of material extracted relative to the amount of ore available. This should be maximized without compromising excavation stability.

$$e = \frac{\text{excavated area}}{\text{total area}} = \frac{(W + B_w)(L + B_L) - (W \cdot L)}{(W + B_w)(L + B_L)} \quad (3)$$

In an educational context, this calculation is particularly useful because it allows students to observe that increasing the ratio generally increases the load transferred to the remaining pillar, which may reduce the safety factors.

105 Although the tributary area theory is widely used, mainly because of its simplicity, it presents a series of assumptions and limitations that must be considered in the pillar design. For example, this theory assumes that the vertical pre-mining stress is



uniformly supported by all pillars in the same manner (Figure 2a). However, this condition is only satisfied when the overburden is weak, shallow (less than or equal to the design span), and the pillars are arranged in homogeneous environments (Figure 2b). When the design is implemented at greater depths, in narrow bodies, or under strong overburden, this distribution changes, with the central pillars tending to bear most of the stress, whereas the pillars near the edge are protected by a pressure arch (Figure 2c) (MSHA, 2021). This difference between the ideal and actual distributions of vertical stress can induce errors or overestimate the safety factor.

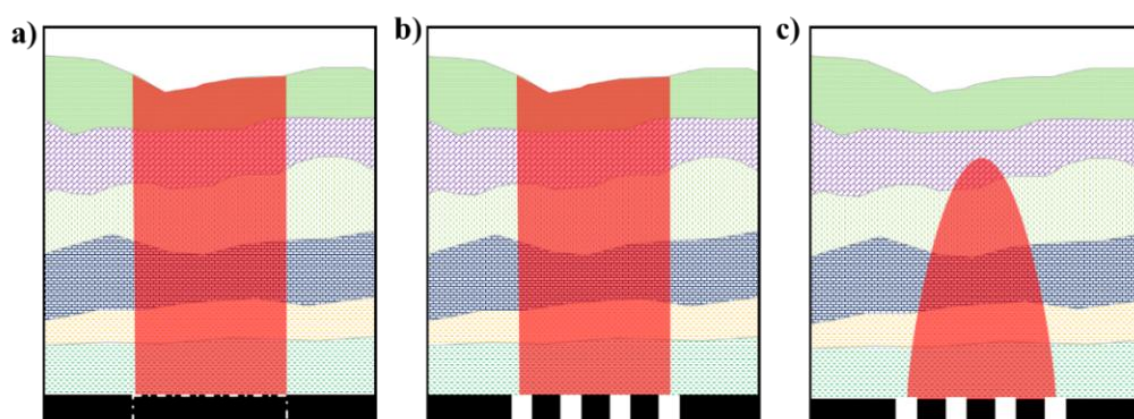


Figure 2: a) Load on the area of interest before excavation; b) stress distribution on the pillar according to TAT, and c) actual distribution (after de Mark and Agioutantis, 2019)

2.3 Pillar strength and empirical models

Pillar strength is controlled by several factors, including pillar geometry, that is, dimensions and shape (width/height ratio), as well as the dimensions of the room (Brady and Brown, 1999). In addition, intact rock strength, rock mass quality, and discontinuity conditions are key factors in pillar stability because pillar behavior depends on the strength of the intact rock and discontinuities, as well as rock mass quality indicators such as the geological strength index (GSI) (Li et al., 2021; Moffat et al., 2021). Experimental, numerical, and empirical techniques have been used to estimate pillar strength (Castro-Caicedo et al., 2019). Various empirical formulas can be found in the literature, most of which are obtained through a back-calculation approach, that is, a retrospective analysis of historical data from stable and failed pillars (Martin and Maybee, 2000). These equations aim to relate pillar strength to geomechanical and geometric parameters for specific terrains.

Lunder and Pakalnis (1997) pointed out that the empirical equations developed generally take two forms: i) shape effect and ii) size effect. Shape effect formulas consider that a pillar with a certain width/height ratio has the same strength, regardless of its volume or dimensions. However, the size effect formulas assume that a pillar of a given shape (width/height ratio) reduces its strength as its size increases.

Equations governed by the shape effect are linear, whereas those governed by the size effect are power-law. Both relationships can be represented by the following unified general equation:



$$\sigma_p = K \left(A + B \frac{W^\alpha}{H^\beta} \right) \quad (4)$$

where K , A , B , α , and β are adjustment factors dependent on the terrain associated with the excavation. It can be assumed that when $\alpha = \beta = 1$, it reduces to the representative equation for the shape effect, and when $A=B=1$ the formulas that follow the size effect methodology are obtained.

Although most of the empirical methods available in the literature allow for the calculation of the strength of square pillars, equations for rectangular pillars have also been developed, for example, Mark-Bieniawski (Mark and Chase, 1997). Another approach, widely used to address the design of rectangular pillars, corresponds to the use of the equivalent width (W_e) proposed by Wagner (1980), which relates the area (A) and perimeter of the pillar (O) (Eq. 5).

$$W_e = \frac{4A_p}{O} = \frac{2WL}{(W + L)} \quad (5)$$

The most common empirical equations for pillar design follow the general unified form, which includes models such as Obert-Duvall, Hedley-Grant, among others (Table 1).

Table 1. Simple models based on the general form.

Models	K	A	B	α	β	Rock type
Obert and Duvall (1967)	σ_c	0.778	0.222	-	-	Coal
Krauland and Soder (1987)	$0.354\sigma_c$	0.778	0.222	-	-	Limestone
Sjoberg (1992)	$0.308\sigma_c$	0.778	0.222			Limestone
Salamon and Munro (1967)	9.115	-	-	0.46	0.66	Coal
Sheorey et al. (1987)	0.27	-	-	0.50	0.86	Coal
Hedley and Grant (1972)	$0.578\sigma_c$			0.50	0.75	Hard rock

While classical empirical equations are mainly based on simple geometric relationships between dimensions, there are more complex models that incorporate parameters and use other foundations, increasing the accuracy and realism, but simultaneously increasing the complexity of the analysis (Table 2).

Table 2. Compilation of pillar design models

Models	Equations	Parameters
Bieniawski (1984)	$\sigma_p = \sigma_1 \left(0.64 + 0.36 \frac{W}{h} \right)$ $\sigma_1 = \sigma_c (D/36)^{0.5}$	σ_1 : Unconfined compressive strength of a large cube of rock (MPa) D : Specimen diameter (in)



Laubscher (1990)	$\sigma_p = DRMS \frac{w^{0.5}}{h^{0.7}}$	DRMS: Design Rock Mass Strength (MPa)
Stacey and Page (1986)	$If: \frac{w}{h} < 4.5 \rightarrow \text{Laubscher [31]}$ $If: \frac{w}{h} > 4.5 \rightarrow \sigma_p = DRMS \frac{2.5}{V^{0.07}} \left[0.13 \left(\left(\frac{4.5 w_{ef}}{h} \right)^{4.5} - 1 \right) + 1 \right]$	DRMS: Design Rock Mass Strength (MPa) V: pillar volume (m3)
Lunder and Pakalnis (1997)	$\sigma_p = 0.44 \sigma_c (0.68 + 0.52 \kappa)$ $\kappa = \tan[\arccos(\frac{1 - C_{pav}}{1 + C_{pav}})]$ $C_{pav} = 0.46 [\log(\frac{w}{h} + 0.75)]^{1.4 / (w/h)}$	κ : internal friction of the pillar C _{pav} : average pillar confinement.
González- Nicieza et al. (2006)	$\sigma_p = \left(\sigma_c e^{\frac{RMR_{89} - 100}{20}} \right) \frac{w^{0.5}}{h^{0.75}}$	RMR89: Rock Mass Rating 1989
NIOSH (Esterhuizen et al., 2011)	$\sigma_p = (0.65 \sigma_c (1 - DDF \cdot FF)) \frac{w^{0.5}}{h^{0.75}}$	DDF: Discontinuity Dip Factor FF: Frequency Factor
Hoek and Brown (1980)	$\sigma_1 = \sigma_3 + \sigma_c (m_i e^{\frac{GSI - 100}{28 - 14D}} \frac{\sigma_3}{\sigma_c} + e^{\frac{GSI - 100}{9 - 3D}})^{\frac{1}{2} + \frac{1}{6} (e^{\frac{-GSI}{15}} - e^{\frac{-20}{3}})}$	GSI: Geological Strength Index m _i : rock parameter D: disturbance factor σ_3 : confinement, obtained from the width/height ratio

145 It is important to note that each of these methods for estimating pillar strength can be adapted to rectangular pillars using Equation 5.

Table 3 summarizes the models grouped according to their general formulation approach. The main input variables, representative examples, and potential educational uses for each group are indicated. This classification aims to help students recognize that empirical pillar models are not interchangeable, as each group emphasizes different controlling factor.

150 **Table 3. Compilation of pillar design models**

Model group	Main variables	Examples	Educational use
Shape-effect models	$\sigma_c, W_e/h$	Obert-Duvall, Krauland-Soder, Sjöberg	Show influence of pillar slenderness
Size-effect models	W_e, h	Hedley-Grant, Salamon-Munro, Sheorey	Show influence of pillar scale
Rock mass-based models	RMR, GSI, m _i , D, DRMS	Laubscher, González, Hoek-Brown-	Show role of rock mass quality
Correction-factor models	DDF, FF, specimen size	USBM, Bieniawski	Show influence of discontinuities and scale
General equation	K, A, B, α, β	General equation	Explore model sensitivity



2.3.1 Safety factor

The stability of the pillar is evaluated using the safety factor (FS), which corresponds to the ratio between the strength of the pillar (σ_p) and the stress to which it is subjected (S_p):

$$FS = \frac{\sigma_p}{S_p} \quad (6)$$

A safety factor of one indicates a limit equilibrium condition, where the strength is equal to the applied load. In practice, an FS greater than 1 is required to ensure stability.

Several authors have proposed acceptable minimum FS values depending on the rock type, geomechanical conditions, and design criticality. For example, Lunder and Pakalnis (1997) suggested an FS of 1.4 as a general criterion for hard rock, Lunder (1994) recommended values greater than 1.6, and NIOSH considered an FS of 1.8 for mining operations in the US.

2.3.2 Use of python and graphical user interfaces in mining applications

Python has become an increasingly useful programming language for the development of computational tools applied in mining, mineral processing, and geoscience applications because of its accessibility, open-source ecosystem, and capacity to integrate numerical calculations, visualizations, and graphical user interfaces. Several studies have demonstrated this trend across different mining-related applications. Manríquez et al. (2020) proposed a simulation-optimization framework for short-term underground mine production scheduling, implemented using Python-based tools such as UDESS, which can be used through scripts or a graphical user interface, and DSIM, a discrete-event simulation software coded in Python using SimPy. In blasting-related applications, Nayak et al. (2022) developed a Python program to compare empirical flyrock prediction models and benchmark their results against Mine Excellence flyrock predictor software. Similarly, Alshibani and Moselhi (2012) developed an optimization-simulation model for fleet selection in earthmoving operations, integrating GPS data, GIS visualization, simulation, optimization, and a GUI module to estimate truck cycle times, evaluate fleet configurations, and support decision-making under uncertainty. Recently, Gu et al. (2026) proposed a hybrid machine learning framework for predicting the peak particle velocity (PPV) induced by blasting in open-pit mines and incorporated an interactive GUI for local data learning, real-time PPV prediction, and dynamic blast design optimization. In geomechanics, Aftab and Moghadam (2025) developed GEOMECHANICS_GUI, an integrated MATLAB-based interface that combines geomechanical parameter calculation, rock brittleness assessment, uncertainty quantification, sensitivity analysis, and real-time visualization in a single user-friendly environment.

Several specialized software tools are available for professional and research applications regarding pillar design. The S-Pillar software, developed by the National Institute for Occupational Safety and Health (NIOSH), assists in the design of stable pillars for room-and-pillar workings in underground stone mines (Esterhuizen et al., 2011). CPillar software, developed by Rocscience, is an interactive tool for assessing the stability of underground crown pillars and roof beds using limit equilibrium



180 methods. In coal mining, the ACPS Analysis of Coal Pillar Stability software proposed by Mark and Agioutantis (2019) integrates previous pillar design tools into a single framework for longwall mining design.

Although these tools offer valuable capabilities for specific pillar design contexts, they are generally geared toward particular applications, such as stone mine, crown, or coal pillars. In contrast, the software proposed in this study was conceived as a preliminary educational and comparative tool. This allows students to evaluate multiple empirical pillar strength models under
185 the same input conditions. Therefore, the contribution of this software lies not in replacing specialized design packages but in providing an accessible environment with a graphical user interface (GUI) for teaching, interpreting, and preliminarily comparing models.

2.4 Educational challenge in teaching pillar design

Teaching pillar design in room-and-pillar mining requires the integration of rock mechanics concepts, mine design, empirical
190 modeling, and economic evaluation. Although the classical design procedure is conceptually straightforward-estimating the pillar load, estimating the pillar strength, and calculating the safety factor-it involves several variables whose effects are not always intuitive to students. The various parameters used, such as depth, rock density, uniaxial compressive strength (σ_c), pillar width and height, equivalent width, room dimensions, and extraction ratio, interact simultaneously and can produce different design interpretations.

195 In many undergraduate and graduate mining engineering courses, pillar strength models are commonly introduced using isolated equations or spreadsheets-based exercises. Although this approach is useful for practicing calculations, it can limit students' ability to visualize how variations in geometry and geomechanical parameters affect pillar stability. For example, students may calculate a safety factor for a single model without clearly understanding why another empirical equation applied to the same case produces a more or less conservative result than theirs.

200 This educational challenge promotes the development of interactive computational tools that allow students to compare empirical pillar design models under identical input conditions. In this context, an educational tool with a graphical user interface (GUI) can foster active learning by enabling students to compare models. Instead of applying equations independently, students can visualize the model variability, perform parametric analyses, and examine the relationship between the equivalent width, stability, and extraction. Thus, the proposed tool can contribute to a more comprehensive conceptual
205 understanding of pillar design and encourage the critical interpretation of empirical methods, rather than being limited to purely mathematical calculations.



3. Software Architecture

3.1 Programming environment

The software was developed in Python because of its accessibility, wide adoption in engineering, and availability of open-source libraries for numerical calculations, graphical interface, and easy data visualization. The graphical user interface (GUI) was implemented using the Tkinter library, which is included in the standard Python distribution and allows the incorporation of the necessary components for creating desktop applications, such as windows, menus, buttons, checkboxes, and tabbed panels. This choice facilitated the development of a lightweight and accessible application without requiring commercial software or complex installation procedures. The main libraries used in the development of the software are summarized below:

- **Math:** this library was used to perform the mathematical operations required in the calculation routines, including vertical stress estimation, pillar load calculation, equivalent width estimation, empirical strength equations, safety factor calculation, and parametric analysis.
- **Matplotlib:** this library was used for graphical outputs, including the room-and-pillar layout representation, safety factor comparison chart, and parametric analysis plots. These visualizations were integrated directly into the GUI, allowing users to interpret the results without exporting the data to an external plotting tool.
- **Pillow:** this library was used to manage image resources such as the software logo and application icon.
- **CSV:** this library was used to export the last calculated results, allowing users to store and review the outputs generated by the application.

The computational routines were organized into independent functions responsible for input validation, pillar load calculation, equivalent width estimation, safety factor calculation, stability classification, and graphical updates. This structure improves code readability and facilitates future extensions, such as the incorporation of additional empirical models, new visualization options, or alternative design criteria.

Finally, the application was packaged as a standalone executable file using PyInstaller. This allows the software to be distributed and used on Windows computers without requiring the end-user to install Python. Therefore, the tool can be used in classroom environments, computer laboratories, or individual student computers with minimal setup requirements.

3.2 Graphical user interface structure

The software was designed with a graphical user interface to facilitate interaction with pillar design models and to reduce reliance on command-line execution or spreadsheet-based calculations. The interface is organized into three main areas: (i) an input panel, (ii) a display panel, and (iii) a results panel. This layout facilitates the visualization of the pillar geometry and graphical results as the user changes the necessary parameters, allowing the interpretation of the calculated results in a single environment. As shown in Figure 3, the interface is organized into input, visualization, and result areas.

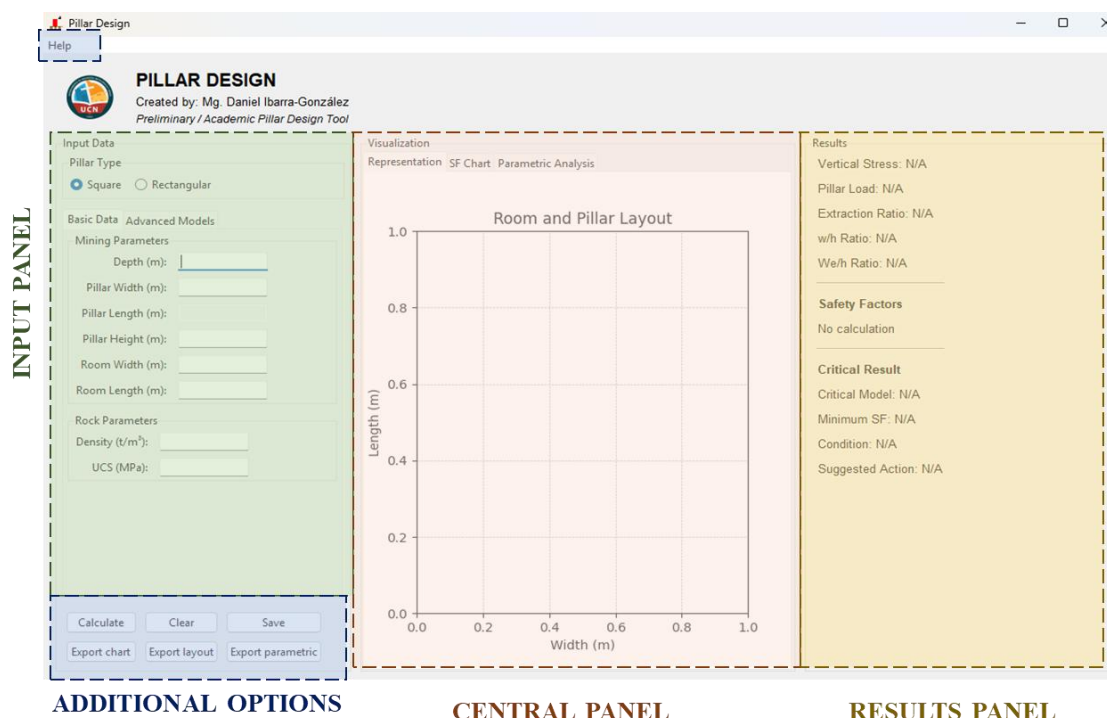


Figure 3: Main graphical user interface of the developed pillar design software. The interface was created by the authors.

The input panel is located on the left side of the interface and contains the main geometric and geomechanical data required for the analysis. These include depth, rock density, uniaxial compressive strength, pillar dimensions, and room dimensions of the rock mass. The user can select whether the pillar geometry is square or rectangular. When a square pillar is selected, the length is automatically set to be equal to the width, which reduces the possibility of inconsistencies in the entered geometric data. This panel is also divided into basic and advanced model sections. The basic section includes the parameters necessary for the general calculation of the pillar load and safety factor, whereas the advanced section allows the user to activate additional empirical or semi-empirical models, previously listed in Table 2, which require specific geomechanical parameters such as GSI, m_i , disturbance factor, RMR, DRMS, discontinuity factors, or sample diameter. This structure allows students to first approach simpler models, while more advanced users can progressively explore formulations that require additional information about the rock mass.

The central panel shows the graphical results obtained using the software. It includes a representation of the room and pillar layout, where the evaluated pillar and its tributary area are shown schematically. This visualization helps users understand how the selected geometry influences the tributary area and, consequently, the load on the pillar. The same panel also allows the visualization of a safety factor chart, in which the results obtained from the selected empirical models are compared using a bar chart. Color classification was used to distinguish between unstable, marginal, and stable conditions. Additionally, this panel includes a parametric analysis module, allowing the pillar width to be varied within a defined range and evaluating the



255 corresponding change in the safety factor as a function of the equivalent width-to-height ratio (W_e/h). A target safety factor can be specified, and the software identifies the minimum required W_e/h ratio for each model to meet this criterion, along with the associated extraction ratio. This function facilitates visual and conceptual learning by enabling students to explore the relationship between stability and mineral recovery through simulation.

The results panel is located on the right side of the interface and summarizes the main numerical data, including the vertical stress, pillar load, extraction ratio, w/h ratio, W_e/h ratio, safety factors, and stability classification. Additionally, the critical model is indicated, which corresponds to the empirical formulation that produces the lowest safety factor for the selected input conditions. This automatic identification is a reference for users regarding the most restrictive model under these conditions. Additional options are offered through buttons and a Help menu. Users can save the last calculated result, export the safety factor chart, export the room-and-pillar design, and export the parametric analysis charts. The Help menu includes a methodology/about window, unit converter, and auxiliary equivalent width calculator for nonrectangular pillars. These auxiliary tools do not modify the main calculations but facilitate interpretation, reduce unit conversion errors, and improve the educational utility of the application. Finally, Table 4 presents a summary of the modules that comprise the software and their respective functions.

Table 4. Summary of modules and their respective functions

Module	Function
Input module	Captures geometric and geomechanical data
Validation module	Checks positive values and model parameter ranges
Calculation module	Computes load, strength, FS and extraction ratio
Visualization module	Generates layout, FS chart and parametric graph
Export module	Saves CSV and figures
Help module	Includes methodology, unit converter and W_e calculator

270 3.3 General workflow

The calculation workflow was designed to follow the classical empirical pillar design procedure: input definition, load estimation, pillar strength calculation, safety factor assessment, and result interpretation. This sequence allows the software to reproduce the typical steps used in preliminary room-and-pillar design while presenting them in an interactive and educational format.

275 The main implemented algorithm allows for the following:

- Capturing user input data corresponding to geomechanical properties (density and σ_c), depth, and geometry of the pillar and room.
- Validate that all entries are complete, positive, and technically consistent.



- Calculate the vertical stress applied to the pillar as a function of the TAT.
- Estimate the equivalent width of the pillar (W_e) depending on whether its shape is square or rectangular.
- Calculate the load acting on the pillar and the corresponding extraction ratio.
- Visualization of the generated room-and-pillar layout and tributary area of the evaluated pillar.
- Enter complex parameters using advanced models or a new model that follows a general unified form (optional).
- Calculate the pillar strength and safety factor for each selected model under the same inputs conditions.
- Classify the calculated safety factors according to reference stability criteria.
- Identify the critical model, defined as the model that provides the lowest safety factor.
- Perform a parametric analysis by varying the pillar width and plotting the safety factor as a function of W_e/h .
- Estimate the minimum W_e/h required to reach the target safety factor and report the associated extraction ratio.
- Export numerical results and graphical outputs, including the safety factor chart, room-and-pillar layout, and parametric analysis plot.

This workflow was designed to help students move from isolated empirical equations to an integrated interpretation of pillar geometry, load redistribution, strength estimation, stability, and extraction ratio. A flowchart summarizing the general workflow of this software is presented in Figure 4. Because the parametric analysis module follows a specific procedure, its workflow is presented separately in Figure 5.

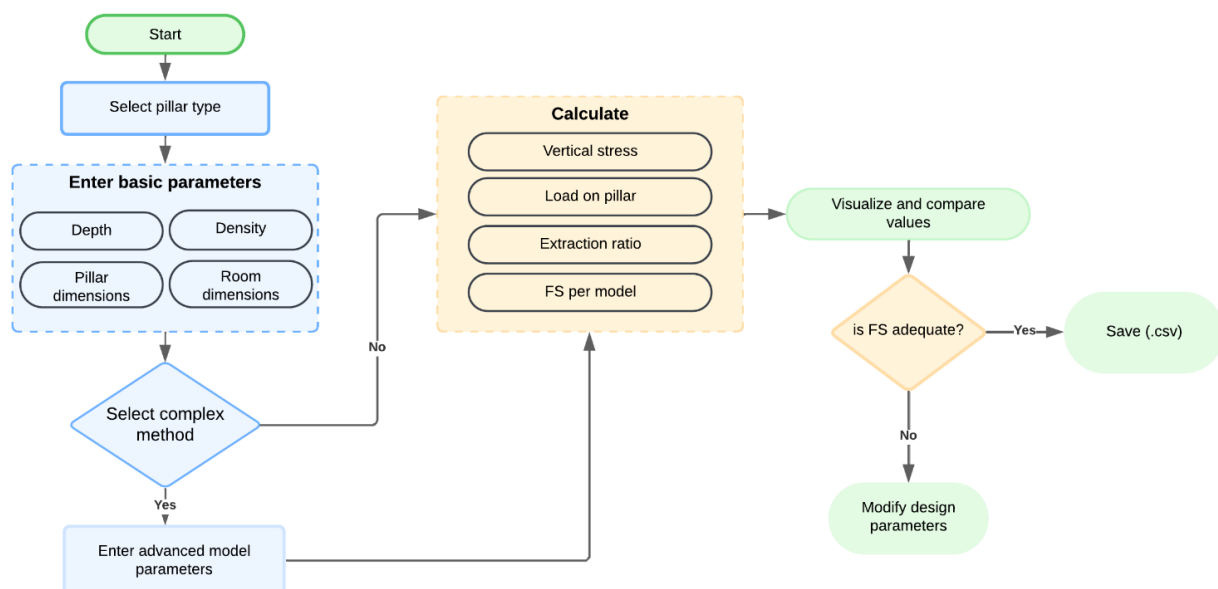


Figure 4: Flowchart of the tool operation for pillar design.

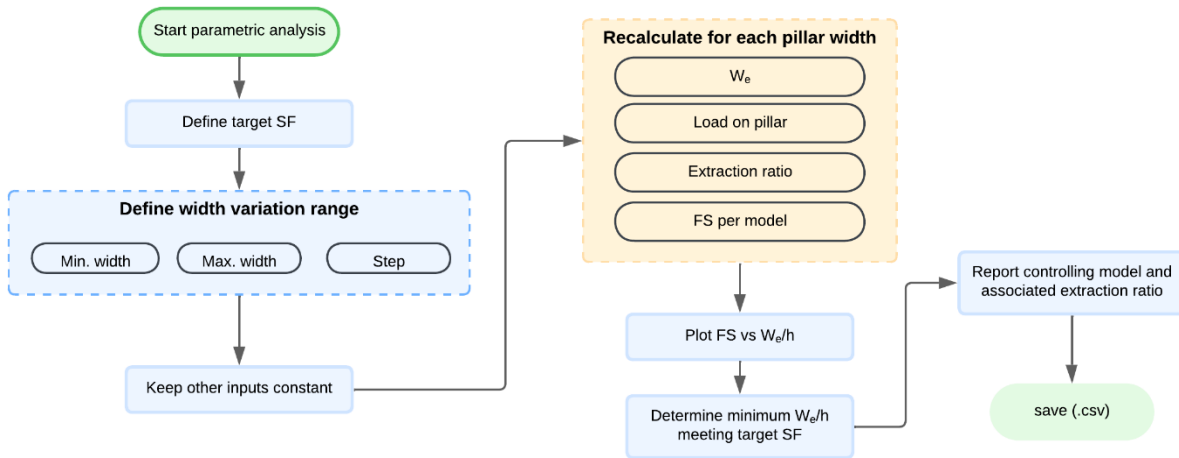


Figure 5: Iterative workflow of the parametric analysis module for evaluating safety factor as a function of W_e/h .

3.4 Educational feedback procedure

To obtain preliminary evidence of the educational usefulness of the tool, a workshop structured as a learning activity was designed and implemented. During the workshop, the students first reviewed the basic concepts of pillar design, tributary area theory, safety factor, and extraction ratio. They then used the software to reproduce the case study, compare empirical pillar design models, interpret the graphical results, and conduct a parametric analysis by varying the pillar width to assess changes in the safety factor and extraction ratio. The activity concluded with a brief discussion on the applicability and limitations of the empirical pillar design models.

After the workshop, a short feedback form was administered to the students. The purpose of the form was to assess the perceived usefulness of the tool as a didactic resource rather than to directly measure learning progress. The student form focused on perceived support for understanding pillar geometry, safety factor, extraction ratio, model comparison, graphical interpretation, parameter sensitivity, ease of use, and potential applicability in mining-related courses. The form included items with a Likert-scale from 1 to 5, where 1 represented “strongly disagree” and 5 “strongly agree,” and open-ended questions. The seven Likert-scale items included in the student form are summarized in Table 5.

Table 5. Likert-scale items included in the student feedback form

Item	Statement
1	The tool helped me understand the relationship between pillar geometry and the safety factor.
2	The tool helped me compare different empirical models for pillar design.
3	The graphical outputs made the results easier to interpret.
4	The tool helped me understand the relationship between the safety factor and extraction ratio.



- 5 The tool allowed me to observe how changes in the input parameters affect the stability of the pillar.
- 6 The tool was easy to use during the activities.
- 7 The tool would be useful in underground mining, rock mechanics, and mine design courses.

Responses were analyzed descriptively using frequency counts for Likert-scale items and thematic grouping for open comments. Owing to the exploratory nature and small sample size of the feedback process, the results are interpreted as preliminary evidence of perceived educational usefulness rather than as a formal measurement of learning outcomes.

4. Case study, results and educational feedback

320 4.1 Case study definition

A case study was developed to demonstrate the effectiveness of the proposed GUI-based educational tool and to compare the behaviors of the different empirical models under the same input conditions. The input parameters considered in this case study are summarized in Table 6. The case represents a preliminary room-and-pillar design scenario in which the user defines geometric and geomechanical parameters. These include mining depth, pillar dimensions, room dimensions, overburden material density, uniaxial compressive strength, and additional parameters required by the selected advanced models when available. The aim of this case study is not to define a final pillar design but to demonstrate how the tool supports comparative analysis, graphical interpretation, and educational discussion of empirical design methods.

Table 6. Parameters of case study

Parameters	Value
Depth (m)	75
Pillar dimensions (m)	3x4
Pillar height (m)	5
Room dimensions (m)	6x7
Overburden material density (t/m^3)	2.5
Uniaxial Compressive Strength - σ_c (MPa)	80
Specimen diameter (mm)	64

Once the input data were entered, the tool automatically calculated the vertical stress, pillar load, extraction ratio, and width-to-height ratio. In addition, the room-and-pillar layout and associated tributary area are displayed graphically. This visualization allows users to relate the entered geometry to the load estimation process based on the TAT.

Base calculation results

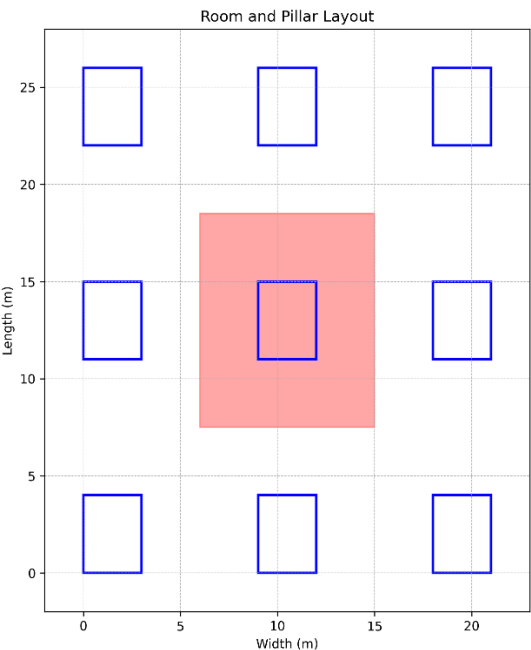
Initially, the application provided stress- and geometry-related parameters. Table 7 presents the vertical stress, load on the pillar, extraction ratio, and width-to-height ratio (w/h and W_e/h).

Table 7. Stress and load results of case study

Parameters	Value
Vertical stress (σ_v) (MPa)	1.84
Load on the pillar (S_p) (MPa)	15.17
Extraction ratio (%)	88
Width/height ratio (w/h)	0.6
Width/height ratio (W_e/h)	0.69

The results provide a basis for empirical model comparison. In particular, the extraction ratio is useful for interpreting the equilibrium between ore recovery and pillar stability. A high extraction ratio generally indicates that a larger proportion of the mineralized body is removed, but it also implies that the remaining pillars must support a greater redistributed load. Therefore, the simultaneous evaluation of the extraction ratio and safety factor is fundamental for discussing the stability-recovery trade-off in room-and-pillar design.

The layout visualization using the tool is shown in Figure 6. The figure represents the selected pillar geometry, surrounding rooms, and tributary area considered for the load calculation. From an educational perspective, this representation helps students connect the numerical calculation of the pillar load with the physical meaning of the TAT.

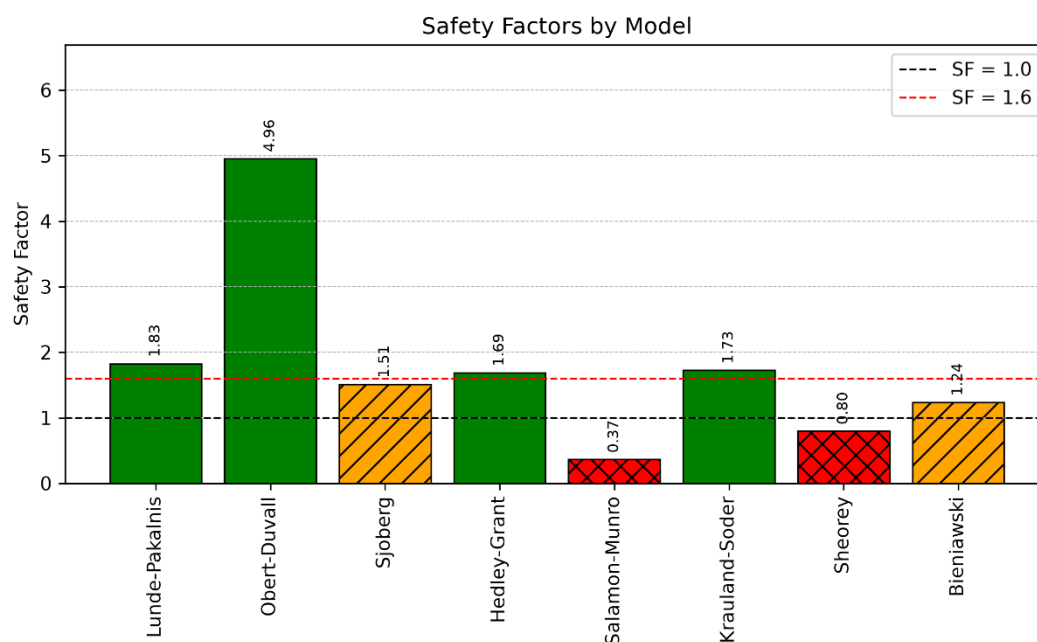




345 **Figure 6: Room-and-pillar layout and tributary area generated by the tool for the case study.**

4.3 Comparison of empirical models

After the base calculations, the software estimates the pillar strength and safety factor for each selected empirical model. The safety factor is calculated using Equation 6. Figure 7 presents a safety factor comparison chart generated by the tool.



350 **Figure 7: Safety factor comparison by empirical model for the case study.**

The graph allows a direct comparison of the variability among the implemented models. Each bar represents the safety factor obtained from an empirical formulation under the same geometric and geotechnical conditions. The chart also includes reference lines corresponding to $SF=1.0$ and $SF=1.6$. These values are used as preliminary reference thresholds, where:

- **$SF < 1.0$:** unstable condition, represented by red bars with cross-hatching
- **$1.0 \leq SF < 1.6$:** marginal condition, represented by orange bars with diagonal hatching
- **$SF \geq 1.6$:** stable condition, represented by green bars without hatching

355 The results show that the empirical models may provide significantly different safety factor values for the same pillar design. This variability is expected because each model was developed using different databases, rock types, assumptions, and calibration procedures. Models based on shape effects tend to respond differently from those based on size effects or rock mass quality parameters. Therefore, the comparison should not be interpreted only as a numerical ranking but also as an opportunity to discuss the model's applicability and limitations.

360 The software also identifies the critical model, which is defined as the model that provides the lowest safety factor among those evaluated. It should be noted that the models by Salamon and Munro and Sheorey deviate significantly from the minimum



design value, delivering considerably lower results, 0.37 and 0.66, respectively, explained by the modified power form used
365 in these equations, which take size effects into account. Moreover, it should be understood that a low safety factor in a model
developed for a different rock type or mining context does not necessarily imply a direct rejection of the design, but it does
indicate the need for careful interpretation of the results.

This comparison illustrates one of the main educational advantages of the tool: students can observe that empirical pillar design
is not a single deterministic calculation. Instead, it requires the interpretation of model assumptions, input parameters, and
370 design objectives.

4.4 Parametric analysis

In addition, the software includes a parametric analysis module designed to evaluate the influence of pillar geometry on the
safety factor. In this module, the user defines the minimum and maximum pillar widths, step size, and target safety factor. The
remaining input parameters were kept constant, and the tool recalculated the equivalent width-to-height ratio, pillar load,
375 extraction ratio, and safety factor for each width value. Parametric analysis was performed by varying the pillar width from 2
to 12 m in steps of 1 m. Considering that the selected pillar height is 5 m, this corresponds to a range of W_e/h from 0.53 to
1.20, allowing the evaluation of slender pillars to squat geometries. Figure 8 shows the resulting safety factor curves as a
function of W_e/h , where each curve represents a different empirical model. The target safety factor is represented by a horizontal
reference line

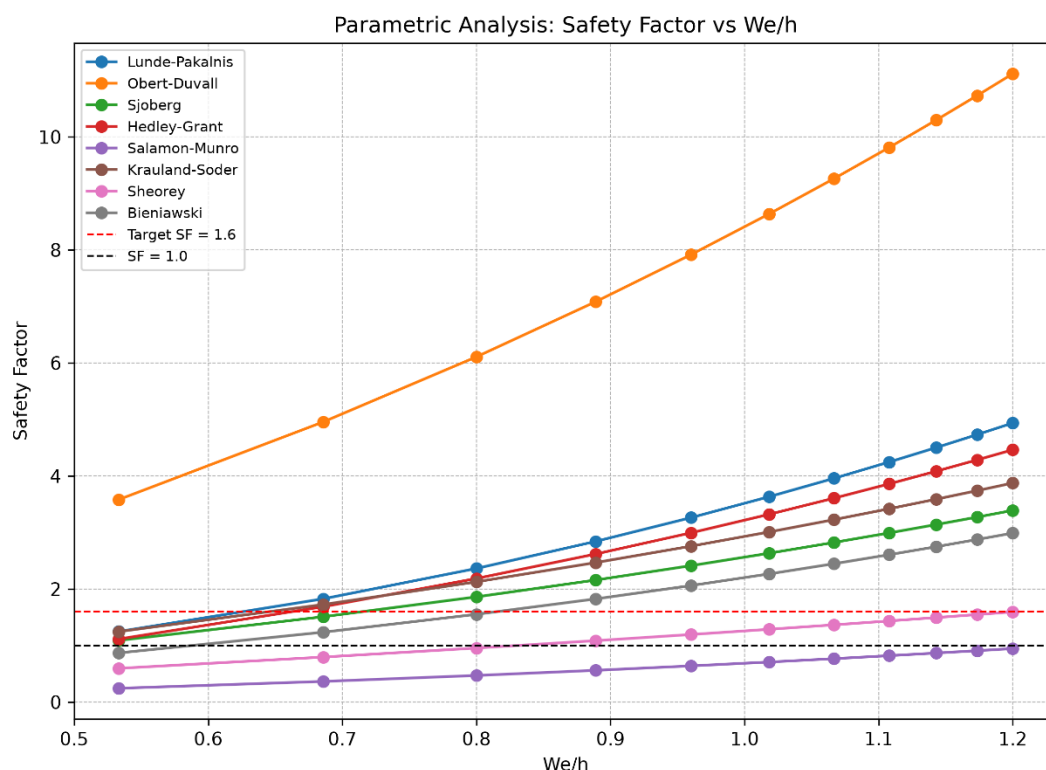


Figure 8: Parametric analysis of safety factor as a function of W_e/h

As shown in Figure 8, the tool allows for the comparison of only those models for which all the required parameters are available. The parametric analysis highlighted a clear increase in the safety factor as the W_e/h ratio increased, which is consistent with the expected improvement in pillar stability for wider pillars. From the safety factor results, it can be observed that the Obert-Duvall model yielded the highest values over the entire W_e/h range. Because this model was originally developed for coal pillars, its results may be less conservative than those of models commonly applied to hard rock pillar design. In contrast, the Sjöberg and Bieniawski models provide comparatively lower safety factors, reflecting a more conservative approach to pillar strength estimation. The Salamon-Munro model gives the lowest safety factors among all the evaluated methods, remaining below the target safety factor within the analyzed range. In contrast, the Sheorey model approaches the target threshold at the upper end of the interval, indicating that the required safety factor could be reached by increasing the W_e/h ratio.

These results demonstrate that the estimated safety factor is highly dependent on the selected empirical model. The tool is useful for educational purposes to illustrate the sensitivity of pillar assessment to the choice of design method. From an educational perspective, parametric analysis is one of the most relevant features of this tool. Instead of calculating a single safety factor for a fixed geometry, students can explore how safety evolves as the pillar width increases or decreases. Table 8 shows the minimum W_e/h required to achieve the target safety factor and the extraction ratio associated with that geometry. It



can be observed that the critical model corresponds to the Bieniawski model and corresponds to the empirical formulae that require the highest W_e/h to satisfy the selected target safety factor. This result provides an additional criterion for preliminary design interpretations.

400 **Table 8. Minimum W_e/h required to reach the target safety factor and the associated extraction ratio.**

Model	Minimum W_e/h required	Associated extraction ratio
Lunder-Pakalnis	0.69	88%
Obert-Duvall	0.53	91%
Sjoberg	0.80	85%
Hedley-Grant	0.69	88%
Salamon-Munro	Not reached within the evaluated range	
Krauland-Soder	0.69	88%
Sheorey et al.	Not reached within the evaluated range	
Bieniawski	0.89	83%

4.5 Preliminary educational feedback

Preliminary educational feedback was collected from 10 undergraduate students who voluntarily participated after interacting with the tool during the workshop. All the participants provided informed consent. The feedback form included seven Likert-scale items and open-ended questions focused on perceived usefulness, usability, graphical interpretation, model comparison, and educational applicability.

410 The quantitative results were generally positive, with mean scores ranging from 4.70 to 4.90 out of 5 (Table 9). All students agreed or strongly agreed that the tool helped them understand the relationship between pillar geometry and the safety factor, compare different empirical pillar design models, interpret graphical results, understand the relationship between the safety factor and extraction ratio, and observe how changes in the input parameters affect pillar stability. The highest-rated item was the perceived usefulness of the tool in underground mining, rock mechanics, and mine design courses, with a mean score of 4.90. The ease of use item obtained a mean score of 4.80, with 9 out of 10 students agreeing or strongly agreeing with this statement.

Table 9. Summary of student feedback on the educational usefulness of the tool

Item	Dimension	Mean	Median	Agree or strongly agree
1	Pillar geometry and safety factor	4.70	5	10/10
2	Comparison of empirical models	4.80	5	10/10



3	Graphical interpretation	4.80	5	10/10
4	Safety factor and extraction ratio	4.70	5	10/10
5	Effect of input parameters on stability	4.70	5	10/10
6	Ease of use	4.80	5	9/10
7	Usefulness in mining-related courses	4.90	5	10/10

Open-ended responses were grouped into four themes: model comparison, graphical visualization, usability, and suggestions for improvement. Students highlighted the simultaneous comparison of empirical models, the safety factor chart, and the rapid visualization of pillar geometry as useful features of the tool. Several comments emphasized that the graphical outputs helped interpret the differences among models in a simple and visual way.

The main limitations identified were the need for prior knowledge of pillar design concepts, tributary area theory, empirical safety factor models, and basic English terminology. Suggested improvements included incorporating multilingual option, adding brief explanations of the empirical equations and input parameters, improving font size and contrast, including automatic examples, adding an option to undo or modify previous entries more flexibly, and considering more interactive visualizations, such as stress representations or three-dimensional views.

4.6 Educational interpretation of results

The case study and preliminary feedback suggest that the tool is practical for use in educational contexts as a support resource for the immediate visualization of the effect of parameter variation and enables a quick and simple preliminary analysis, which is ideal for the pre-feasibility or conceptual engineering phases. When evaluating various models under the same geometric and geomechanical conditions, the software clearly indicates that pillar design does not depend on a single deterministic equation. However, the estimated safety factor can vary significantly depending on the selected empirical formulation, original database, and assumptions. A comparative chart of the safety factors provides a clear visualization of this variability. Models such as Obert-Duvall may produce higher safety factor estimates, whereas others, such as those of Salamon-Munro or Sheorey, may provide more conservative results depending on the geometry and rock conditions considered. This contrast is useful in an educational context, as it allows students to analyze the applicability of each model.

The automatic identification of the critical model also facilitates this interpretation. Because the critical model corresponds to the lowest safety factor among the evaluated formulations, students can immediately identify the most restrictive response for the selected design scenario. However, this result should be interpreted with caution, as the lowest safety factor does not necessarily indicate that the corresponding model is the most appropriate for every rock type or mining condition. Therefore, the tool encourages critical analysis of model selection, conservative design, and limitations of empirical approaches.



The parametric analysis further reinforces the conceptual understanding of pillar behavior. By varying the pillar width while keeping the other input parameters constant, students can visualize how the safety factor changes as a function of W_e/h . This enables the identification of the minimum W_e/h required to meet a target safety factor and to assess the associated extraction ratio. Thus, the tool helps illustrate the relationship between stability and ore recovery, which is one of the central aspects of room-and-pillar design.

In general, the case study demonstrates that the software can be used as a teaching support tool in courses related to underground mining, mine design, rock mechanics and geomechanics. This tool does not replace detailed geotechnical analyses or professional judgment. Instead, it provides an interactive environment in which students can compare empirical models, visualize the influence of geometry, and interpret the relationship between pillar load, strength, safety factor, and extraction ratio.

5. Discussion

5.1 Educational contribution of the tool

The main contribution of the developed software is its ability to integrate empirical pillar design calculations, graphical visualization, and parametric analysis within a single educational environment. Traditional teaching of pillar design is usually based on manual calculations or exercises using spreadsheets, where each empirical model is addressed separately. While these approaches are useful for understanding the mathematical form of the equations, they may limit students' ability to compare models simultaneously and visualize the effect of design parameters on pillar stability.

The structure of the tool, based on a graphical user interface (GUI), overcomes this limitation by allowing users to input a single set of geometric and geomechanical parameters and evaluate multiple empirical models under identical conditions. This facilitates a more consistent comparison among models and helps students recognize that different formulations can lead to different design interpretations. Consequently, the tool promotes the critical analysis of empirical models rather than the mechanical application of equations.

Another relevant educational contribution is the graphical representation of the layout of rooms and pillars, as well as the tributary area, which helps in understanding the concept of the load redistribution. Instead of interpreting the pillar load only as a numerical value, students can visualize how the pillar and room dimensions define the area of influence and affect the load acting on the evaluated pillar.

The safety factor graph also aids in the model interpretation by displaying the results visually and comparatively. The use of reference lines and stability colors allows for the rapid identification of unstable, marginal, and stable conditions. Additionally, the automatic identification of the critical model facilitates the observation of the most restrictive result and allows for a discussion on the applicability and uncertainty of the model.

The parametric analysis module represents an additional educational advantage, as it allows the exploration of the sensitivity of the safety factor to changes in the pillar geometry. By graphically representing the safety factor as a function of W_e/h and



470 reporting the extraction ratio required to reach a target safety factor, the tool helps students understand the relationship between stability and ore recovery. This type of interactive exploration is difficult to achieve using static equations in isolation. Therefore, the software can be used as a supplementary resource in theoretical classes, practical sessions or computer-assisted laboratory activities. It can support activities in which students compare empirical models, assess the influence of geometry, identify critical design conditions, and analyze the limitations of preliminary pillar design methods. Its purpose is not to replace
475 specialized geotechnical software, numerical modeling, or site-specific design procedures but rather to provide an accessible educational platform for conceptual understanding and preliminary comparative analysis.

Regarding RQ1, the tool facilitates the comparison of empirical pillar design models by allowing the evaluation of various formulations under identical input conditions. This helps students move from isolated equation-based exercises to comparative interpretations, where differences among models can be discussed in terms of assumptions, input parameters, calibration
480 contexts, and limitations.

Regarding RQ2, the case study and parametric analysis demonstrated that the tool enabled the visualization of the relationship between the pillar geometry, tributary area load, safety factor, and extraction ratio. In particular, the graphical outputs allow students to observe how changes in the pillar width and equivalent width-to-height ratio influence both stability and recovery, making the trade-off between safety and extraction more explicit.

485 Regarding RQ3, preliminary feedback provided exploratory evidence of the tool's perceived educational usefulness. The Likert-scale responses suggested positive perceptions regarding graphical interpretation, model comparison, understanding the relationship between pillar geometry and safety, and the usefulness of the tool in mining-related courses. Open-ended comments also highlighted the value of comparing several empirical models simultaneously, visualizing room-and-pillar layout, and graphically interpreting safety factor results. Simultaneously, students identified areas for improvement, such as
490 multilingual support, clearer explanations of the equations and parameters, better font size and contrast, and more interactive visualizations. However, this evidence should be interpreted as exploratory because it is based on perceived usefulness rather than a direct measurement of learning outcomes. Therefore, the results support the tool's potential as a complementary teaching resource for educators.

Limitations and future work

495 The proposed tool is intended for educational purposes and preliminary analyses. Therefore, the results should not be interpreted as definitive design recommendations or substitutes for detailed geotechnical assessments, site-specific calibrations, numerical modeling, or the judgment of a professional engineer.

The calculation of the pillar load is based on the tributary area theory, which assumes regular configurations and a simplified redistribution of stresses. Furthermore, the implemented strength equations are empirical models developed for specific rock
500 types, mining methods, and assumptions. Consequently, their applicability must be carefully evaluated before being applied under other geological or operational conditions.



The educational evaluation presented in this study was exploratory and based on a limited number of participants. The feedback reflects perceived usefulness rather than a direct measurement of learning outcomes. Therefore, the results should not be interpreted as conclusive evidence of improved learning. Future studies should include larger samples, structured classroom implementation, pre- and post-intervention instruments, and comparisons with traditional teaching methods to assess the tool's effect on conceptual understanding. Future technical developments should include additional pillar strength models, enhanced sensitivity analysis, automated reports, solved examples, multilingual support, and a public repository with instructional materials.

Conclusions

The room-and-pillar method is one of the simplest, safest, and most economical mining methods. A proper pillar design is fundamental to ensuring the success of operations using this method; otherwise, both safety and feasibility are compromised. This study presents the development of an educational software tool with a graphical user interface (GUI) for the comparison of empirical pillar design models in underground mining. The software, developed in Python, integrates pillar load estimation, equivalent width calculation, pillar strength models, factor of safety evaluation, stability classification, extraction ratio calculation, graphical visualization, and parametric analysis into a single interactive environment.

The proposed tool allows users to evaluate various pillar strength models, both empirical and semi-empirical, under identical geometric and geomechanical conditions. This enables a systematic comparison of the safety factors obtained from different formulations and helps illustrate the variability that may arise from model selection. The automatic identification of the critical model provides an additional reference for preliminary interpretation, although the applicability of each model should always be assessed based on its original assumptions, rock type, and calibration data.

The case study demonstrated the usefulness of the software for comparing empirical design approaches and visualizing the influence of pillar geometry on stability. The safety factor chart enables a direct comparison between models, whereas the parametric analysis module allows users to evaluate the relationship between W/h and the safety factor. By reporting the extraction ratio associated with the minimum W/h required to achieve a target factor of safety, the tool also facilitates the analysis of the trade-off between stability and ore recovery in designing rooms and pillars.

From an educational perspective, the software provides a practical environment to move beyond isolated empirical equations and towards an integrated interpretation of pillar geometry, tributary area loading, strength estimation, safety factor, and extraction ratio. Its graphical interface, visualization tools, and auxiliary modules can support teaching activities in underground mining, rock mechanics, geomechanics, and mine design courses. The students' preliminary comments supported the perceived educational utility of the tool, particularly for comparing empirical models, interpreting graphical results, and understanding the relationships between pillar geometry, safety factor, and extraction ratio. However, these results should be interpreted as exploratory, as they reflect students' perceptions rather than the direct measurements of learning progress.

Nevertheless, the tool is intended for educational and preliminary analysis purposes and should not be used as a substitute for detailed geotechnical design, numerical modeling, site-specific calibration, or the judgement of a professional engineer. Future



535 work should focus on incorporating additional empirical models, enhancing sensitivity analysis capabilities, developing automated reporting, preparing instructional materials, and formally assessing the educational impact of the tool in classrooms and laboratories.

Code and data availability

540 The input data used in this study are described in detail in the case study section of this manuscript. The results used to generate the figures and tables were derived from these input data and the empirical models described in this manuscript. Additional information can be obtained from the corresponding author upon justified requests. The software code is not publicly available at this stage because the application is still under development.

Author contributions

545 DI-G: conceptualization, methodology, software, formal analysis, investigation, visualization, and drafting of the original manuscript. MC: conceptualization, methodology, validation, supervision, review, and editing. JA: Methodology, validation, supervision, and review and editing. EM-B: validation, review and editing, supervision. All authors have discussed the results and have approved the final version of the manuscript.

Competing interests

The authors declare that they have no conflict of interest.

550 Ethical statement

This study did not involve the collection, processing, or publication sensitive personal data from students or other human participants. After the educational workshop, student feedback was collected using a voluntary and confidential form. No sensitive personal data, grades, demographic information, or academic performance data were collected from the participants. The responses were analyzed in aggregate, and no student was identified in the manuscript. Participation did not influence the students' grades, course involvement, or academic evaluation. The authors adhered to the principles of confidentiality, 555 voluntary participation, responsible teaching practices, and academic integrity throughout the study.

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