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ESD Ideas: A "Butterfly" on Orbital Timescales: Weak Millennial Forcing or "Brief" Human Impact May Dramatically Change the Rhythmicity of the Long-Memory Ice-Climate System

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Abstract. We demonstrate that (a) the patterns of the Pleistocene climate history as different as classic reconstructions of Lisiecki and Raymo (2005) and the new one of Clark et al (2025) may both be produced by the same long-memory ice-climate system under different initial conditions or under slightly different millennial forcing, and (b) even single "brief" (on orbital timescales, i.e., a few hundred years) pulse-perturbation of the forcing, consistent with a possible human impact, may fundamentally change the dynamics of the global ice-climate system for hundreds of thousands years, making the time series we have not observed in the past.

Main Text.

To explain the records of continuous variability from millennial to orbital periods (e.g., Huybers and Curry, 2006), Verbitsky et al (2019) applied the millennial forcing to a *short-memory* ice-climate system and demonstrated that it may propagate upscale and essentially "highjack" the ice ages. To have a noticeable effect though, the amplitude of the millennial forcing has to be about order-of-magnitude higher than the amplitude of the orbital forcing. We will now demonstrate that in the case of the initial-value-dependent *long-memory* ice-climate system, the millennial forcing of much weaker amplitude may play a dominant role in shaping the ice-age variability.

Recently, Verbitsky and Volobuev (2025) described a dynamical long-memory ice-climate system that responds to the orbital forcing with the periodicity controlled by the following conglomerate similarity parameter:

$$C = \frac{\hat{H}}{\hat{a}^{1/2}\varepsilon^{1/2}T} \quad (1)$$

where $\hat{H}$ (m) is the initial thickness of the ice sheet that is related to the initial glaciation area $\hat{S}$ (m^2) as $\hat{H} = \zeta\hat{S}^{\,1/4}$ (Verbitsky, 1992), $\hat{a}$ is the snow mass influx (m/s), ε (m/s) and T (s) are the amplitude and period of the orbital forcing. Figure 1 presents five numerical experiments with this system.

Figure 1(A) is the reference case adopted from Verbitsky and Volobuev (2025, Figure 9). It describes the first million years of the system's response to pure obliquity-period 41-kyr forcing with initial area $\hat{S}$ =12.0666 10^6 km^2. The pattern of the response has a Pearson correlation of 0.72 with the last 500 kyr of the late Pleistocene Lisiecki and Raymo (2005) data. Consistently with the same data, the spectral diagram is dominated by the P ~100-kyr period, and 41-kyr response amplitudes A are significantly (~2.5 times) lower than the amplitudes of ~100-kyr variations. Thus $A \sim P^n$, where $n \sim 1$.

Without any changes to model parameters, we apply, additively to $\hat{a}$, millennial forcing with the period of 5 kyr and the amplitude $\varepsilon_m = 2\varepsilon$. The system response is shown in Figure 1(B) and it is dramatically different from the reference one. It now depicts a periodicity-relaxation process from dominant P ~ 40 kyr periodicity to P ~ 80-kyr periods. The amplitudes of 40-kyr variations and 80-kyr variations are close in a way that is very reminiscent of Clark et al (2025) reconstructions, i.e., $n \sim 0$.

Remarkably, an almost identical pattern can be obtained if, instead of applying millennial forcing of $\varepsilon_m = 2\varepsilon$, we reduce the initial area to about one-half of it to $\hat{S}$ = 5.9 10^6 km^2 as it is presented in Figure

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1 (C). This finding means that the similarity parameter $\frac{\varepsilon_m}{\varepsilon}$ and the initial area $\hat{S}$ are components of the same conglomerate parameter C, and since $\hat{H} = \zeta \hat{S}^{1/4}$, $\frac{\varepsilon_m}{\varepsilon}$ enters this parameter as $\left(\frac{\varepsilon_m}{\varepsilon}\right)^{1/4}$. Thus, with the presence of the millennial forcing:

$$C = \frac{\hat{H}}{\hat{a}^{1/2}\varepsilon^{1/4}\varepsilon_m^{1/4}T} \qquad (2)$$

The implication of this finding (2) is significant. Certainly, the history of climate is given to us as a single time series. At the moment though, there are two competing reconstructions of it that are both based on the oxygen isotopic composition ($^{18}O/^{16}O$) of benthic foraminifera shells. These are the already classic reconstruction of Lisiecki and Raymo (2005) and the latest one of Clark et al (2025). While in the Lisiecki and Raymo (2005) time series the ~100-kyr variations have much stronger amplitude than ~40-kyr variations, in the Clark et al (2025) these amplitudes are of about the same value. Without making a judgement which of these reconstructions is closer to the reality (see ongoing discussion in, e.g., Shackleton et al, 2026), for the purpose of our presentation, it is important to note that the time series of both types may emerge from the same set of equations, derived from the ice media conservation laws, under different initial values or under slightly different millennial forcing. In other words, the Clark et al (2025) time series *is not "less physical"* than the Lisiecki and Raymo (2005) time series if we accept that the climate system has a long memory.

Edward Lorenz used the "butterfly" metaphor to illustrate that in initial-value-sensitive systems the prediction is fundamentally unreliable because of our inability to precisely measure the initial values. Though the glacial rhythmicity we are studying here is not a deterministic chaos, the unpredictability of ice ages is extreme because the initial values $\hat{S}$ enter the equations as a component of the conglomerate similarity parameter (2), and the uncertainty in the initial extent of ice sheets $\hat{S}$ is aggravated by the combined uncertainties of mass influx, orbital forcing amplitude, and millennial forcing amplitude $\hat{a}^{1/2}\varepsilon^{1/4}\varepsilon_m^{1/4}$. For the very same reason, even brief pulse-type forcing change, in terms of snow influx or radiative forcing, is extremely consequential. To illustrate this, we compare two time series in Figures 1(D, E). The time series in Figure 1(D) is the same reference time series as in Figure 1(A), but extended for 3,000 kyr. As we have already mentioned above, its first 500 kyr can be associated with the late Pleistocene history (Pearson correlation of 0.72 with Lisiecki and Raymo (2005) data) and the rest of it is undisturbed relaxation until complete phase-locking with the obliquity forcing. The second time series is exactly the same but at *t* = 0 (current interglacial) a single 500-***years*** pulse-perturbation forcing of the same amplitude as in the experiment of Figure 1(B), i.e., $\varepsilon_m = 2\varepsilon$ is added. This brief intervention (of hypothetically anthropogenic nature) has radically changed the dynamics of the system making the time series *we have not observed in the past* as it does not resemble neither Lisiecki and Raymo (2005) nor Clark et al (2025) time series. Instead, it makes a sequence of ~80-kyr glaciations with higher than late Pleistocene amplitudes and long, 80-kyr, interglacials. Remarkably, the system remembers (preserves) this rhythmicity for about 1 million years until it returns to the undisturbed pattern thereafter.

The result of Figure 1 (E) is not an actual forecast (as $\varepsilon_m = 2\varepsilon$ is not linked to any CO_2 level) but the important lesson may still be learned. We can see that the second "butterfly" at *t* = 0 creates a variability that has not been observed in the past. This leads us to the question: Can models that have not been derived from the laws of physics but were merely designed to fit empirical data be used to predict the future? While the resilience of natural ice-ages has been a goal of many studies (see e.g., Harteg *et al,* 2026, for a review), they have been based on phenomenological, short-memory models, heavily tuned on the geological records. Their conclusions may have an imprint of the past (e.g., critical glaciation levels or Boolean statements forcing a model behavior to fit the data) that may be irrelevant for the future. Physical, long-memory models open a new, almost unexplored, perspective in which the memory of the past plays even bigger role, i.e., the power of the "butterfly" (2) is overwhelming, but customary habits of "copying" the past to predict the future are cautiously avoided.

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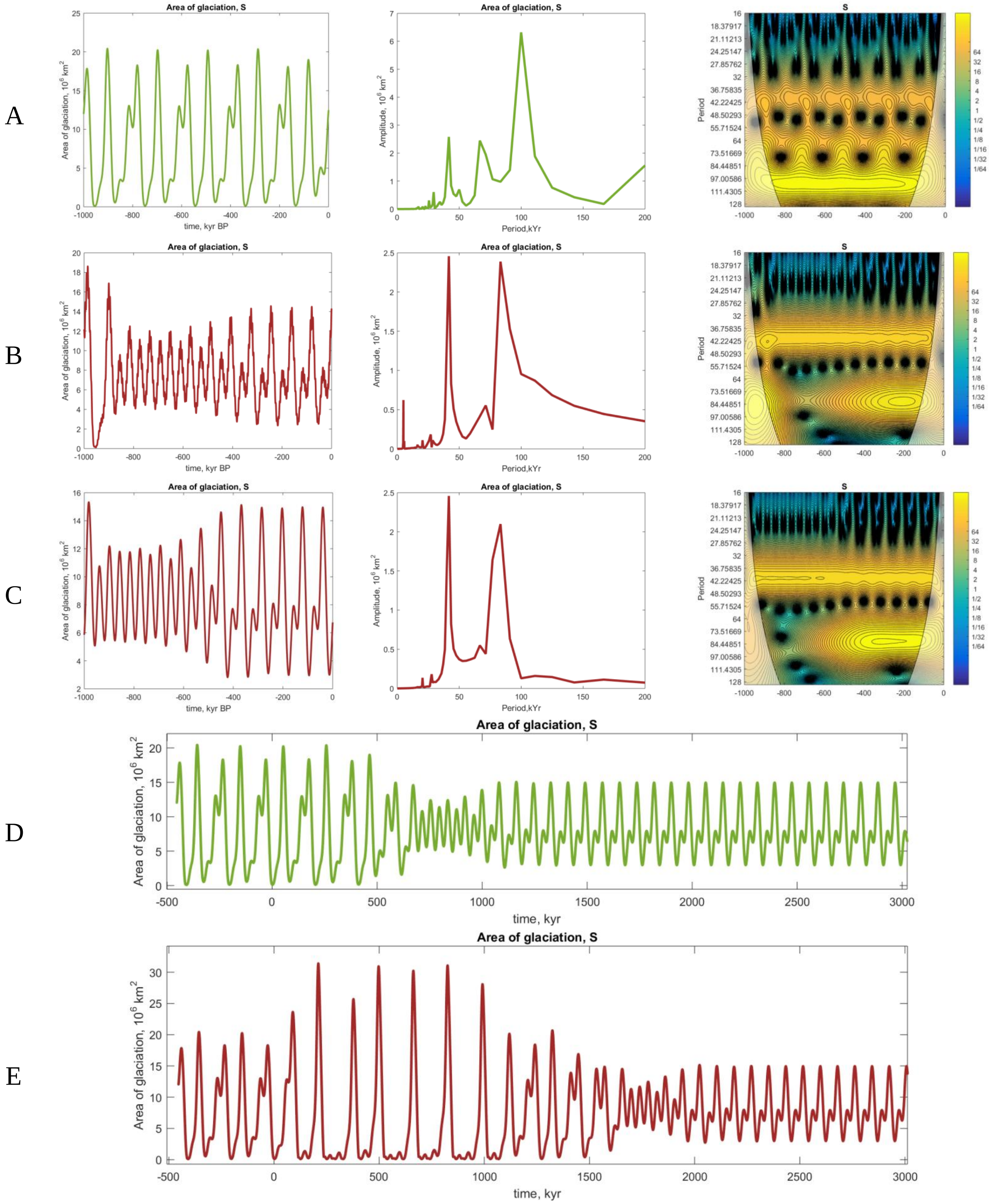

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Figure 1: The ice-climate system response (area-of-glaciation time series, spectral diagrams, and the evolution of the wavelet spectra) to 41-kyr forcing for different initial values or under different millennial forcing. The vertical axis of wavelet spectra is the period (kyr); the horizontal axis is time (kyr before present).The color scale shows the continuous Morlet wavelet amplitude, the thick line indicates the peaks with 95 % confidence, and the shaded area indicates the cone of influence for the wavelet transform. **A**: $\hat{S}$=12.0666 10^6 km^2, $\frac{\varepsilon_m}{\varepsilon} = 0$; **B**: $\hat{S}$=12.0666 10^6 km^2, $\frac{\varepsilon_m}{\varepsilon} = 2$; **C**: $\hat{S}$=5.9 10^6 km^2, $\frac{\varepsilon_m}{\varepsilon} = 0$; **D**: $\hat{S}$=12.0666 10^6 km^2, $\frac{\varepsilon_m}{\varepsilon} = 0$; **E**: $\hat{S}$=12.0666 10^6 km^2, $\varepsilon_m = 2\varepsilon$ for $0 < t < 500$ years.

Code availability: The MATLAB R2021b code and data to reproduce the results as they are presented in Fig. 1(A, D) are available at https://doi.org/10.5281/zenodo.17272864 (Volobuev and Verbitsky, 2025).

Competing interests: The author has declared that there are no competing interests.

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